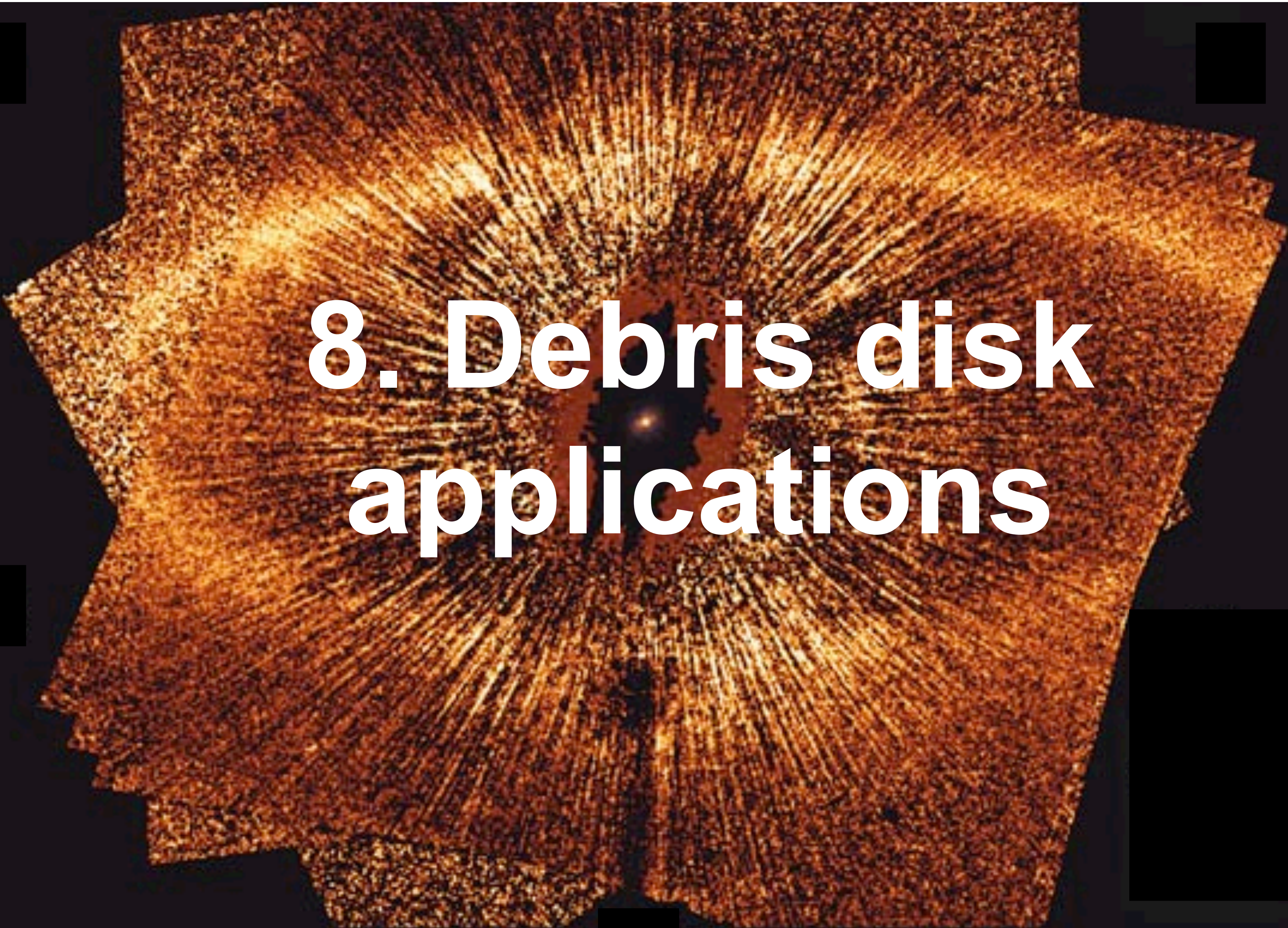
8. Debris disk applications



Planetary perturbations

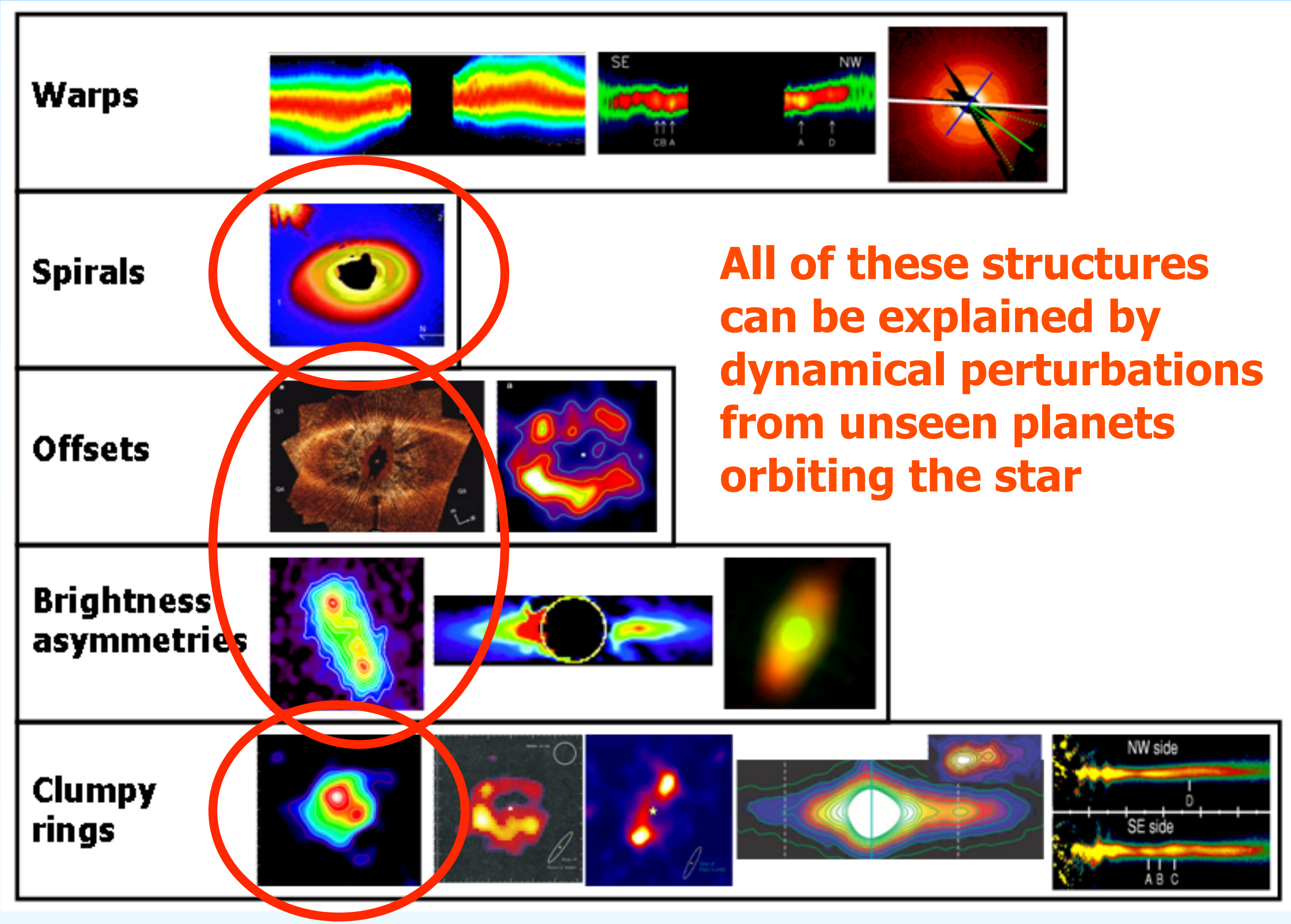
Planetesimal belt theory provides a solid model with which to interpret disk structure, because it explains

- ring structure
- extended dust distributions
- emission spectrum
- dust mass evolution

Study of the solar system shows that the most important perturbation to the structure and evolution of a debris disk is the formation of massive planets within the disks. Here I will show the effect of planetary perturbations, and how they explain:

- spiral structure
- offsets and brightness asymmetries
- clumps

Observed debris disk asymmetries

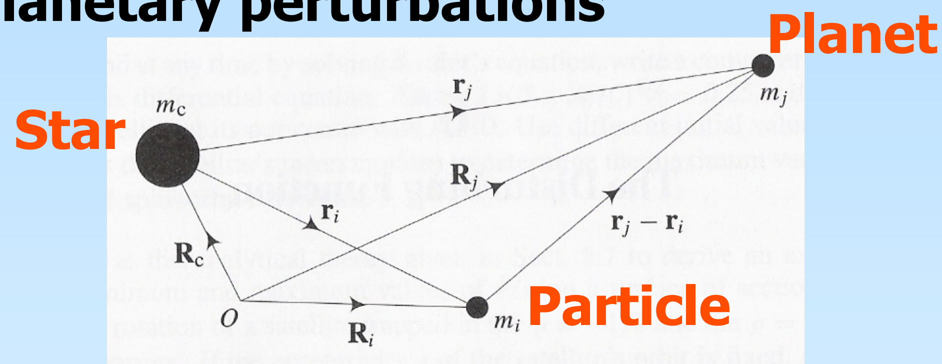


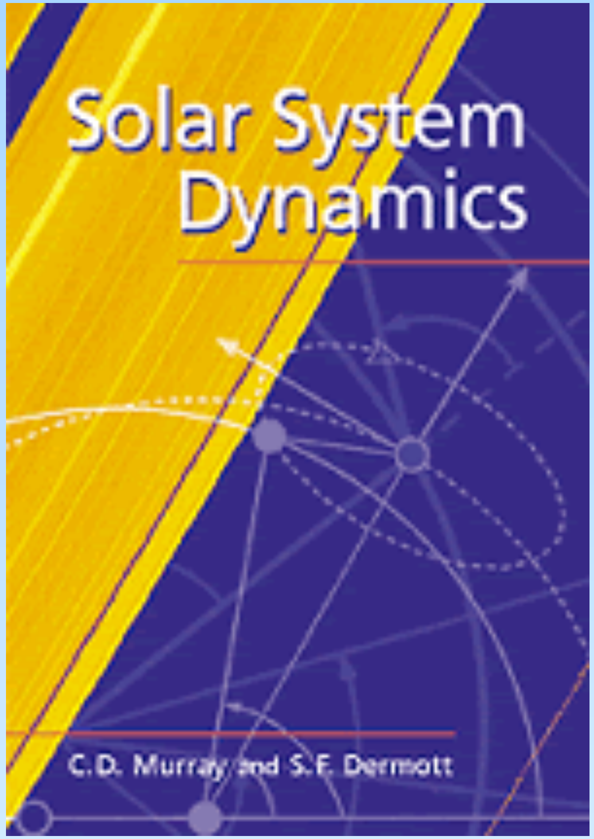
All of these structures can be explained by dynamical perturbations from unseen planets orbiting the star

Gravity!

Actually it is exactly this set of features which are predicted from planetary system dynamics

Planetary perturbations





Equation of motion for M_i is: $d^2\mathbf{r}_i/dt^2 = \nabla_i(U_i + \mathcal{R}_i)$
where $U_i = G(M_c+M_i)/r_i$ is the 2 body potential
and $\mathcal{R}_i = GM_j/|\mathbf{r}_j-\mathbf{r}_i| - GM_j\mathbf{r}_i.\mathbf{r}_j/r_j^3$ is the disturbing function

The disturbing function can be expanded in terms of standard orbital elements to an infinite series:

$$\mathcal{R}_i = \mu_j \sum S(a_i,a_j,e_i,e_j,I_i,I_j)\cos(j_1\lambda_i+j_2\lambda_j+j_3\varpi_i+j_4\varpi_j+j_5\Omega_i+j_6\Omega_j)$$

Different types of perturbations

Luckily for most problems we can take just one or two terms from the disturbing function using the **averaging principle** which states that most terms average to zero over a few orbital periods and so can be ignored by using the averaged disturbing function $\langle \mathcal{R} \rangle$

only time dependence, $\lambda = n(t-\tau)$

$$\mathcal{R}_i = \mu_j \sum S(a_i, a_j, e_i, e_j, I_i, I_j) \cos(j_1\lambda_i + j_2\lambda_j + j_3\varpi_i + j_4\varpi_j + j_5\Omega_i + j_6\Omega_j)$$

Terms in the disturbing function can be divided into three types:

- **Secular**
  Terms that don't involve λ_i or λ_j which are slowly varying
- **Resonant**
  Terms that involve angles $\phi = j_1\lambda_i + j_2\lambda_j + j_3\varpi_i + j_4\varpi_j + j_5\Omega_i + j_6\Omega_j$
  where $j_1 n_i + j_2 n_j = 0$, since these too are slowly varying.
- **Short-period**
  All other terms, average out

Lagrange's planetary equations

The disturbing function can be used to determine the orbital variations of the perturbed body due to the perturbing potential using Lagrange's planetary equations:

$$da/dt = (2/na)\partial\mathcal{R}/\partial\varepsilon$$
$$de/dt = -(1-e^2)^{0.5}(na^2e)^{-1}(1-(1-e^2)^{0.5})\partial\mathcal{R}/\partial\varepsilon - (1-e^2)^{0.5}(na^2e)^{-1}\partial\mathcal{R}/\partial\varpi$$
$$d\Omega/dt = [na^2(1-e^2)\sin(I)]^{-1}\partial\mathcal{R}/\partial I$$
$$d\varpi/dt = (1-e^2)^{0.5}(na^2e)^{-1}\partial\mathcal{R}/\partial e + \tan(I/2)(na^2(1-e^2))^{-1}\partial\mathcal{R}/\partial I$$
$$dI/dt = -\tan(I/2)(na^2(1-e^2)^{0.5})^{-1}(\partial\mathcal{R}/\partial\varepsilon + \partial\mathcal{R}/\partial\varpi) - (na^2(1-e^2)^{0.5}\sin(I))^{-1}\partial\mathcal{R}/\partial\Omega$$
$$d\varepsilon/dt = -2(na)^{-1}\partial\mathcal{R}/\partial a + (1-e^2)^{0.5}(1-(1-e^2)^{0.5})(na^2e)^{-1}\partial\mathcal{R}/\partial e + \tan(I/2)(na^2(1-e^2))^{-1}\partial\mathcal{R}/\partial I$$

where $\varepsilon = \lambda - nt = \varpi - n\tau$

Tip: as with all equations, these can be simplified by taking terms to first order in e and I

Secular perturbations between planets

• To second order the secular terms of the disturbing function for the j^{th} planet in a system with N_{pl} planets are given by:

$$\mathcal{R}_j = n_j a_j^2[0.5A_{jj}(e_j^2 - I_j^2) + \Sigma^{Npl}_{i=1,\, i\neq j} A_{ij} e_i e_j \cos(\varpi_i - \varpi_j) + B_{ij} I_i I_j \cos(\Omega_i - \Omega_j)]$$

where $A_{jj} = 0.25 n_j \Sigma^{Npl}_{i=1, i\neq j} (M_i/M_*) \alpha_{ji} \underline{\alpha}_{ji} b^1_{3/2}(\alpha_{jj})$

$A_{ji} = -0.25 n_j (M_i/M_*)\, \alpha_{ji} \underline{\alpha}_{ji} b^2_{3/2}(\alpha_{ji})$

$B_{ji} = 0.25 n_j (M_i/M_*)\, \alpha_{ji} \underline{\alpha}_{ji} b^1_{3/2}(\alpha_{ji})$

α_{ji} and $\underline{\alpha}_{ji}$ are functions of a_i/a_j and $b^s_{3/2}(\alpha_{ji})$ are Laplace coefficients

• Converting to a system with $z_j = e_j \exp(i\varpi_j)$ and $y_j = I_j \exp(i\Omega_j)$ and combining the planet variables into vectors $\mathbf{z} = [z_1, z_2, \ldots, z_{Npl}]^T$ and for $\mathbf{y}$ gives for Lagrange's planetary equations

$da_j/dt = 0$, $d\mathbf{z}/dt = iA\mathbf{z}$, $d\mathbf{y}/dt = B\mathbf{y}$, where A,B are matrices of A_{ji},B_{ji}

• This can be solved to give:

$z_j = \Sigma^{Npl}_{k=1} e_{jk} \exp(ig_k + i\beta_k)$ and $y_j = \Sigma^{Npl}_{k=1} I_{jk} \exp(if_k t + i\gamma_k)$

where g_k and f_k are the eigenfrequencies of A and B and β_k γ_k are the constants

