Table of Contents

2.7 Tidal Disruption

Consider the tidal disruption of a star that

approaches $r_{peri} < r_t$ from a black hole

at the centre of a cluster

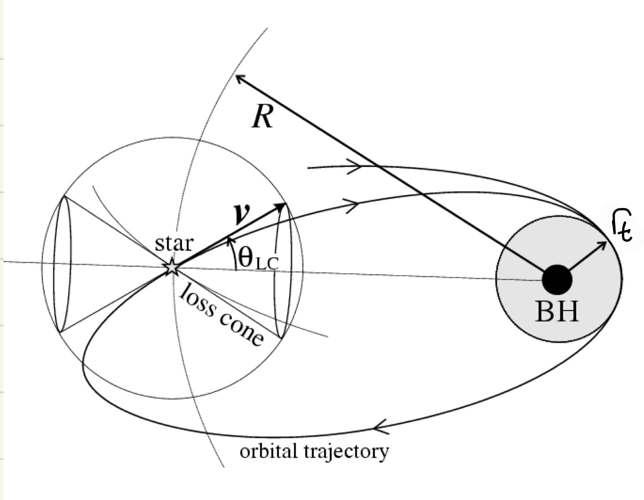
→ close encounter / fills Roche Lobe

What sets the rate of such TDEs?

(tidal disruption events)

What are the details of the disruption process?

2.7.1 Tidal Disruption Event (TDE) Rate



Remember, for tidal disruption:

$$a_{peri} < r_t = \left(\frac{3M_{bh}}{M_*}\right)^{1/3} R_*$$

A star at R with velocity V will be disrupted if

$$\theta < \theta_{LC}$$

where θ_{LC} defines the "loss cone" at R for tidal disruption → stars disrupted in next pericentre

Angular momentum conservation: $R V \sin\theta_{LC} = \sqrt{2GM_{bh} r_t}$

$\therefore$ loss cone is smaller at larger R (as $\sin\theta_{LC} \propto 1/R$)

Encounters with other stars in the cluster lead to 2-body relaxation

Remember, this leads to a random walk in velocity on a timescale of $\tau_{2br} = v^3 / (G^2 m^2 n)$

Thus stars are scattered into, or out of, the loss cone on a timescale of τ_{2br}

At which point they are lost by disruption on a timescale of τ_{cross}

There is a critical radius, R_{crit}, at the location where $\tau_{cross} = \tau_{2br}$

At $R \gg R_{crit}$ loss cones are full but stars are scattered out before disrupting

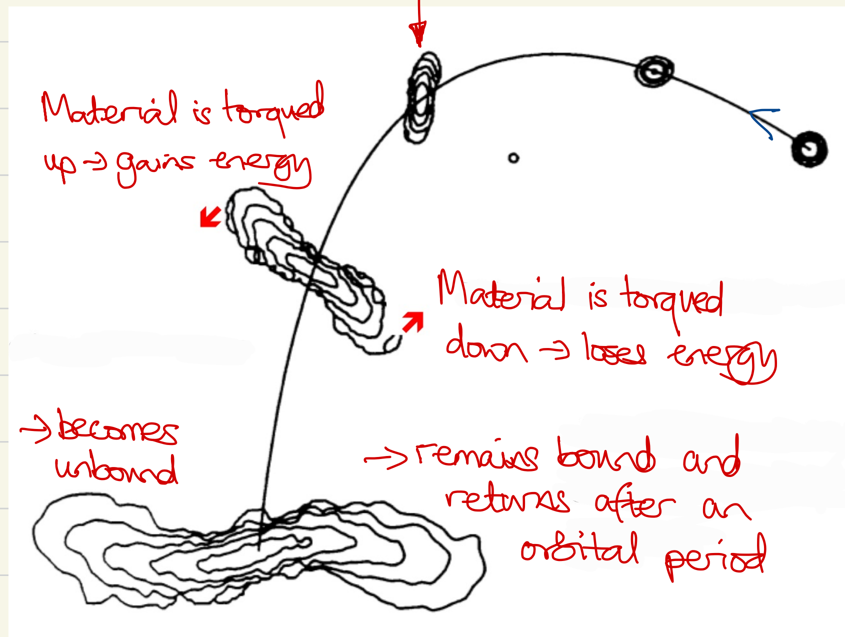
$R \ll R_{crit}$ stars are rapidly disrupted so loss cones are empty

$R \sim R_{crit}$ stars are perturbed into LC at the rate they are disrupted (steady state)

$\rightarrow$ TDE rate is set by R_{crit}

2.7.2 Details of Tidal Disruption Process

Consider an initially parabolic encounter of a star with a black hole



∴ Half of the mass is unbound by the encounter

Alternatively imagine what happens to the different sides in the absence of the star's gravity

What happens to the half of the material that remains bound?

This returns to the disruption point after one orbital period, but material from different parts of the star suffer different levels of energy loss ΔE and so different periods t_{orb} , returning at different times

Energy dissipation from crossing streams results in energy dissipation and the formation of an accretion disk

What is the rate at which material is incorporated into this disk?

Consider a portion of the star that suffers energy loss ΔE

This sets the new semi-major axis: $\Delta E = \frac{1}{2} G M_{bh} / a$

$$\therefore \; a \propto \Delta E^{-1}$$

And so orbital period: $t_{orb} \propto a^{3/2} \propto \Delta E^{-3/2}$

Define $n(\Delta E)\, d\Delta E$ the fraction of the star's mass that loses energy in the range $\Delta E \to \Delta E + d\Delta E$

And assume that $n(\Delta E) = \text{const}$.

