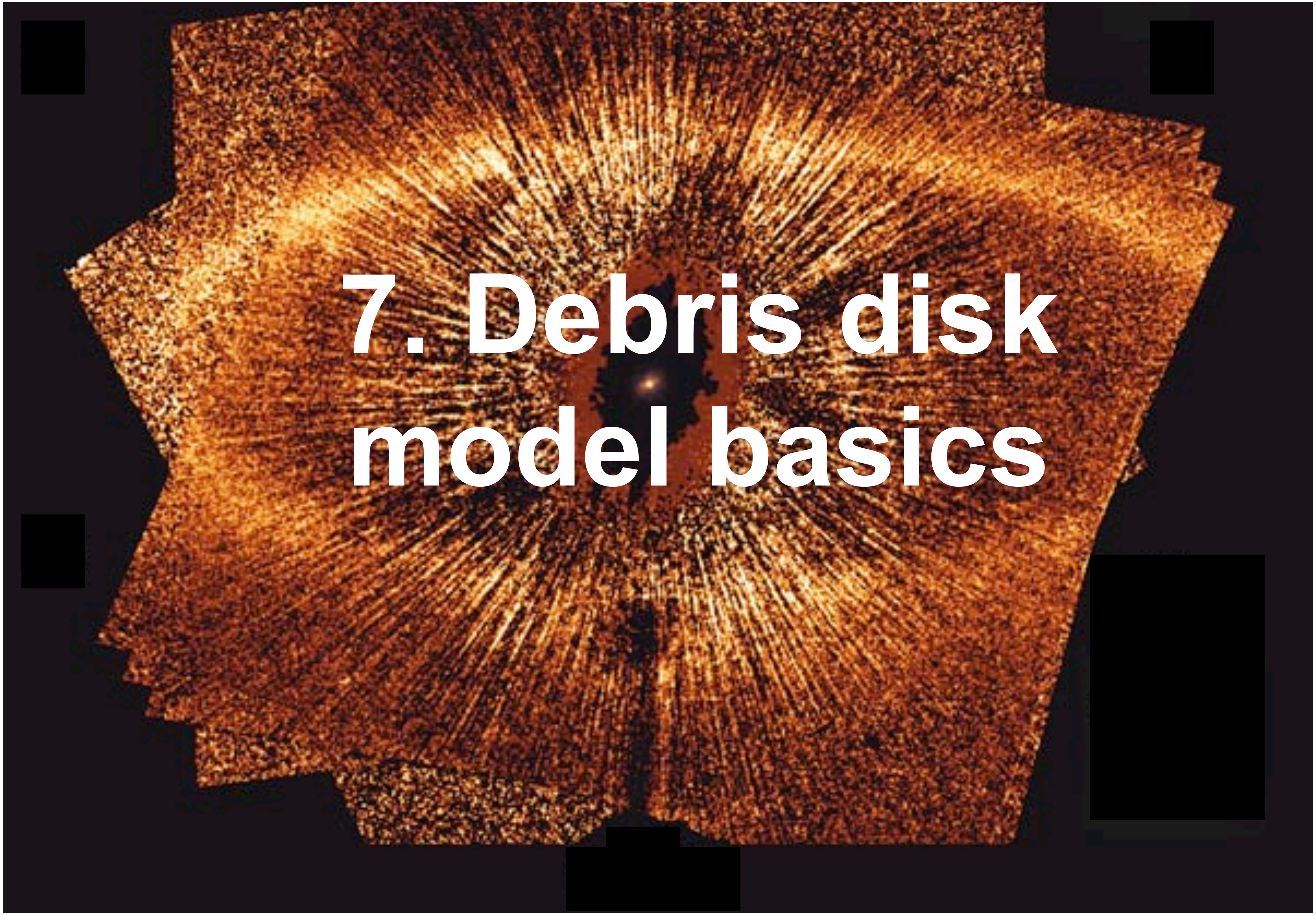