Secular perturbations of eccentric planet on planetesimal orbit

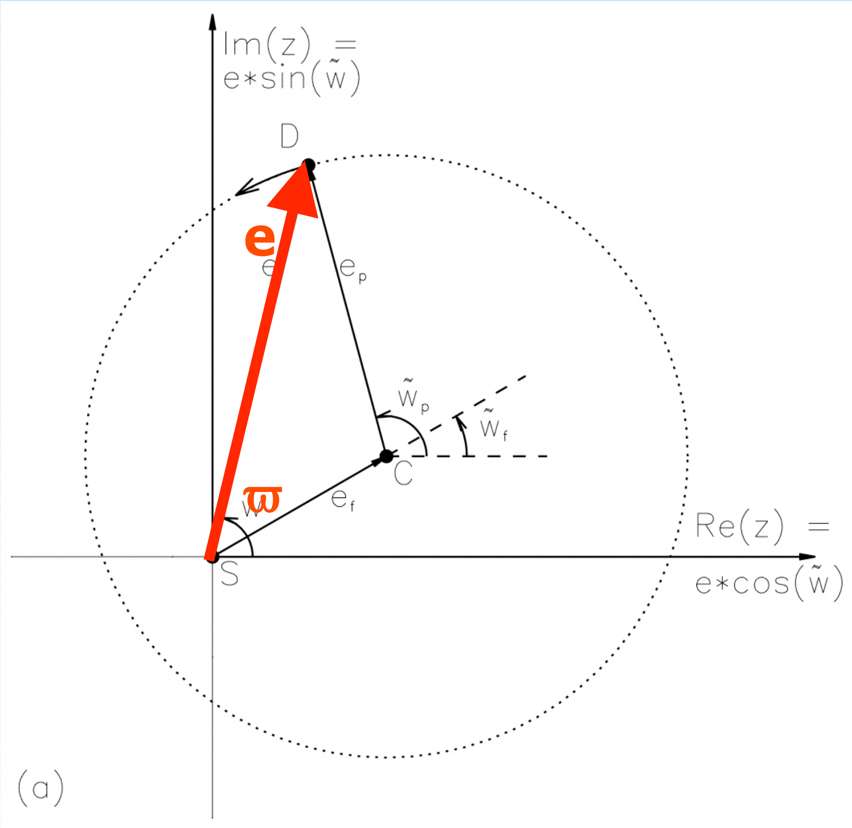
Taking terms to second order in e and I, Lagrange's planetary equations are:

$$dz/dt = iAz + i\Sigma^{Npl}_{j=1} A_j z_j$$

where $z=e*\exp[i\varpi]$
with a similar equation for $y=I*\exp[i\Omega]$.

$$z = z_f + z_p$$

$$= \sum^{Npl}_{k=1}\left[\sum^{Npl}_{j=1}\left[A_j e_{jk}\right]/(g_k-A)\exp(ig_k t+i\beta_k)\right]$$
$$+ e_p \exp(iAt+i\beta_0)$$

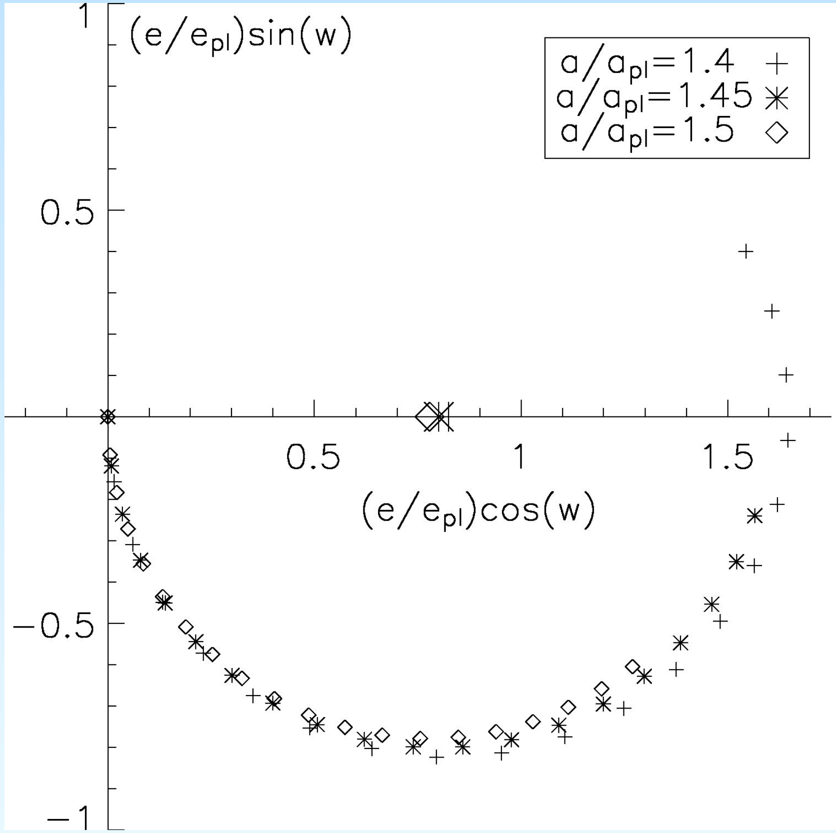


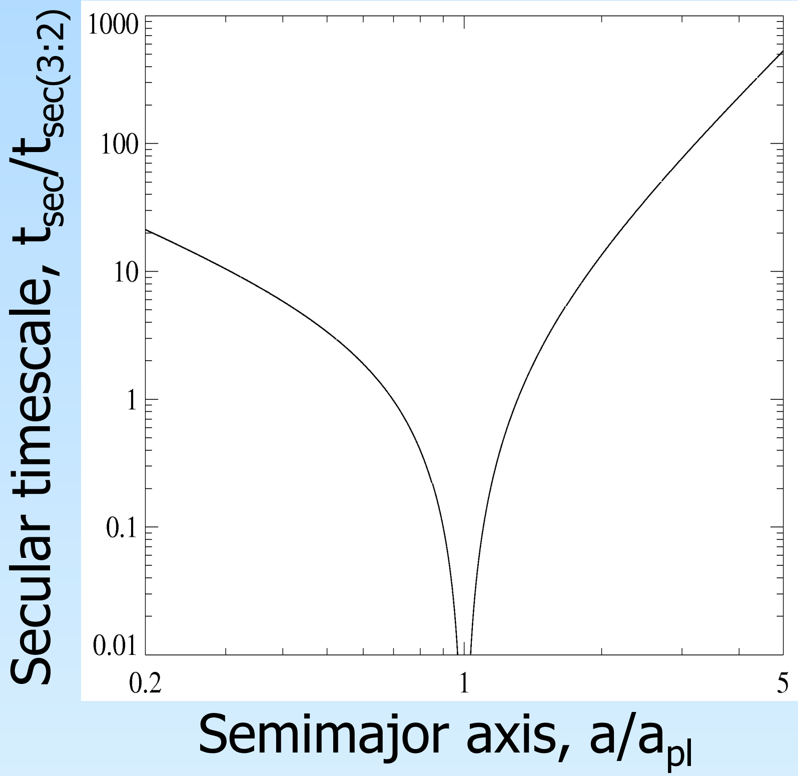
Meaning the orbital elements of planetesimals precess around circles centred on forced elements imposed by planetary system

Murray & Dermott (1999)

Post planet formation evolution

Consider impact of sudden introduction of planet on eccentric orbit on extended planetesimal belt for which eccentricity vectors start at origin

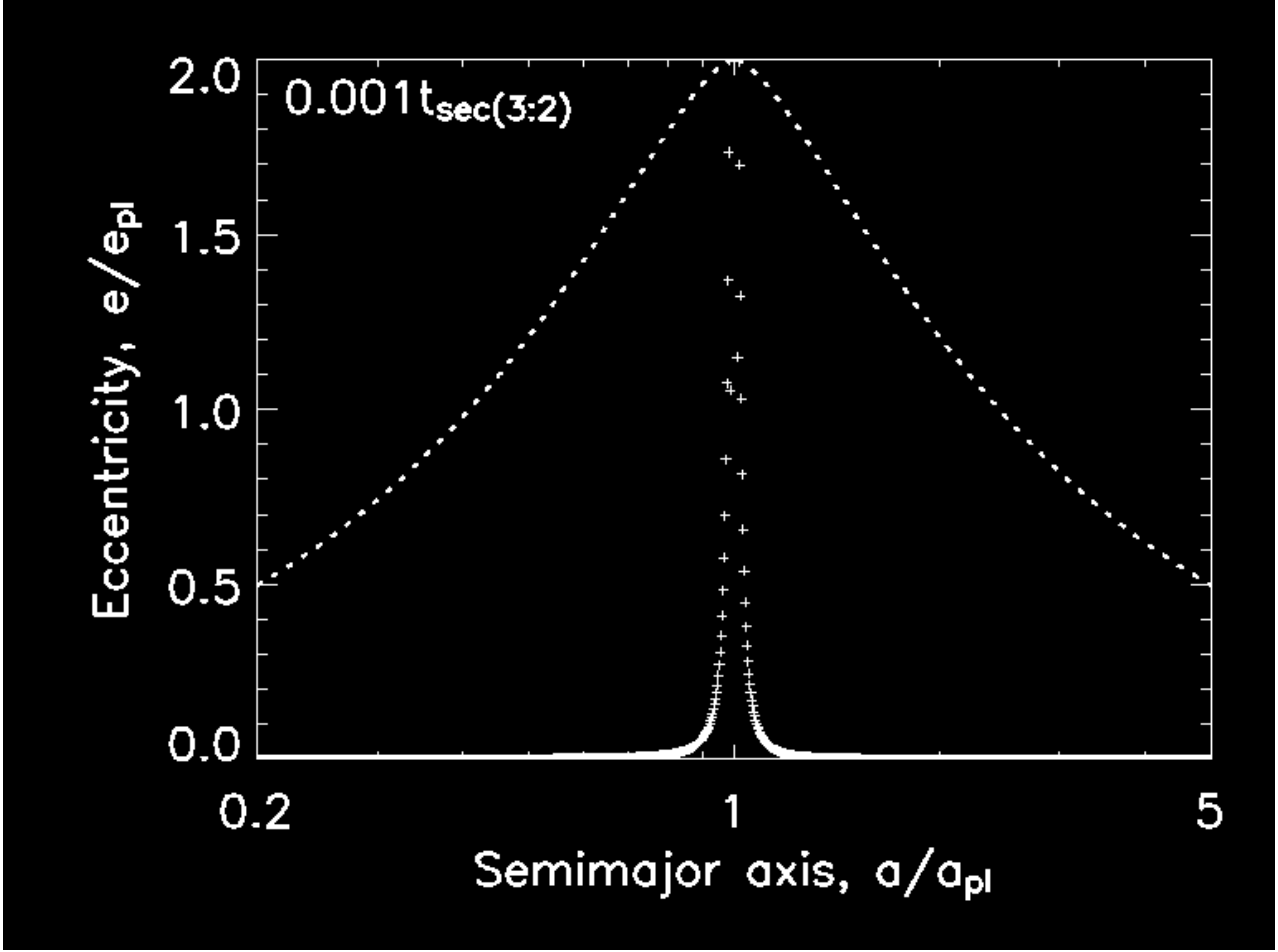




Precession rates are slower for planetesimals further from planet which means dynamical structure evolves with time

$t_{sec(3:2)} = 0.651 t_{pl}/(M_{pl}/M_{star})$

Wyatt (2005)



0.001t_sec(3:2)
2.0
1.5
1.0
0.5
0.0
Eccentricity, e/e_pl
0.2
1
5
Semimajor axis, a/a_pl

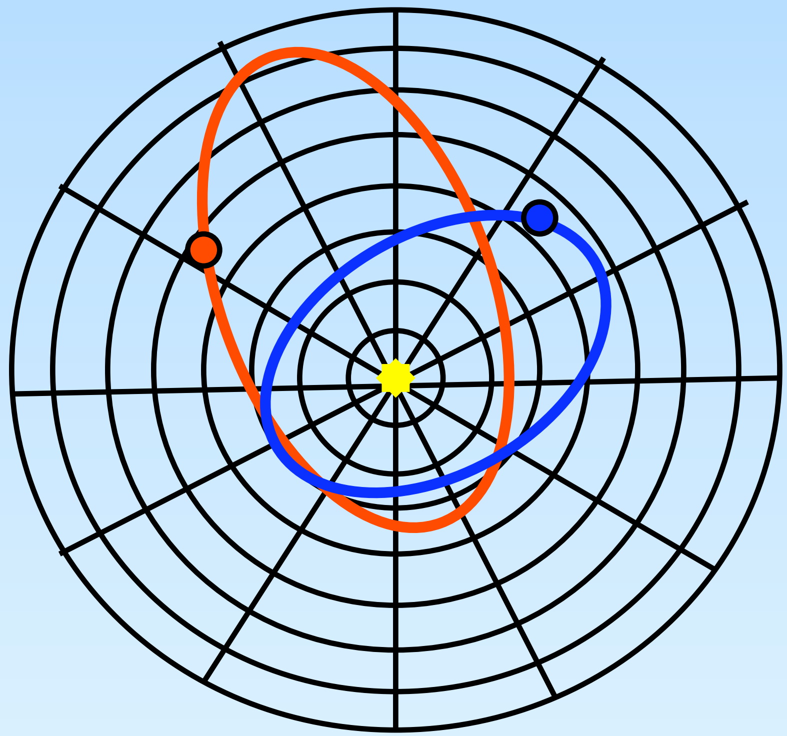
Converting dynamical structure to spatial distribution

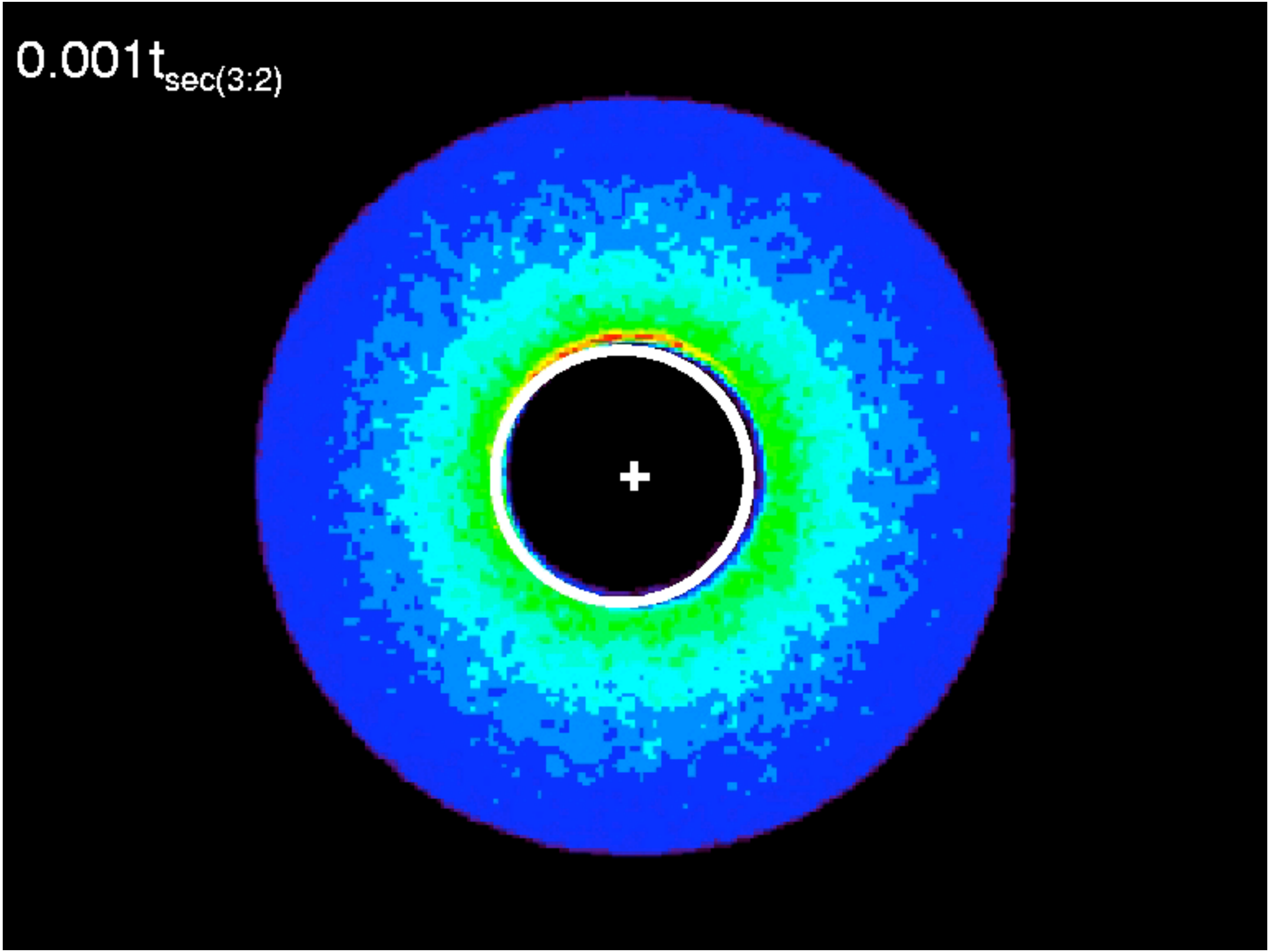
(1) Make a grid in r and θ

(2) Choose N particles on orbits with
- Semimajor axis, a
  between a_1 and a_2
- Eccentricity, e
  where e depends on a and t
- Pericentre orientation, ϖ
  where ϖ depends on a and t
- Mean longitude, λ
  between 0^0 and 360^0