Since $t_{orb} \propto \Delta E^{-3/2}$, material with $\Delta E \to \Delta E + d\Delta E$ returns in a time window

$$dt \propto \Delta E^{-5/2}\, d\Delta E$$

Since this material has a mass $\propto d\Delta E$

This means that mass returns to the black hole at a rate $\propto \Delta E^{5/2} \propto t^{-5/3}$

$\rightarrow$ luminosity predicted to fall off $\propto t^{-5/3}$

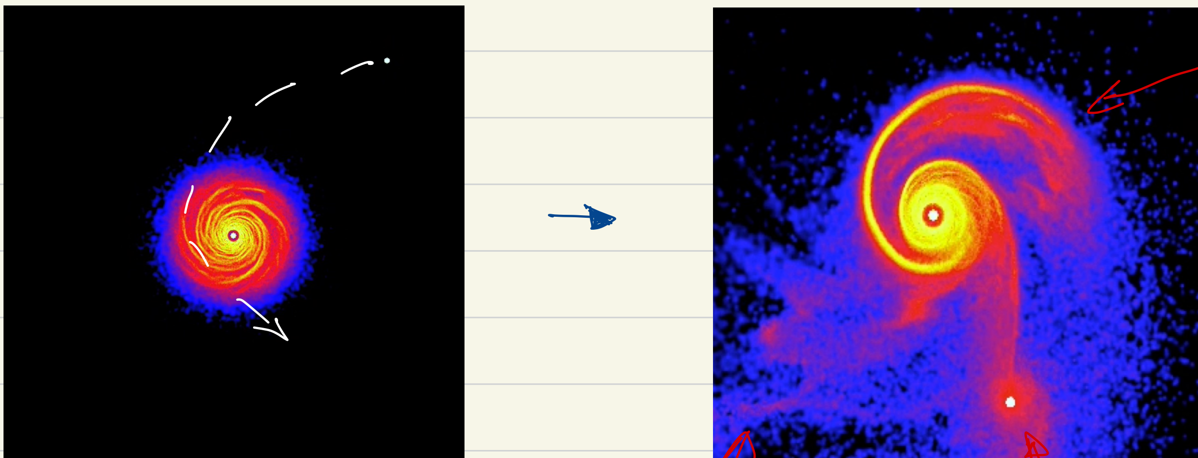
$\rightarrow$ See Chris Reynold's guest lecture on TDEs around a BH.

2.8 Tidal Interactions with Disks

2.8.1 Effect on Disk

E.g., “Smoothed Particle Hydrodynamics” (SPH) simulation of a star-disk encounter

Note difference between treating fluids in Eulerian grid vs Lagrangian particles, requiring care to get dp/dr terms, but common in PPD/AGN/cosmo sims



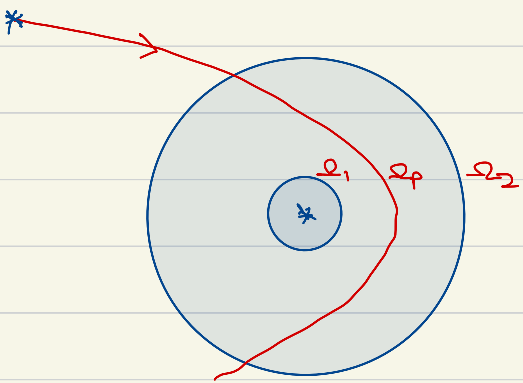
Spiral launched in bound material

Part of disc becomes unbound

Some material is captured by the star

2.8.2 Effect on Object Encountering Disk

Consider a star undergoing a parabolic encounter with a star that hosts a protoplanetary disk



Angular velocity in the disk: $\Omega \propto r^{-3/2}$

$\therefore \Omega_1 > \Omega_2$

For a strong interaction, the pericentre lies within the disk

$\rightarrow r_1 \ll r_p \ll r_2$

$\rightarrow \Omega_1 \gg \Omega_p \gg \Omega_2$

The arrow of time argument: $\rightarrow$ material at 1 gives E and J to perturber

2 takes E and J from perturber

Overall, simulations show that the amount of energy that is transferred from the perturber is approximately that required to unbind the part of the disk outside the pericentre

$$\therefore \Delta E = G M \Delta m / r_p$$

M ↑ mass of disc-hosting star

Δm ↖ mass of disc outside r_p

The perturber can become bound if $\Delta E > \frac{1}{2} M v_\infty^2$

Since many stars form in high density clusters surrounded by protoplanetary disks

→ this mechanism can form binary star systems from initially unbound single stars

2.8.3 Creating Binaries in Clusters

modification due to gravitational focussing

How many binaries are created in a cluster in this way?

First, determine the collision rate, remembering this is $\Gamma = n\sigma v$

where $\sigma = \pi R^2 \left[1 + \frac{2GM}{RV_\infty^2}\right]$

radius at which "collision" occurs, $R = R_{disc}$ here

For a binary-forming encounter: $GM\Delta m / R_{disc} \gg \frac{1}{2} M V_\infty^2$

$\therefore 2GM / R_{disc}V_\infty^2 \gg M/\Delta m \gg 1$ as $\Delta m \ll M$ for stability

Thus gravitational focussing dominates and $\sigma \simeq 2\pi R_{disc} GM / V_\infty^2$

The rate at which a star with a disk undergoes binary-forming encounters is $\Gamma = n \, 2\pi R_{disc} GM / V_\infty$

2.8.4 Examples

(just below belt, illuminated by massive stars, obscured by dust in optical)

2.8.4.1 Example 1: Orion Nebula Cluster

$N \sim 500$ stars

$n \sim 10^4$ stars pc^{-3}

$v \sim 1$ km/s

$R_{disc} \sim 100$ au

Remember $\Gamma = n\, 2\pi R_{disc}\, GM / v_\infty$

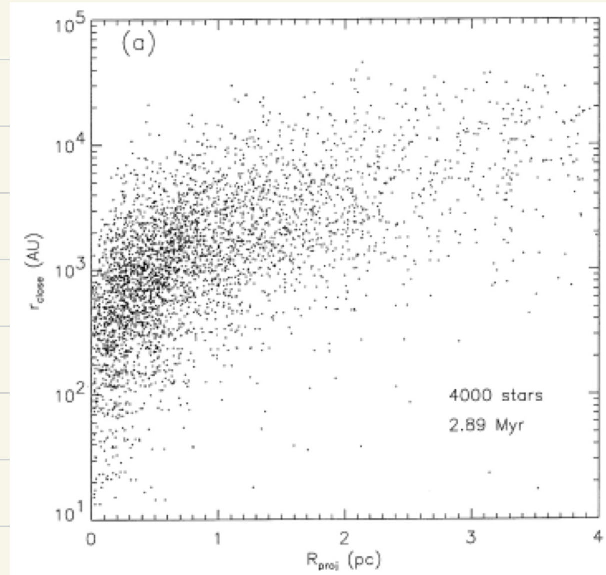
$\simeq 0.1$ / Myr per star

$\therefore$ expect ~ 50 star-disc encounters over ~ 1 Myr lifetime of cluster

Compare with N-body simulations of the cluster:

