

# Planetary Migration to Large Radii

R.G. Martin<sup>1</sup>, S.H. Lubow<sup>2</sup>, J.E. Pringle<sup>1,2</sup> & M.C. Wyatt<sup>1</sup>

<sup>1</sup>*Institute of Astronomy, Madingley Road, Cambridge, CB3 0HA, UK*

<sup>2</sup>*STScI, 3700 San Martin Drive, Baltimore, MD 21218, USA*

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## ABSTRACT

There is evidence for the existence of massive planets at orbital radii of several hundred AU from their parent stars where the timescale for planet formation by core accretion is longer than the disc lifetime. These planets could have formed close to their star and then migrated outwards. We consider how the transfer of angular momentum by viscous disc interactions from a massive inner planet could cause significant outward migration of a smaller outer planet. We find that it is *in principle* possible for planets to migrate to large radii. We note, however, a number of effects which may render the process somewhat problematic.

**Key words:** accretion, accretion discs - planets and satellites: formation - planetary systems: protoplanetary discs.

## 1 INTRODUCTION

Most stars at an age of about  $10^6$  yr are surrounded by circumstellar discs which are optically thick at optical and infrared wavelengths (Strom et al. 1989, Kenyon & Hartmann 1995, Haisch, Lada & Lada 2001). At earlier times, since these discs are part of the star formation process, it seems likely that they contained a significant fraction of the stellar mass (e.g. Lin & Pringle, 1976). At this age of around  $10^6$  yr, the discs are found to be as massive as a few per cent of a solar mass and have typical sizes of a few hundred AU (Beckwith et al. 1990). It must be within these discs that planets form. Most of the detections of extrasolar planets are based on Doppler techniques and a smaller number of detections come from occultations<sup>1</sup> (Butler et al. 2006). For obvious reasons these detection methods are biased toward finding planets at relatively small distances from their parent stars compared to the typical disc sizes, and compared to the planets on the Solar System (e.g. Beer et al., 2004). In addition the planets discovered are almost all high mass, and therefore most likely, all gas-rich giants. Current ideas on the formation of such planets, involving the initial formation of a metal-rich core and subsequent accretion of a gaseous envelope from the disc, suggest that these planets formed at larger radii (around a few AU so that core-formation can occur) and then migrated inwards to their current positions (e.g., Lin, Bodenheimer, & Richardson 1996). However, in addition to these planets at small radii, there is evidence, albeit somewhat indirect, for several massive planets at very large radii, much greater than

the putative formation radius of around a few AU. For example, one model for the formation of spiral structure seen in the disc of HD141569 involves the presence of a planet of mass  $0.2 - 2 M_J$  orbiting at 235 – 250 AU and a Saturn-mass planet at 150 AU (Wyatt 2005a). Gaseous discs appear to dissipate on a timescale of 5 – 10 Myr (Haisch, Lada & Lada 2001) and clearly the gas giant planets must form before this occurs. In the standard core accretion models (Safronov 1969), the timescale for building a giant planet core scales with the square of the distance from the central star (Pollack et al. 1996). The formation time for Jupiter (at 5 AU) is estimated to be 1 – 10 Myr. Hence, to form a  $1 M_J$  planet at 200 AU would take 1.6 – 16 Gyr, by which time the gaseous disc has long since disappeared. It is therefore thought highly unlikely that gas giant planets can form *in situ* at large distances from the central star.

One obvious possible explanation for the presence of such planets at large radii is that they form at a standard radius and migrate outwards. The key issue is then what provides the outward torque. Inward migration seems a more natural possibility, since the disc, which facilitates the migration, is part of the accretion process. For example, consider a planet whose mass is small compared to the disc but sufficiently large to open a gap in the disc. Such a planet follows the radial mass flow of the disc in what is commonly called Type II migration (Lin & Papaloizou 1986). Generally, disc material flows inward as an accretion disc carrying the planet inward towards the central star. However, in order for accretion to proceed, the disc must move angular momentum outwards. Thus some material in the outer parts of an accretion disc moves outwards in order to conserve the angular momentum lost by most of the disc. The fraction of

<sup>1</sup> See <http://exoplanet.eu> for a recent catalogue of exoplanets.

outward moving material decreases with time. A planet located in such a disc region would similarly move outward, at least for a limited time. Veras & Armitage (2004) considered this possibility. They model a situation in which disc photoevaporation at large radii causes a planet at (small) standard radii to effectively experience outer edge effects which in turn causes outward migration. For the models they considered, planets were able to migrate to about 20 AU and in a few cases as far as 50 AU.

There are two main problems to be addressed in trying to drive a planet to large distance. First it is necessary to provide a large enough source of angular momentum. And second, it is necessary to give the planet that angular momentum on a short enough timescale. In the models of Veras & Armitage (2004) the angular momentum was provided by the disc. Here we build on these ideas, but consider the possibility that the source of the angular momentum is an inner, more massive, planet. As a simple example, suppose initially we have two planets of masses  $M_1$  and  $M_2$  orbiting a star in circular orbits at radii  $a_1$  and  $a_2$  respectively with  $a_2 > a_1$ . Then if the inner planet could somehow be induced to give all its angular momentum to the outer one, the outer one would move out to a final radius

$$a_f = \left( \frac{M_1}{M_2} a_1^{1/2} + a_2^{1/2} \right)^2 \quad (1)$$

where we have assumed, as is reasonable if the angular momentum transfer is effected through tidal interactions with a gaseous disc, that the outer planet remains in a circular orbit. Suppose we have a planet of mass  $5 M_J$  that begins at  $a_1 = 5$  AU and a planet of mass  $1 M_J$  that begins at  $a_2 = 10$  AU. If the inner planet manages to give up all its angular momentum, and so ends up at the central star with radius  $R \ll a_1$ , then the outer planet can migrate to a distance  $a_f = 206$  AU. This demonstrates that mutual interaction within a two (or more) planet system can, *in principle*, drive a planet out to the required distances of a few hundred AU.

In this paper we investigate what is required in practice for this to be achieved. It is clear that there are two main requirements. First we need some efficient mechanism for transferring angular momentum from the inner planet to the outer one. Second, we need to effect the transfer on a sufficiently short timescale before the means of transferring the angular momentum, presumably tied to the disc, has vanished.

The outline of the paper is as follows. In Section 2 we consider the evolution of a region of a steady accretion disc that lies exterior to a newly formed massive planet or companion star. We show that the disc changes format from accretion to decretion and in doing so changes its surface density profile on a viscous timescale. In Section 3 we consider the evolution of a steady accretion disc in which two planets are permitted to form and consider the circumstances necessary for the outer planet to be forced to migrate outwards to large radius. By an age of  $10^7$  yr it appears that massive circumstellar discs are no longer present. All that remains is possibly a debris disc. Photoevaporation of the disc is considered a possible gas dispersion mechanism. In Section 4 we investigate the effects of mass loss from the disc caused by photoevaporation. In Section 5 we discuss the applicability

of our findings to some observed systems. We summarise our conclusions in Section 6.

## 2 THE CHANGE FROM ACCRETION DISC TO DECETION DISC

An accretion disc occurs when the disc has a mass sink at its inner edge (allowing accretion onto the central object) while providing no torque at the inner edge. A decretion disc occurs when the central object does not accrete, but instead provides a central torque which prevents inflow. For a decretion disc, the torque exerted by the central object results in radial outflow. Decretion discs can arise when the central torque is provided by a binary star or star-planet system. We consider in this Section how an accretion disc adjusts when conditions at the inner boundary change so that accretion there no longer occurs. We consider the idealised situation in which, for example, a massive planet, or binary companion, forms suddenly (compared to a viscous timescale) near the inner disc edge. Then the tidal influence of the inner binary is assumed to prevent further accretion, and to provide the disc with angular momentum through a tidal torque. What we are interested in here is the resulting change to the disc structure caused by such an event.

The equation that describes the evolution of a flat Keplerian accretion disc with surface density  $\Sigma(R, t)$  where  $R$  is the radius from centre of the star and  $t$  is the time, is

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left[ 3R^{1/2} \frac{\partial}{\partial R} (\nu \Sigma R^{1/2}) \right] \quad (2)$$

(Pringle 1981) where  $\nu(R, \Sigma)$  is the kinematic viscosity. In general we shall model the effect of the planet or companion star on the disc as an extra angular momentum source term. The governing equation for such a disc is

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left[ 3R^{1/2} \frac{\partial}{\partial R} (\nu \Sigma R^{1/2}) - \frac{2\Lambda \Sigma R^{3/2}}{(GM_\star)^{1/2}} \right], \quad (3)$$

where  $\Lambda(R, t)$  is the rate of input of angular momentum per unit mass (Lin & Papaloizou 1986, Armitage et al. 2002). We analyse here the evolution of disc material that resides outside the orbit of the companion.

