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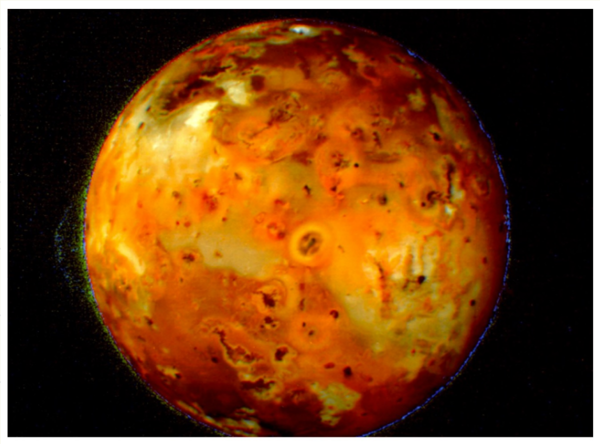
2 Tides

Locally familiar

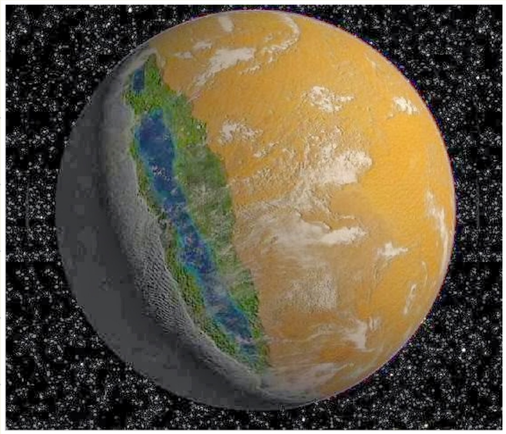
2.1 Astrophysical Examples of Tides

And astrophysically important on scales from planets…

Volcanism on Io

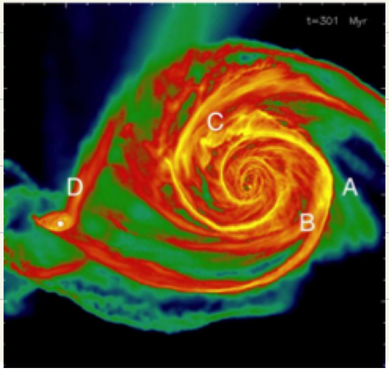


Tidally locked exoplanets



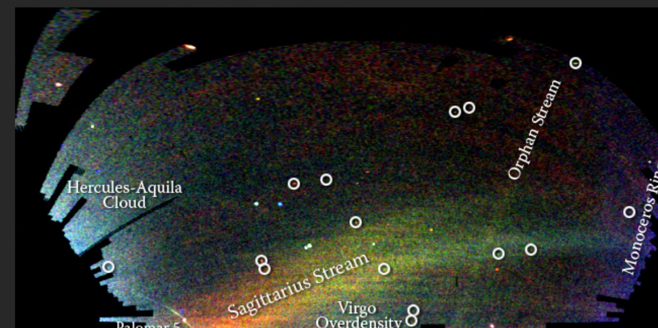
… to galaxies

Large-scale structures from interacting galaxies



Remnants of tidally shredded satellite galaxies in the Milky Way

SDSS Field of Streams - Gallery



A map of stars in the outer regions of the Milky Way Galaxy, derived from the SDSS images of the northern sky, shown in a Mercator-like projection. The color indicates the distance of the stars, while the intensity indicates the density of stars on the sky. Structures visible in this map include streams of stars torn from the Sagittarius dwarf galaxy, a smaller 'orphan' stream crossing the Sagittarius streams, the 'Monoceros Ring' that encircles the Milky Way disk, trails of stars being stripped from the globular cluster Palomar 5, and excesses of stars found towards the constellations Virgo and Hercules. Circles enclose new Milky Way companions discovered by the SDSS; two of these are faint globular star clusters, while the others are faint dwarf galaxies.

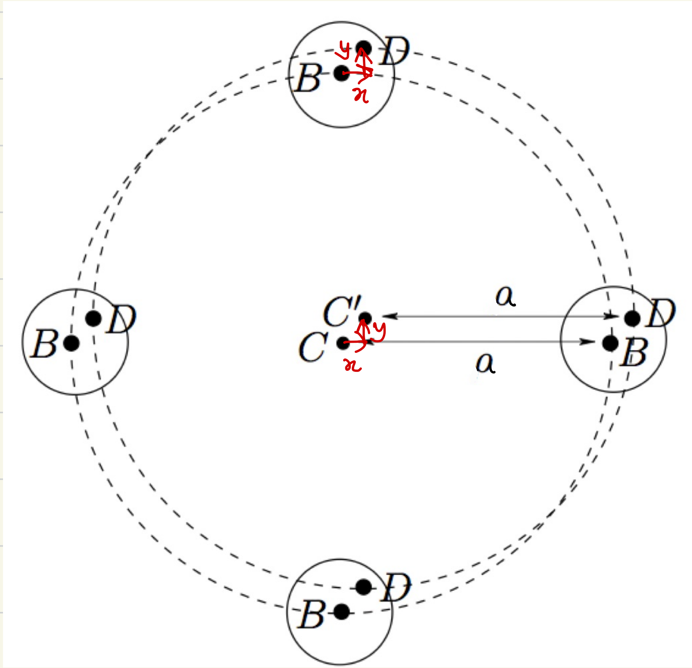
Credit: V. Belokurov and the Sloan Digital Sky Survey.

2.2 Tidal Acceleration and Tidal Potential

Consider an object with centre at B on a circular orbit around C of radius a

The motion of point D that is offset from B by some fixed x, y is a circle of radius "a" but centred at C' offset by x, y from C

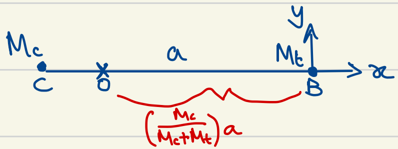
B and D follow circle of same radius and angular velocity → same centrifugal acceleration



Two masses M_c and M_t are in circular orbit about their centre of mass O separated by a

Consider the mass M_t centred at B which is being tidally affected by the mass M_c centred at C

(note no assumption about M_t / M_c)



reduced as circle is about c.o.m.