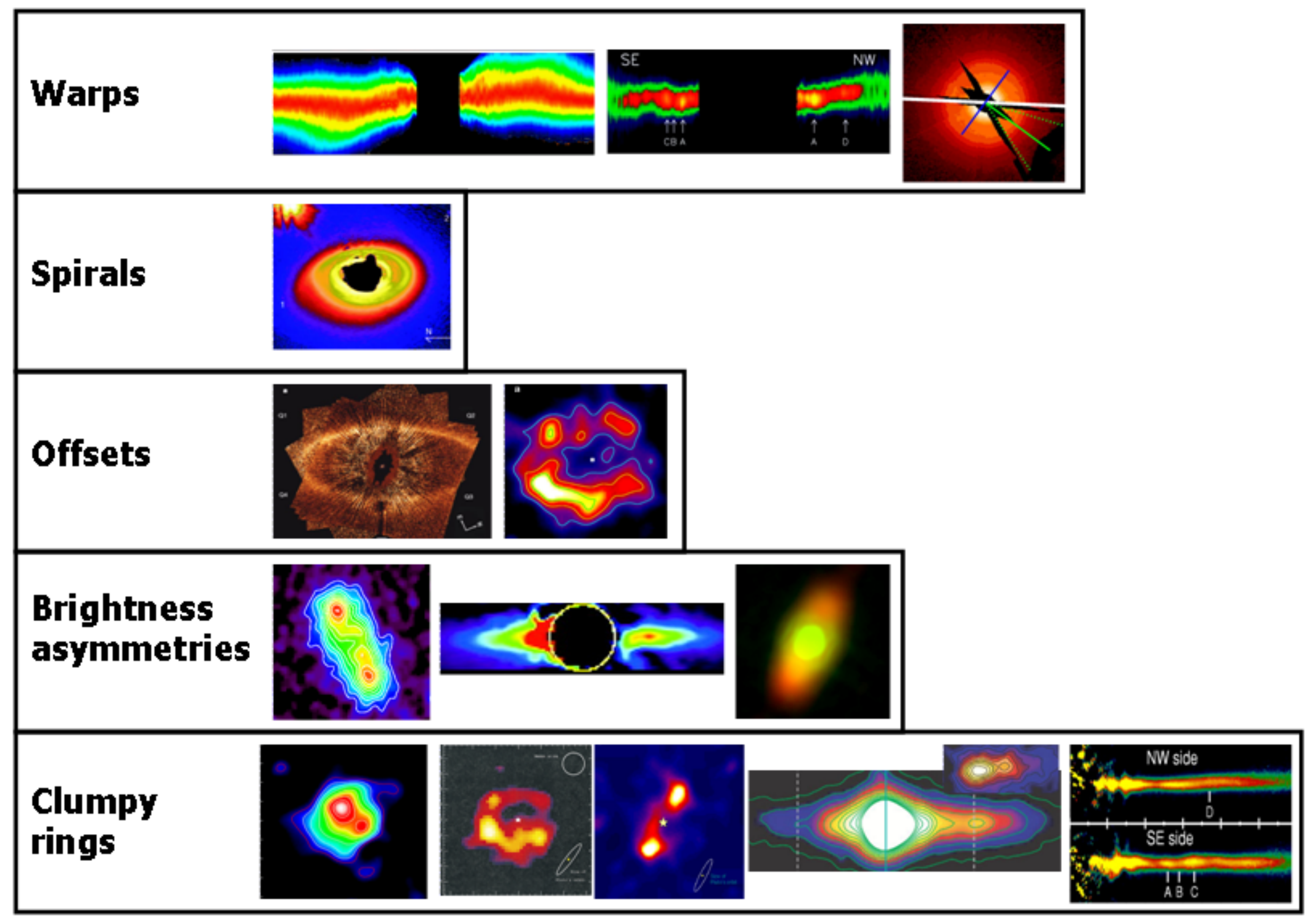
7. Debris disk model basics