In this Section, for the purposes of illustration, the companion is assumed sufficiently massive that its angular momentum is much greater than that of the disc, so that the orbital evolution of the central objects can be ignored. We consider a high mass planet or companion star which orbits at fixed radius and prevents mass from passing interior to the disc inner edge. In this situation, the effect of this extra torque term involving  $\Lambda$  is equivalent to imposing a zero radial velocity inner boundary condition at  $R = R_{\text{in}}$  (Pringle 1991) where the radial velocity in the disc is given by

$$V_R = -\frac{3}{\Sigma R^{1/2}} \frac{\partial}{\partial R} (R^{1/2} \nu \Sigma). \quad (4)$$

We initially take an accretion disc with constant accretion rate but truncated at some outer radius  $R_t$ . Thus for the initial surface density we take

$$\Sigma(R) = \Sigma_0 \left( \frac{R_{\text{in}}}{R} \right)^\beta \quad (5)$$

in the range  $R_{\text{in}} < R < R_t$  where  $\Sigma_0$  is a constant which

determines the initial mass of the disc and  $R_t$  is the initial outer edge of the disc. We take  $\beta = 3/2$  (Weidenschilling 1977, Hayashi 1981) in most of our numerical simulations, but also investigate the effect of varying  $\beta$ .

In order that  $\nu\Sigma = \text{const.}$  in the initial accretion disc, and to keep things simple at later times, we choose the kinematic viscosity of the disc to be

$$\nu = \nu_0 \left( \frac{R}{1 \text{ AU}} \right)^\beta \quad (6)$$

with  $\nu_0 = 2.466 \times 10^{-6} \text{ AU}^2 \text{ yr}^{-1}$  to give a reasonable time scale for the evolution of the disc of a few Myr. In terms of the  $\alpha$ -prescription for the viscosity  $\nu = \alpha c_s H$  where  $\alpha$  is dimensionless,  $c_s$  is the sound speed and  $H$  is the scale height of the disc, and for a typical value  $H/R = 0.05$  (see below), this implies

$$\alpha = 1.57 \times 10^{-4} \left( \frac{R}{\text{AU}} \right)^{(\beta-1/2)}. \quad (7)$$

We solve equation (3) on a fixed, uniform mesh in the variable  $x = R^{1/2}$  by using a simple first order explicit numerical method. We use 4000 grid points with a zero radial velocity inner boundary condition at  $R_{\text{in}} = 5 \text{ AU}$ ,  $V_R(R_{\text{in}}, t) = 0$  for all time  $t$  and a zero torque outer boundary at  $R_{\text{out}} = 1500 \text{ AU}$ . Due to the inner boundary condition on the radial velocity, the gravitational torque term  $\Lambda$  in equation (3) is ignored. The outer boundary condition does not effect the disc evolution because it is well outside the disc over the course of the evolution we consider.

In the simulations, we trace the radial movement of disc particles by integrating the radial velocity because

$$\frac{da}{dt} = V_R(a(t), t), \quad (8)$$

where  $a(t)$  is the radial position of the particle. The imposition of a non-absorbing boundary at an inner radius  $R_{\text{in}}$  transforms the initial accretion disc into a decretion disc. To show the outcome of imposing such a boundary condition, we plot in Figure 1 the radial motions of particles in two discs, one initially truncated at  $R_t = 20 \text{ AU}$  (dashed line in Figure 1) and the other initially truncated at  $R_t = 100 \text{ AU}$  (solid lines in Figure 1). In both discs, the particles initially follow the inward accretion, before reversing and being expelled to large radii. The smaller disc makes the initial adjustment more quickly and expels the particles more quickly than the larger disc.

To understand the disc flow analytically, we consider the behaviour of self-similar solutions to the disc flow that are valid at large times based on Pringle (1991). We first consider the case of an accretion disc. In a steady accretion disc, the left-hand side of equation (2) is zero. This condition is satisfied for

$$V_R = -\frac{3\nu}{2R} \quad (9)$$

and the mass flux is given by

$$\dot{M} = -3\pi\nu\Sigma \quad (10)$$

which is independent of radius. The latter provides the radial variation  $\Sigma(R)$  in the steady-state. Over time, an arbitrary initial disc which evolves with absorbing (mass flow) inner

boundary conditions will approach this steady-state accretion disc. The self-similar solution valid at large times for an accretion disc implies that

$$V_R = -\frac{\nu_0}{R_0} \left( \frac{3r^{\beta-1}}{2} - \frac{r}{(2-\beta)\tau} \right), \quad (11)$$

where  $\nu_0$  is given by equation (6) with  $R_0 = 1 \text{ AU}$ , dimensionless radius  $r = R/R_0$  and dimensionless time  $\tau = t\nu_0/R_0^2$ . Consequently, at fixed radius and for sufficiently large  $t$ , the radial velocity in equation (11) approaches the steady state value given in equation (9) which provides the Type II migration velocity and the motion is always eventually inward.

In contrast, a steady decretion disc has  $V_R = 0$  everywhere and

$$\nu\Sigma \propto R^{-1/2} \quad (12)$$

(Pringle 1991). The decretion disc simulations in Figure 1 start with a steady-state accretion disc density profile. Initially, the disc is unaware of the inner boundary where the outward torque is exerted, and behaves as a standard accretion disc. Over time the mass builds up near the inner boundary as a consequence of the blocked inflow. The outward torque increases and its effects propagate outward. Over time, more of the disc material flows radially outward to become a decretion disc. The disc gains angular momentum at the expense of the central system.

We discuss below whether/how a decretion disc approaches its steady-state density and velocity over time. We also determine the particle drift velocity  $V_R$  for a decretion disc and thereby obtain its Type II migration rate. It can be shown from the self-similar solutions for a decretion disc (using the equations of Pringle (1991) for a viscosity which is non-linear in  $\Sigma$  and applying the limit of the  $\Sigma$  variation disappearing) that the density and velocity evolves at large times as

$$\Sigma(r, \tau) = \frac{\Sigma_0 \exp[-r^{2-\beta}/(b\tau)]}{\tau^c r^{\beta+1/2}} \quad (13)$$

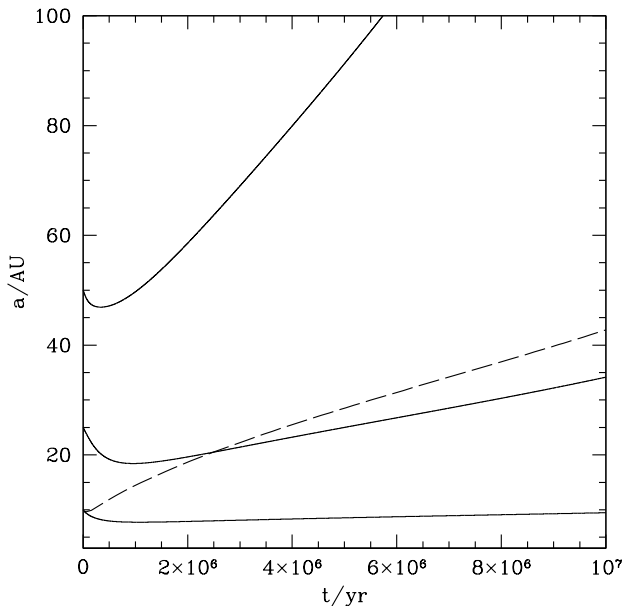
and

$$V_R(r, \tau) = \frac{\nu_0}{R_0} \frac{r}{(2-\beta)\tau}, \quad (14)$$

where  $\nu_0$ ,  $R_0$ ,  $r$  and  $\tau$  are as defined below equation (11),  $\Sigma_0$  is the density normalisation constant,  $b = 3(2-\beta)^2$  and  $c = 1 - 1/(4-2\beta)$ . This result shows that for fixed  $r$  the density at large time varies approximately as a power law in time. At fixed  $r$  and large  $\tau$  the density decreases in time for  $\beta < 3/2$ , remains constant in time for  $\beta = 3/2$  and increases in time for  $3/2 < \beta < 2$  as a consequence of the mass flux variation in radius. Therefore, the density approaches a nonzero steady-state value only for  $\beta = 3/2$ . For  $0 < \beta < 2$ , the spatial variation of the density at large fixed time does satisfy equation (12) where the proportionality constant in that equation generally depends on time. For  $0 < \beta < 2$ , at fixed radius the radial velocity  $V_R$  at all radii approaches the steady-state value of zero at large time.

The particle trajectory  $a(t)$  at large times is determined by applying equation (14) in equation (8). We find that

$$a(\tau) = d\tau^{1/(2-\beta)} \quad (15)$$



**Figure 1.** Particle paths as a disc changes from accretion to decretion. The vertical axis is the particle orbital radius in AU and the horizontal axis is the time in years. The disc viscosity is given by equation (6) with  $\beta = 3/2$  and the disc inner edge is located at  $R_{in} = 5$  AU. At time  $t = 0$  the disc is taken to be a steady accretion disc. The solid lines are for paths located in a disc that is initially truncated at  $R_t = 100$  AU. The dashed line is for a path in a disc with  $R_t = 20$  AU. The effect of a central torque is provided by the boundary condition on the radial velocity  $V_R = 0$  at the disc inner edge.

and

$$\frac{da}{dt} = V_R(a(t), t) = \frac{\nu_0 d \tau^{\frac{\beta-1}{2-\beta}}}{R_0^2 (2-\beta)}, \quad (16)$$

where  $d$  is a constant with units of length along each particle path which is determined by its initial radius and radial velocity. Equation (15) shows that the evolution at large times proceeds as the particles in general move to large radii for  $0 < \beta < 2$ . However if  $d = 0$ , the path remains fixed at small radii. This situation occurs for example at the disc inner boundary. Even small values of  $d$  eventually lead to outflow to large distances if the disc survives for a sufficiently long time.