Centrifugal balance at B :
$$\frac{GM_c}{a^2} = \left(\frac{M_c}{M_c+M_t}\right) a\,\Omega^2$$

$$\rightarrow \Omega^2 = \frac{G(M_c+M_t)}{a^3}$$

$$\frac{GM_c}{a^2} = \left(\frac{M_c}{M_c+M_t}\right) a\,\Omega^2$$

Consider the accelerations at D, for now only the contributions of gravity from M_c and centrifugal terms

In the $\hat{x}$ direction:

$$a_x\big|_D = \frac{-GM_c(a+x)}{[(a+x)^2+y^2]^{3/2}} + \left(\frac{M_c}{M_c+M_t}\right) a\,\Omega^2$$

binomial expansion

If $x/a,\ y/a \ll 1$:

$$= -\frac{GM_c}{a^2} + 2x\frac{GM_c}{a^3} + \left(\frac{M_c}{M_c+M_t}\right) a\,\Omega^2 + O\left(\left(\frac{x}{a}\right)^2, \left(\frac{y}{a}\right)^2\right)$$

cancel

$$\simeq \frac{2GM_c}{a^3}x \rightarrow \text{repulsive}$$

In the $\hat{y}$ direction:

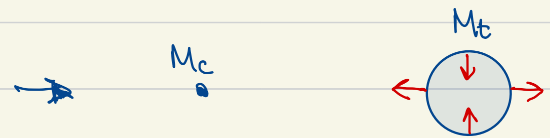
$$a_y\big|_D = \frac{-GM_c\,y}{[(a+x)^2+y^2]^{3/2}} \simeq -\frac{GM_c}{a^3}y \rightarrow \text{attractive}$$

Tidal Potential

Remember

$$a_x|_D = 2\left(\frac{GM_c}{a^3}\right)x$$

$$a_y|_D = -\left(\frac{GM_c}{a^3}\right)y$$



These are the tidal accelerations experienced at D due to not being at the centre of an orbiting body which is in centrifugal balance

The form of the tide is that of a repulsive harmonic potential in x and an attractive harmonic potential in y and so can be written

$$\Phi_{tidal} = \left(\frac{GM_c}{a^3}\right)\left[-x^2 + \frac{1}{2}y^2\right]$$

2.3 Tidal Bulge

e.g. the sea

A fluid surface conforms to the equipotential of the tide plus the gravity of M_t

$$\Phi_{tidal} = \left(\frac{GM_c}{a^3}\right)\left[-x^2 + \frac{1}{2}y^2\right]$$

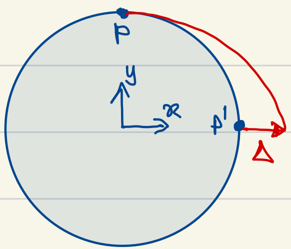
$$\Phi_{grav} = -GM_t / [x^2+y^2]^{1/2}$$

The tidal bulge is the change in equipotential surface due to companion

2.3.1 Height of the Tidal Bulge

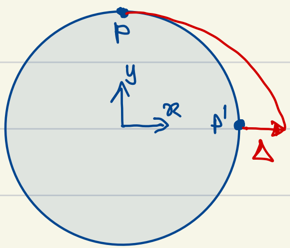
Remember $\Phi = \left(\frac{GM_c}{a^3}\right)\left[-x^2 + \frac{1}{2}y^2\right] - GM_t / \left[x^2 + y^2\right]^{1/2}$

Consider points P and P' both a distance R_t from the centre



Tides cause the equipotential passing through P to pass a height Δ above P' = tidal bulge

$$\therefore \left(\frac{GM_c}{a^3}\right)\frac{1}{2}R_t^2 - \frac{GM_t}{R_t} = -\left(\frac{GM_c}{a^3}\right)(R_t+\Delta)^2 - \frac{GM_t}{(R_t+\Delta)}$$



$$\left(\frac{GM_c}{a^3}\right)\frac{1}{2}R_t^2 - \frac{GM_t}{R_t} = -\left(\frac{GM_c}{a^3}\right)(R_t+\Delta)^2 - \frac{GM_t}{(R_t+\Delta)}$$

Either: (i) Do binomial expansion in $\Delta / R_t \ll 1$

Or: (ii) Equate the difference in G.P.E. at the two points $\sim$ "g"$\Delta = \frac{GM_t}{R_t^2}\Delta$

to the difference in tidal potential $\sim \frac{GM_c}{a^3}\left[\frac{3}{2}R_t^2\right]$

$$\therefore \Delta = \frac{3}{2}\left(\frac{M_c}{M_t}\right)R_t^4 / a^3$$

$$\Delta / R_t = \frac{3}{2}\left(\frac{M_c}{M_t}\right)\left(\frac{R_t}{a}\right)^3$$

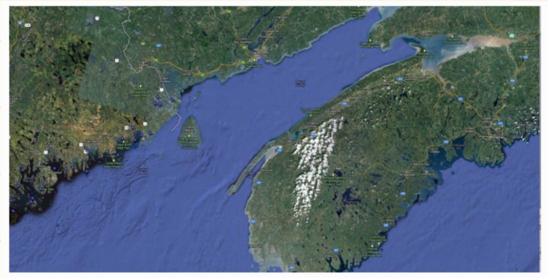
slightly easier to remember in dimensionless form

2.3.2 Tidal Bulge on Earth

Remember $\Delta / R_\oplus \approx \frac{3}{2}\left(\frac{M_\mathbb{C}}{M_\oplus}\right)\left(\frac{R_\oplus}{a}\right)^3$

For Earth + Moon: $\Delta \sim 0.6\,m$

This is a good estimate for the mid-ocean tide, but this can be amplified at coasts, e.g., Bay of Fundy



Also get tidal bores where a river flows into an estuary, which is a jump in height analogous to a shock wave due to a disturbance moving faster than information can propagate

but due to incompressible

eg. Kings Lynn (a few cm but can be m).