r_{close} tracks distances of close approaches

R_{proj} tracks where these occur in the cluster



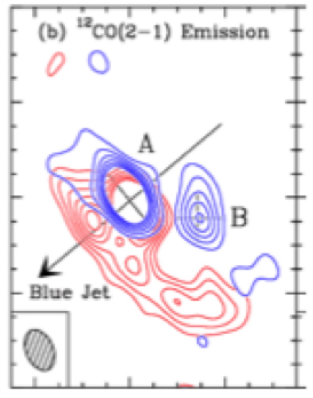
So, there are ~100 close approaches within 100au

that mostly occur in the centre of the cluster

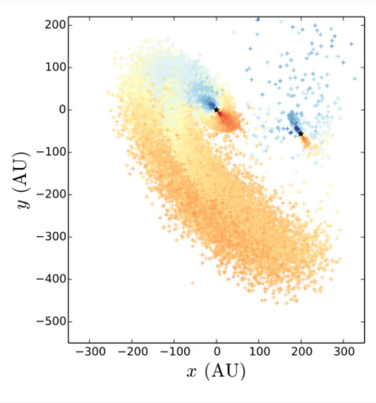
2.8.4.2 Example 2: Taurus-Auriga Star Forming Region

The CO map of the protoplanetary disk around the star RW Aur seems to show evidence for a “tidal tail”

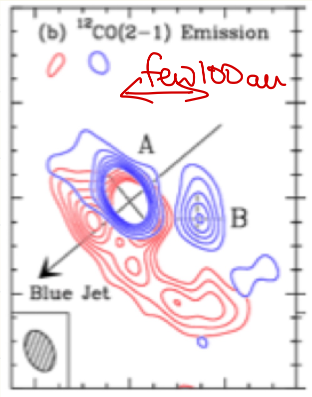
Observation:



SPH simulation of a star-disk encounter:



Yet, this cluster is less dense than Orion



$N \sim 100$ stars

$n \sim 100$ stars / pc^3

$v \sim 1$ km/s

$\therefore \Gamma = 0.001$ / Myr per star

and probability of an interaction in 1 Myr is ~ 0.1

But proximity of B to A → interaction occurred in last few 100 yr

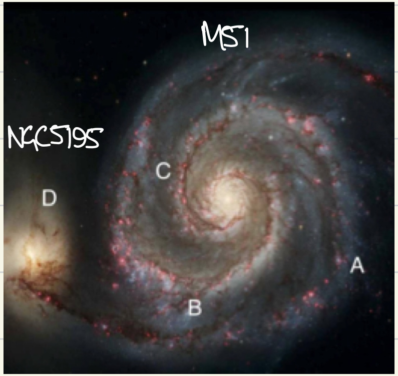
→ very unlikely ($p \sim 10^{-5}$)

∴ probably formed as a binary and undergo repeated encounters

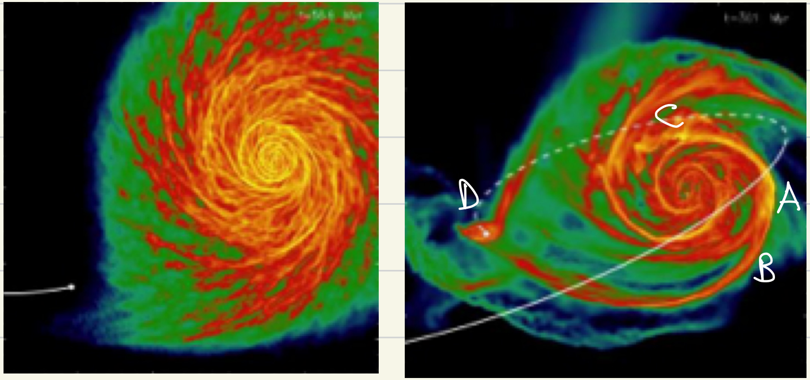
2.8.4.3 Example 3: Galaxy - Galaxy Interactions

The Physics discussed is scale-free and so also applies to galaxy interactions (albeit requiring a dark matter potential)

Observation

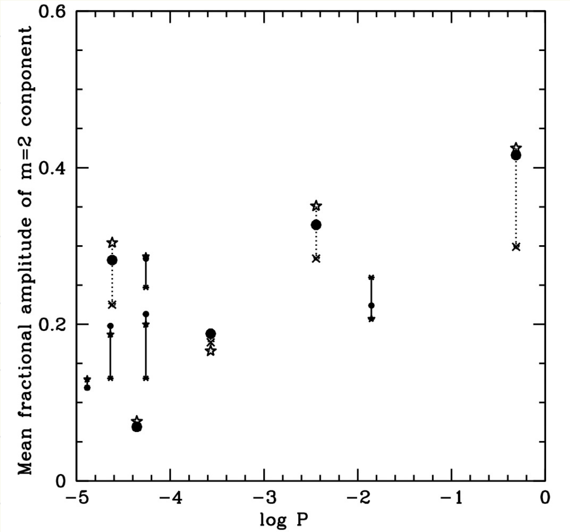


SPH Simulations of Interaction



Note similarity to observations and to star-disc encounters

Do all galaxy spirals form in tidal interactions?