(3) Convert particle location into r and θ

(4) Add up number of particles in each grid cell

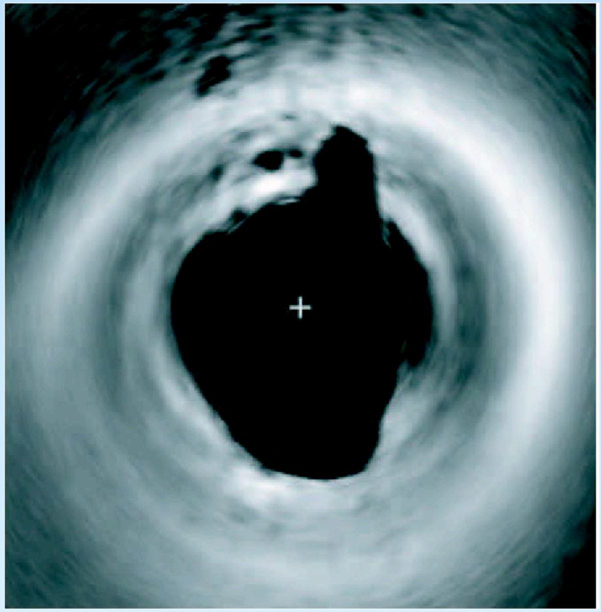



$0.001t_{sec(3:2)}$

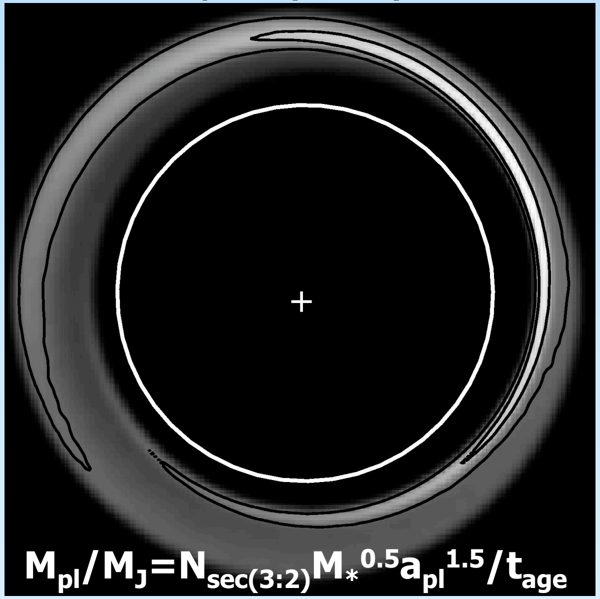
Spiral Structure in the HD141569 Disk

- HD141569A is a **5 Myr**-old **B9.5V** star at **99 pc**
- Dense rings at 200 and 325 AU with tightly wound spiral structure (Clampin et al. 2003)

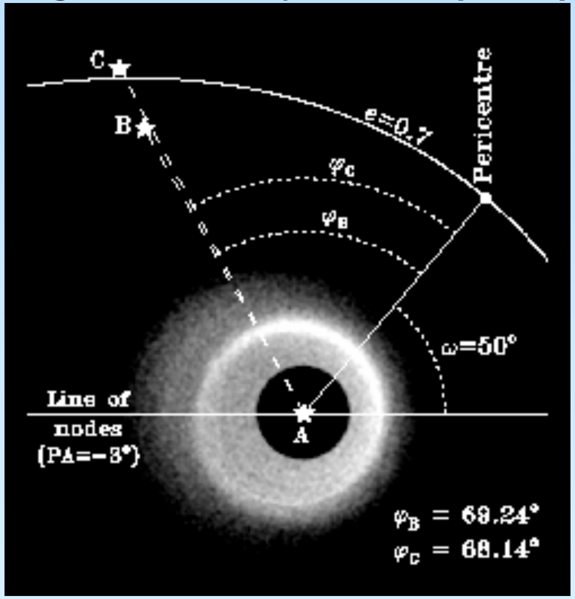
Observation



Wyatt (2005)



Augereau & Papaloizou (2004)

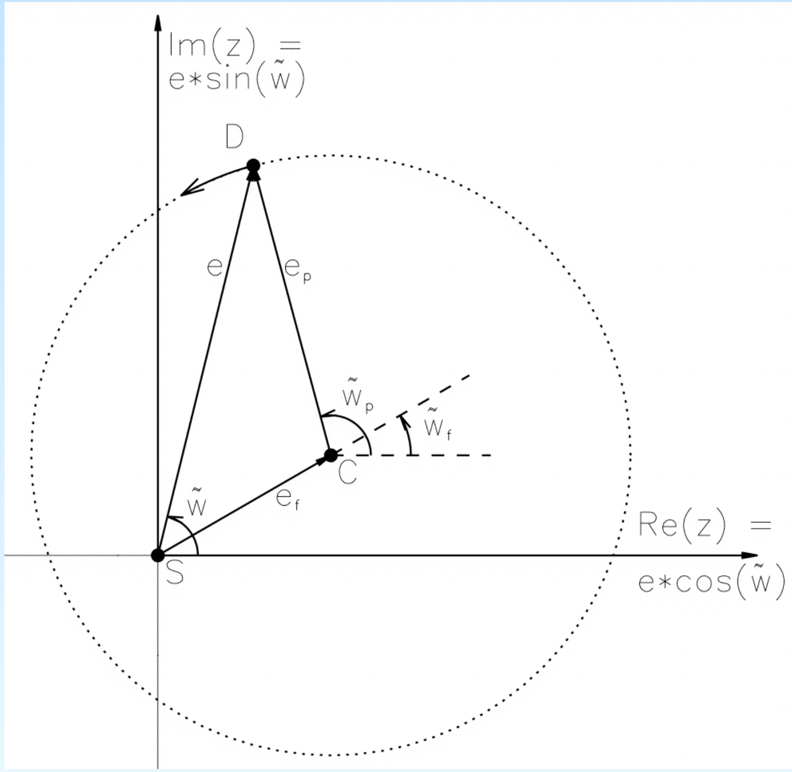


- Spiral structure at 325 AU can be explained by:
  - $0.2M_{Jupiter}$ planet at 250AU with e=0.05 (Wyatt 2005)
  - binary companion M stars at 1200AU (Augereau & Papaloizou 2004)
- Spiral structure at 200 AU implies planet at 150 AU
- Same structure seen in Saturn's rings (Charnoz et al. 2005) but for different reason…

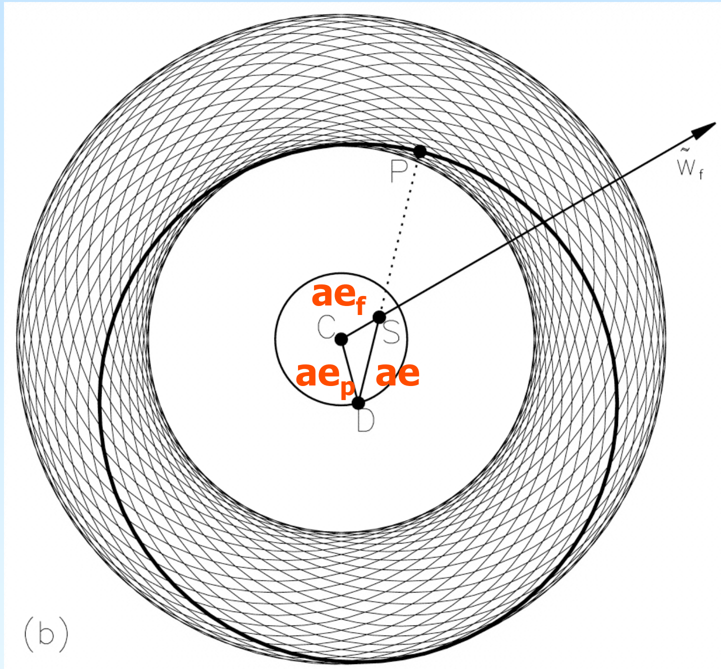
Perturbations at late times in narrow ring

Consider planetesimals with same proper eccentricities e_p at semimajor axis a

After many precession periods, orbital elements distributed evenly around circle centred on z_f



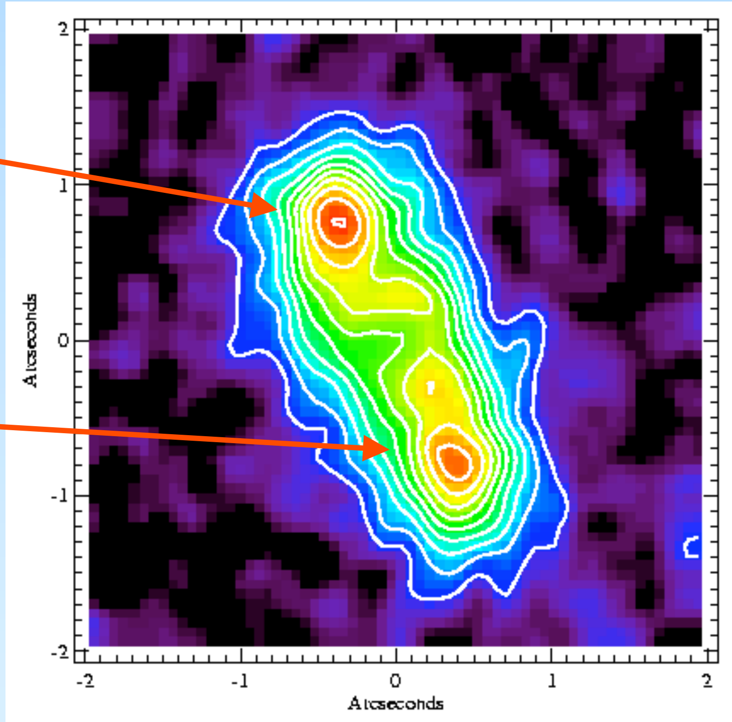
This translates into material in a uniform torus with centre of symmetry offset from star by ae_f in direction of forced apocentre

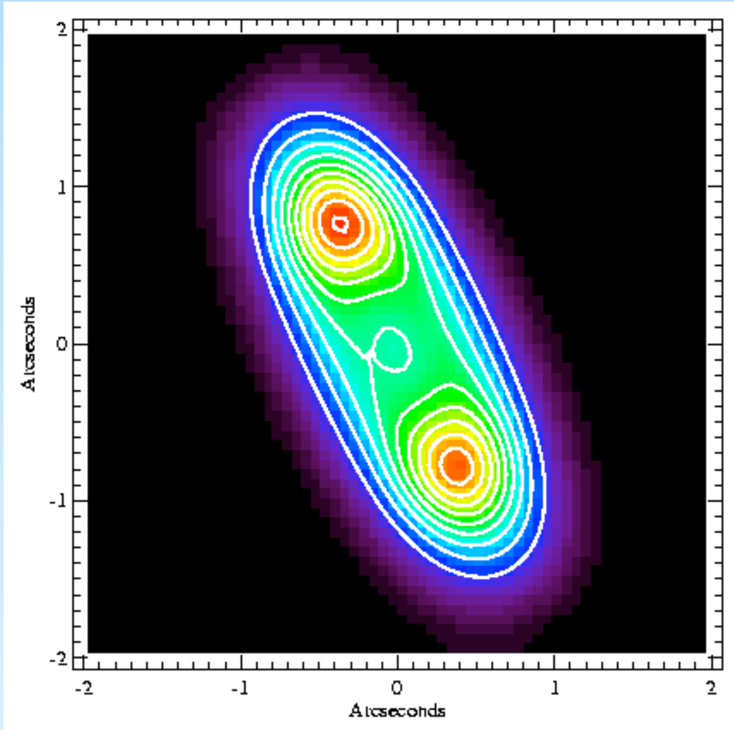


Pericentre glow in HR4796

Phenomenon predicted based on observations of the dust ring around HR4796 (A0V , 10Myr) (Wyatt et al. 1999)

NE lobe is 5% brighter than SW lobe





Model fitting like Fomalhaut with:

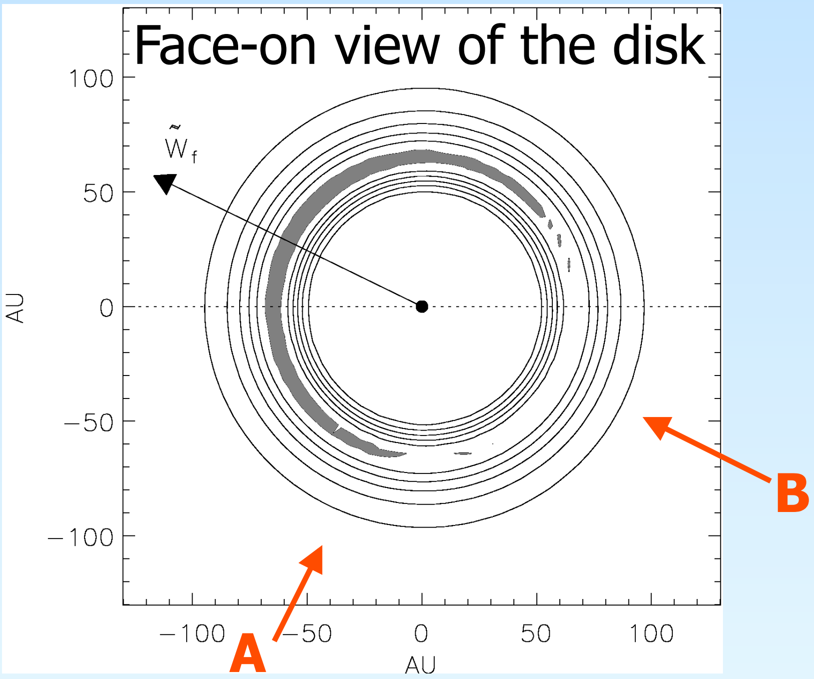
• four observables (lobe distance, brightness, vertical distribution, brightness asymmetry)

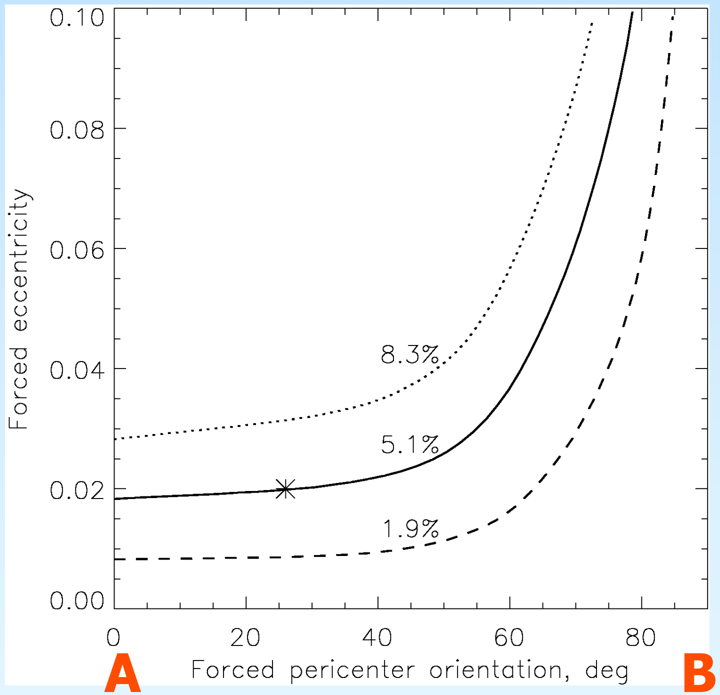
• five free parameters (radius, total area, inclination, forced eccentricity, pericentre orientation)

Interpretation of HR4796

The forced eccentricity causes the forced pericentre side to be closer to the star and so hotter and brighter than the opposite side

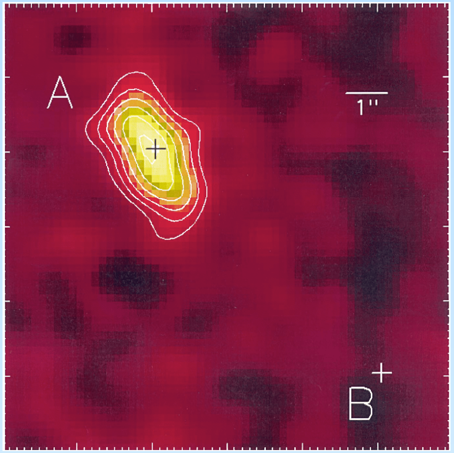
The forced eccentricity required to give 5% asymmetry is dependent on the orientation of the forced pericentre to the line-of-sight