Models have to explain...

(1) Radial structure (2) Asymmetric structure (3) Evolution



Planetesimal belt dynamical theory

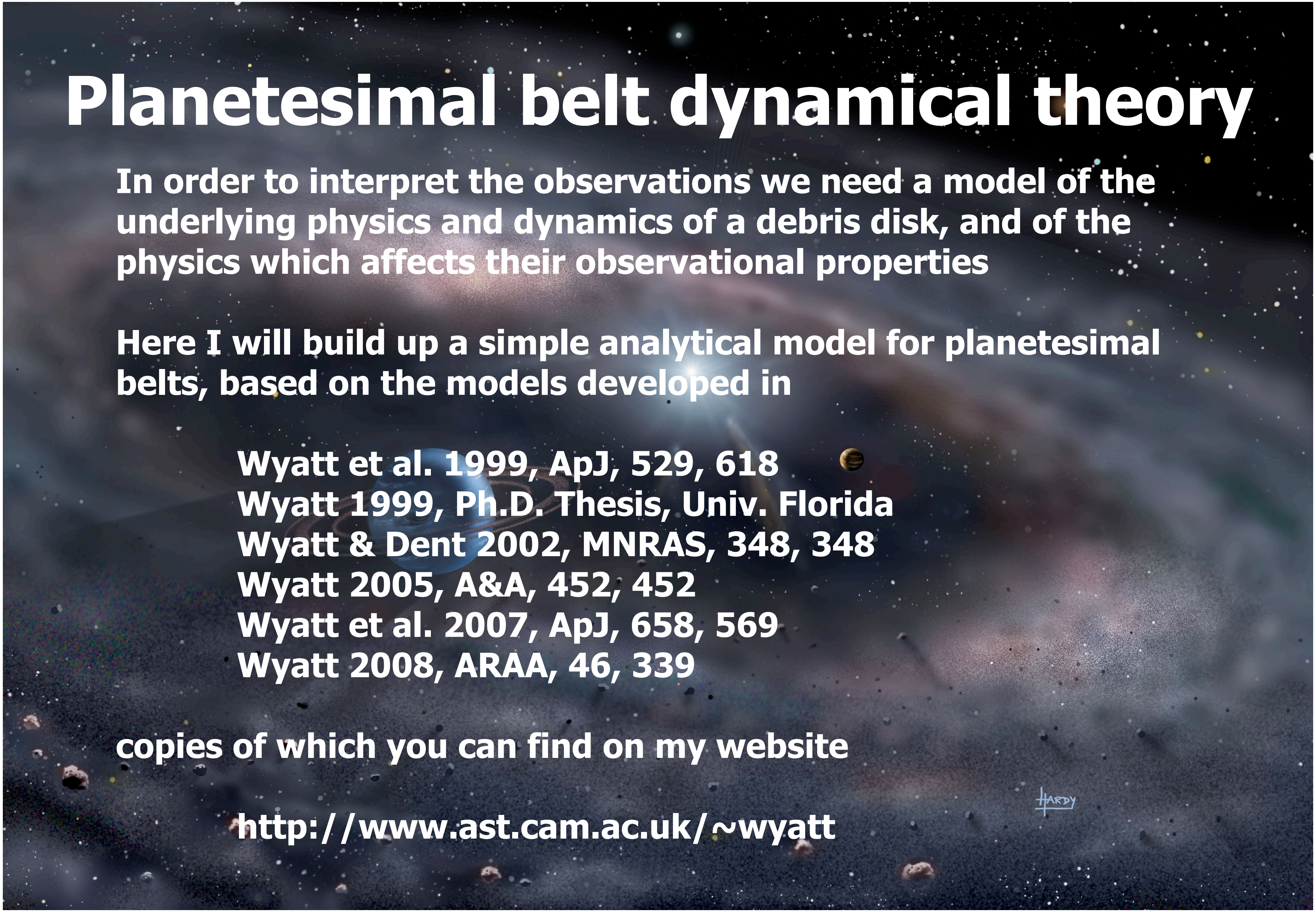
In order to interpret the observations we need a model of the underlying physics and dynamics of a debris disk, and of the physics which affects their observational properties