Test by determining the amplitude of the spiral (y-axis), as measured in the near-IR to follow the mass distribution

And plotting against a measure of the tidal pull, P

$P = M/R^3$

$= \text{mass} / \text{distance to companion}^3$

→ amplitude increases with P

No, as spirals can form in other ways, likewise in protoplanetary discs

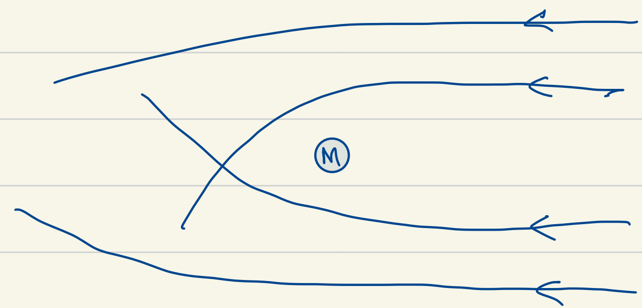
2.9 Evolutionary Effects in Clusters

Cluster evolution is driven by internal energy transfer between stellar orbits via 2-body relaxation (a.k.a. dynamical drag)

This leads to mass segregation and gravothermal catastrophe

2.9.1 Mass Segregation

In the frame of a star of mass M:



In the frame of the cluster: deflection leads to energy transfer

How are different mass stars affected?

The rate at which a star of mass M encounters mass: $n m \cdot \sigma \cdot v \propto \sigma \propto b_{crit}^2$

The relevant impact parameter for energy transfer is set by that causing a large deflection:

$$\frac{1}{2} V_\infty^2 \sim GM / b_{crit}$$

$$\therefore \quad b_{crit} \sim GM / V_\infty^2$$

Thus the force acting on the star: $\propto M^2$

So the timescale for momentum transfer: $\propto$ Momentum / Force

$$\propto M / M^2 \propto 1/M$$

→ more massive stars are more rapidly affected and sink to the core

2.9.2 Gravothermal Catastrophe

The gravothermal catastrophe arises as self-gravitating systems have negative heat capacities

(clusters don't start like this but evolve towards this).

Start with the virial theorem (from AFD): $2T_{kin} + W = 0$ (kinetic energy: T_{kin}; G.P.E.: W)

Thus total energy $E = T_{kin} + W = -T_{kin}$

T_{kin} relates to the mean square speed of stars and so temperature of system

As $\partial E / \partial T_{kin} = -1$ → negative heat capacity

→ energy loss leads to system heating up

From Statistical Physics, we know that energy flows from hot to cold

Conventionally $(dE/\partial T > 0)$

If $E \to E - \Delta E$

$T \to T - \Delta T$

$\therefore$ energy flows into sub-system to restore thermodynamic equilibrium

Self-gravitating system $(\partial E/\partial T < 0)$

If $E \to E - \Delta E$

$T \to T + \Delta T$

$\therefore$ energy flows out of subsystem (down the temperature gradient)

$\therefore$ unstable!

The **Gravothermal Catastrophe:**

In a star cluster, a sub-system that has lost energy tends to collapse

In the process of that collapse it acquires more kinetic energy

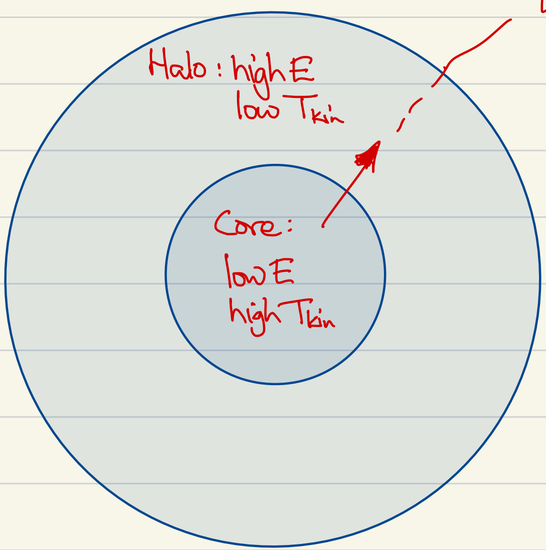
→ the central regions of clusters become ever hotter and denser and implode !?

The process is similar to why satellites speed up when they encounter atmospheric friction

→ energy is removed from orbit but kinetic energy increases as satellite falls into potential well

Timescale for the gravothermal catastrophe:

Energy flows to halo as stars flung out and core contracts



Timescale for energy transfer is set by 2-body relaxation

For equal mass stars:

$$t_{2br} \sim N \times \text{crossing time} \sim 0.1 N \left(\frac{R^3}{GNm}\right)^{1/2}$$

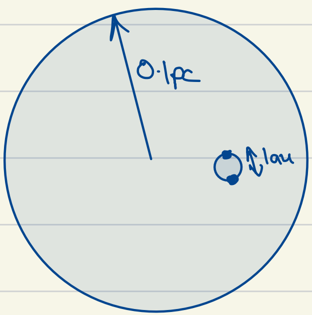
See 1st example sheet eg.

For globular clusters: (large N) $t_{2br} \leq t_{Hubble} \sim t_{age}$

Why have their cores not imploded?

2.9.3 Averting the Gravothermal Catastrophe with Binaries

Consider a 0.1pc, N=100 cluster of $1 M_\odot$ stars containing a 1au separation binary



Compare the gravitational potential energy in the cluster with that in the binary

$$|E| = GM^2/r$$

$$\therefore E_{cluster}/E_{bin} = (100^2/0.1\,pc) \times 1\,au$$

$$\sim 0.5, \text{ i.e. comparable}$$

Remember: $E_{bin} \sim E_{cluster}$

Thus binary can act as a heat source

i.e., it can eject stars by reducing the size of its orbit

Remember: the arrow of time means that energy is transferred from faster- to slower-moving objects

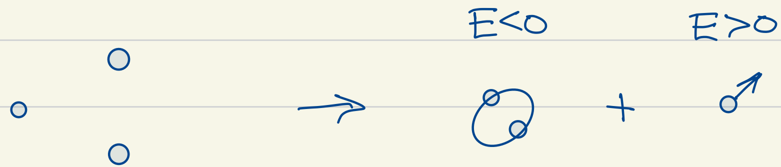
i.e., binary moves fast and gives energy to slower-moving stars

→ Core collapse is prevented by energy transfer from a tight binary into surrounding core