The 5% asymmetry is likely caused by $e_f \sim 0.02$

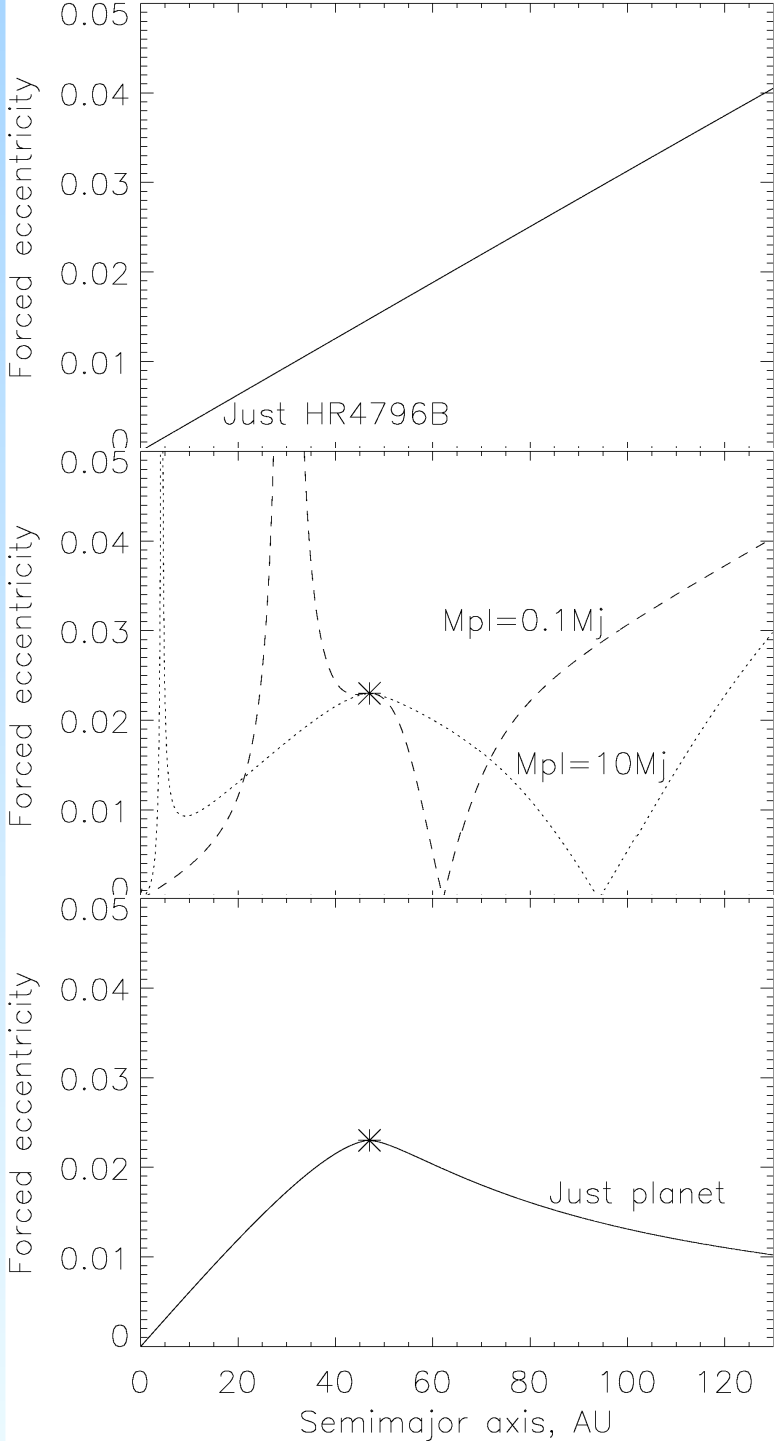
Origin of forced eccentricity



There is an M star binary companion at 517AU the orbit of which is unknown, but an eccentricity of ~0.13 could have imposed this offset

But then again, so could a planet with e_{pl}=0.02

Most likely both binary and planet are perturbing the disk leading to a complex forced eccentricity distribution



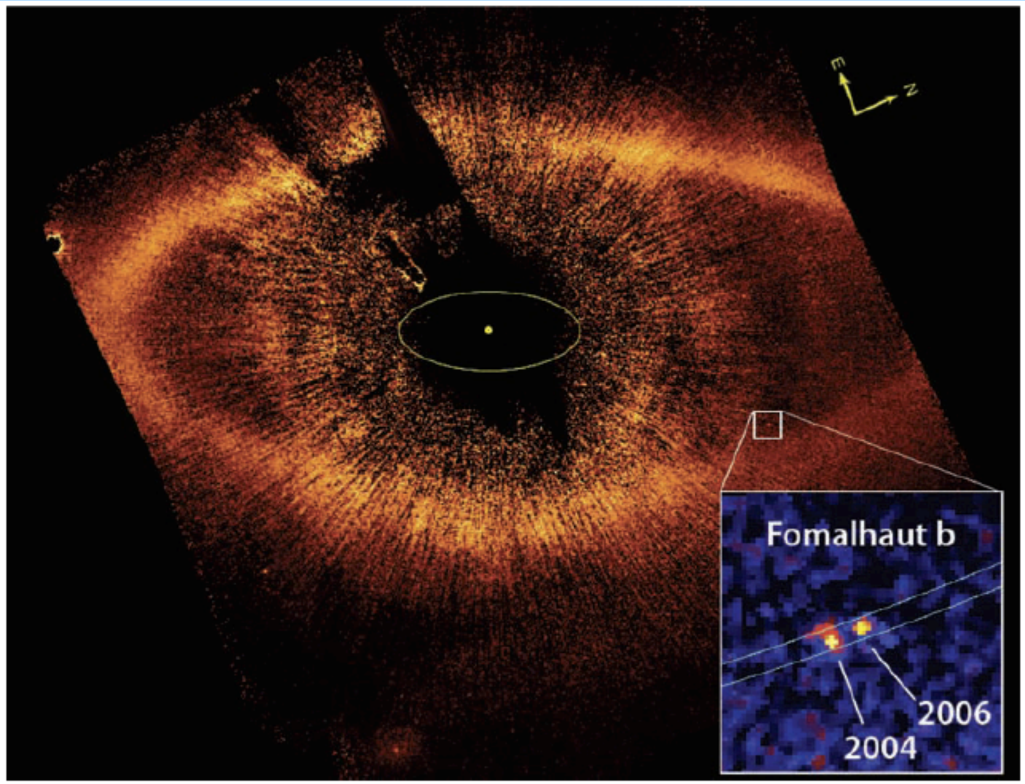
Offset in Fomalhaut

While the offest in HR4796 remains unconfirmed, though tentatively detected, it has been confirmed in HST imaging of Fomalhaut (Kalas et al. 2005)

Image shows a ~133AU radius ring with a centre of symmetry offset by 15 AU from the star, implying a forced eccentricity of ~0.1

Fomalhaut has an M star binary companion at 2^{o} (50,000AU, 0.3pc), which could perturb the ring, but not that much (a_B>25,000AU so $a_{ring}/a_B<5\times10^{-3}$ and $e_{ring}<5\times10^{-3}$)

However, the sharp inner edge implies presence of another perturber (Quillen 2006)



Discovery images of this planet are described in Kalas et al. (2008)

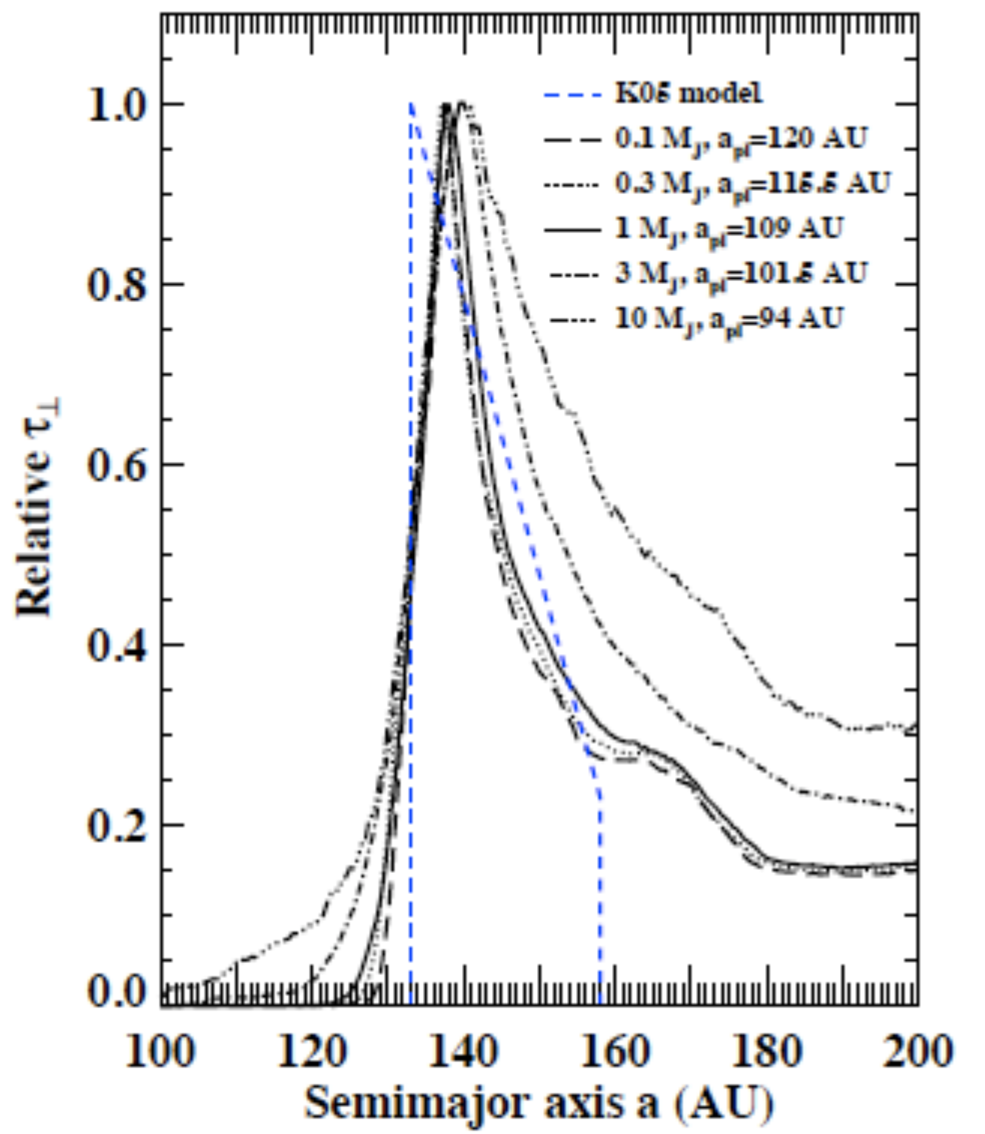
Constraints on M_{pl} in Fomalhaut

The radial profile was modelled in Chiang et al. (2008) putting the inner edge of the ring at the edge of the planet's chaotic resonance overlap region to show that:

• planet mass must be below $3M_{jup}$, otherwise the inner edge is too shallow (a similar conclusion to Quillen 2006, though see Chiang for discussion of this point)

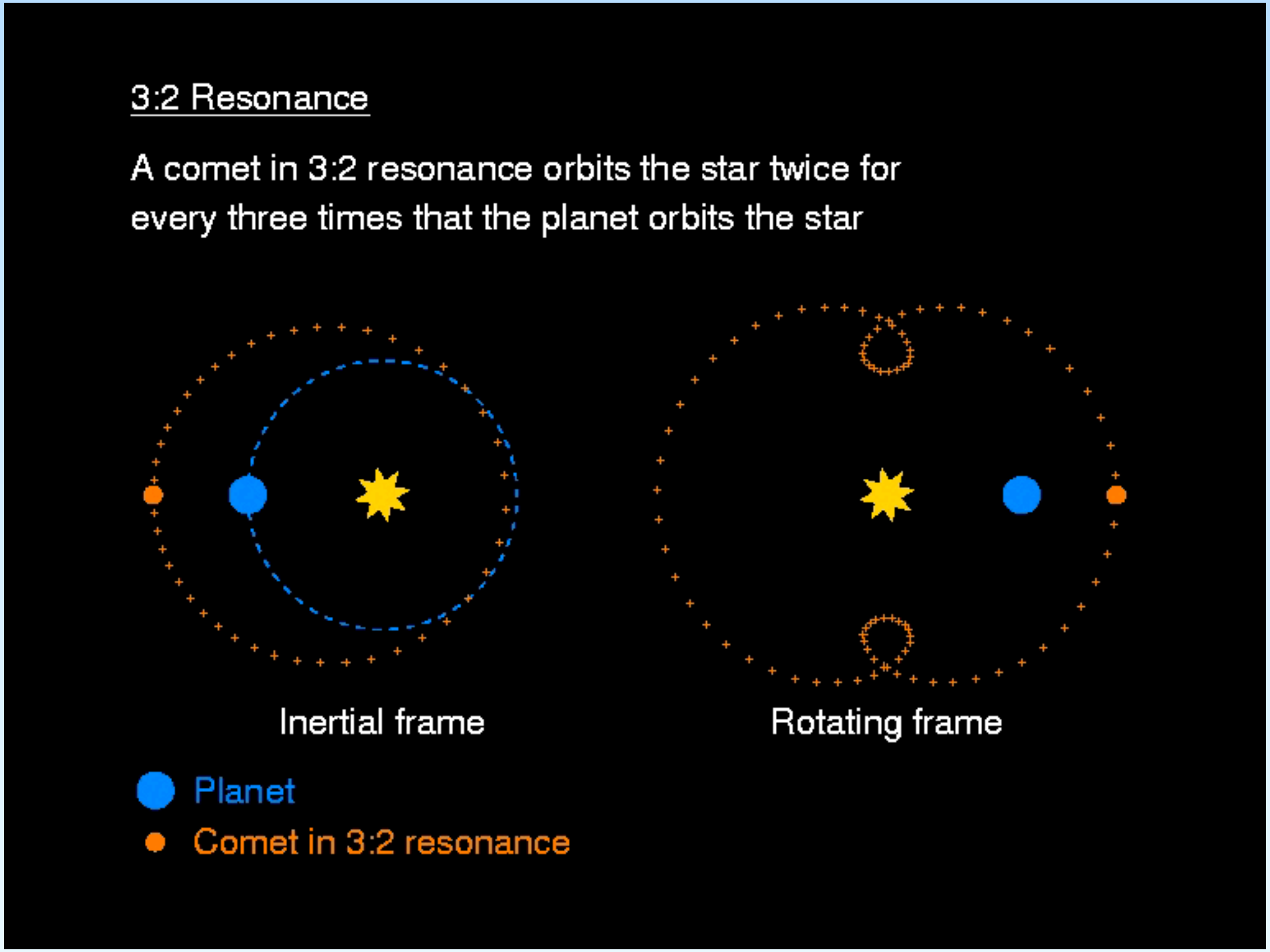
• proper eccentricities are low

The latter point is a problem, or rather a clue to how the system formed



Geometry of resonance

- A resonance is a location where a planetesimal orbits the star **p** times for every **p+q** planet orbits, which occurs at $\mathbf{a_{res}}=a_{pl}\,[\,(p+q)/p\,]^{2/3}$



- Resonances are special because of the periodic nature of the orbits and the way that planet and planetesimal have encounters