Here I will build up a simple analytical model for planetesimal belts, based on the models developed in

Wyatt et al. 1999, ApJ, 529, 618
Wyatt 1999, Ph.D. Thesis, Univ. Florida
Wyatt & Dent 2002, MNRAS, 348, 348
Wyatt 2005, A&A, 452, 452
Wyatt et al. 2007, ApJ, 658, 569
Wyatt 2008, ARAA, 46, 339

copies of which you can find on my website

http://www.ast.cam.ac.uk/~wyatt



The planetesimal belt

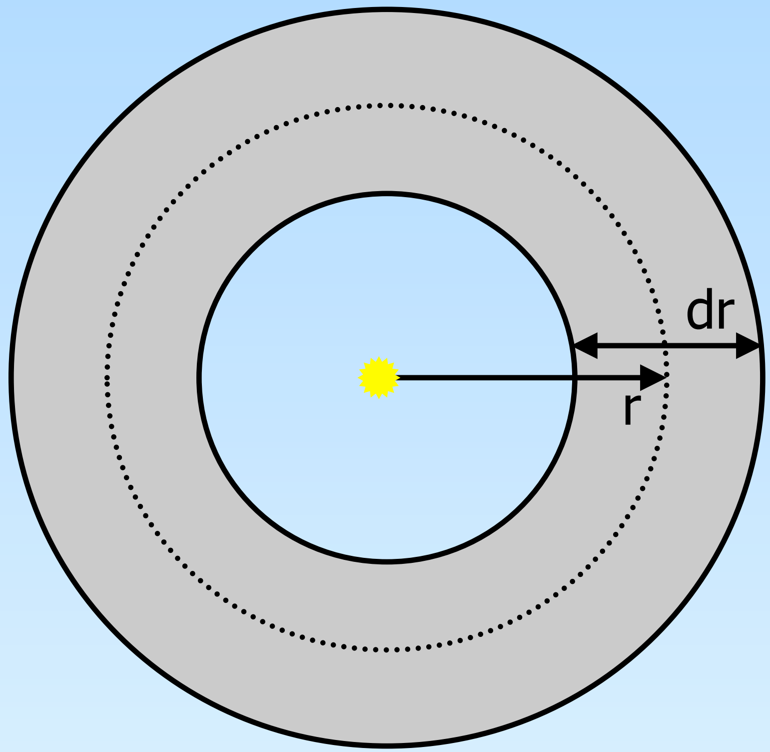
Consider planetesimals orbiting the star at a distance r in a belt of width dr

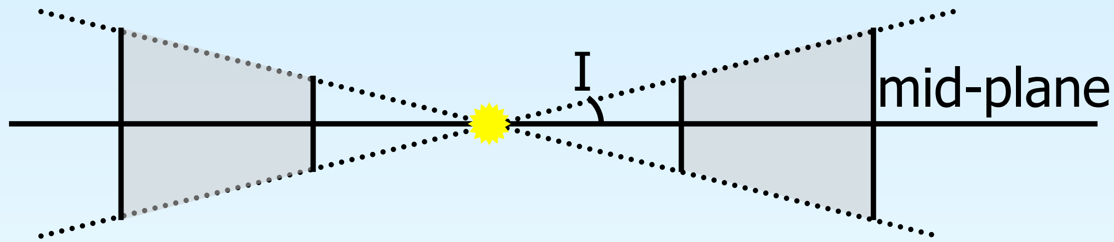
Face-on area of belt is:
$2\pi\ r\ dr$

Volume of belt is:
$4\pi\ r^2\ I\ dr$

Cross-sectional area of material in belt:
σ_{tot} in AU^2

Surface density of the belt:
$\tau_{eff} = \sigma_{tot} / (2\pi r\ dr)$, AU^2/AU^2