The Earth is affected by tidal perturbations from both the Moon and the Sun, so which dominates its tidal bulge?

Remember $\Delta / R_t = \frac{3}{2}\left(\frac{M_c}{M_t}\right)\left(\frac{R_t}{a}\right)^3$

So for fixed M_t, R_t : $\Delta / R_t \propto M_c / a^3 \sim$ mean density by spreading companion around orbit

angular diameter of Moon from Earth D

$\therefore \Delta_{moon} / \Delta_{sun} = \left(\frac{M_L}{a_{EL}^3}\right)\left(\frac{a_{ES}^3}{M_\odot}\right) = \left(\frac{\rho_L}{\rho_S}\right)\left(\frac{R_L}{a_{EL}}\right)^3\left(\frac{a_{ES}}{R_\odot}\right)^3 = \left(\frac{\rho_L}{\rho_S}\right)\left(\frac{D_L}{D_\odot}\right)^3$

~ 1 from eclipse

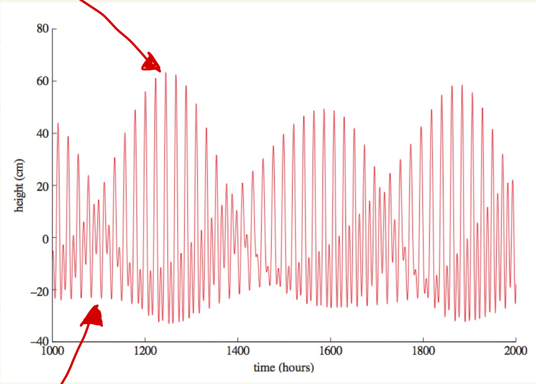
$\sim \rho_L / \rho_S$

$\therefore$ Moon is dominant by a factor of a few

NB Isaac Newton used this + tidal record to infer that ρ_S slightly less than ρ_L

The Sun causes the neap / spring tide cycle

Spring = Sun + Moon tides aligned

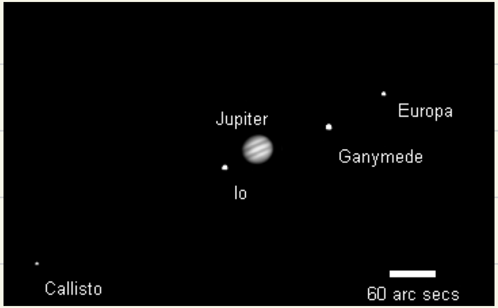


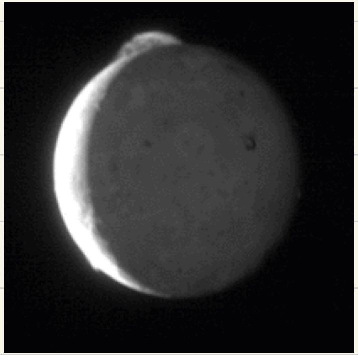
Neap = Sun + Moon tides orthogonal

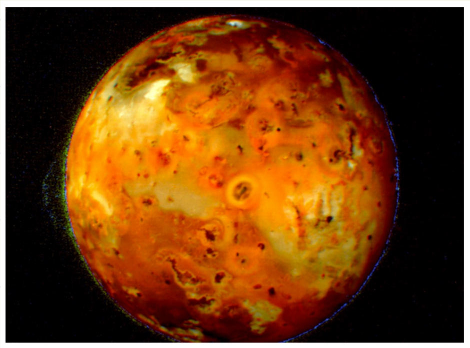
Did this drive evolution by forcing tetrapods out of high pools (the earliest fossils are found at extremes of tidal amplitude)?

2.3.3 Tidal Bulges on Solid Bodies

Solid material cannot respond to the change in potential, so it becomes cracked and heated leading to vulcanism







e.g., Io's orbit is forced to be eccentric by Jupiter's other moons

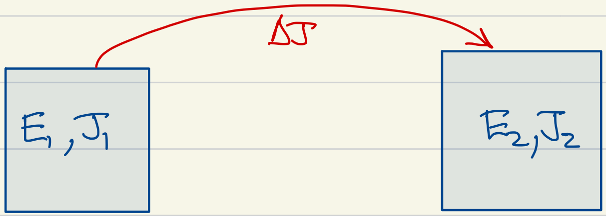
Tidal forcing at pericentre is evidenced by the volcano Tvashtar spewing material up 330 km and molten sulphur pools on the surface

2.4 Transfer of Energy (E) and Angular Momentum (J)

Tides cause transfer of angular momentum J due to dissipation of energy E

2.4.1 General Considerations

Consider a transfer of angular momentum ΔJ between systems 1 and 2



The change in total energy $\Delta E = \left[\partial E / \partial J |_2 - \partial E / \partial J |_1 \right] \Delta J$

Remember: $\Delta E = \left[\partial E/\partial J|_2 - \partial E/\partial J|_1 \right] \Delta J$

The system could be:

(i) A rigid body of fixed moment of inertia $I = \frac{2}{5} M R^2$ (for constant ρ) spinning with angular velocity Ω

$E = \frac{1}{2} I \Omega^2$

$J = I \Omega$

$\therefore E = \frac{1}{2} J^2 / I$

$\therefore \partial E/\partial J = J/I = \Omega$

(ii) Two masses M and m separated by a orbiting about a centre of mass at angular velocity Ω

$E = -\frac{1}{2} G M m / a \quad = -\frac{1}{2} G \mu M_{tot} / a$ (reduced mass)

$J = \left(\frac{Mm}{m+M}\right) \sqrt{G(M+m)a} \quad = \mu \sqrt{G M_{tot} a}$

$\therefore E = -\frac{1}{2} \mu^3 G^2 M_{tot}^2 / J^2$

$\therefore \partial E/\partial J = \mu^3 G^2 M_{tot}^2 / J^3$

$= \sqrt{G M_{tot} / a^3} \quad = \Omega$

In both cases $\Delta E = \left[\Omega_2 - \Omega_1 \right] \Delta J$

Remember: $\Delta E = [\Omega_2 - \Omega_1] \Delta J$

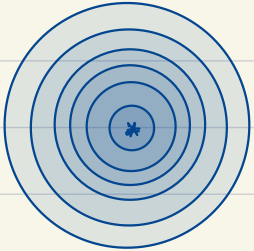
The arrow of time means that the energy of the combined system must decrease