Geometry of resonance

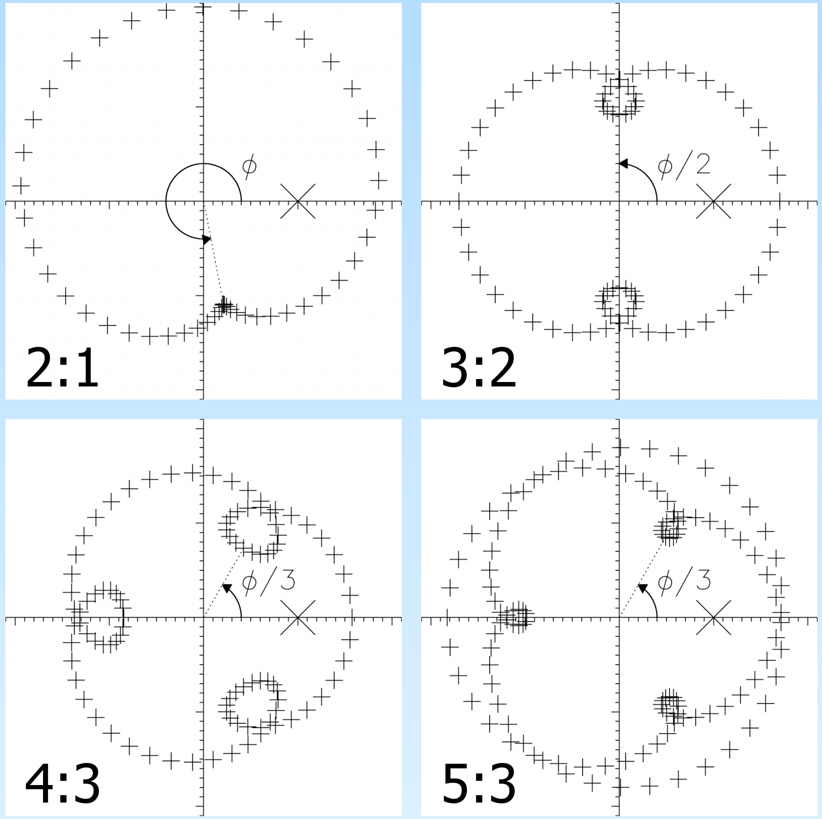
- Orientation of the loopy pattern is defined by the resonant angle, e.g.

$$\phi = (p+q)\lambda - p\lambda_{pl} - q\varpi$$
$$= p[\varpi - \lambda_{pl}(t_{peri})]$$

which is the appropriate term in the disturbing function

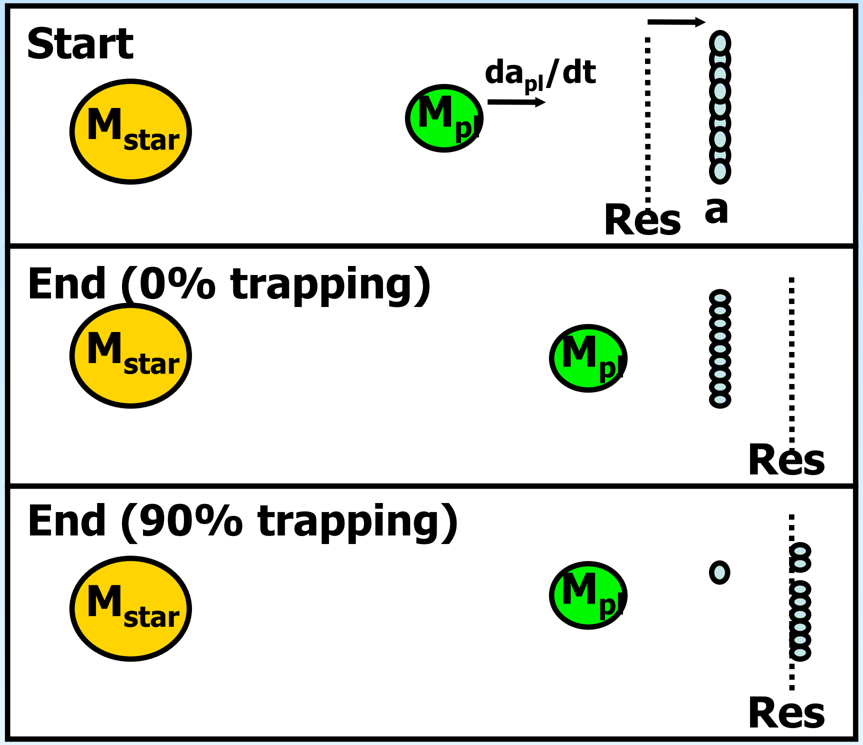
- ϕ librates about 180^o for all but the n:1 resonance for which this is function of eccentricity (=**asymmetric libration**)

Each resonance has its own geometry

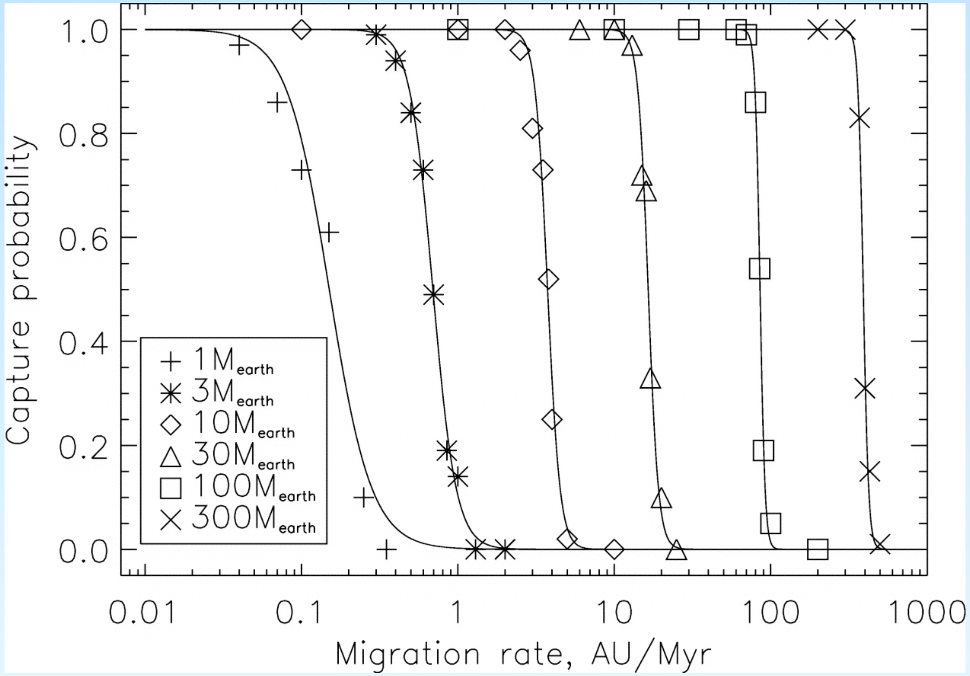


Capture by migrating planet

Planetesimals can become captured into the resonances of a migrating planet



Numerical integrations of star, migrating planet, 200 planetesimals giving capture probabilities for 3:2 resonance:



Capture probability dependencies

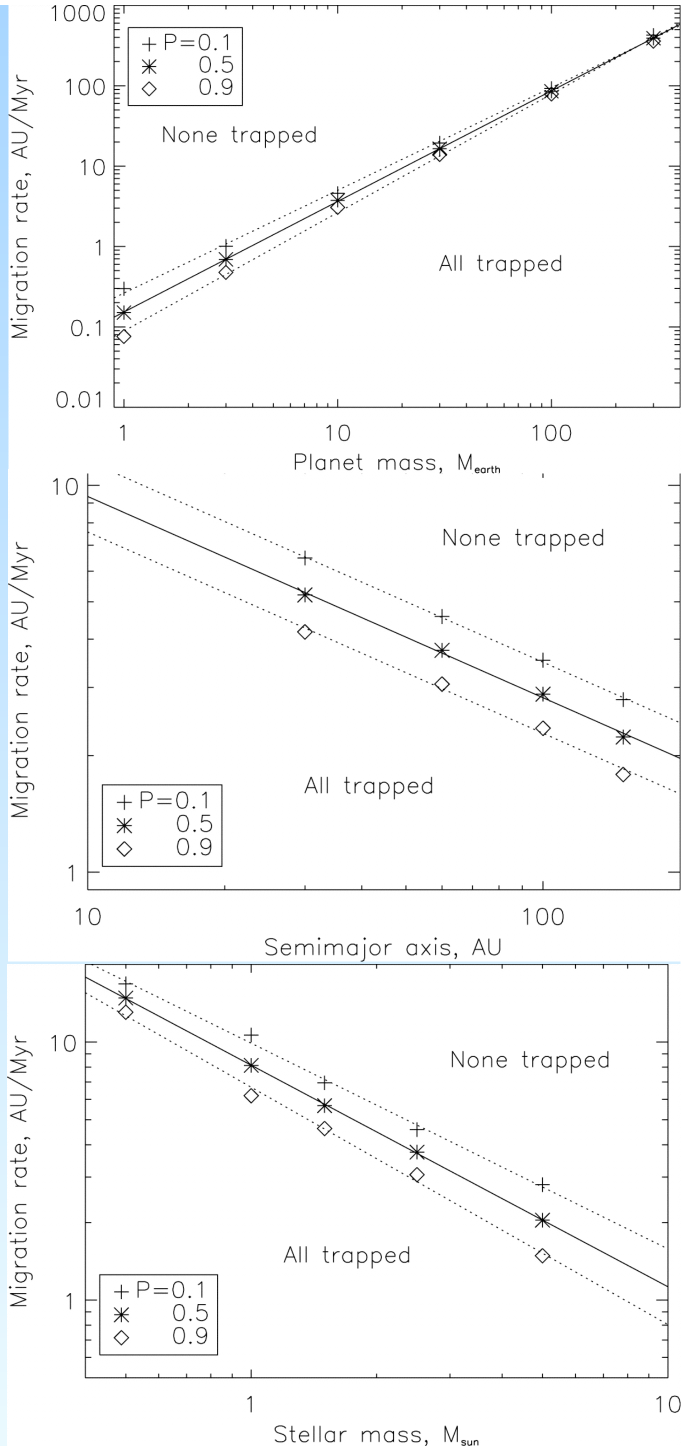
The runs were performed changing planet mass, planetesimal semimajor axis and stellar mass

Probability of capture into a resonance as it passes is a function only of (Wyatt 2003)

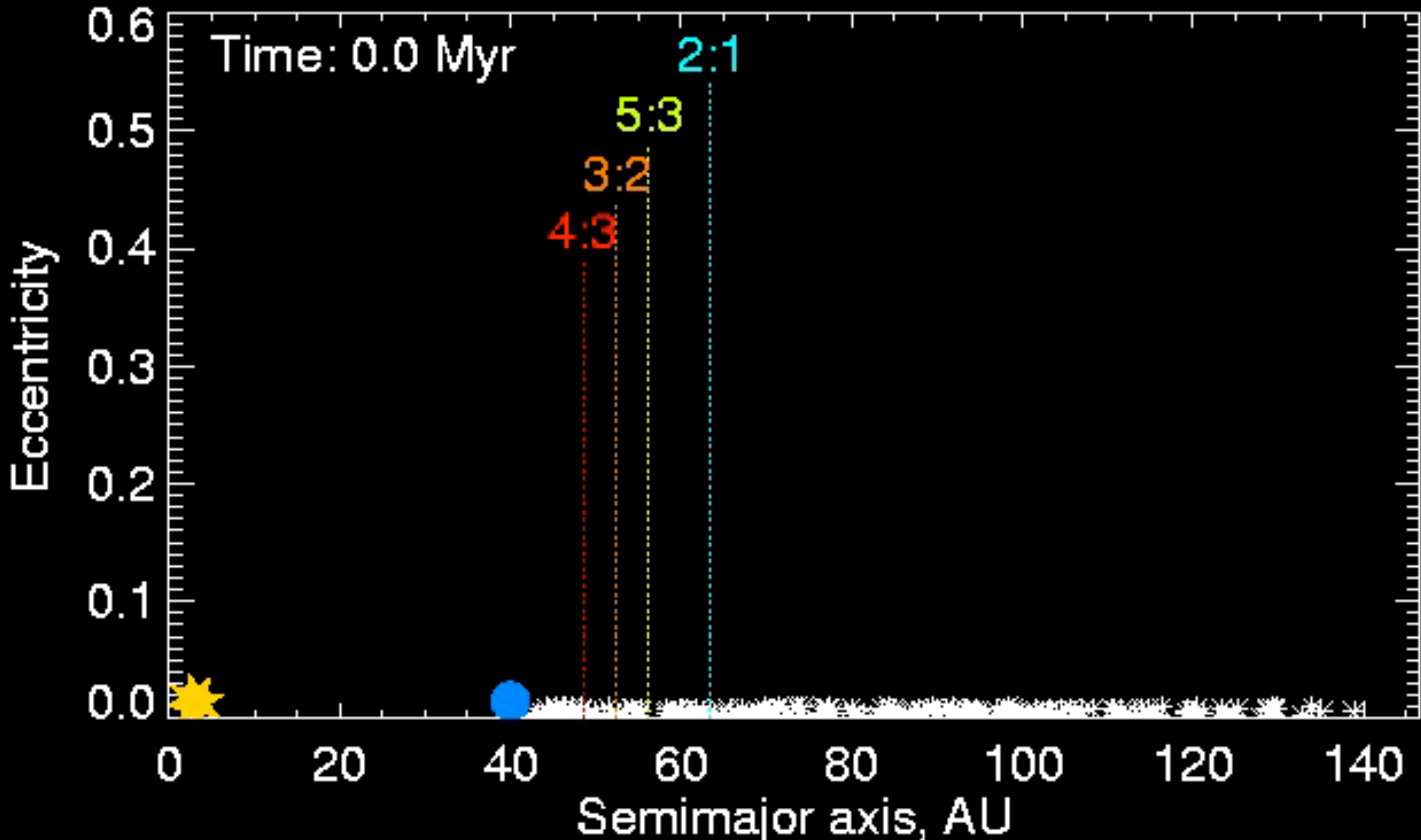
$$\mu = M_{pl}/M_*$$
$$\theta = (da_{pl}/dt)(a/M_*)^{0.5}$$
$$P = [1+(0.37\mu^{-1.37}\theta)^{5.4\mu\wedge 0.38}]^{-1}$$

Following capture the eccentricity of a planetesimal is pumped up according to the relation:

$$e^2 = e_0^2 + [q/(p+q)]\ln(a/a_0)$$

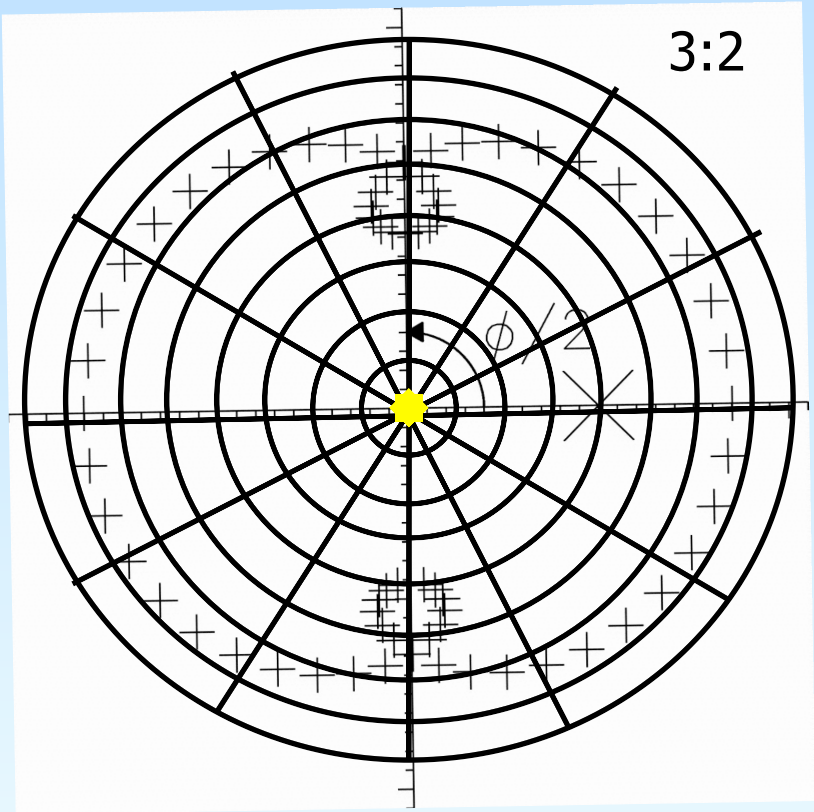


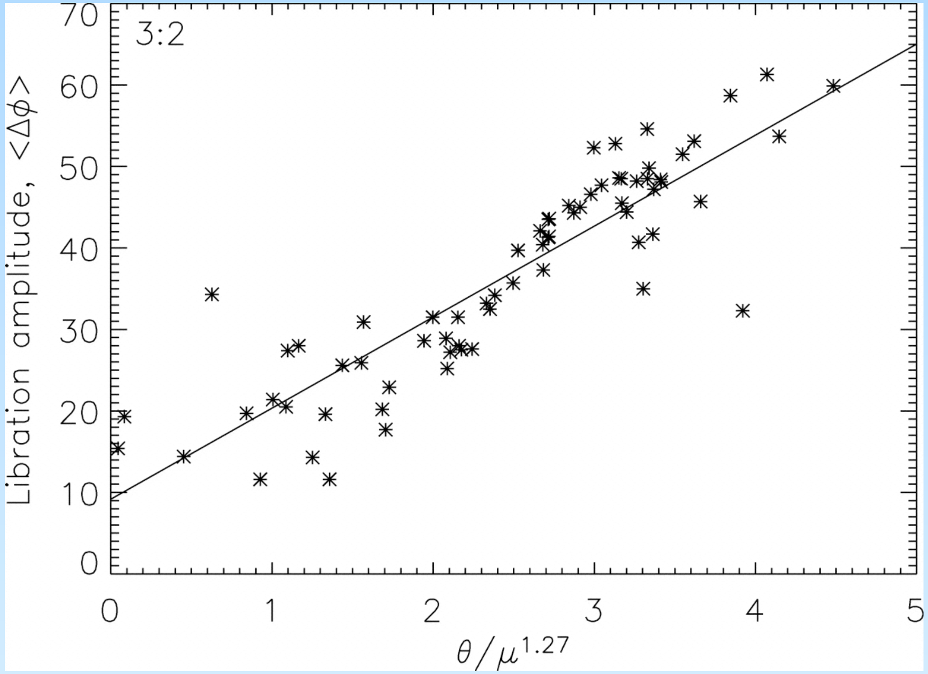
The outward migration of a Neptune mass planet (●) around Vega sweeps many comets (*) into the planet's resonances