NB doesn't need infinite energy, just enough

2.9.4 Origin of Binaries

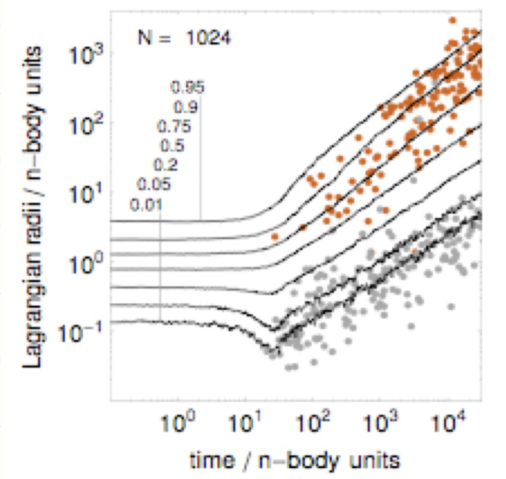
- Primordial (formed from cloud collapse)
- Tidally captured (rare but possible)
- Three-body capture



Requires 3 objects within GM/v^2 so they know about each other then undergo gravitationally focussed energy exchange

2.9.5 Cluster Evolution Summary

Core collapses until the density in the core is high enough to form binaries



Note lines are straight in log-log space → power law

The cluster is then re-inflated by energy that is transferred from binaries

Remember: $t_{2br} \propto R^{3/2}$

- Expect $dR/dt \sim R/t_{2br}$ → $dR/dt \propto R^{-1/2}$ → $R \propto t^{2/3}$
- Get same result by setting evolution time = current time (self similar)

2.9.6 Cluster Evolution in Galactic Environment

The cluster expands until it fills its Roche Lobe due to Galactic tides

$$R_t = \left(\frac{M_{cluster}}{3 M_{gal}} \right)^{1/3} R_{gal}$$

then loses mass and so R_t shrinks and cluster dissolves

Remember: $t_{2br} \sim 0.1 N \left(\frac{R^3}{GNm} \right)^{1/2} \propto N^{1/2}$

→ smaller clusters dissolve faster, only see massive globular clusters today

Tidal tails demonstrate that clusters are not embedded in a dark matter halo (which prevent tidal dissipation)

Dissolved clusters populate the Galactic halo, but abundance differences show these are not dominant

2.10 Origin of the Moon – Example application of Topics material

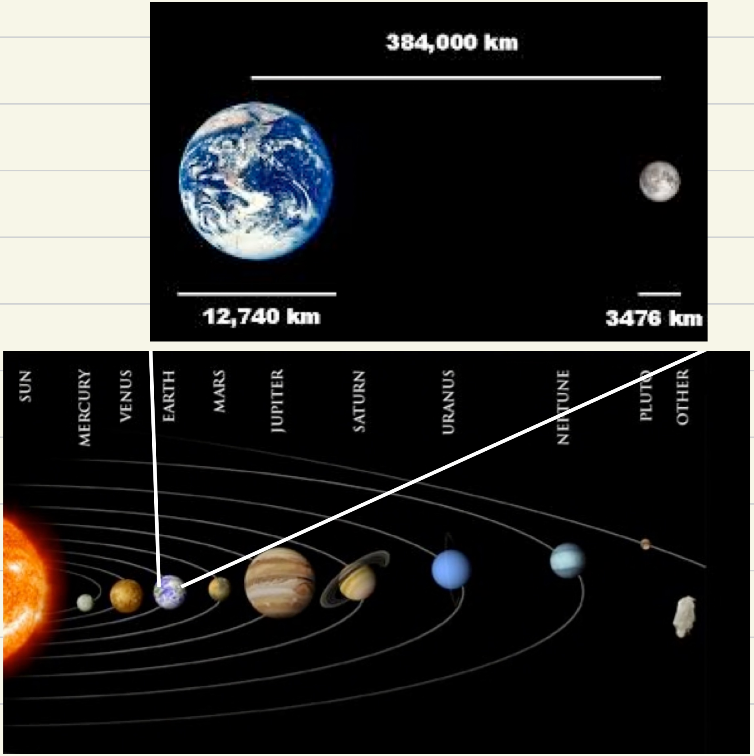
2.10.1 Basic Parameters

Relevant distance scales in the system:

- Moon orbits Earth at 384,000 km

  ($e \sim 0.05$, $I \sim 5^\circ$ to ecliptic)

- Earth orbits Sun at 150,000,000 km



Thus the Earth's Hill radius (beyond which circumplanetary orbits become unbound)

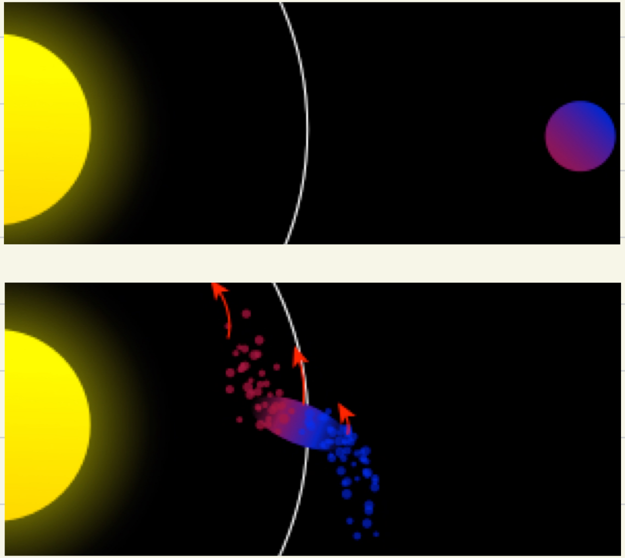
$$R_H = a_\oplus \left(\frac{M_\oplus}{3 M_\odot} \right)^{1/3}$$

$$= 1{,}500{,}000 \text{ km}$$

→ Moon is within this!