Equation (16) provides the Type II migration velocity in a decretion disc. Notice that  $V_R$  goes to zero at large time for  $0 < \beta < 1$ , is constant for  $\beta = 1$  and increases in time for  $1 < \beta < 2$ . There is a marked difference in behaviour between the (Lagrangian) particle velocity and the (Eulerian) velocity at fixed  $R$ , as discussed earlier which always vanishes at large  $t$ . The reason for the difference is that a particle having  $d > 0$  will never experience viscously relaxed conditions as it moves outwards while the velocity at fixed radius declines to zero over several local viscous times. This is a consequence of the particle velocity being of order the characteristic viscous propagation speed  $\sim \nu(a(t))/a(t)$  for  $d \sim R_0$ .

The computations described in this Section can be taken as the limiting case of what happens if a stellar com-

panion or massive planet is introduced and truncates the outer disc at an inner radius of 5 AU. The paths traced by the particles represent the radial motion of a light planet undergoing Type II migration in the outer disc. In this case direction of motion of the light planet is reversed on the local viscous timescale and the planet can eventually be expelled to a large radius by the presence of the inner objects. The simplifying assumptions made in this Section imply that there is an infinite source of angular momentum at the inner disc edge. In the next Section we describe a more realistic calculation with the two planets and the disc all of finite and comparable, mass.

### 3 TWO PLANETS

In this section we consider the migration of two planets in the disc. We shall take the inner planet to have mass  $M_1 = 5 M_J$  and the outer planet to have mass  $M_2 = 1 M_J$ . We use a model of planetary migration in a disc similar to that of Armitage et al. (2002). This is a one-dimensional model evolving by internal viscous torques and external torques from planets embedded in the disc (Goldreich & Tremaine 1980, Lin & Papaloizou 1986, Trilling et al. 1998, Trilling, Lunine & Benz 2002). We consider only planets massive enough to open a clean gap, i.e. pure type II migration, and cases where the planets are sufficiently well separated that there is gas between them. If they are too close, this is not the case and the evolution proceeds in a different manner from that found in this paper (e.g. Kley, Peitz & Bryden, 2004).

The criterion given by Lin & Papaloizou (1986) for the opening of a clean gap, and so for the validity of simple Type II migration, can be written in terms of the local Reynold's number  $Re = R^2 \Omega / \nu$ , where  $R$  is the radius of the planet's orbit and  $\Omega$  the angular velocity of the disc flow there. The condition for gap opening is that

$$Re \geq 40 \left( \frac{M}{M_p} \right)^2 \left( \frac{H}{R} \right)^3, \quad (17)$$

where  $M$  is the mass of the star and  $M_p$  the mass of the planet. The detailed protoplanetary disc models of Bell et al. (1997) suggest that to a reasonable approximation we may assume  $H/R = 0.05$ , and so we make this assumption throughout. For a planet mass  $M_p = 1 M_J$  and for this value of  $H/R$  this implies that Type II migration occurs when  $Re \geq 5.5 \times 10^3$ . If  $\nu = \alpha c_s H$  then the Reynold's number can be written as

$$Re = \left( \frac{R}{H} \right)^2 \frac{1}{\alpha}. \quad (18)$$

For the value of  $\alpha$  given by equation 7 with  $\beta = 3/2$ , we find  $Re = 2.55 \times 10^6 (R/\text{AU})^{-1}$ . If these estimates hold, then we conclude that we may safely assume simple Type II migration for the calculations in this paper.

In practice, there may be some flow through the gap when full 2D or 3D effects are taken into account in the simulations (Artymowicz & Lubow, 1996; Lubow & D'Angelo, 2006). D'Angelo, Lubow & Bate (2006) find that for planets on circular orbits, and with  $Re \approx 10^5$ , the 1D migration rate agrees to within a few percent with 2D simulations for planets whose masses are of order  $1 M_J$  and greater.

Recently Crida, Morbidelli & Masset (2007) and Crida & Morbidelli (2007) have pioneered a more sophisticated approach which involves modelling the gas flow close to the planet using an evolving 2D grid, combined with the 1D approach for the rest of the disc. For a planet with mass  $M_p = 1 M_J$  and a disc with  $H/R \approx 0.05$ , they find that clean Type II migration occurs only when the local Reynold's number exceeds  $10^5$ . For Reynold's numbers  $10^4 \leq Re \leq 10^5$  the torque felt by the planet is reduced relative to the usual Type II torque by a factor of order  $Re/10^5$ . For lower Reynold's numbers the torque is found to reverse.

In this paper we are interested in the migration of a nominal  $1 M_J$  planet from around 10 AU to 200 AU, for which our Reynold's numbers are in the range  $2.5 \times 10^5$  to  $10^4$ . If the work of Crida & Morbidelli (2007) is correct, then the results given here overestimate the efficiency of outward migration by as much as an order of magnitude. However, since the critical Reynold's number varies as the square of planetary mass (equation 17), the results, suitably scaled, would still be valid for the outward migration of a planet of  $\sim 3 M_J$ .

In view of the uncertainty of the actual time-dependent properties of planet forming discs, in particular with regard to disc thickness and magnitude of viscosity in comparison to the simple formulae employed here, in the following we shall simply use the standard Type II migration torque formulae. We shall find that the conditions under which outward migration can be achieved are limited, and it therefore needs to be borne in mind that they may in fact be more limited still.

In this case, the governing equation becomes

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left[ 3R^{1/2} \frac{\partial}{\partial R} (\nu \Sigma R^{1/2}) \right] - \frac{1}{R} \frac{\partial}{\partial R} \left[ \frac{2(\sum_{i=1}^2 \Lambda_i) \Sigma R^{3/2}}{(GM_\star)^{1/2}} \right] - \dot{\Sigma}_w. \quad (19)$$

where  $\Lambda_i(R, a_i)$ , for  $i = 1, 2$ , is now the rate of angular momentum transfer per unit mass from planet  $i$  to the disc where  $a_i(t)$  is the distance of the planet from the star of mass  $M_\star$ . In Section 4 we shall consider the possibility of mass loss from the disc caused by a wind, represented by the term  $\dot{\Sigma}_w(R, t)$  on the right-hand side of the equation. In this Section we consider the case for  $\dot{\Sigma}_w(R, t) = 0$ .

We take the torque distribution to be of the form

$$\Lambda_i(R, a_i) = \begin{cases} -\frac{q_i^2 GM_\star}{2R} \left( \frac{R}{\Delta_p} \right)^4 & \text{if } R \leq a_i, \\ \frac{q_i^2 GM_\star}{2R} \left( \frac{a_i}{\Delta_p} \right)^4 & \text{if } R \geq a_i, \end{cases} \quad (20)$$

(Armitage et al. 2002) where  $q_i = M_i/M_\star$  is the ratio between the mass  $M_i$  of planet  $i$  and that of the star and

$$\Delta_p = \max(H, |R - a|), \quad (21)$$

where  $H$  is the scale height of the disc. As remarked above, we assume  $H = 0.05R$ . By Newton's third Law, the orbital migration of planet  $i$  occurs at a rate

$$\frac{da_i}{dt} = - \left( \frac{a_i}{GM_\star} \right)^{1/2} \left( \frac{4\pi}{M_{p_i}} \right) \int_{R_{in}}^{R_{out}} \Lambda_i \Sigma R dR, \quad (22)$$

(Lin & Papaloizou 1986). Note that we neglect the gravitational interactions between the two planets.

As before, we solve equation (19) on a fixed mesh which is uniform in the variable  $x = R^{1/2}$ , by using a simple first order explicit numerical method. We use 4000 grid points with an inner boundary at  $R_{in} = 0.01$  AU and an outer boundary at  $R_{out} = 900$  AU. We take zero torque boundary conditions at both  $R_{in}$  and  $R_{out}$ . The outer boundary is large enough to not affect the disc. The inner boundary condition allows all material arriving there to be accreted by the central star. Because the viscous torque is given by

$$G = -2\pi\nu\Sigma R^3 \frac{d\Omega}{dR} = 3\pi\nu\Sigma(GM_\star R)^{1/2}, \quad (23)$$

assuming  $\Omega = (GM/R^3)^{1/2}$  for a Keplerian disc, the boundary conditions are implemented by taking  $\Sigma = 0$  at  $R_{in}$  and at  $R_{out}$ .

We assume that the planets are initially located at radii  $a_1 = 5$  AU and  $a_2 = 10$  AU and that the viscosity is described by equation (6) with  $\beta$  generally equal to  $3/2$ . Because our interest is in discovering what conditions are required to enable a sufficient degree of outward migration, we adopt a highly simplified initial disc density distribution and gap sizes which we describe below.

Over time, the disc gap sizes in the simulations adjust, since they are determined dynamically by a competition between tidal and viscous torques. In each of the three ranges of radii,  $(R_{in}, a_1)$ ,  $(a_1, a_2)$  and  $(a_2, R_t)$ , we assume that the initial surface density profile is as given by equation (5), except that we may adjust the amount of mass in each region by setting the local value of  $\Sigma_0$ . We consider a form of the initial density distribution that permits the specification of the disc mass interior to the planets (inner disc), between the planets (between-planet disc) and exterior to the outer planet (outer disc).