Resonant spatial distribution

The location of planetesimals in the grid depends on their semimajor axis, a, eccentricity, e, and resonant angle, ϕ, with random longitude, λ:





Since the resonant angle librates

$$\phi = \phi_m + \Delta\phi \sin(2\pi t/t_\phi)$$

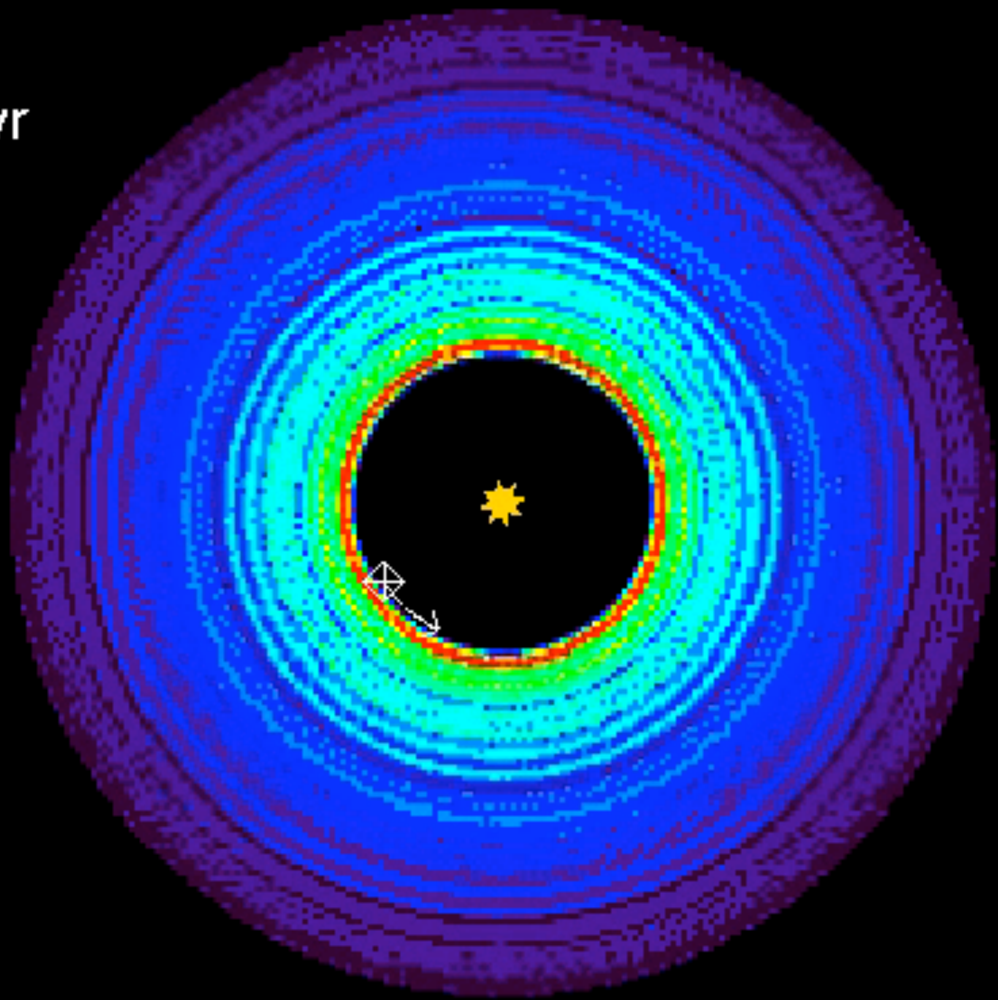
To determine the spatial distribution we need to know:

$$\phi_{m3:2} = 180^o + 7.5(\theta/\mu) - 0.23(\theta/\mu)^2$$

$$\Delta\phi_{3:2} = 9.2^o + 11.2(\theta/\mu^{1.27})$$

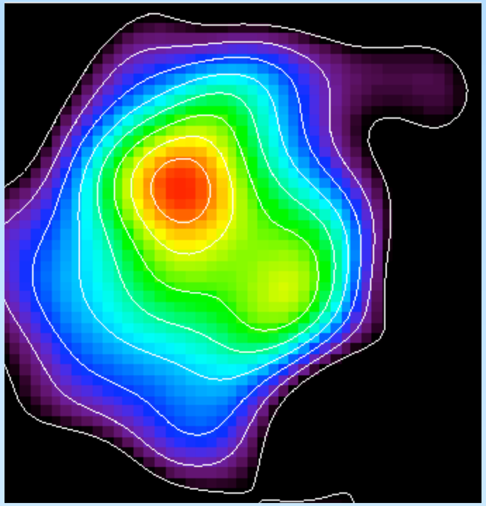
The trapping of comets in Vega's disk into planetary resonances causes them to be most densely concentrated in a few clumps

Time: 0.0 Myr

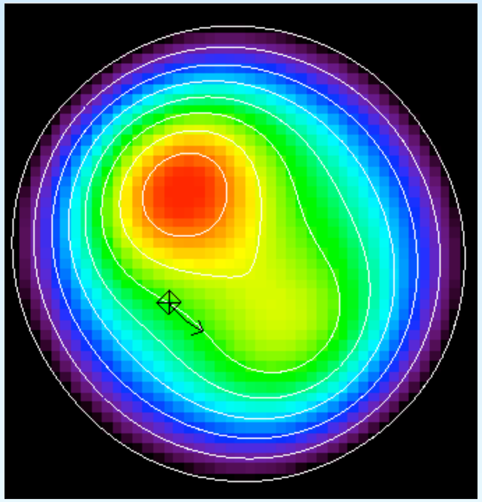


Constraints on Vega's planetary system

Observation

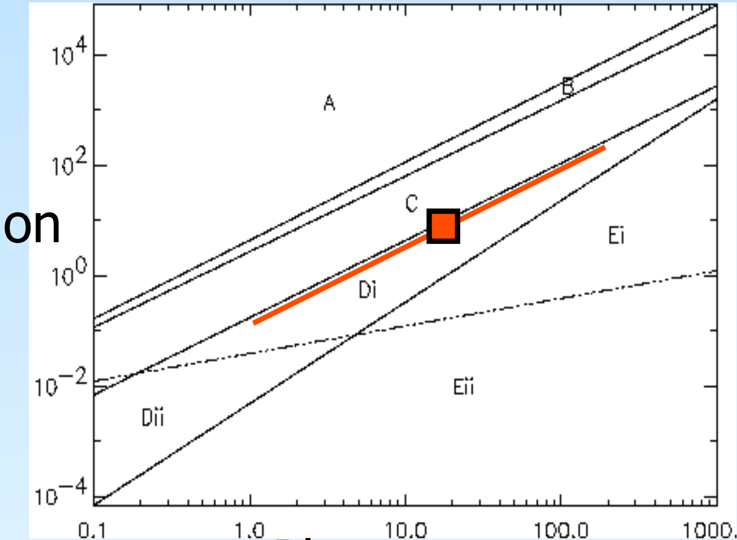


Model



The two clumps of asymmetric brightness in sub-mm images of Vega's debris disk (Holland et al. 1998) can be explained if planet mass and migration rate fall in a certain region of parameter space ( ) (Wyatt 2003)

Planet migration rate

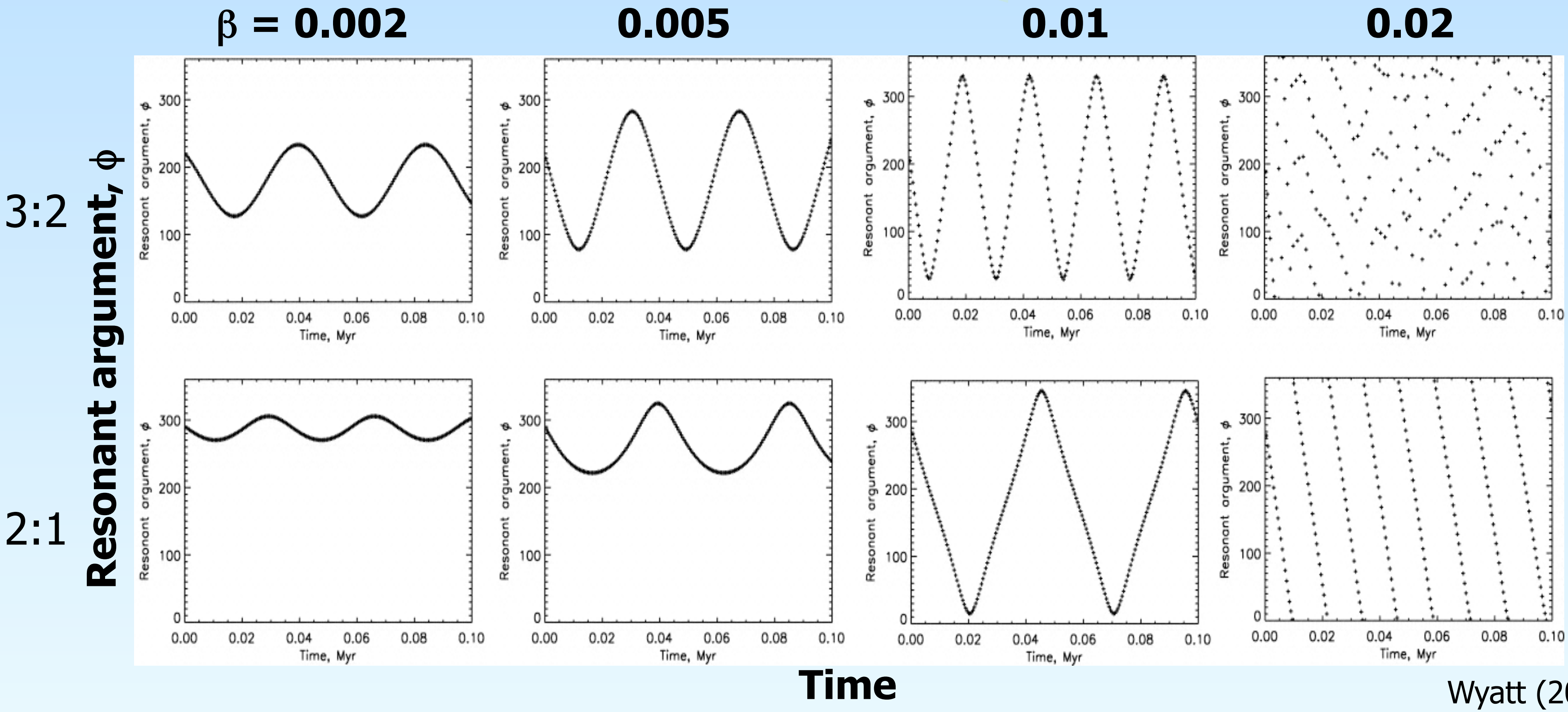


Planet mass

At $1M_{neptune}$ and 56Myr migration timescale ( ), implies Vega system formed and evolved like solar system

Dynamics of small bound grains

- Radiation pressure alters orbital period of dust and so its relation to resonance; $\Delta a = a_d - a_{rd} = a_r\beta(4/3 \pm 2e)$
- Small grains have higher libration widths than planetesimals
- Particles smaller than **200μm $(L_*/M_*)\mu^{-0.5}$** fall out of resonance



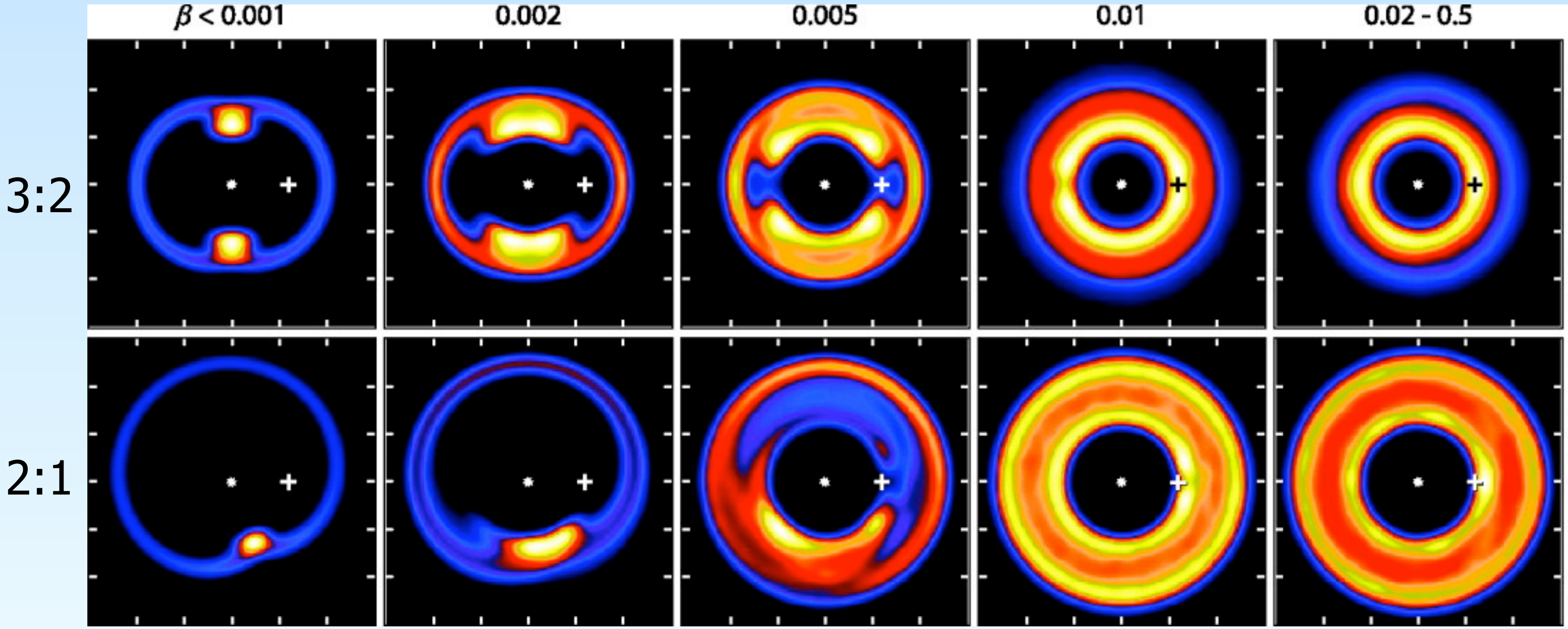
Wyatt (2006)

Distribution of small bound grains

• Large particles have the same clumpy distribution as the parent planetesimals

• The increased libration width of moderate sized grains smears out clumpy structure

• The smallest bound grains have an axisymmetric distribution



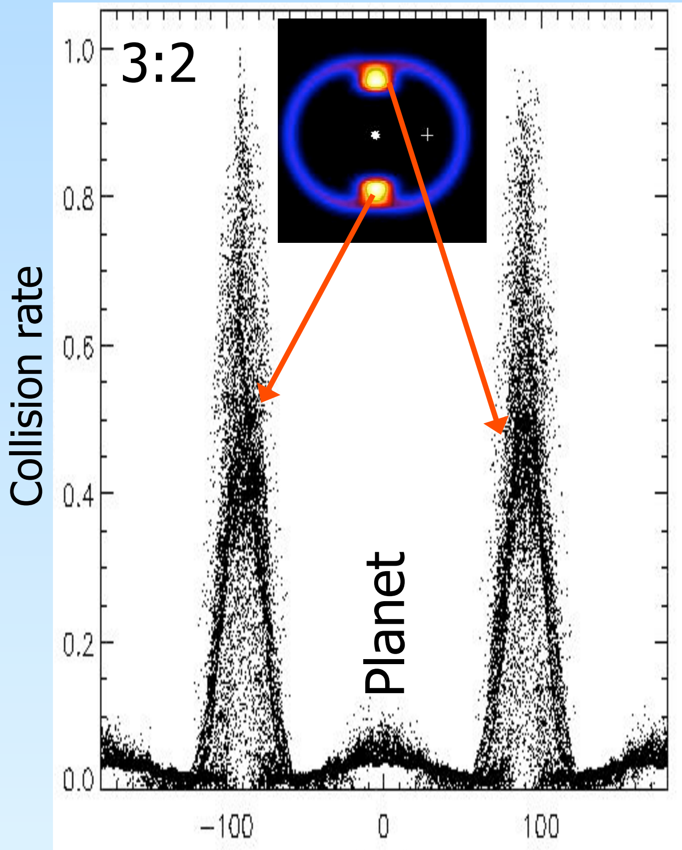
Wyatt (2006)