Gravity

• The dominant force on all planetesimals is gravitational attraction of star

• The force between two massive bodies, M_1 and M_2 is given by

$$F = GM_1M_2/r^2,$$

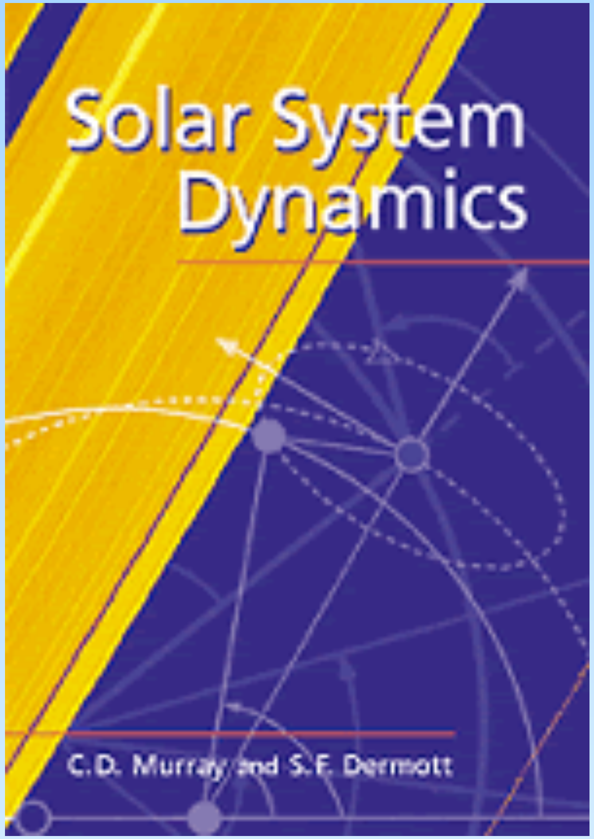
where $G=6.672\times10^{-11}$ Nm^2kg^{-2}

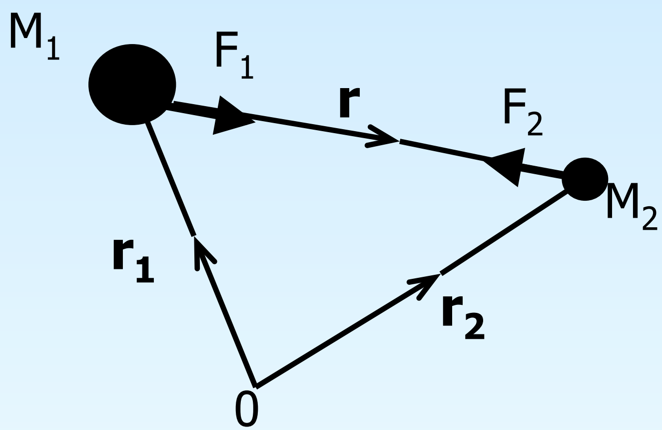
• Expressing in terms of vector offset of M_2 from M_1, **r** gives the equation of motion as