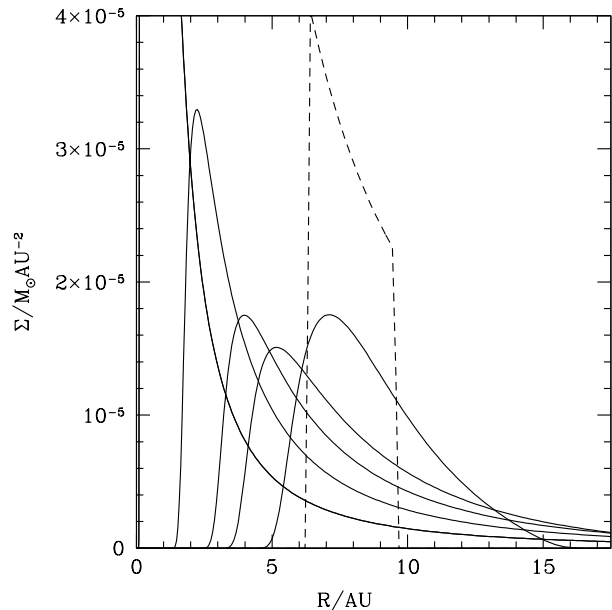
We take the initial surface density distribution to be of the form

$$\Sigma_0 = \begin{cases} \Sigma_{0in} & \text{if } R_{in} \leq R \leq a_1 - \Delta a_{1-}, \\ 0 & \text{if } a_1 - \Delta a_{1-} < R < a_1 + \Delta a_{1+} \\ \Sigma_{0bet} & \text{if } a_1 + \Delta a_{1+} \leq R \leq a_2 - \Delta a_{2-} \\ 0 & \text{if } a_2 - \Delta a_{2-} < R < a_2 + \Delta a_{2+} \\ \Sigma_{0out} & \text{if } a_2 + \Delta a_{2+} \leq R \leq R_t. \end{cases} \quad (24)$$

Here the gap widths  $\Delta a_{i\pm}$  are chosen so that the mass removed from the disc (inner, intermediate, or outer discs) to form the gap is equal to the half mass of corresponding planet, provided the disc mass is nonzero. We shall find below only in some circumstances can sufficient outward migration be achieved. For this reason we do not assume the surface density is the same across gaps. In addition, this disc is initially truncated at some radius  $R_t \ll R_{out}$ .

### 3.1 Disc only between the planets

Disc mass that is located outside a planet's orbit provides a negative torque on the planet which pushes it inwards. The higher the outer disc mass the stronger the torque and so the faster the inward migration. Similarly, disc mass located inside a planet's orbit creates a positive torque which pushes it outwards. A minimum requirement for the model under consideration here is that the inner planet can transfer angular momentum to the outer one. For this to be able to

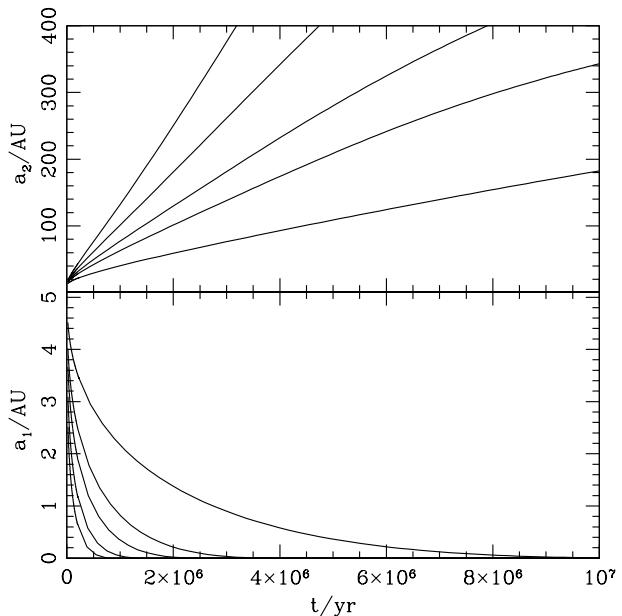


**Figure 2.** Plot of disc surface density as a function of radius. At time  $t = 0$  there is a  $5 M_J$  planet at  $a_1 = 5$  AU and a  $1 M_J$  planet at 10 AU. The gaseous disc has  $\beta = 3/2$  and consists of  $5 M_J$  distributed only between the planets. The density distribution at  $t = 0$  is shown by the dashed line. The subsequent evolution of the surface density is shown at times  $4.0 \times 10^4$ ,  $1.4 \times 10^5$ ,  $2.5 \times 10^5$ ,  $5.5 \times 10^5$  and  $1.8 \times 10^6$  yr with the peak of the distribution moving monotonically inwards.

happen there must be disc mass between the planets. We consider first what happens if there is only a disc between the two planets, i.e.,  $\Sigma_{0in} = \Sigma_{0out} = 0$  in equation (24). In Figure 2 we plot the surface density evolution for the system with two planets, a  $5 M_J$  inner planet and a  $1 M_J$  outer planet with a  $5 M_J$  gaseous disc between them. As seen in Fig 2, the peak of the density distribution moves inwards in time and simultaneously the density spreads outward. Since the disc lies at all times between the two planets, it is evident that as the inner one moves inwards, the outer one moves outwards.

Figure 3 shows the inward and outward migration of the two planets in this case. We also show the effect of varying the amount of disc mass between the two planets. It is evident that the more mass there is between the two planets, the faster they migrate, even though the viscous evolution timescale  $\tau_\nu \sim R^2/\nu \propto R^{2-\beta}$  is a fixed function of radius, independent of surface density. The dependence comes about because a higher mass disc exerts stronger tidal torques on the planets. In strict Type II migration, the migration rate is independent of the disc mass. However, Type II conditions do not hold here because there is no disc interior to the inner planet and the planet mass is comparable to the disc mass. For higher mass initial discs, there is more angular momentum in the system which aids the outer planet's migration to larger distances.

In practice the inner planet would eventually fall into the star. At an age of  $5 \times 10^6$  yr, a  $1 M_\odot$  protostar has a radius of  $1.5 R_\odot$  (Tout, Livio & Bonnell 1999). Using their model (Tout, private communication) and equation (6) of



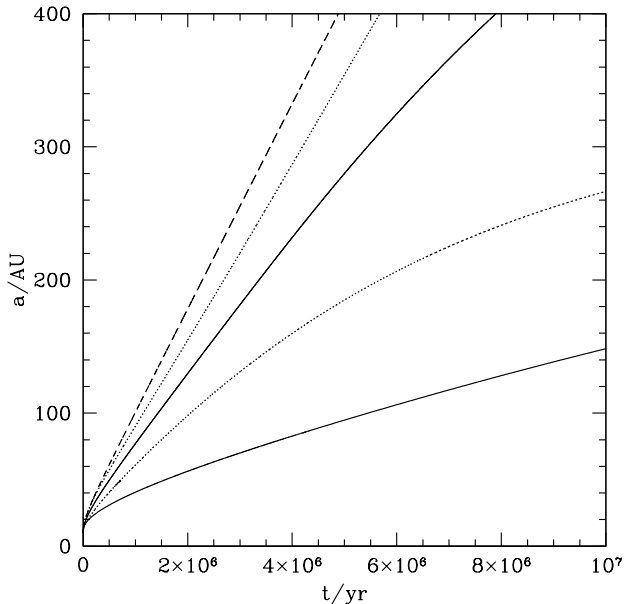
**Figure 3.** Migration of the planets as a function of time for the case when the disc mass is distributed only between them according to equation (24) with  $\Sigma_{0in} = \Sigma_{0out} = 0$ . The initial configuration of the planets is as described for Figure 2. The five lines drawn in each plot correspond to different amounts of gas distributed between the planets. With increasingly rapid rates of migration, the amounts of gas are  $1 M_J$ ,  $3 M_J$ ,  $5 M_J$ ,  $10 M_J$  and  $20 M_J$ . The upper plot is for the outer planet (moving outwards from 10 AU) and lower plot for the inner planet (moving inwards from 5 AU).

Rasio et al. (1996) we find that tides in the star capture the planet when it is about  $2.7 R_\odot = 0.0125$  AU from the star. At this radius, the tides cause the planet to fall into the star faster than the disc pushes it in. This radius is in line with our choice of inner disc boundary at  $0.01$  AU. Once the inner planet has moved in this far it has surrendered all of its angular momentum and so is no longer driving any decretion or outward migration of the outer planet.

### 3.2 The effect of an inner disc

We now consider the effect of disc material located at radii  $R < a_1$ , interior to the inner planet, so that  $\Sigma_{0in}$  is nonzero in equation (24). In Figure 4 we compare models having  $5 M_J$  between the planets with and without disc matter located inside the inner planet. The migration of planets with no inner disc is reproduced from Figure 3. For comparison we show the case (dashed line) where the initial surface density between the planets is the same but the surface density profile is then continued inwards to the inner radius with  $\Sigma_{0in} = \Sigma_{0bet}$ . For this case with mass interior to the inner planet, the inner disc mass is  $15.9 M_J$ .

We see that the mass interior to the planets causes the outer planet to migrate outwards faster. The inner planet also migrates inwards more slowly. This is a consequence of the inner disc providing a positive torque on the inner planet and an additional source of angular momentum.