(as energy is dissipated as heat increasing overall entropy)

$\rightarrow \Delta E < 0$

$\rightarrow$ J is passed from high Ω to low Ω systems

E.g., consider an accretion disk, for which the two systems could be neighbouring annuli



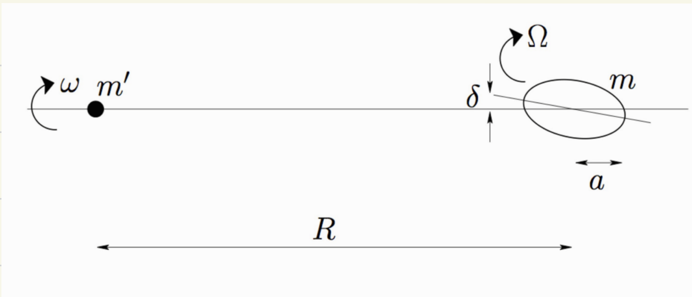
$\Omega = \sqrt{GM/r^3}$

$\therefore$ energy dissipation transfers angular momentum outward

and is why the Sun does not have all J in the SS

2.4.2 How Angular Momentum is Transferred by Tides

E.g., Consider the Earth+Moon system



Spin of Earth = Ω > ω = Orbit of Moon

Hence, J is transferred from rapidly spinning Earth to slowly orbiting Moon, and this is causing both the day and the month (lunar cycle) to get longer

eventually day and month will be equal.

But how? friction on sea floor drags the bulge causing offset δ

→ high tide is after Moon is overhead,

magnitude of δ depends on frictional coupling at ocean floor

so Moon exerts torque on bulge slowing Earth's spin

listening to sea is dissipating E from tides!

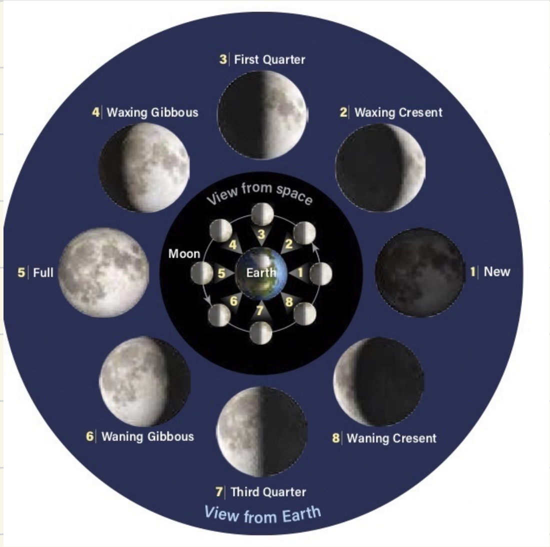
2.4.3 Tidal Locking

Remember: $\Delta E = [\Omega_2 - \Omega_1] \Delta J$

→ no energy dissipation when $\Omega_1 = \Omega_2$

→ synchronous rotation is ultimate end-state

→ at this point $\delta = 0$



Earth's Moon: $\Omega_{moon} = \omega$ hence we only see one side

Note that at New Moon we only see Earth-shine

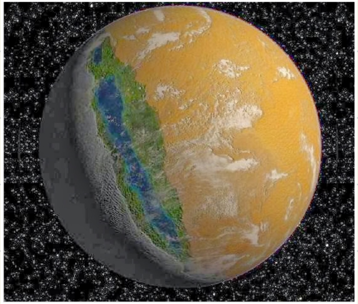
(light reflected from Earth to Moon and back)

Pluto and its moon Charon: $\Omega_{pluto} = \Omega_{charon} = \omega$

used to compare with exoplanets

Many close-in exoplanets are tidally locked to their stars, $\Omega = \omega$,

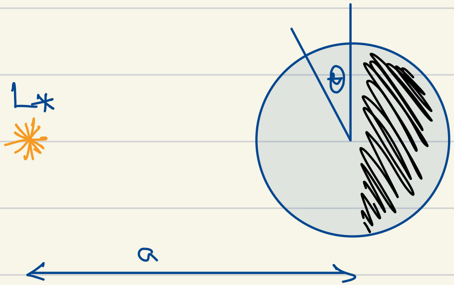
always showing the same face to the star



One hemisphere is in permanent night, the other permanent daylight,

with implications for atmosphere dynamics and habitability

What is the size of the habitable limb on a tidally locked planet a distance a from a star of luminosity L_* ?



Incident flux = $\left(\frac{L_*}{4\pi a^2}\right) \sin\theta = \sigma T_*^4 \left(\frac{R_*}{a}\right)^2 \sin\theta$

Radiated flux = $\sigma T(\theta)^4$

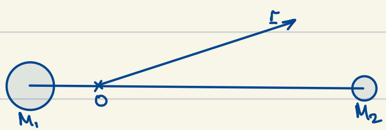
Ignoring day-night side heat transfer: $T(\theta) = T_* \sqrt{R_*/a}\, \sin^{1/4}\theta$

eg for $1 L_\odot$, 0.1au, 300K requires $\theta \simeq 0.3^\circ$

2.5 Binary Potential

Two masses M_1 and M_2 are separated by a forming a binary system that orbits its centre of mass O

Consider the frame co-rotating with the binary's angular motion $\underline{\Omega} = \sqrt{G(M_1 + M_2)/a^3}$



2.5.1 Roche Potential

coriolis terms for moving particles

The gradient of $\Phi_{roche}(\underline{r})$ gives the acceleration in this frame on a stationary particle at $\underline{r}$