$$d^2\mathbf{r}/dt + \mu\mathbf{r}/r^3 = 0,$$

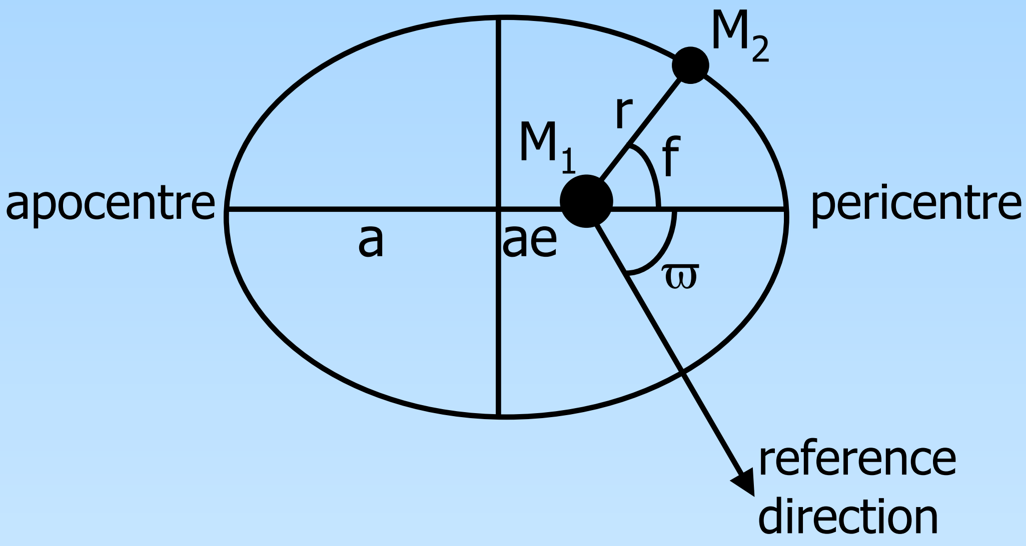
where $\mu=G(M_1+M_2)$

• Which can be solved to show that the orbit of M_2 about M_1 is given by an ellipse with M_1 at the focus (or, e.g., a parabola)





Orbits in 2D



- The orbit is given by:
  $r = a(1-e^2)/[1 + e\cos(f)]$,
  a=semimajor axis,
  e=eccentricity,
  f=true anomaly

- Angular momentum integral:
  $h = r^2 df/dt = [\mu a(1-e^2)]^{0.5} = \text{const}$
  Orbital period $t_{per} = 2\pi(a^3/\mu)^{0.5}$

- Energy integral:
  $0.5v^2 - \mu/r = \text{const} = C = -0.5\mu/a$
  $V_p = [(\mu/a)(1+e)/(1-e)]^{0.5}$
  $V_a = [(\mu/a)(1-e)/(1+e)]^{0.5}$

- Mean angles:
  Mean motion: $n = 2\pi/t_{per}$
  Mean anomaly: $M = n(t-\tau)$
  Mean longitude: $\lambda = M + \varpi$

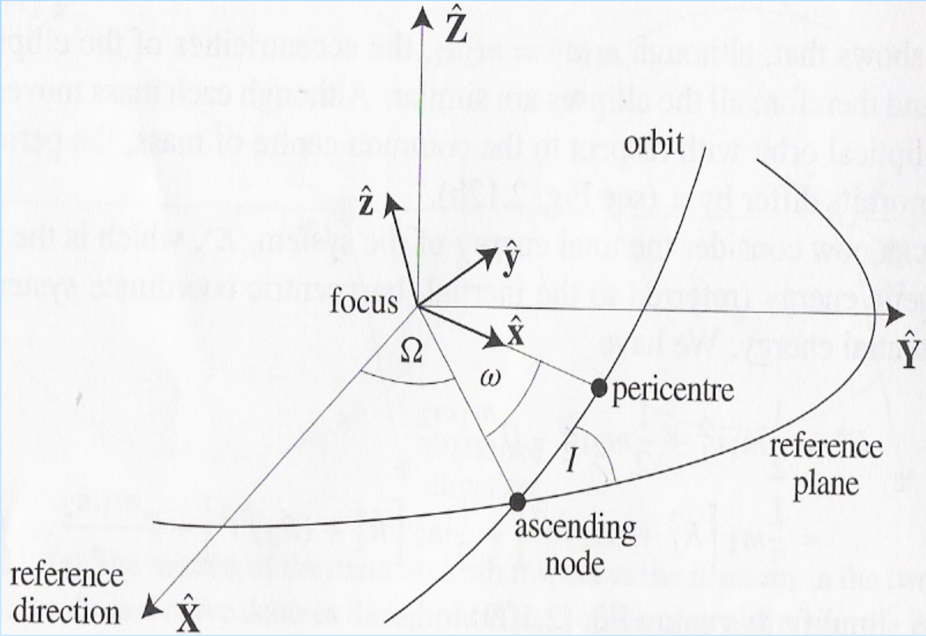
- Eccentric anomaly, E
  $\tan(E/2) = [(1-e)/(1+e)]^{0.5}\tan(f/2)$
  $M = E - e\sin(E)$