**Figure 4.** Migration of the outer planet as a function of time. In each case the inner planet starts at 5 AU and the outer planet at 10 AU. The fiducial configuration (upper solid line) consists of an inner  $5 M_J$  planet, an outer  $1 M_J$  planet and a  $\beta = 3/2$  gaseous disc consisting of  $5 M_J$  distributed between the planets. The fiducial configuration is the same as the  $5 M_J$  disc case plotted in Fig 3. The plots described below refer to configurations that consist of a single variant on the fiducial case. The lower solid line has the viscosity law with  $\beta = 1$ . The dashed line is for a configuration with gas interior to the inner planet (see Section 3.2). The effect is to speed the outward migration of the outer planet and to slow the inward migration of the inner planet. The dotted lines show the effect of changing the mass of the inner planet. When the inner planet has a mass of  $10 M_J$  (upper dotted line) the outer planet is forced outwards more rapidly and the inner planet moves inwards more slowly. Conversely, when the inner planet has a mass of  $1 M_J$  (lower dotted line), the outer planet is forced outwards more slowly and the inner planet moves inwards more rapidly.

### 3.3 Effect of the mass of the inner planet

In Figure 4 we also show the effect of changing the mass of the inner planet. The two dotted lines show the difference in migration for the inner planet mass of  $1 M_J$  versus  $10 M_J$ . We see that with a more massive inner planet, the outward migration of the outer planet proceeds on a shorter timescale. As we increase the mass of the inner planet, it migrates more slowly towards the star. In strict Type II migration, the migration rate is independent of the planet mass. However, Type II conditions do not hold here because there is no disc interior to the inner planet and the planet mass is comparable to the disc mass. The migration rate of a single planet with similar disc conditions decreases with planet mass but is accurately described by 1D models for circular orbit planets with  $Re = 10^5$  (D’Angelo, Lubow, & Bate 2006). The result is that the gas between the planets moves in more slowly and hence that the outer planet migrates outwards more rapidly, closer to the case described in Section 2.

### 3.4 Effect of $\beta$

In Figure 4 we also show the effect of varying  $\beta$ , the exponent of  $R$  in the assumed viscosity law while maintaining a constant viscosity value at radius 1 AU in equation (6). We note that for  $\beta = 1$  the Reynold’s number  $Re \propto R^{-1/2}$  falls off more slowly with radius, thus increasing the range of validity of the Type II migration torque formulae. For  $\beta = 1$ , the viscosity is smaller at large radii throughout the fiducial disc in Figure 4 and so the overall viscous timescale  $\propto R^2/\nu$  increases. As a consequence we see that migration to large radii occurs on a much longer timescale with smaller  $\beta$ .

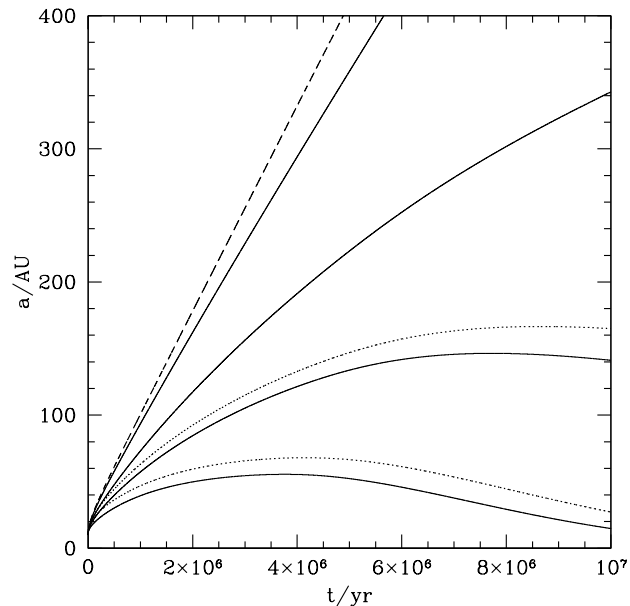
### 3.5 Effect of an outer disc

Finally in this section, we consider the more realistic case of the surface density distribution extending both interior to and exterior to the pair of planets, i.e.,  $\Sigma_{0in}$  and  $\Sigma_{0out}$  are both nonzero in equation (24). The disc is initially truncated at  $R_t = 20$  AU unless otherwise stated. We consider several values for the disc mass exterior to the outer planet while  $5 M_J$  of disc mass resides between the two planets and  $15.9 M_J$  interior to the inner planet. The mass distribution follows equation (24) and the outer disc mass is varied by changing parameter  $\Sigma_{0out}$ . In Figure 5 we plot the migration of the planets for various values of the outer disc mass.

As we would expect with more mass in the outer disc, the outward migration of the outer planet is slower and sometimes reverses. With  $0.1 M_J$  in the outer disc the migration is not very different from the case of no outer disc. With  $2 M_J$  in the outer disc, the planet migration is initially outwards. However, the torque from the outer disc is strong enough to reverse the migration back towards the star. Once the inner planet has fallen into the star, the outer planet continues to migrate inwards and the disc behaves as a simple accretion disc.

In each of these cases, there is a jump in surface density (from  $\Sigma_{0ins}$  to  $\Sigma_{0out}$ ) across the radius of the outer planet. If there were no jump then for  $R_t = 20$  AU and  $M_{bet} = 5 M_J$ , the outer disc mass would be  $16 M_J$ . This mass is greater than the outer disc mass for the plotted models. For this outer disc mass it is clear that the outer planet would not migrate out very far before it gets pushed back in by the large torque exerted by the outer disc. We also investigated the case of a smaller disc that had no surface density jump across the outer planet. This configuration has a  $1 M_J$  outer disc with initial truncation radius  $R_t = 11.12$  AU. The evolution of this model is not very different from our model truncated at  $R = 20$  AU with the same mass outer disc. We conclude that the outer disc mass has more influence on outer planet migration than the initial disc truncation radius. To demonstrate this further, we also show in Figure 5 the effect of increasing the truncation radius of the disc while keeping the outer disc mass fixed.

The torque from the outer disc acting on the planet is determined by the surface density close to the planet. Suppose we fix the initial mass of the outer disc but vary its distribution. Because the outer disc is a decretion disc, within one of its own viscous times it relaxes to have  $\Sigma \propto R^{-(\beta+1/2)}$ . If  $\beta > 1/2$ , then the mass is concentrated at the inner edge and so for a given mass, the surface density close to the planet is roughly the same.



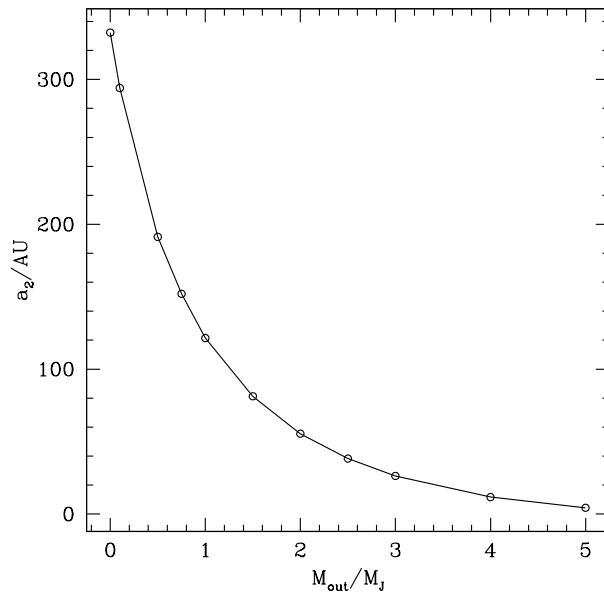
**Figure 5.** Migration of the outer planet as a function of time for different mass outer discs. Each case consists of a  $5 M_J$  inner planet that starts at 5 AU, a  $1 M_J$  outer planet that starts at 10 AU, a gaseous disc  $5 M_J$  that is distributed between the planets and an inner gaseous disc of  $15.9 M_J$ . The amount of gas in the outer disc, exterior to the outer planet, is varied in the plots. The gaseous discs have  $\beta = 3/2$ . The solid lines plot cases when the outer disc is initially truncated at 20 AU. The uppermost solid line corresponds to the outer disc containing only  $0.1 M_J$  and is almost indistinguishable from the case of there being no mass in the outer disc, shown in the dashed line in Figure 4 and here. As the mass in the outer disc increases with masses of  $0.1 M_J$ ,  $0.5 M_J$ ,  $1 M_J$  and  $2 M_J$ , the inner planet migrates inwards more quickly and outward migration of the outer planet is increasingly slowed and even reversed. The dotted lines plot cases with 1 and  $2 M_J$  in the exterior disc but with the matter in the outer disc now distributed out to a much larger truncation radius of  $R_t = 100$  AU. This demonstrates that the migration behaviour depends predominantly on the total mass in the outer disc, rather than on its distribution for a  $\beta = 3/2$  disc.

The time the outer disc takes to relax depends on the viscous timescale of the initial mass distribution. The longest viscous timescale at that time depends on  $R_t$  (for  $\beta < 2$ ). What really matters is the viscous timescale at the radius where most of the mass is initially. For  $\beta > 1$  most of the mass is initially at the inner edge. So for  $\beta > 1$  the initial transient timescale does not depend on  $R_t$ . Conversely, for  $\beta < 1$ , the mass is predominantly initially at  $R_t$  and so the relevant timescale does depend on  $R_t$ .

Consider a decretion disc with the  $V_R = 0$  inner boundary condition. It can be shown that the torque is given by Equation 23 evaluated at  $R = R_{in}$  which follows from the azimuthal force equation. The initial surface density for steady accretion with  $\beta = 1$  corresponds to a disc mass

$$M_{disc} = 2\pi\Sigma_0 R_{in}(R_t - R_{in}). \quad (25)$$

At early times the density is the same as the initial one so that for fixed  $R_{in}$ ,  $G \propto M_{disc}/(R_t - R_{in})$  at early times. At late times  $\Sigma$  becomes independent of  $R_t$  as the disc forgets its initial conditions and approaches the self-similar solution.