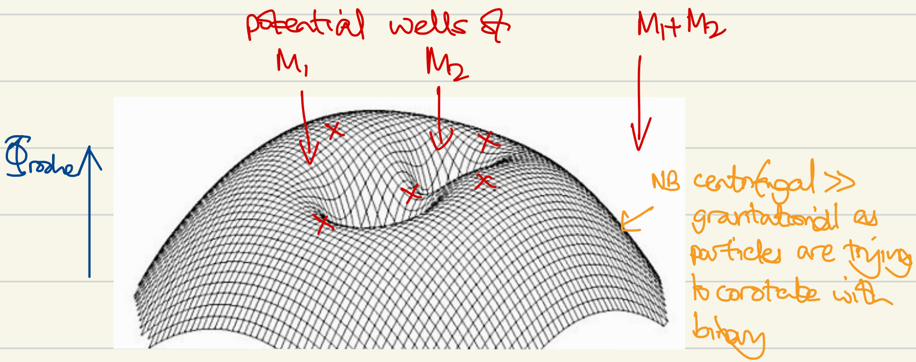
$$\Phi_{roche}(\underline{r}) = -\frac{GM_1}{|\underline{r}-\underline{r}_1|} - \frac{GM_2}{|\underline{r}-\underline{r}_2|} - \frac{1}{2}(\underline{\Omega} \wedge \underline{r})^2$$

gravitational potential + centrifugal potential

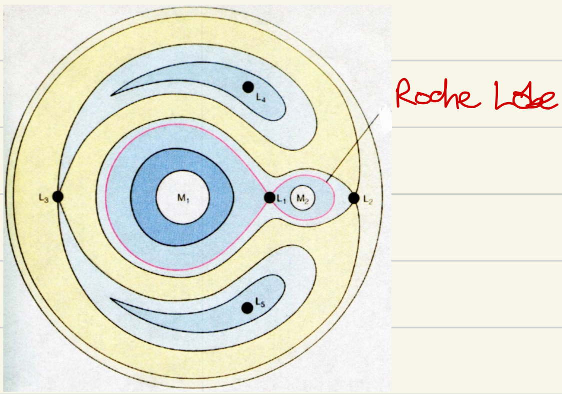
different to analysis in 2.2 as centrifugal acceleration dep. on $|\underline{r}|$

Remember: $\Phi_{roche}(\underline{r}) = -\dfrac{GM_1}{|\underline{r}-\underline{r}_1|} - \dfrac{GM_2}{|\underline{r}-\underline{r}_2|} - \frac{1}{2}(\underline{\Omega} \wedge \underline{r})^2$

Consider Φ_{roche} in the orbital plane



Contours of Φ_{roche}

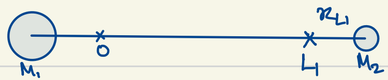


The extremes of $\Phi_{roche}(\underline{r})$ are the Lagrange equilibrium points; e.g., between M_1 and M_2 lies L_1

This separates the region of influence around each star known as the **Roche Lobe**

Very important for binary star evolution, cluster evolution etc.

2.5.2 Size of Roche Lobe



To work out the location of the L_1 point, let x_{L1} be the distance from M_2 and assume $M_2 \ll M_1$, then equate the gravitational acceleration from M_2 with that from M_1 plus the centrifugal acceleration due to $\Omega = \sqrt{G(M_1+M_2)/a^3}$

$$GM_2/x_{L1}^2 = GM_1/[a-x_{L1}]^2 - \Omega^2 |\underline{r}_{L1}|$$

$$\text{where } |\underline{r}_{L1}| = a\left(\frac{M_1}{M_1+M_2}\right) - x_{L1}$$

Substituting in for Ω and doing a binomial expansion in x_{L1}/a: $GM_2/x_{L1}^2 \simeq 3GM_1 x_{L1}/a^3$

$$\therefore x_{L1} = \left(\frac{M_2}{3M_1}\right)^{1/3} a$$

Same analysis predicts $x_{L2} = -x_{L1}$ ie on other side of M_2

$$x_{L_1} = \left(\frac{M_2}{3M_1}\right)^{1/3} a$$

This is the radius of a secondary's Roche Lobe in the limit $M_2 \ll M_1$

For larger M_2 the exansion is not valid but not bad as for $M_2 = M_1$, $x_{L_1} = 0.7a$ cf $0.5a$ expected

Quick estimate using densities:

Density of M_2 if spread through Roche Lobe: $M_2 / x_{L_1}^3$

Density of M_1 if spread through a: M_1 / a^3

equating these $\rightarrow x_{L_1} \simeq \left(\frac{M_2}{M_1}\right)^{1/3} a$

Implication of this calculation in terms of densities is that an object will overflow its Roche Lobe and be tidally disrupted if its density $\rho < 3 \times$ mean density of system in which it is orbiting

2.5.3 Examples of Roche Lobe Overflow

2.5.3.1 Nomenclature

Binary star: Roche radius

Planet: Hill radius

Cluster: Tidal radius

all the same

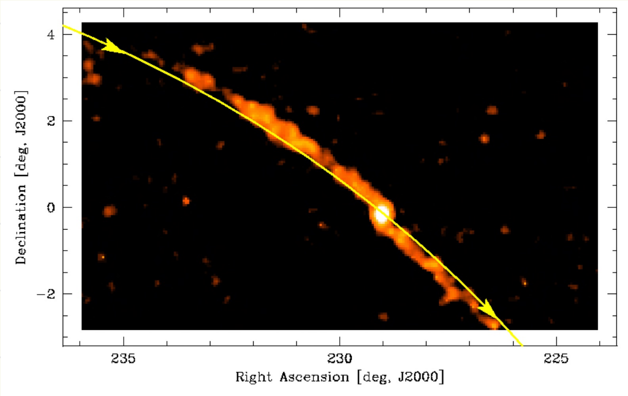
an object "overflows"

or is "tidally stripped"