$t_{per}=(a^3/M_*)^{0.5}$ yrs and $v_k=30(M_*/a)^{0.5}$ km/s, where M_* is in M_{sun} and a in AU

Orbits in 3D

• In 3D just need to define the **orbital plane**, which is done with:
I = inclination
Ω = longitude of ascending node

• Also need to define the direction to pericentre:
ω = argument of pericentre
ϖ = longitude of pericentre = Ω+ω

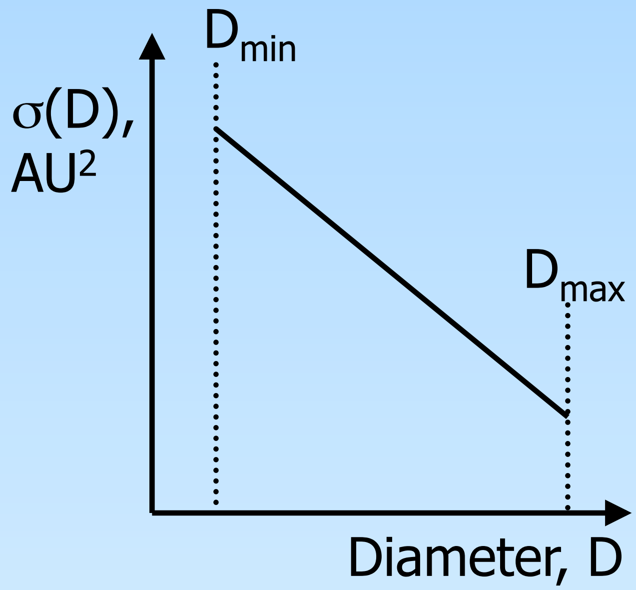


• So, the orbit is defined by five variables: **a, e, I, Ω** and **ϖ** (or ω)

• One time dependent variable describes location in orbit: λ (or f, M or E)

• Method for converting between [**X,Y,Z,V_x,V_y,V_z**] and [**a,e,I,Ω,ϖ,λ**] is given in Murray & Dermott (1999)

Size distributions



Planetesimals have a range of sizes

Define a size distribution such that n(D) dD is the number of planetesimals in size range D to D+dD

$n(D) = K D^{2-3q}$ between D_{min} and D_{max}

Assuming spherical particles so that $\sigma=\pi D^2/4$ gives

$\sigma_{tot} = [0.25K\pi/(5-3q)][D_{max}^{5-3q} - D_{min}^{5-3q}]$

$\sigma(D_1,D_2) = \sigma_{tot} [(D_1/D_{min})^{5-3q} - (D_2/D_{min})^{5-3q}]$

Similar relations for $m(D_1,D_2)$ (assuming $m=\pi D^3\rho/6$) and $n(D_1,D_2)$ meaning that number mass and area in the distribution is dominated by large or small particles depending on q

q	$n(D_1,D_2)$	$\sigma(D_1,D_2)$	$m(D_1,D_2)$
<1	large	large	large
1 to 5/3	small	large	large
5/3 to 2	small	small	large
>2	small	small	small

Collisional cascade

When two planetesimals collide (an **impactor** D_{im} and **target** D) the result is that the target is broken up into fragments with a range of sizes

If the outcome of collisions is self-similar (i.e., the size distribution of fragments is the same for the the same D_{im}/D regardless of whether D=1000km or 1μm), and the range of sizes infinite, then the resulting size distribution has an exponent (Dohnanyi et al. 1969; Tanaka et al. 1996)

$q = 11/6$

This is known as a **collisional cascade** because mass is flowing from large to small grains

Shattering and dispersal thresholds

The outcome of a collision depends on the **specific incident kinetic energy**
$Q = 0.5\,(D_{im}/D)^3\,v_{col}^2$

Shattering threshold, Q_S^*: energy for largest fragment after collision to have $(0.5)^{1/3}D$

- Impacts with $Q<Q_S^*$ result in cratering (ejection of material but planetesimal remains intact)
- Impacts with $Q>Q_S^*$ result in catastrophic destruction