**Figure 6.** The position of the outer planet at a time  $t = 4$  Myr is plotted against the amount of mass in the exterior disc for the cases (solid lines) shown in Figure 5. It is evident that in order for the outer planet to be able to migrate to large radii, the amount of mass in the exterior disc must be less than, or comparable to, the mass of the outer planet ( $1 M_J$  in this case).

The torque becomes independent of  $R_t$  but remains linear in  $M_{disc}$ . Hence we see that the migration of the outer planet depends on the amount of mass outside it and not the distribution.

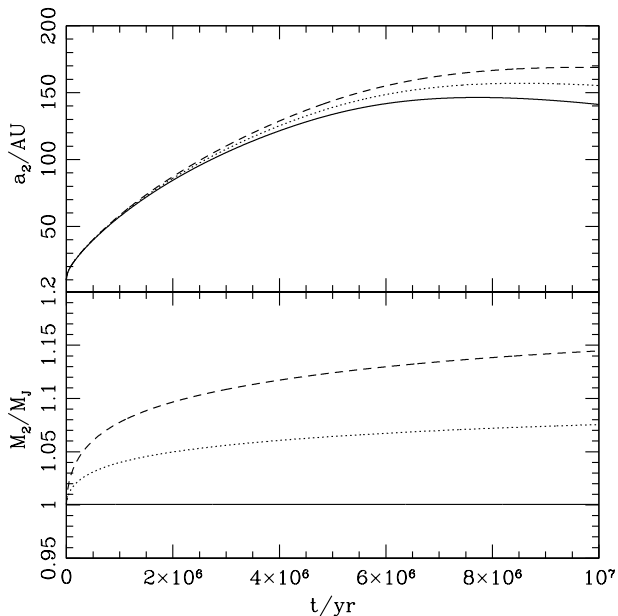
We plot in Figure 6 the orbital radius of the outer planet at a time  $t = 4$  Myr against the exterior disc,  $M_{out}$ . As we remarked before, the lower the mass in the outer disc, the further the outward migration of the outer planet. For the particular disc model we discuss here with  $\beta = 3/2$ , in order for the outer planet to migrate to large radii (say  $> 100$  AU), the amount of mass remaining in the outer disc after the formation of the outer planet needs to be less than, or of order, the mass of the planet.

### 3.6 Effect of Planetary Accretion

Because the gap around the planet is in general not completely cleared, it is possible for accretion onto the planet to occur. This effect is not always included in numerical computations of disc torques. This is partly because high numerical resolution is required in order to make sure that the details of the flow of material onto the planet is properly converged, and partly because of uncertainties about the reaction of the planet to accretion in terms of dissipation and radiation of accretion energy. Thus in torque computations the planet is often represented either as a softened potential or as a sink particle of fixed size.

There are however a number of estimates of accretion rates onto planets in such discs. These have been considered by (Veras & Armitage 2004), who give a fit to the accretion





**Figure 7.** Migration (upper plot) and mass (lower plot) of the outer planet as a function of time for different accretion rates on to the outer planet. Each case consists of a  $5 M_J$  inner planet that starts at 5 AU, a  $1 M_J$  outer planet that starts at 10 AU, a gaseous disc  $5 M_J$  that is distributed between the planets and an inner gaseous disc of  $15.9 M_J$ . The amount of gas in the outer disc, exterior to the outer planet is initially  $1 M_J$  but decreases as mass is accreted on to the outer planet. The gaseous discs have  $\beta = 3/2$ . The outer disc is initially truncated at 20 AU. The solid lines have no accretion on to the planet. The dotted lines have  $f = 0.5$  and the dashed lines have  $f = 1$ .

rates derived from numerical simulations in the form

$$\dot{M}_p = 1.668f\dot{M}_{\text{th}} \left( \frac{M_p}{M_J} \right) \exp \left( -\frac{M_p}{1.5M_J} \right) + 0.04, \quad (26)$$

where  $f$  is a constant parameter which we vary with  $0 \leq f \leq 1$  and  $\dot{M}_{\text{th}} = 3\pi\nu\Sigma$  is the accretion rate through the disc further out from the planet. We consider the effect of applying this formula to the outer planet. We neglect accretion on to the inner planet because the inner planet is significantly more massive.

We remove the required amount of mass from the first zones with non-zero mass outside the planet's gap and accrete the mass and angular momentum of this material on to the planet. We consider the case of a  $5 M_J$  inner planet that starts at 5 AU, a  $1 M_J$  outer planet that starts at 10 AU, a gaseous disc  $5 M_J$  that is distributed between the planets and an inner gaseous disc of  $15.9 M_J$ . The amount of gas in the outer disc, exterior to the outer planet is  $1 M_J$ . We run models with  $f = 0, 0.5$  and  $1$ . We see in figure (7) that the accretion has little effect on the planetary migration, in fact, the planets migrate further out with this accretion. There are two effects here. First, the outer planet extracts angular momentum from the outer disc and so moves out more quickly. Second, the mass of the outer disc is steadily reduced. We conclude that if the Veras & Armitage (2004) formula is correct, then the effect accretion is small and all our previous migration results would not be significantly affected had we included accretion on to the outer planet.

#### 4 EFFECT OF PHOTOEVAPORATION

We have seen in the previous section that outward migration to large radii is possible only if the outer disc, at radii  $R > a_2$ , is for some reason depleted of gas. One means of doing this which we investigate here, is by photoevaporation.

The surface layers of a disc can be heated by the central star to sufficiently high temperature that the gas can escape the gravity of the star. Shu, Johnstone & Hollenbach (1993) suggested that protoplanetary discs are dispersed because of heating by ultraviolet radiation from the central star. The ionizing flux creates a photoionized disc atmosphere where the gas can become hot enough to be gravitationally unbound and escape from the system. The thermal energy of the gas is greater than the gravitational binding energy beyond the gravitational radius

$$R_G = \frac{GM_\star}{c_s^2} \approx 10 \left( \frac{M_\star}{M_\odot} \right) \text{AU}, \quad (27)$$

where  $c_s$  is the speed of sound in the disc's HII atmosphere at  $10^4$  K.

##### 4.1 Photoevaporation only in $R \geq R_G$

Hollenbach et al. (1994) used detailed models of the density at the base of the photoevaporating atmosphere and found a wind mass flux as a function of radius given by

$$\dot{\Sigma}_w = \begin{cases} 0 & \text{if } R \leq R_G, \\ \dot{\Sigma}_0 \left( \frac{R}{R_G} \right)^{-5/2} & \text{if } R \geq R_G, \end{cases} \quad (28)$$

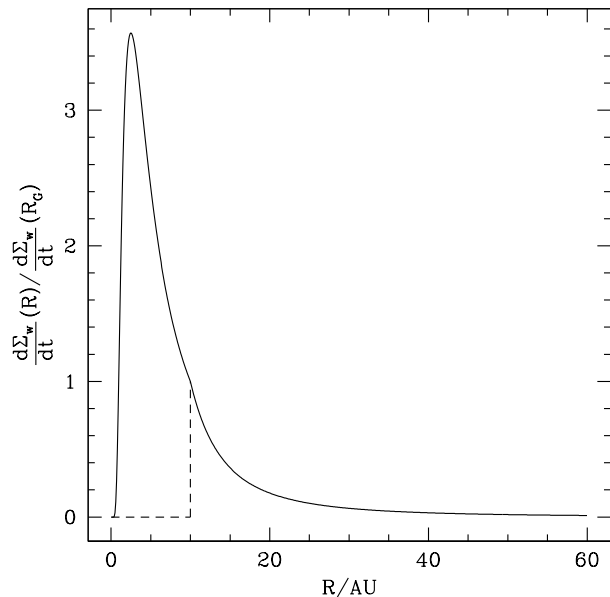
where

$$\dot{\Sigma}_0 = 1.16 \times 10^{-11} \left( \frac{\phi}{10^{41} \text{s}^{-1}} \right)^{1/2} \left( \frac{R_G}{\text{AU}} \right)^{-3/2} M_\odot \text{AU}^{-2} \text{yr}^{-1} \quad (29)$$

and  $\phi$  is the rate of ionizing photons coming from the star. This rate of change of surface density is plotted against the distance from the star in Figure 8 as the dashed line. Alexander, Clarke & Pringle (2005) analysed emission measures and found rates of ionizing photons from the chromospheres of five classical TTs in the range  $10^{41} - 10^{44}$  photon  $\text{s}^{-1}$ . We calculate the total wind mass loss rate from the disc to be

$$\begin{aligned} \dot{M}_w &= \int_0^\infty 2\pi R \dot{\Sigma}_w dR = 4\pi \dot{\Sigma}_0 R_G^2 \\ &= 1.45 \times 10^{-10} \left( \frac{\phi}{10^{41} \text{s}^{-1}} \right)^{1/2} \left( \frac{R_G}{\text{AU}} \right)^{1/2} M_\odot \text{yr}^{-1}. \end{aligned} \quad (30)$$

We now consider the evolution of an initial setup as described in Section 3.5 with a gas disc of mass of  $5 M_J$  between the planets, an inner disc of mass  $15.9 M_J$  and an outer disc of  $0.5 M_J$  but now allow for disc depletion according to the formula given in equation (28) for various values of the photon flux  $\phi$ . The results are shown in Figure 9. The migration initially proceeds on a faster timescale than with no mass loss by photoevaporation. This is because initially most of the mass lost arises from the outer disc. This decrease in mass means the negative torque on the planets is smaller and so they migrate outwards faster. However, once the outer planet has moved significantly outwards, the main effect of photoevaporation is to remove the gas from between



**Figure 8.** The rate of change of surface density scaled to the rate at  $R_G$  is plotted against radius. The dotted line corresponds to photoevaporation only in the range in  $R \geq R_G$ , as given by equation (28). The solid line takes account of possible outflow from within that radius and is given by equation (31).