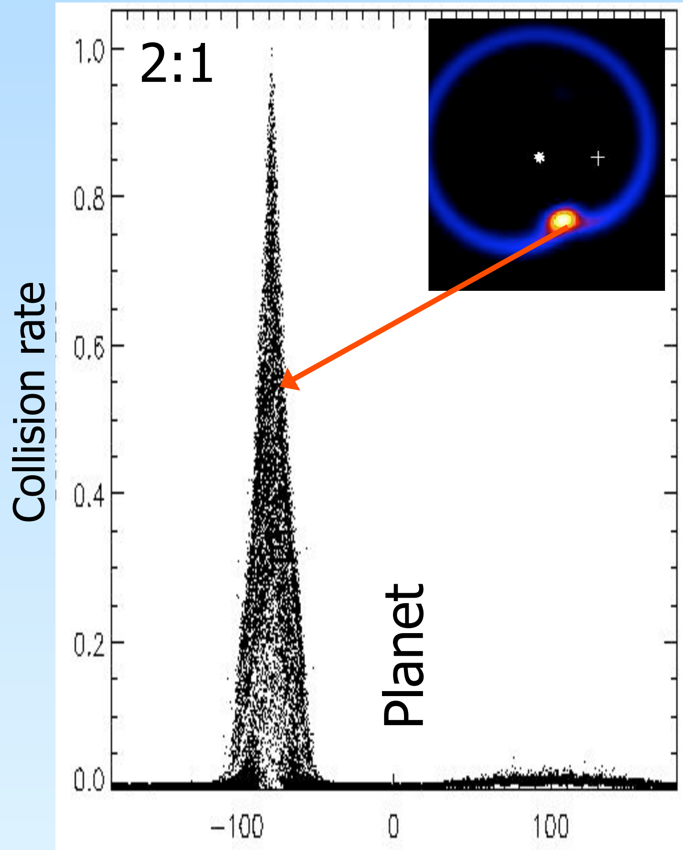
Dynamics of small unbound grains

- Radiation pressure puts small (β>0.5) grains on hyperbolic trajectories

- The collision rate (R_{col}) of resonant planetesimals is higher in the clumps

- In model, work out R_{col} by looking at number of planetesimals within 4AU and average relative velocity

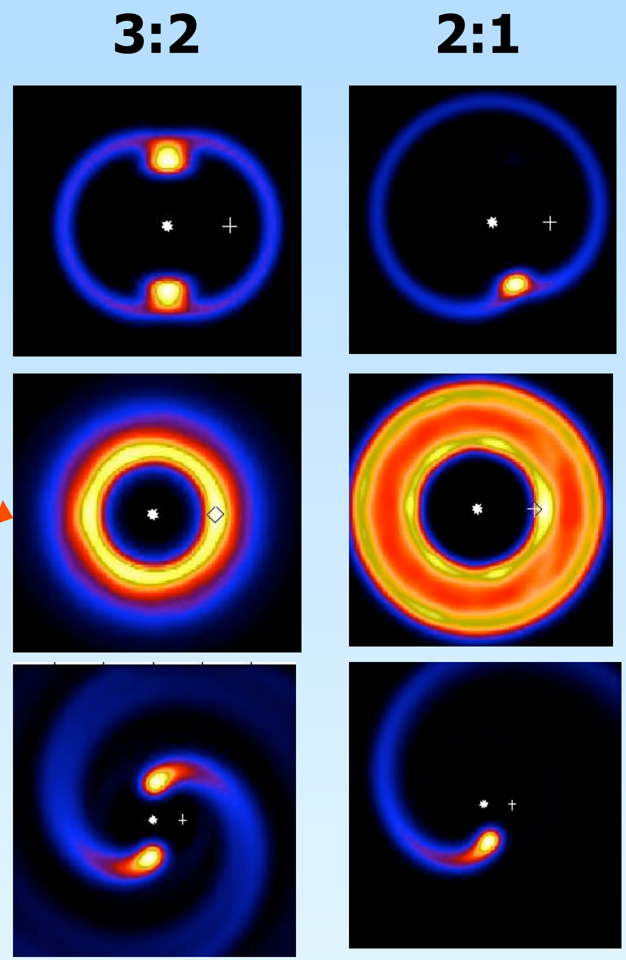




Blow-out grains exhibit spiral structure if created from resonant planetesimals

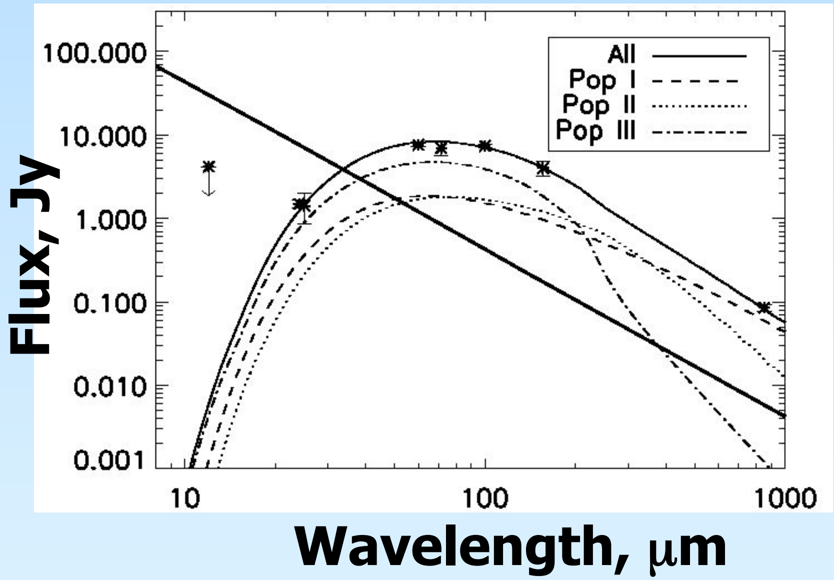
Particle populations in a resonant disk

Grain size	Population	Spatial distribution
Large	I	Same clumpy distribution as planetesimals
Medium	II	Axisymmetric distribution
Small	III	$\tau \propto r^{-1}$ distribution
	IIIa	Spiral structure emanating from resonant clumps
	IIIb	Axisymmetric distribution



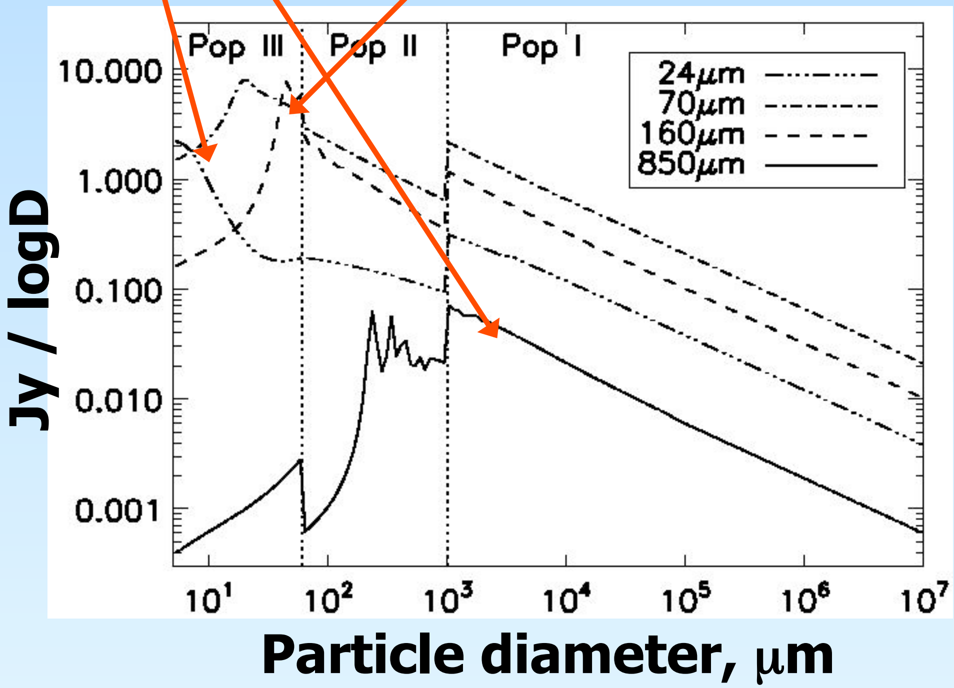
What does this mean for Vega?

SED modelling used to determine the size distribution…



… then used to assess contribution of grain sizes to observations:

- Sub-mm samples pop I
- Mid- and far-IR sample pop III

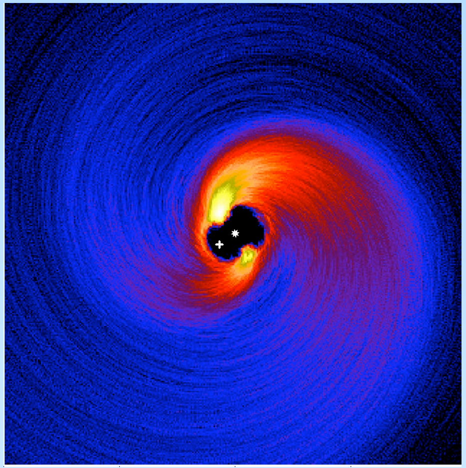


Observations in different wavebands sample different grain sizes and so populations, thus multi-wavelength images should show different structures and can be used to test models

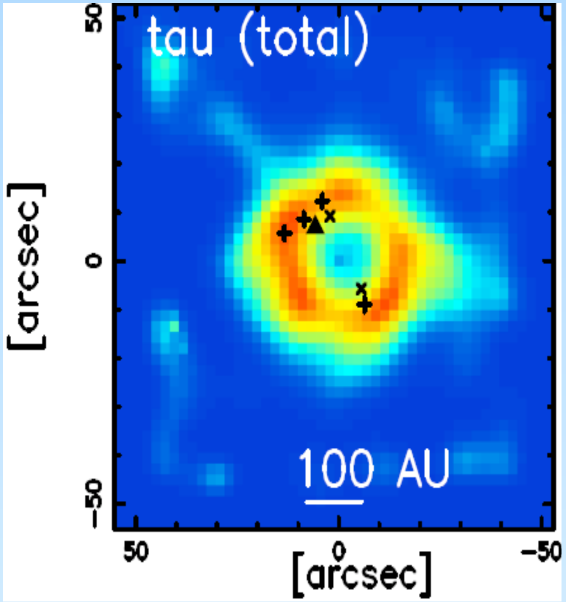
Wyatt (2006)

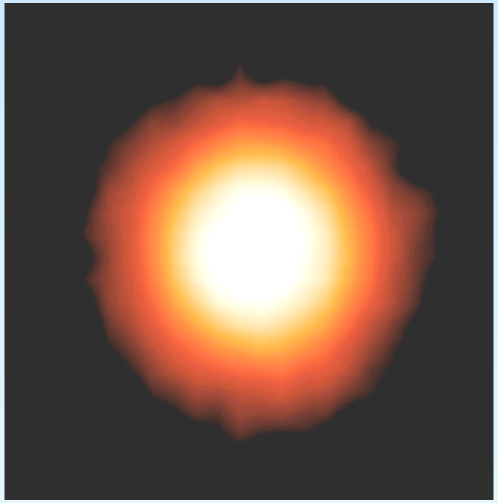
Comparison with observations

Mid- to far-IR images should exhibit spiral structure emanating from clumps

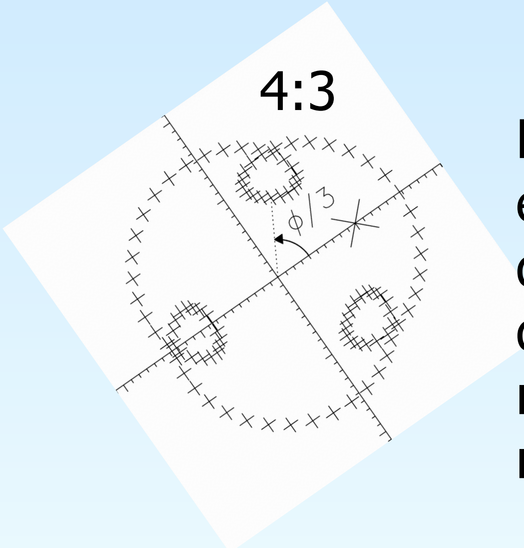


Meanwhile 350μm imaging shows evidence for 3 clump structure (Marsh et al. 2006)





Not detected at present, but resolution of published Spitzer observations may not have had sufficient resolution to detect this (Su et al. 2005)

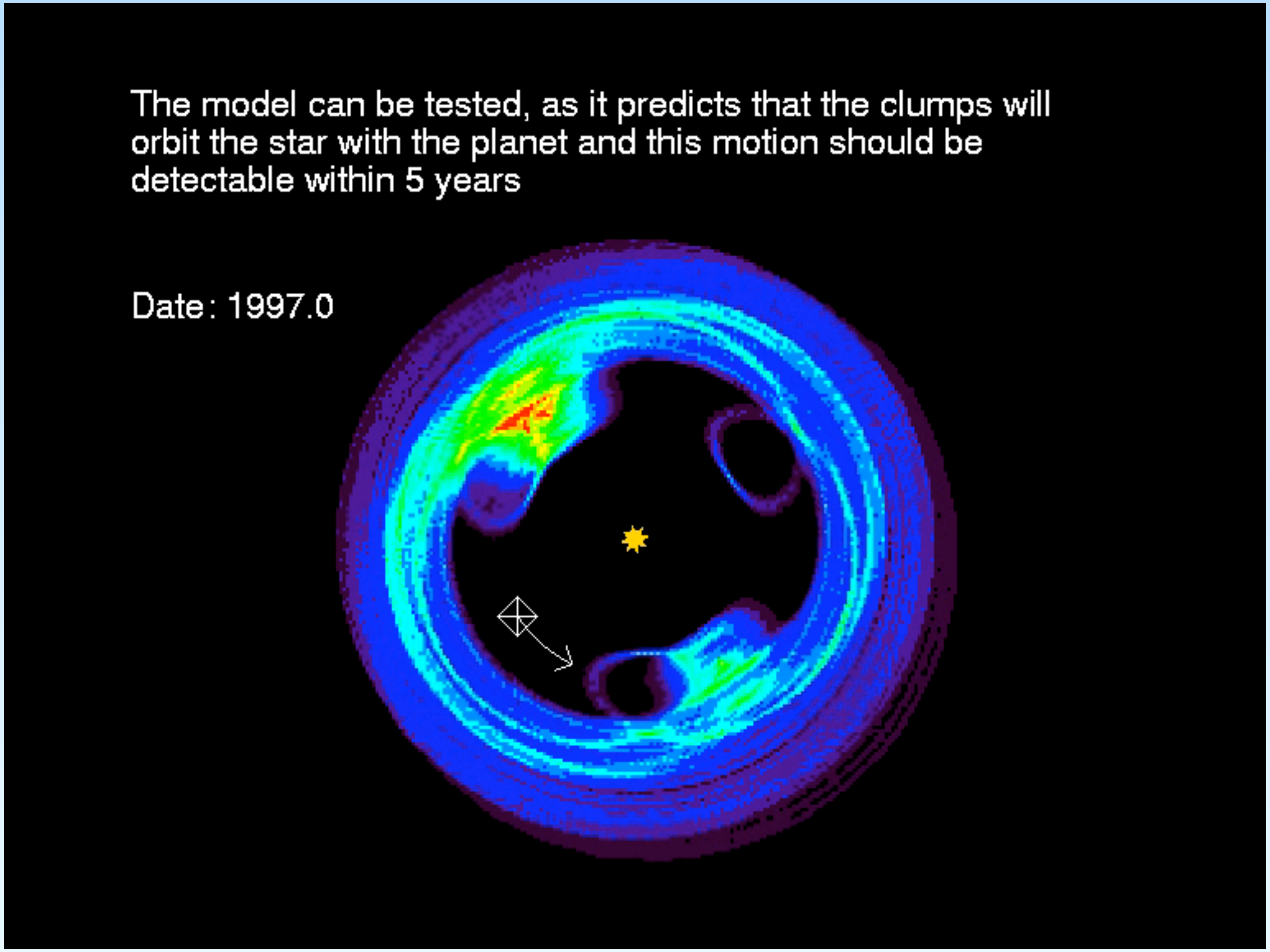


Possible evidence for a different size distribution of material in 4:3 resonance?