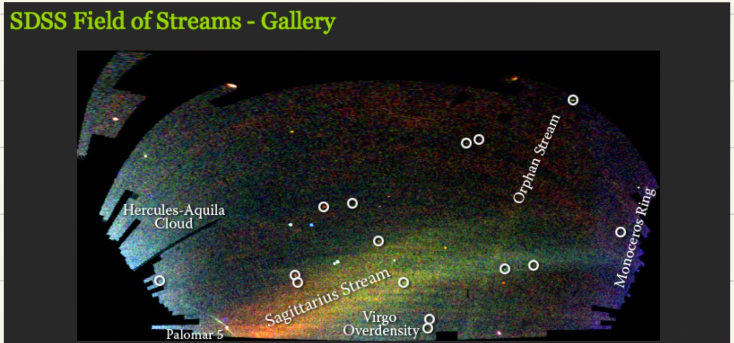
if $R_2 > (M_2/3M_1)^{1/3} a$

2.5.3.2 Tidal Stripping of Star Clusters and Dwarf Galaxies

Tidally disrupted star cluster



Dwarf galaxies are tidally stripped when they fall onto the onto the Milky Way as $\langle \rho \rangle < 3 \langle \rho_{MW} \rangle$



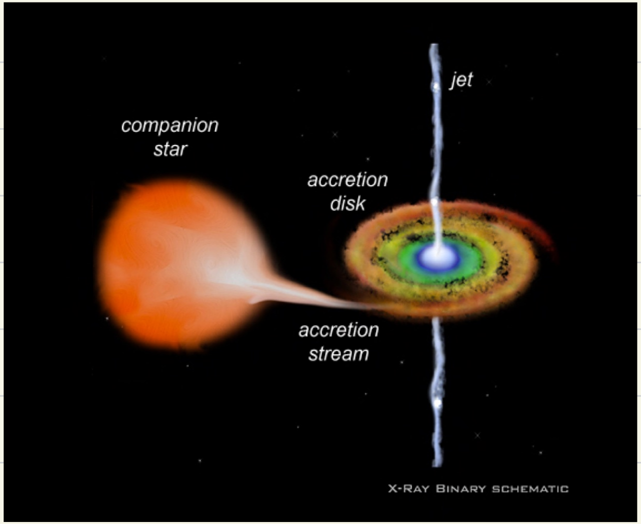
A map of stars in the outer regions of the Milky Way Galaxy, derived from the SDSS images of the northern sky, shown in a Mercator-like projection. The color indicates the distance of the stars, while the intensity indicates the density of stars on the sky. Structures visible in this map include streams of stars torn from the Sagittarius dwarf galaxy, a smaller 'orphan' stream crossing the Sagittarius streams, the 'Monoceros Ring' that encircles the Milky Way disk, trails of stars being stripped from the globular cluster Palomar 5, and excesses of stars found towards the constellations Virgo and Hercules. Circles enclose new Milky Way companions discovered by the SDSS; two of these are faint globular star clusters, while the others are faint dwarf galaxies.
Credit: V. Belokurov and the Sloan Digital Sky Survey.

How do we know the direction of motion?

= angular momentum conservation

2.5.3.3 X-Ray Binaries

A main sequence star expands as it evolves into a giant, overfilling its Roche Lobe and feeding material onto a WD or NS companion, with angular momentum causing an accretion disk that accretes onto the compact object



For $M_2/M_1 \ll 1$ overflow is symmetric

For larger M_2/M_1, L_2 is further than L_1

→ overflow is through L_1

→ material falls onto the primary

2.5.4 Stable Equilibria in the Roche Potential

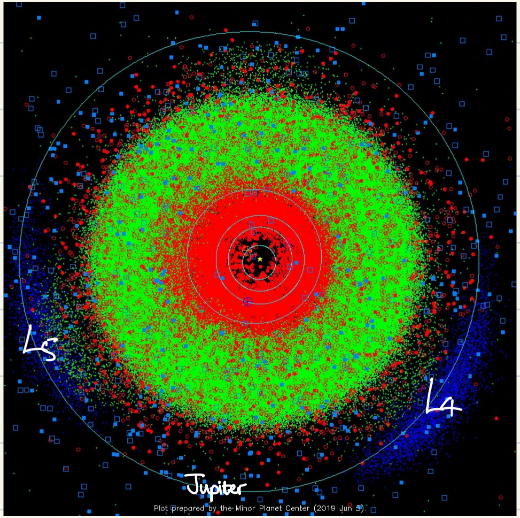
While L_1, L_2, L_3 are unstable, L_4 and L_5 are stable

Why, if they are maxima in the Roche potential?

stabilised by Coriolis forces

E.g., Trojans of Jupiter at its L_4, L_5

Trojans of Earth 2010 TK7 and 2020 XL5



2.6 Outcomes of Close Encounters

2.6.1 Tidal Capture

Mild tidal interactions can put enough energy into tides to cause two objects to become bound

Consider a mass M_2 on a highly eccentric or hyperbolic orbit around M_1

- M_2 is tidally distorted at pericentre
- E_{tidal} is put into tide
- As they separate M_2 relaxes and E_{tidal} is dissipated
- Energy comes from 2-body orbit

  → if E_{tidal} is large enough orbit can become bound

  or at least semi major axis will shrink

What is the energy in the tide when 2 object approach each other at a_{peri} ?

Remember, the height of the tide raised on M_2 is $\Delta / R_2 \simeq \left(\frac{M_1}{M_2}\right)\left(\frac{R_2}{a_{peri}}\right)^3$

(This assumed a circular orbit but is a good estimate)

$$\therefore E_{tidal} = m.g.h \simeq \left(\frac{R_2^2 \Delta}{R_2^3}\right) M_2 \cdot \frac{GM_2}{R_2^2} \cdot \Delta$$

$$\simeq GM_1^2 R_2^5 / a_{peri}^6$$

note dependence on M_1, R_2, a_{peri}

Tidal capture requires: $E_{tidal} > \frac{1}{2} M_2 V_\infty^2$

$$\therefore a_{peri} / R_2 < \left(\frac{M_1}{M_2}\right)^{1/3} \left(\frac{GM_2}{V_\infty^2 R_2}\right)^{1/6}$$

This is necessary but not sufficient as there are other possible outcomes

2.6.2 Tidal Disruption

Remember: $E_{tidal} \sim G M_1^2 R_2^5 / a_{peri}^6$

$x_{L1} \sim a \left(M_2 / 3M_1 \right)^{1/3}$

$\therefore \quad E_{tidal} \sim \left(\frac{G M_2^2}{R_2} \right) \left(R_2 / x_{L1} \right)^6$