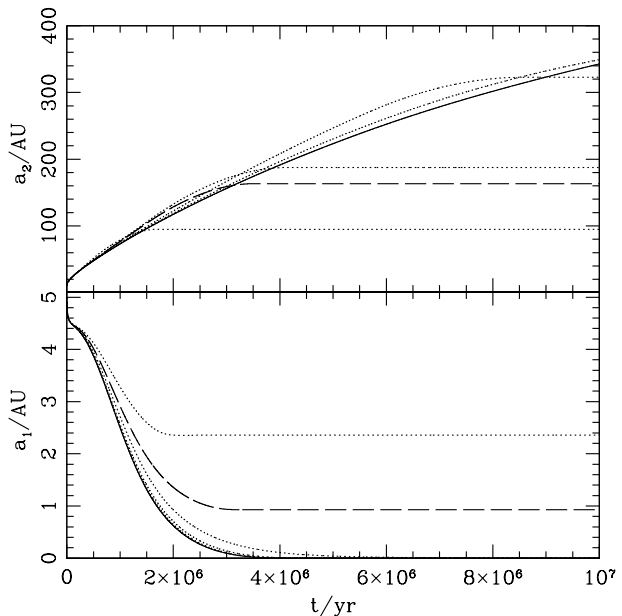
the planets. Once sufficient gas has been removed, all contact is lost between the planets and the inner planet is no longer able to give up angular momentum to the outer one and migration stops.

In Figure 10 we show the results of an identical set of computations, except that initially the amount of gas in the outer disc is increased to  $1 M_J$ . With stronger photoevaporation, the distance the outer planet moves decreases and its migration ceases due to disc dispersal.

#### 4.2 Varying the radial dependence of the evaporation rate.

There have been more recent suggestions that photoevaporation happens as far in as  $0.1 - 0.2 R_G$  (Liffman 2003, Adams et al. 2004, Font et al. 2004). To simulate the effects of this form of photoevaporation, we have taken the model from the previous section and changed the gravitation radius to  $R_G = 1$  AU with the same range of  $\phi$ . In this case, we find the photoevaporation has little effect on the disc. This is because now photoevaporation mostly removes mass from radii well within the planetary orbits. We have already seen in Section 3.3 that the presence or absence of an inner disc has only a limited effect on the migration.

Alternatively, in line with the ideas of Dullemond et al. (2006) who model the photoevaporation as a Bernoulli flow, we take the radial dependence of the rate of change of



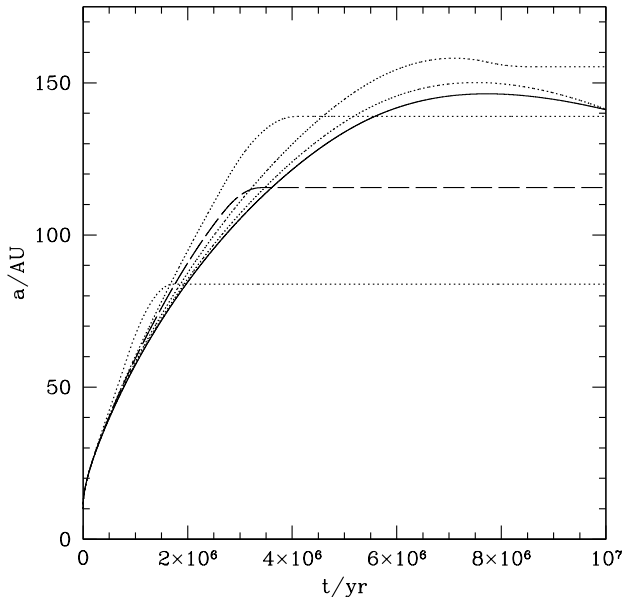
**Figure 9.** Migration of the planets shown as a function of time accounting for the effects of photoevaporation. The upper plot is for the outer planet and the lower plot for the inner planet. The  $5 M_J$  planet starts at 5 AU and the  $1 M_J$  planet at 10 AU. There is initially a gas disc of mass of  $5 M_J$  between the planets, an inner disc of mass  $15.9 M_J$  and an outer disc of  $0.5 M_J$  which is truncated at 20 AU. The dotted lines show the effects of photoevaporation according to equation (28). The solid lines are identical to the  $0.5 M_J$  exterior disc case shown in Figure 5 without photoevaporation,  $\phi = 0$ . The line corresponding to the lowest photon rate of  $\phi = 10^{40} \text{ s}^{-1}$  lies above the solid line for  $\phi = 0$ . This is because photoionization in this case is able to remove some of the mass in the outer disc and so enables faster outward migration. The line corresponding to  $\phi = 10^{41} \text{ s}^{-1}$  lies above the  $\phi = 0$  line for some time initially for the same reason but eventually photoevaporation removes enough disc material from between the two planets to cut communication and so to halt further outward migration. The same applies to the lines corresponding to  $\phi = 10^{42} \text{ s}^{-1}$  and  $10^{43} \text{ s}^{-1}$  with the halt in migration occurring sooner for the larger photoevaporation rate. The dashed lines use photoevaporation model described in equation (31) with  $\phi = 10^{42} \text{ s}^{-1}$ .

surface density due to photoevaporation as

$$\dot{\Sigma}_w(R) = \begin{cases} \dot{\Sigma}_0 \exp\left(\frac{1}{2}\left(1 - \frac{R_G}{R}\right)\right) \left(\frac{R}{R_G}\right)^{-2} & \text{if } R \leq R_G, \\ \dot{\Sigma}_0 \left(\frac{R}{R_G}\right)^{-5/2} & \text{if } R \geq R_G. \end{cases} \quad (31)$$

The spatial form is the same as equation (28) for  $R \geq R_G$  but extends further inwards to smaller radii. We show this graphically as the solid line in Figure 8. We see that in this case, the peak surface density loss occurs at  $0.25 R_G$  whereas for equation (28) the peak loss occurs at  $R_G$ .

The dotted and dashed lines in Figure 9 show the migration of planets with discs undergoing photoevaporation described by equation (28) and equation (31), respectively. We see that with the inner photoevaporation described by equation (31), the outer planet does not migrate out as far. This is because the total rate of loss of mass is increased



**Figure 10.** This plot describes the same configuration as Figure 9 except that here there is a higher initial outer disc mass  $1.0 M_J$ . We plot the migration of the outer planet. As before, the solid line corresponds to the case  $\phi = 0$  shown in Figure 5. The dotted line corresponding to  $\phi = 10^{40} \text{ s}^{-1}$  lies at all times just above the  $\phi = 0$  line for the same reason as discussed in Figure 9. The other dotted lines correspond, in order of decreasing final radius in this diagram, to  $\phi = 10^{41}, 10^{42}, 10^{43} \text{ s}^{-1}$ . By comparison with Figure 9, we see that for a given photon flux, or photoevaporation rate, a larger initial mass in the exterior disc leads typically to a larger final radius for the outwardly migrating planet. The dashed lines have the photoevaporation model described in equation (31) with  $\phi = 10^{42} \text{ s}^{-1}$ .

and so the disc is removed on a shorter timescale. The peak in  $\dot{\Sigma}_w$  is now inside of the inner planet. Mass removed from the inner disc has little effect on the migration of the planets (see Figure 4).

## 5 OBSERVED SYSTEMS

We now consider whether the mechanism discussed here for moving planets out to large distances from the star can explain the structure of systems inferred to have planets at  $R > 10 \text{ AU}$ . Such systems have been discovered through direct imaging (Chauvin et al. 2004) and interpreted in terms of planet perturbations on dust discs (Wyatt et al. 1999).

Planets discovered through direct imaging have a relatively high mass ratio. For example, the system 2MASS 1207334-393254 (2M1207) has a  $5 M_J$  planet at 55 AU from a  $25 M_J$  brown dwarf (Chauvin et al. 2004). The mass ratio is closer to those in binary stars than for known star planet systems (Lodato et al. 2005). Lodato et al. (2005) argue that the planet could have formed *in situ* by gravitational instability but not through core accretion owing to the prohibitively long planet formation timescales at this distance. The planet could however have formed in a reasonable time at 0.6 AU (Lodato et al. 2005). They suggest that this is too near to the star for outward migration to

provide the current separation. Using our simple model and equation (1) we find that to drive out the planet to 55 AU the inner planet would have to be comparable in mass to the star 2M1207. In other words, for this scenario to work we would require 2M1207 to have, or to have had, a close binary companion, and the  $5 M_J$  planet must have formed in a circumbinary disc. Assuming the companion is of similar mass to 2M1207, equation (1) implies that the current planet parameters are compatible with a stellar companion originally at 1.3 AU, and a formation location for the planet of 2.9 AU.