but note orbits beyond $R_H/2$ are unstable

The Roche radius, inside which tidal forces would disrupt orbiting satellites:



$$R_H = a_R \left(\frac{M_s}{3M_\oplus} \right)^{1/3} = R_s$$

$$\therefore a_R = 3^{1/3} R_\oplus \left(\frac{M_\oplus / R_\oplus^3}{M_s / R_s^3} \right)^{1/3}$$

$$= C R_\oplus (\rho_\oplus / \rho_s)^{1/3}$$

where $C = 1{\cdot}26 - 2{\cdot}44$

$$\therefore a_R = 10{,}000 - 18{,}000 \text{ km}$$

→ the Moon is ~ 20 times beyond these limits

2.10.2 What's Unusual About the Moon?

2.10.2.1 Mass

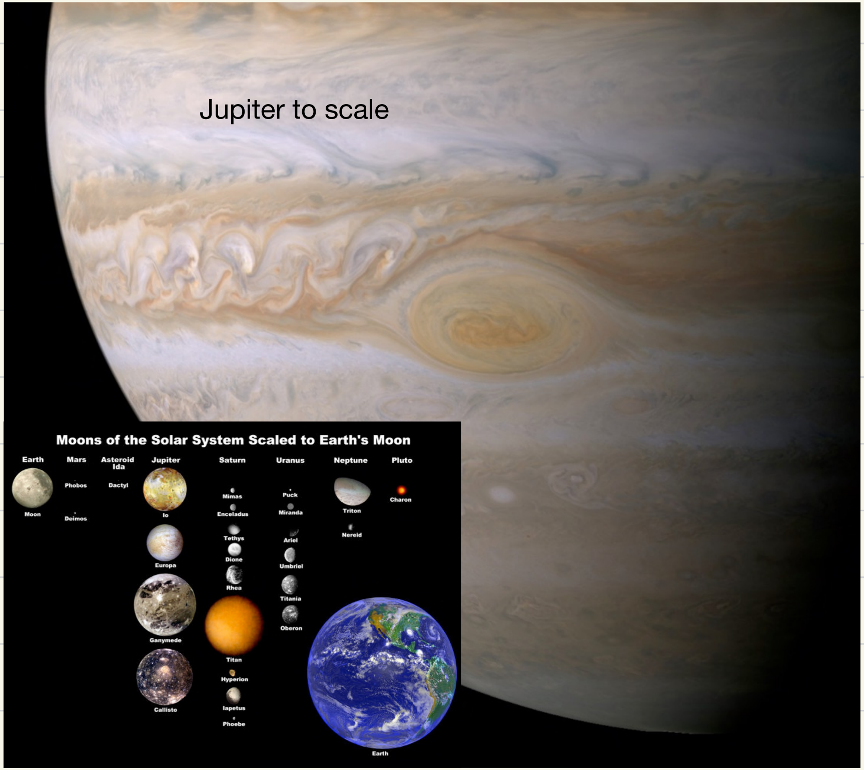
Not in absolute terms

However, relative to its planet

$M_L / M_\oplus \sim 1/80$

Other moons are $< 1/4000$

except $M_{Charon} / M_{Pluto} \sim 1/8$



2.10.2.2 Angular Momentum

Orbit: $J_{orb} \sim M_L \sqrt{G M_\oplus a_L} = 2.9 \times 10^{34}$ kg m^2/s

Spin of Earth: $J_{rot\oplus} = I\Omega \simeq \frac{4\pi}{5} M_\oplus R_\oplus^2 / P_{rot} = 7.1 \times 10^{33}$ kg m^2/s

Spin of Moon: $J_{rot L} \simeq J_{rot\oplus} / (80 \times 3.7^2 \times 28) = J_{rot\oplus} / 30{,}000$ → negligible

→ most angular momentum is in orbit, in contrast to other moons

e.g. $J_{orb} = J_{rot}/100$ for Jupiter

2.10.2.3 Past Tidal Evolution

Remember: $\Delta E = [\Omega_2 - \Omega_1] \Delta J$

Thus, tidal evolution explains current low $J_{rot,L}$ → moon has been tidally despun

It also explains why J is currently being passed from $J_{rot\oplus} \rightarrow J_{orb}$

from a rapidly spinning Earth → slower orbit

∴ Earth's spin is slowing and days lengthening by 23 μs/yr

Moon's orbit is receding by 38 mm/yr

Thus, in the past: the Earth was spinning faster, and Moon was closer

Tidal Catastrophe

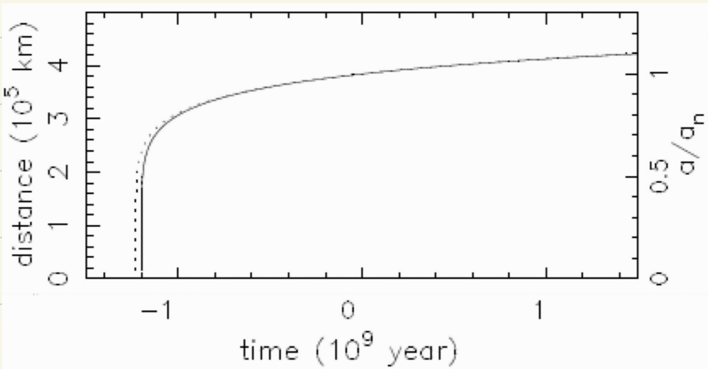
If the recession rate was constant over 4.5Gyr… → Moon started at 214,000 km

But, $E_{tidal} \sim GM_L^2 R_\oplus^5 / a_L^6 \quad \propto a_L^{-6}$

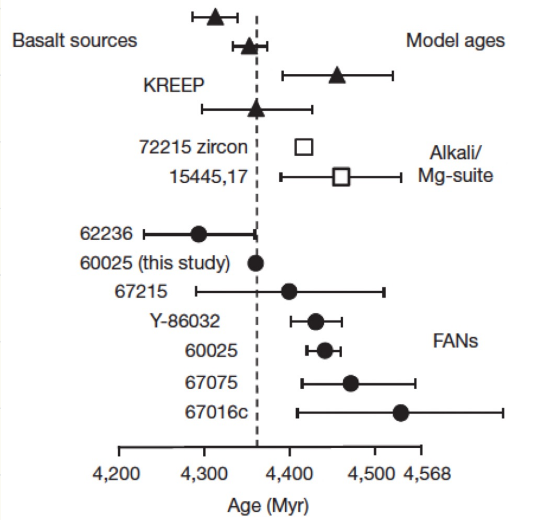
so expect tides to have been stronger when the Moon was closer → recession not constant