Thus the potential energy in the tide is of order the binding energy of the object $\left(\frac{3}{5} \frac{G M_2^2}{R_2} \right)$ if it fills its Roche Lobe

$\therefore$ Object tidally destroyed at pericentre if $E_{tidal} > G M_2^2 / R_2$

2.6.3 Summary of Outcomes

Physical collision if $a_{peri} < R_1 + R_2$

Tidally destroyed if $E_{tidal} > GM_2^2 / R_2$

Tidally captured intact if $\frac{GM_2^2}{R_2} > E_{tidal} > \frac{1}{2} M_2 V_\infty^2$

→ to be captured intact requires $\frac{GM_2^2}{R_2} > \frac{1}{2} M_2 V_\infty^2$

$\therefore \; V_\infty < \sqrt{2GM_2/R_2} = V_{esc}$ ↙ escape velocity from surface of M_2

ie encounters faster than the escape velocity can't result in capture

2.6.4 Which Object Dominates Tidal Dissipation

Remember: $E_{tidal} \sim G M_1^2 R_2^5 / a_{peri}^6$ = energy in the tide raised on M_2 by M_1

$\therefore E_{tidal2} / E_{tidal1} = \dfrac{M_1^2 R_2^5}{M_2^2 R_1^5}$

Which dominates depends on the mass-radius relation $R(M)$

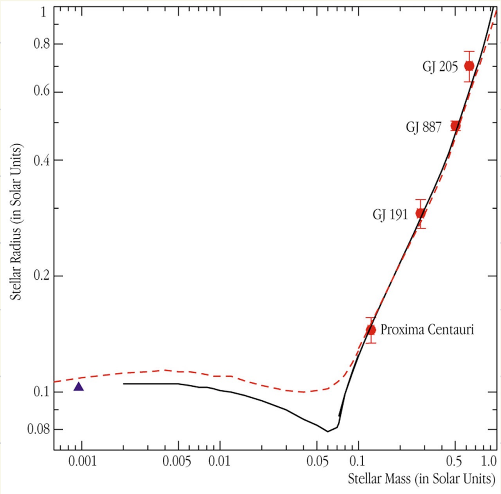
For stars $M \propto R \rightarrow E_{tidal2} / E_{tidal1} = (M_2 / M_1)^3$

$\therefore$ tidal dissipation is in primary

For planets $R \sim$ constant

$\rightarrow$ for fixed M_1, R_1 $E_{tidal2} / E_{tidal1} \propto M_2^{-2}$

$\therefore$ dissipation is in planet for small M_2



2.6.5 Scenarios

2.6.5.1 Tidal Capture of Stars in Clusters into Binaries

Remember, tidal capture requires $a_{peri}/R_2 < \left(\frac{M_1}{M_2}\right)^{1/3}\left(\frac{GM_2}{V_\infty^2 R_2}\right)^{1/6}$

Consider $M_1 = M_2 = 1M_\odot$, $R_1 = R_2 = 1R_\odot$

And that in a typical globular cluster $V_\infty \simeq 10$ km/s

$\therefore \sqrt{GM_2/R_2} \simeq 400$ km/s

$\therefore a_{peri}/R_\odot < 3\text{–}4$ for capture

→ tidal capture binaries are rare and if achieved are very tight

2.6.5.2 Tidal Capture of Free-Floating Planets by Stars

Remember, tidal captures requires $a_{peri} / R_2 < \left(\frac{M_1}{M_2}\right)^{1/3} \left(\frac{GM_2}{V_\infty^2 R_2}\right)^{1/6}$

$\therefore a_{peri} / R_1 < \left(\frac{\rho_1}{\rho_2}\right)^{1/3} \left(\frac{GM_2}{V_\infty^2 R_2}\right)^{1/6}$

Consider $M_1 = 1 M_\odot, R_1 = 1 R_\odot$ and $M_2 = 10^{-3} M_\odot, R_2 = 0.1 R_\odot$

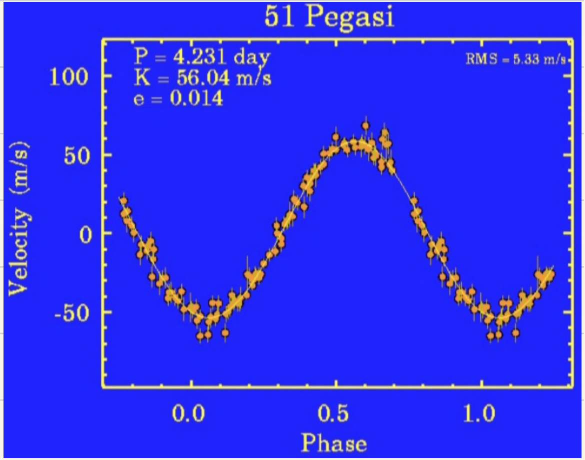
And that stars and free-floating planets encounter each other at $V_\infty \sim 30$ km/s

$\therefore \sqrt{GM_2 / R_2} \sim 40$ km/s

$\therefore a_{peri} / R_\odot < 1$ for capture

→ hard to achieve capture without suffering a collision

2.6.5.3 Tidal Origin of Hot Jupiters



Hot Jupiters are $\sim 10^{-3} M_\odot$ planets on circular orbits at <0.1au

found to 1% of stars

Unlikely to form in situ or to have been tidally captured

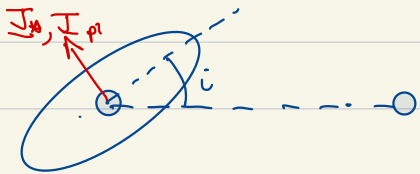
Thus likely to have formed further out and migrated in

- by interaction with a disk
- by high eccentricity migration
  = tides raised at pericentre of high-eccentricity orbit dissipate energy causing planet to become more tightly bound

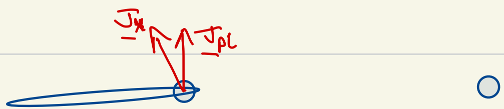
A wide binary companion could be the origin of the eccentricity