The inferred debris-disc planets are found in two different types of systems, old systems and young systems. Most are found in relatively old systems (much older than 10 Myr) and inferred to lie close to the inner edge of a planetesimal belt at greater than 30 AU. No gas is seen in these systems (Dent et al. 2005) and the inner regions are devoid of dust and planetesimals (Wyatt 2005b). The existence of planets are inferred from asymmetries in the structure of the dust discs which have been imaged for the closest systems. This type of system is typified by the debris disc around 350 Myr old Vega. The disc's clumpy structure (Holland et al. 1998, Wilner et al. 2002) has been used to infer the presence of a Neptune-mass planet currently located at 65 AU from the star, since that structure can be explained by the planet having migrated outward from 40 AU over 56 Myr while trapping planetesimals at its resonances (Wyatt 2003, Wyatt 2006). The modelling also permitted different planet masses to have caused the same clumpy resonant structure as that observed, as long as the migration rate is changed accordingly.

It seems reasonable to assume that the migration scenario proposed here could explain distant planets in systems like Vega, since, if their existence is confirmed, the outward migration of such planets would naturally explain both their large orbital radii and the clumpy structure of their debris discs. For this scenario to work, there are certain requirements on the outer planet:

- (i) the migration must occur before the gas disc dissipates for which we require a migration timescale of less than 6 Myr (Haisch, Lada & Lada 2001), which means the planet mass must be greater than  $0.25 M_J$  to cause the observed clumpy dust structure (Wyatt 2003);
- (ii) to migrate by Type II migration at 40 AU the planet must be greater than roughly  $1 M_J$  in order to open a gap in the disc (Bryden et al 1999);
- (iii) to have avoided detection in direct imaging surveys the planet must have a mass less than  $7 M_J$  (Macintosh et al. 2003, Hinz et al. 2006).

Thus we consider that a  $2 M_J$  planet that migrated from 40 to 65 AU over 0.3 Myr could explain the observed disc structure, parameters which can be reproduced with the migration mechanism proposed here. The planet could also have started closer in if there is some mechanism to remove material from the resonances at high eccentricity.

This interpretation would make the prediction that there was at one time (and possibly still is if it has not already been accreted) a more massive planet close to Vega. The fact that this system is close to pole-on (Aufdenberg et al. 2006) indicates that such a planet would be hard to detect by radial velocity measurements.

However, we cannot escape from the requirement of the proposed model that the outer disc be deficient in mass, in order to provide outward migration. If the deficiency were due to truncation, then the truncation would have to apply to the gas disc and not the solids. The reason is that clumpy structure in the debris disc is modelled as planetesimals that were present during the migration process, and which extended beyond the 2:1 resonance of the outer planet at  $1.59a_2$ . This makes problematic the possibility that the deficiency might have been caused by physical truncation due to stellar encounters in a high density star formation environment. However, photoevaporation of gas remains a possible mechanism.

Planets have also been inferred from structures in young 5 – 20 Myr systems. Such systems have both dust and gas at a wide range of distances. These are typified by 5 Myr old HD141569, for which tightly wound spiral structure at 325 AU (Clampin et al. 2003) has been used to infer the presence of a planet of  $0.2 - 2 M_J$  at 235 – 250 AU (Wyatt 2005b). Spiral structure in a ring at 185 AU (Clampin et al. 2003) may also be indicative of a Saturn mass planet at 150 AU (Wyatt 2005b). Like the planet in 2M1207, the planets in this system are unlikely to have formed *in situ* unless by gravitational instability. However, the migration scenario proposed here provides a viable mechanism through which these planets could migrate out to their current locations following formation closer to the star, since an inner planet in this scenario could equally drive out more than one planet. If this interpretation is correct, then we would make the prediction that a more massive planet exists, or existed, at a distance of less than a few AU from HD141569. The migration must also have occurred on timescales much shorter than 5 Myr, since some time is required after the migration for the spiral structure to be imprinted on the disc (Wyatt 2005b).

The current distributions of gas and dust in this system provide some clues as to whether this scenario is feasible. There certainly appears to be a radially extended gas distribution which might in the past have contained enough mass interior to the planet at 150 AU to cause rapid outward migration (Jonkheid et al. 2006), and there is no evidence that the mass exterior to the planets would have been sufficient to prevent that migration. However, there are a number of density variations in the radial distributions of gas and dust which would be hard to explain within the context of this model (Marsh et al. 2002; Merin et al. 2004; Goto et al. 2006; Jonkheid et al. 2006). Such variations are likely caused by processes not included in the current scenario, such as the dynamical interaction between gas and dust (e.g., Takeuchi & Artymowicz 2001, Krauss & Wurm 2005), photoevaporation of gas near the gravitational radius (Clarke, Gendrin & Sotomayer 2001), and grain growth which may have occurred after the planetary migration.

## 6 CONCLUSIONS

We have shown that it is possible *in principle* for planets to form at small radii and then to migrate out to large radii from their star by means of planet-disc interactions. The possible mechanism we discuss here involves the formation of a massive inner planet and of a less massive outer

planet suitably spaced in radius that there is enough gas between them to effect angular momentum transfer and suitably spaced in time that the first to form has not migrated too far before the formation of the second. In view of the uncertainties surrounding the planet formation process, we have not here addressed the plausibility of setting up such a configuration.

We also require that tidal torques act efficiently on the planet, even at large radii. This leads to a conflict of requirements on the size of the disc viscosity in that a large outward torque requires the opening of a clean gap and so a small viscosity (e.g. Crida & Morbidelli 2007), whereas too small a viscosity implies that the migration takes too long and cannot take place before the disc has been dispersed. Whether there is a finite range of viscosity between these two constraints depends on the detailed properties of such discs, and in particular on the unknown nature and magnitude of the disc viscosity (e.g. King, Pringle & Livio, 2007).

To compute the evolution of disc and planetary orbits we have used a 1D disc approximation, used an idealised model of the disc structure, and have used a standard formulation which attempts to estimate the interactive disc-planet torques, under the assumption that the planets are massive enough to open a clean gap. In reality it would be better, but much more computer-intensive, to undertake the disc/planet evolution while the torques are being computed using 2D, or better 3D, hydrodynamics simulations for the gas in the neighbourhood of the planet (c.f. Crida et al., 2007), and to solve simultaneously for the local disc structure (for example assuming thermodynamic equilibrium, e.g. Bell et al., 1997) and in addition taking account of heating of the disc by the central star (e.g. Garaud & Lin, 2007). But even so, there still remain sufficient uncertainties about the basic physics involved that it is not always easy to assess the degree to which such simulations reflect physical reality. Even within a given set of simulations, with a fixed set of assumptions, it is necessary to take some trouble to ensure that the calculations have enough resolution that the torque estimates have converged (see, for example, D’Angelo, Bate & Lubow, 2005). Although we have made an attempt to estimate its effects, it is clear that the degree, nature and effects of accretion onto the planet have yet to be fully understood. And in addition, almost all simulations so far assume that the disc viscosity is some kind of fixed form of Navier-Stokes viscosity, whereas in reality the viscosity is most likely due to some form of magneto-hydrodynamics turbulence (e.g. Nelson & Papaloizou, 2004), and may in the outer, cooler parts of the disc be spatially confined to a small part of the disc (Gammie, 1996).

Once the planet formation is complete, we find that for significant outward migration to occur, it is necessary for there to be very little or no disc mass exterior to the outer planet: the lower the surface density exterior to the outer planet, the faster and further the migration. One possible way of achieving this is through dynamical truncation of the disc, either by a fly-by in a dense environment or through the presence of a distant binary companion or through a multi-body interaction during the formation process (Clarke & Pringle 1991). We have investigated the possibility that the outer disc is reduced by photoevaporation. If the source of photoevaporation is external, due for example to the nearby presence of a hot star, then the disc can be evaporated from

the outside as we require (and as modelled in effect by Veras & Armitage, 2004). In this paper we have investigated the possibility of photoevaporation being caused by the central star. We find that, since such photoevaporation depletes the disc predominantly at around  $R_G \approx 10$  AU, the general effect is in this case to hinder rather than enhance migration to large radii.

The main conclusion from these calculations is that while outward migration is indeed a feasible mechanism for the production of gas giants at large distance from the central star, the parameter space in which its occurrence is possible may, for the reasons given above, be somewhat limited.

Outward planet migration is also possible in circumbinary discs. We consider the case that the binary's angular momentum is at least comparable to that of the surrounding disc. Planets which open gaps undergo outward migration in such discs, such as shown in Figure 1. The formation of the planet can occur after the formation of the binary while the disc is present. Unlike the case involving an inner planet discussed above, the disc mass external to the planet need not be small. The main issue is whether the disc behaves as acretion disc. Two-dimensional simulations indicate that some material can accrete past the disc inner edge and onto the binary (Artymowicz & Lubow 1996; Gunther & Kley 2002). The radial distribution of planets around binary stars could be quite different from the single star case.

We have discussed briefly the possibility of applying these ideas to observed systems. The major uncertainties in making such applications are the timescales on which planets form, the radial positions at which they form and the timescales on which gaseous discs evolve and are dissipated. In this paper we have used a simple and idealised description of the viscosity whereas in reality the viscosity and in consequence the migration timescales are functions of the local disc properties as they evolve. Nevertheless we note that at least some of the observed systems might be compatible with the migration scenario discussed here.

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