Resonant structure follows the planet

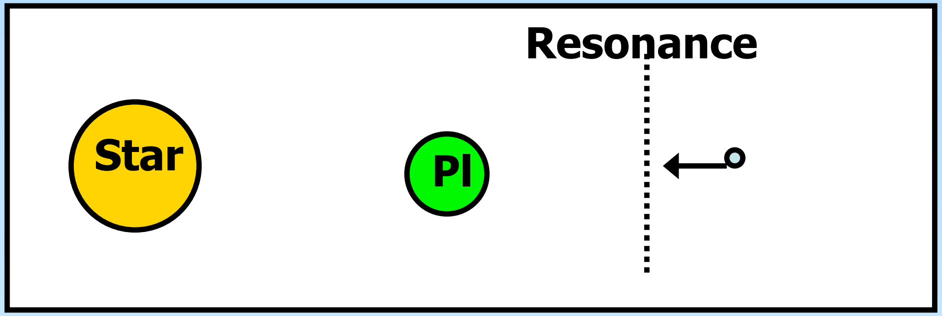
- The model can be tested by multi-epoch imaging of the clumpy sub-mm structure, since resonant structures orbit with the planet

- Decade timescales for confirmation, and there is already a 2σ detection of rotation in disk of ε Eri (Poulton et al. 2006)

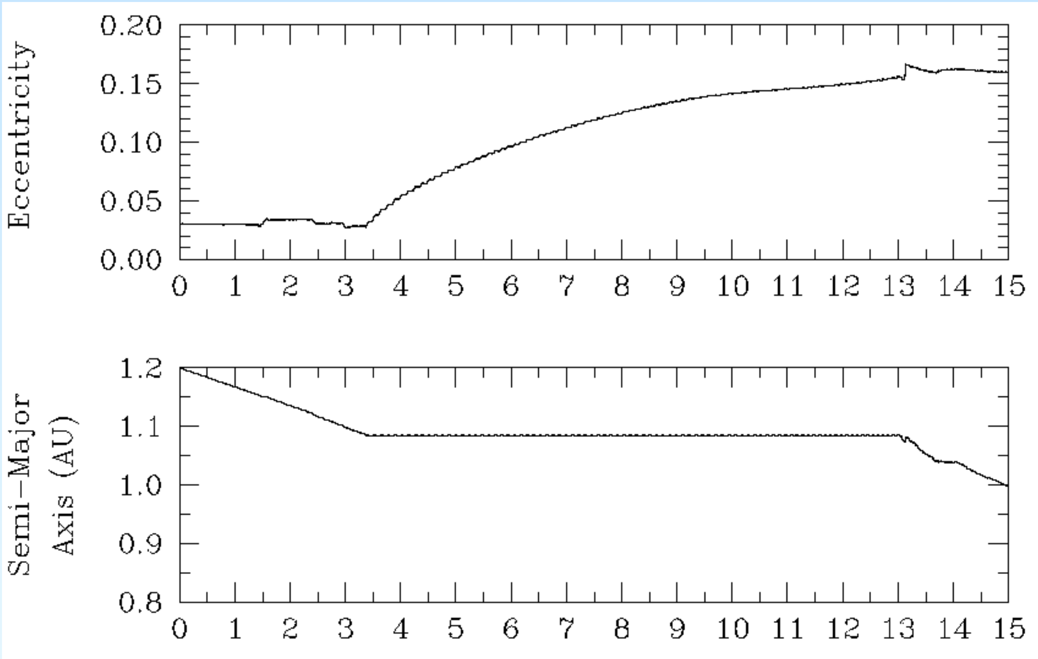


Dust migration into planetary resonances

Resonances can also be filled by inward migration of dust by P-R drag, since resonant forces can halt the migration



For example dust created in the asteroid belt passes the Earth's resonances and much of it is trapped temporarily (~10,000yrs)



Time, 1000 years

Trapping timescale is of order t_{pr} meaning ring forms along Earth's orbit

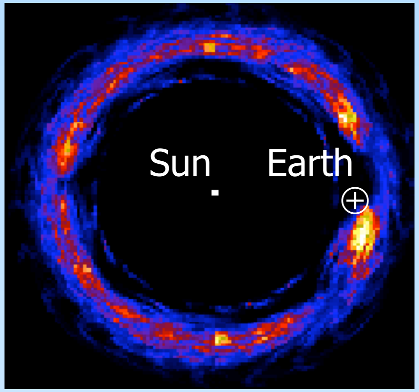
Structures of resonant rings

The structure expected when dust migrates into planetary resonances depends on the planet's mass and eccentricity (Kuchner & Holman 2003)

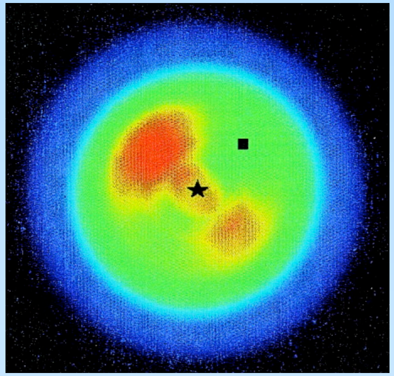
However, this ignores that P-R drag is not important in detectable debris disks

low M_{pl} | high M_{pl}

low e_{pl}

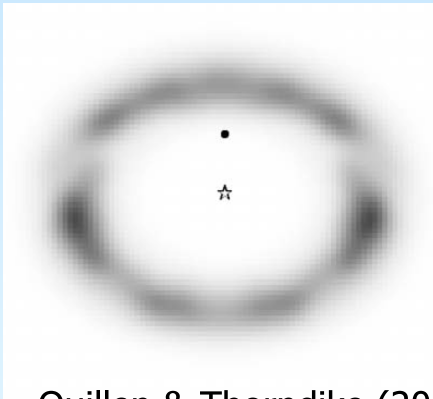


Dermott et al. (1994)

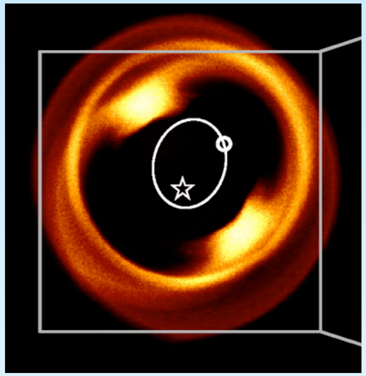


Ozernoy et al. (2000)

high e_{pl}



Quillen & Thorndike (2002)



Wilner et al. (2002)

Why P-R drag is insignificant

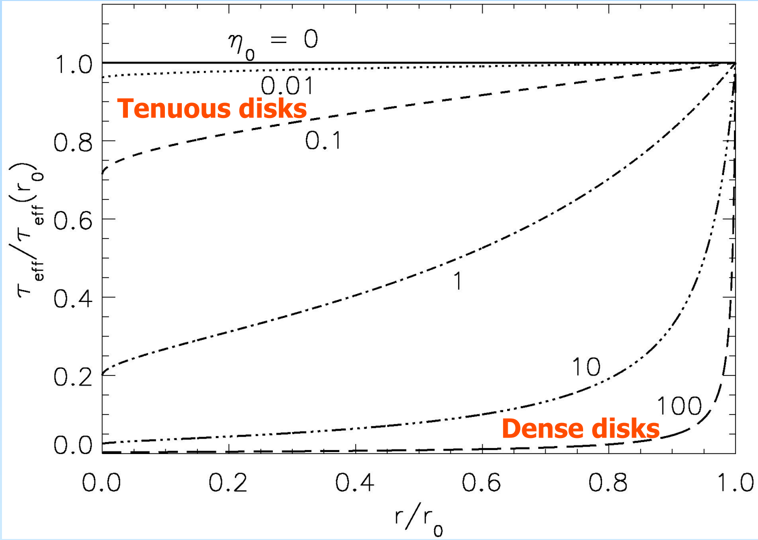
Remember that the surface density of a disk evolving due to collisions and P-R drag is only determined by the parameter

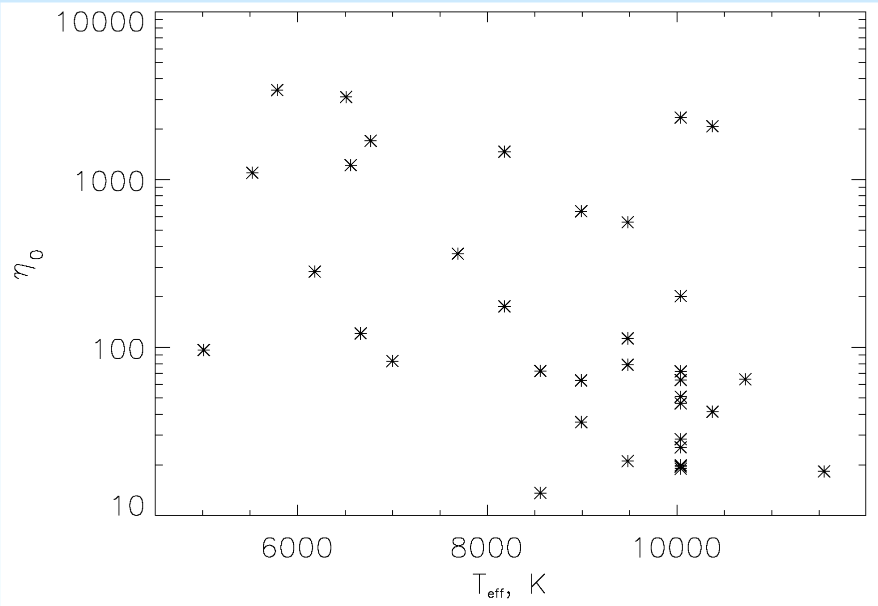
$\eta_0 = 5000\tau_{eff}(r_0)[r_0/M_*]^{0.5}/\beta$

This is an observable parameter, since r_0 can be estimated from dust temperature, $\beta<0.5$, and

$\tau_{eff} \approx 6.8 \times 10^9\ d^2 F_\nu/[r dr B_\nu(T)]$

Wyatt (2005)





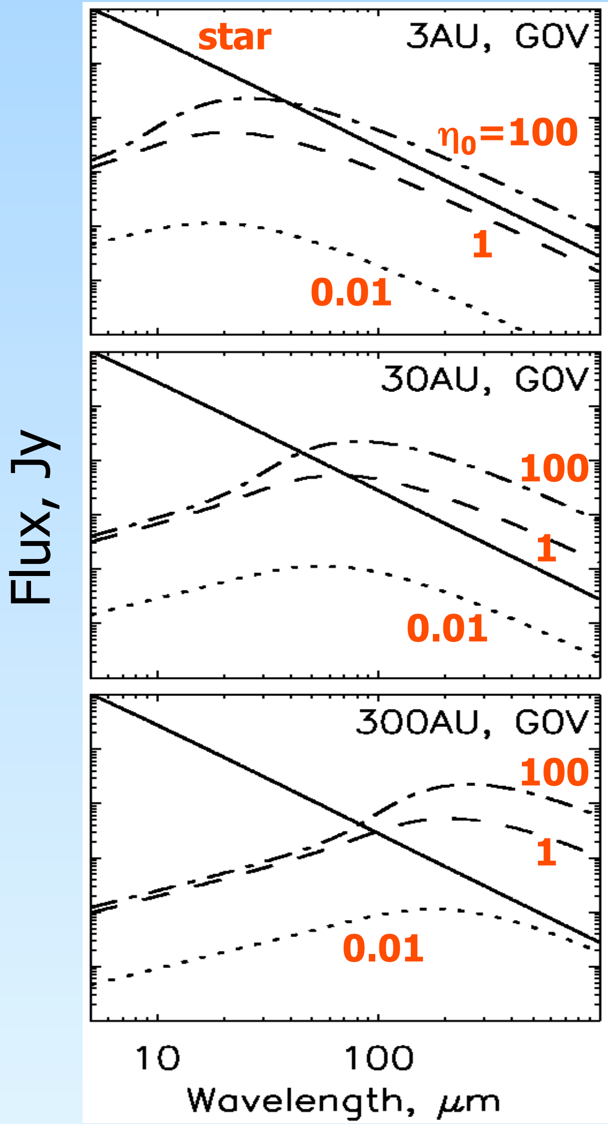
For the 38 disks detected at more than one wavelength (for which T can be estimated) P-R drag is insignificant

When P-R drag becomes important

The reason is that detectable disks have to be dense to have a flux that exceeds that of the photosphere

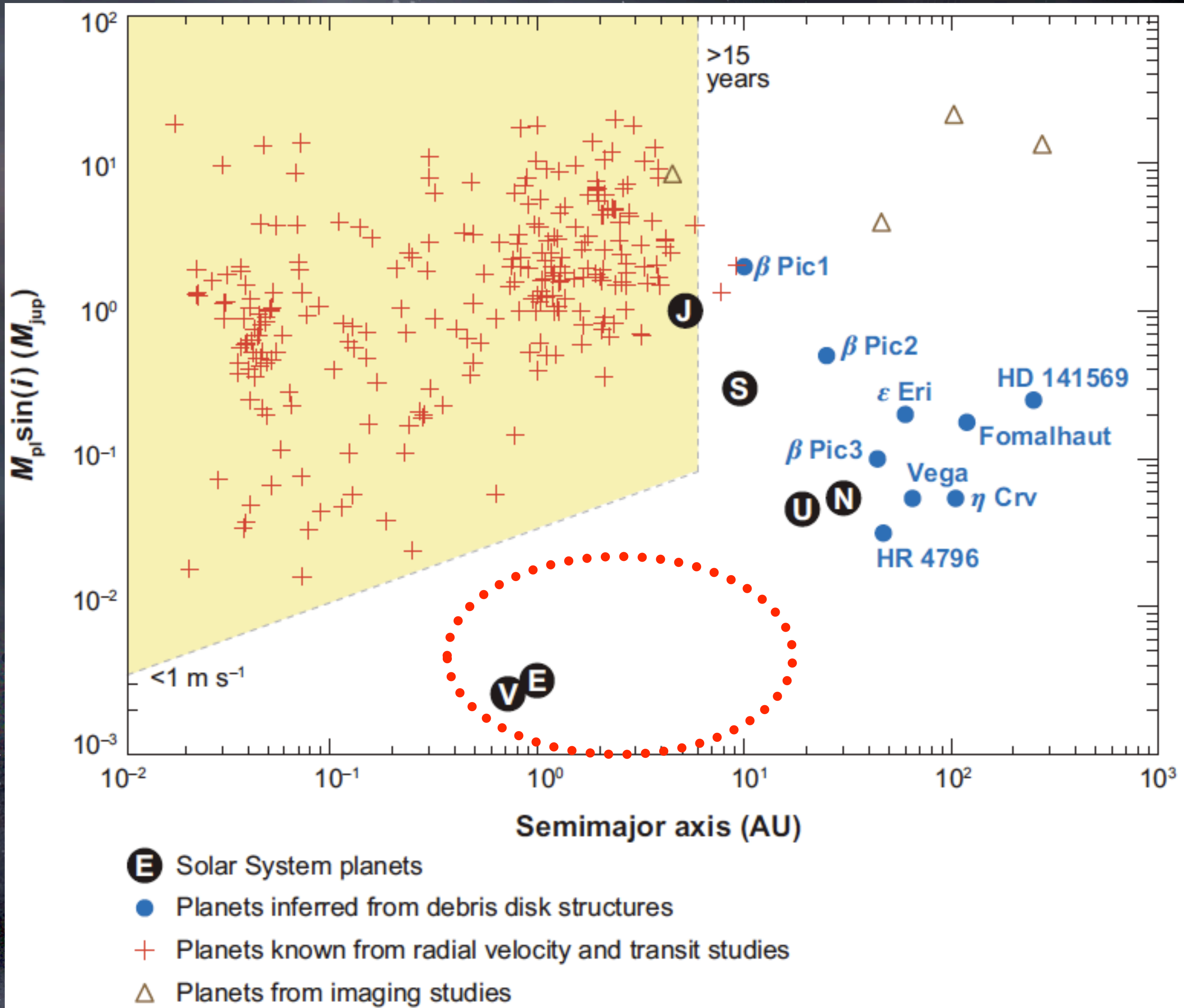
This is only possible for η_0>1 for distant belts around high mass stars detected at long wavelengths…

… although low η_0 disks can be detected if they are resolved (ALMA/ JWST/ TPF/ DARWIN)…



… at which point we may be able to detect the resonant rings of Earth-like planets more readily than the planets themselves!

Conclusions



Modelling debris disks provides information about unseen planetary systems

These currently occupy the Saturn, Uranus, Neptune region of parameter space

Future observations will probe the Earth, Venus regions