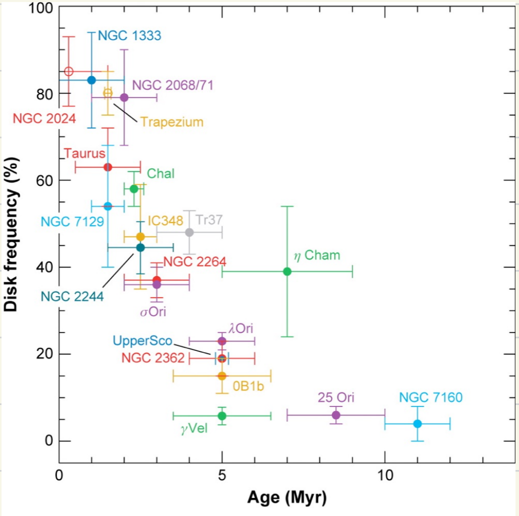
→ Moon started very close to Earth
and formed recently?

→ tidal dissipation would have
melted Earth



2.10.2.4 Age





~ 50 Myr after the Sun formed → after the protoplanetary disk dispersed

2.10.2.5 Composition

Lack of iron - 3.3 g/cm^3 implies 0.25x cosmic abundance of Fe

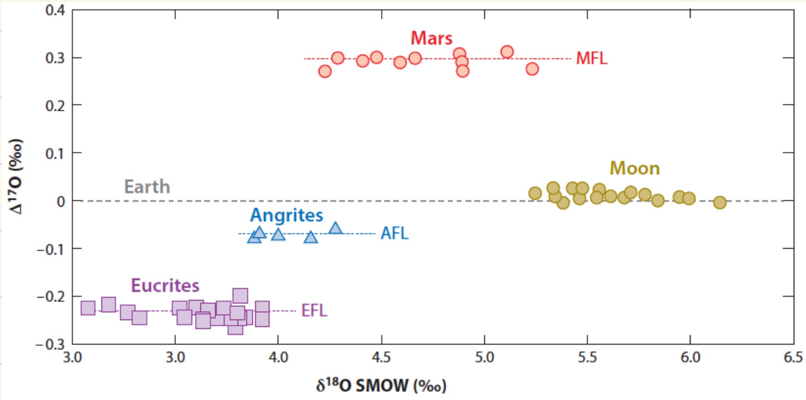
Lack of volatiles - no water except from comets?

Oxygen isotopes ratios - $^{17}O/^{18}O$

identical to Earth

Magma ocean - Apollo rocks show evidence

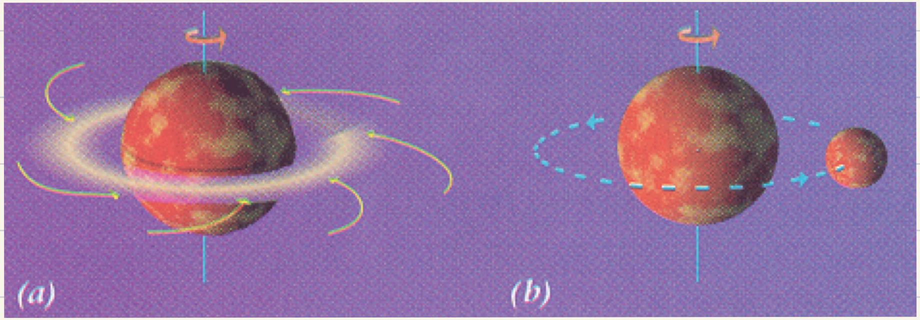
for melting early in history



2.10.3 Formation Scenarios

2.10.3.1 Formation Scenario 1: Co-Accretion

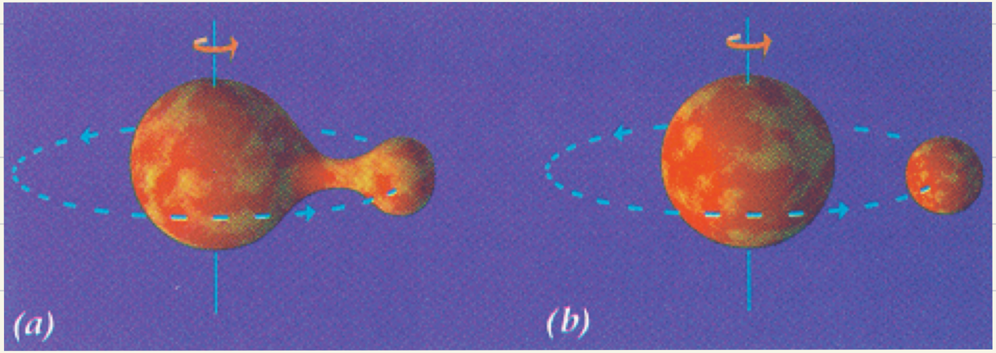
Moon formed out of a circum-terrestrial disk



But… high J, age, composition

2.10.3.2 Formation Scenario 2: Fission

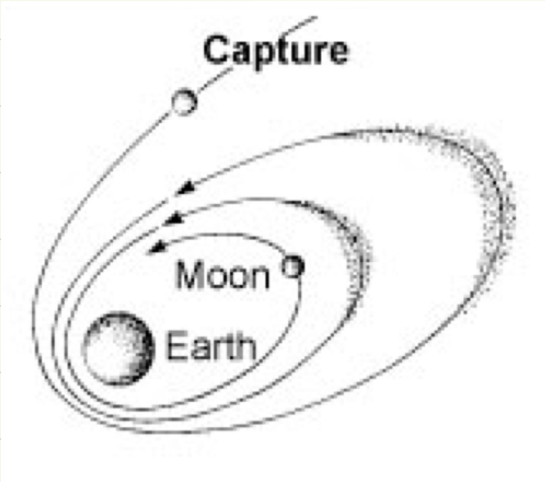
Rapidly spinning Earth undergoes fission