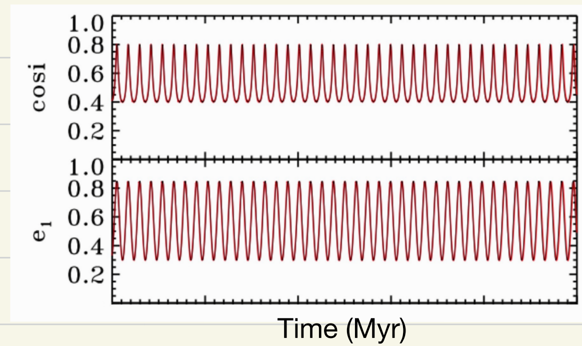
E.g., assume the planet forms on an orbit aligned with the stellar spin, but the outer binary's orbital plane is inclined by i

Start



Mid-cycle

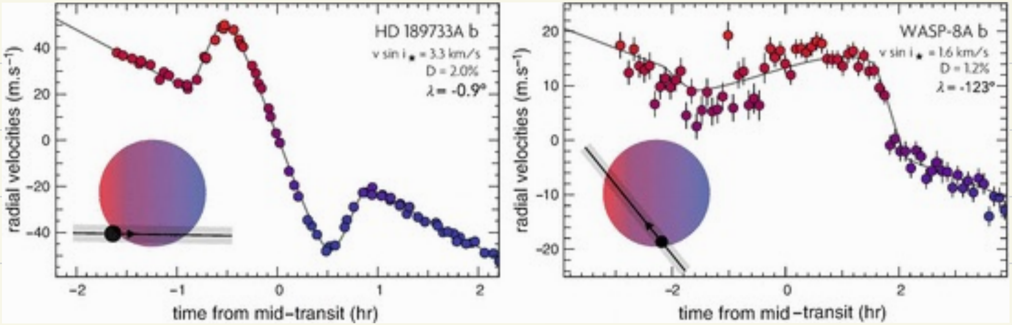




The gravity of the binary companion causes the planet's orbit to undergo Kozai-Lidov cycles, reaching high eccentricity and bringing the pericentre close to the star

Tides at pericentre can dissipate energy circularising the planet's orbit near pericentre, resulting in an orbit that is misaligned with the stellar spin axis

This potentially explains misaligned Hot Jupiters observed via the Rossiter McLaughlin effect:



A star's radial velocity is modified during transit as a planet blocks the stellar surface

2.6.5.4 Tidal Capture of a Star by a Compact Object

white dwarf / neutron star

This is similar to the scenario of capture of stars in clusters into binaries, and so capture requires $a_{peri} / R_\odot < 3-4$

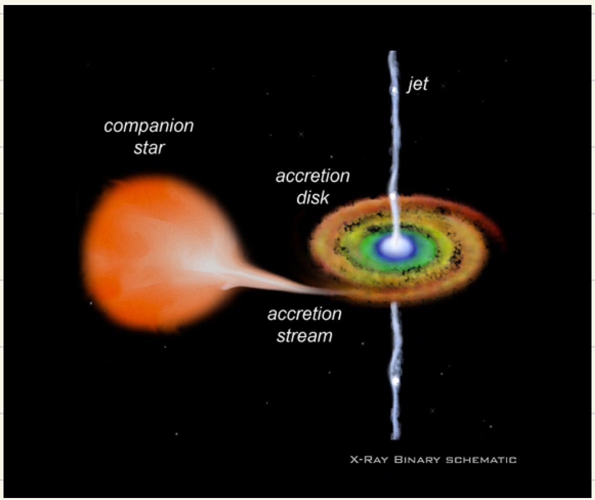
However, a collision is unlikely as $R_1 \ll R_\odot$

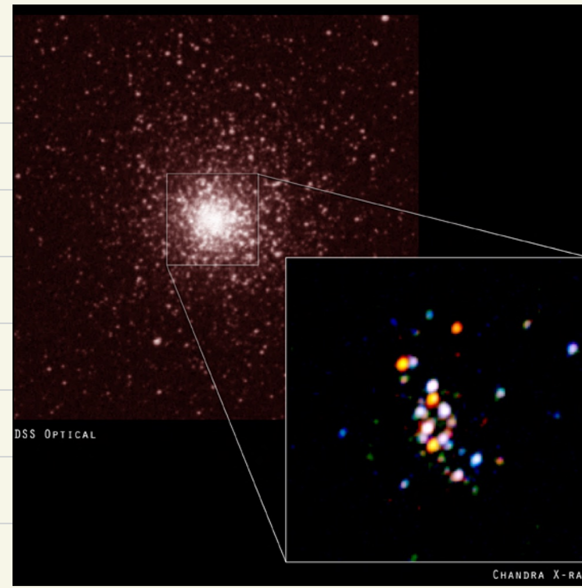
Thus can get capture intact if $V_\infty < V_{esc}$ ($V_\infty \sim 10$ km/s, $V_{esc} \sim 100$s of km/s)

→ neutron stars can capture stars onto orbits of a few $R_\odot$

Tidal capture binaries evolve into X-ray binaries in which a star overfills its Roche Lobe when it swells as a red giant creating a disk that accretes onto the companion star

Tides explain the origin of X-ray binaries and why these are preferentially found in the cores of globular clusters





E.g., Fabian, Pringle & Rees (1975)

2.6.5.5 Tidal Disruption of Stars by a Black Hole in Galactic Nucleus

Stars are tidally destroyed if they overflow their Roche Lobe

$$\rightarrow \quad \left(\frac{M_*}{3M_{bh}}\right)^{1/3} a_{peri} < R_*$$

$$\therefore \quad a_{peri} < r_t = \left(\frac{3M_{bh}}{M_*}\right)^{1/3} R_*$$

But a star is swallowed whole if the pericentre is inside the Black Hole's Schwarzschild radius

$$\rightarrow \quad a_{peri} < r_s = 2GM_{bh}/c^2$$

$\rightarrow$ can't be tidally disrupted if $2GM_{bh}/c^2 > \left(\frac{3M_{bh}}{M_*}\right)^{1/3} R_*$ (ie $r_s > r_t$)

$$\therefore \quad M_{bh} > \sim 10^9 M_\odot$$