But… Viscosity damps triggering processes

2.10.3.3 Formation Scenario 3: Capture

Moon formed elsewhere, becoming bound via tides or a 3-body interaction

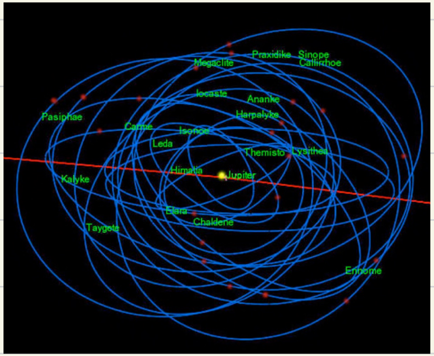


But… composition, no heating, wide orbit expected

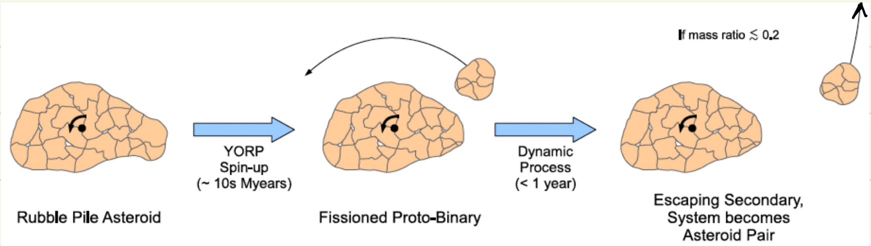
2.10.3.4 Precedents in the Solar System

Jupiter's regular moons formed in a circum-Jovian disk

Jupiter's irregular satellites are captured asteroids and comets



Binary asteroids like formed by fission



2.10.3.5 Formation Scenario 4: Giant Impact

A circum-terrestrial disk was created in a collision with

a Mars-sized impactors (Theia) at ~50 Myr

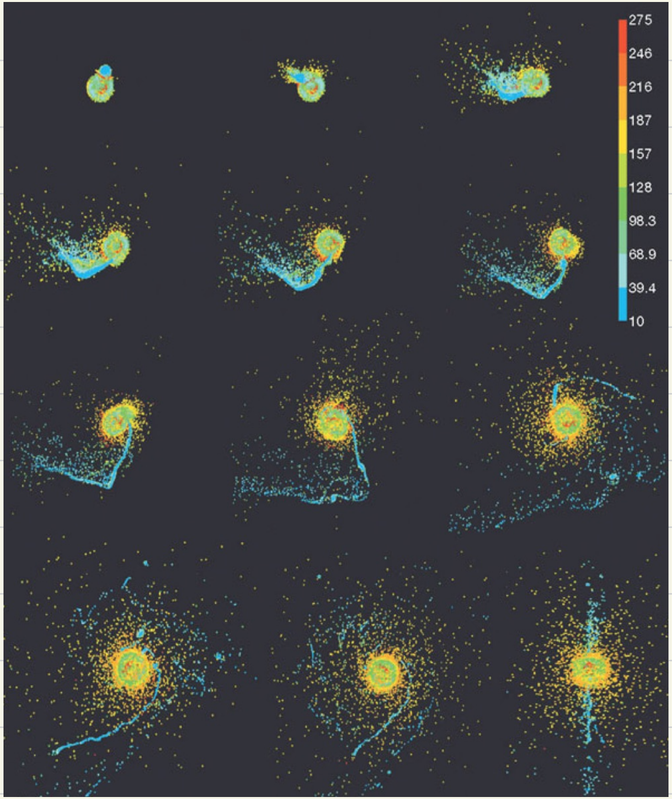
(as expected in planet formation models)

SPH simulations show the plausibility of achieving a

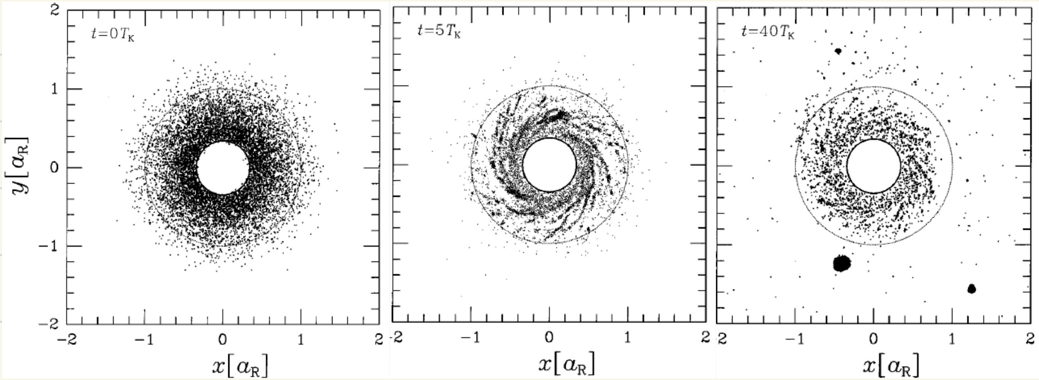
disk with the correct angular momentum

If the Earth was differentiated this explains the Moon's

low iron content



Evolution of a circum-terrestrial disk:



- Disk contracts via collisional damping
- Particle clumps grow inside the Roche radius, but shear out to form spiral structure
- Gravitational torques push particles beyond the Roche radius where moonlets form
- Moonlets coalesce, and a single moon sweeps up all particles that are pushed beyond the Roche radius
- When the Moon is large enough, it pushes the inner disk onto the Earth

→ formation of Moon robust from $\sim 3 M_L$ disk inside Roche

2.10.4 Ongoing Work

Plausibility of the collision? Estimate ~1% probability of a Theia-like collision

→ appeal to anthropic principle? (if Moon's existence favours life, we see Moon)

→ different collision parameters, eg involving two $0.5 M_{\oplus}$ bodies

use evection resonance to remove J from Earth-Moon system

(by exchanging with Earth's orbit around Sun)

Why is the composition of the Moon so similar to the Earth if part of the impactor goes into the Moon?

→ protolunar disk physics

→ composition measured is of a "late veneer"