Dispersal threshold, Q_D^*: energy for largest fragment after reaccumulation to have $(0.5)^{1/3}D$

Strength regime: $Q_D^* \approx Q_S^*$ for $D<150$m
Gravity regime: $Q_D^* > Q_S^*$ for $D>150$m

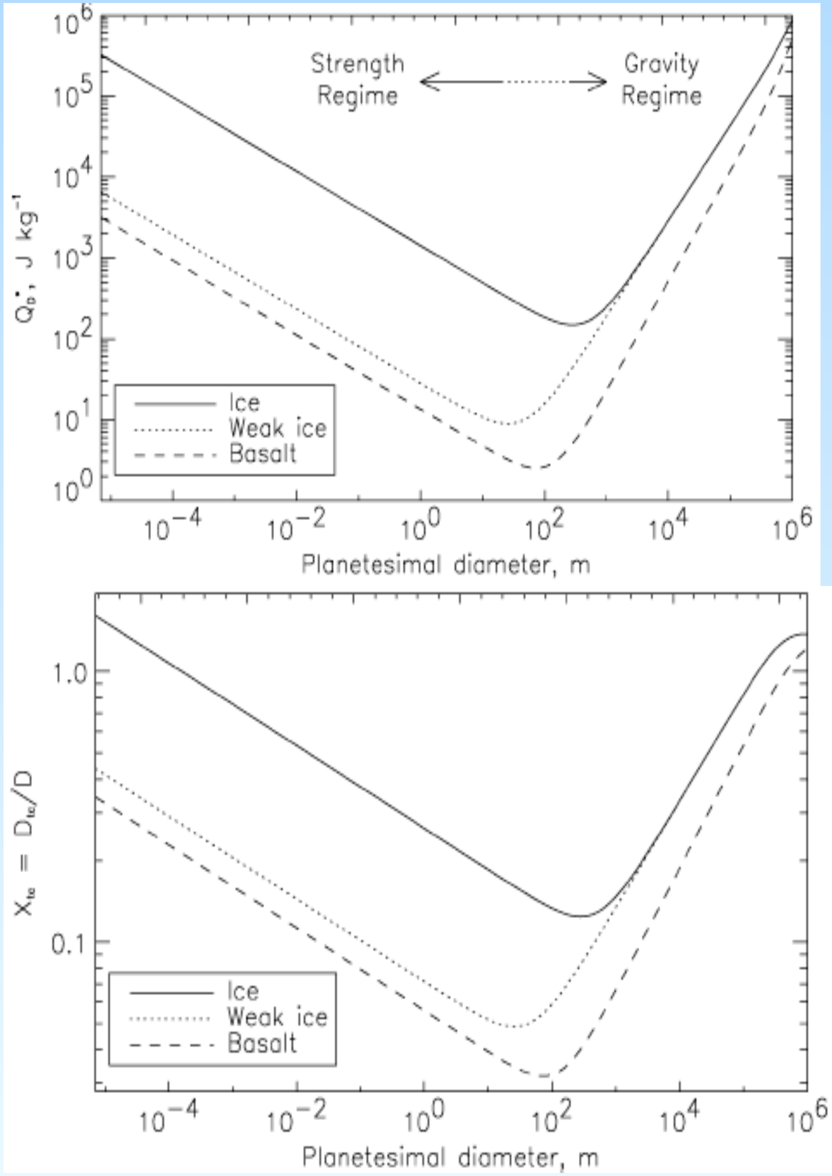
Catastrophic collisions

One study which generalises the outcome of collisions for a range of energies of interest in debris disks is the SPH simulations of Benz & Asphaug (1999)

They parametrised Q_D^* as a function of composition (basalt/ice) and for a range of v_{col} (two which can be interpolated between, or extrapolated)

For catastrophic collisions: $Q>Q_D^*$ so
$D_{im}/D > X_{tc} = (2Q_D^*/v_{col}^2)^{1/3}$

For collisions at v_{col}=1 km/s this means
X_{tc}=0.01 to 1

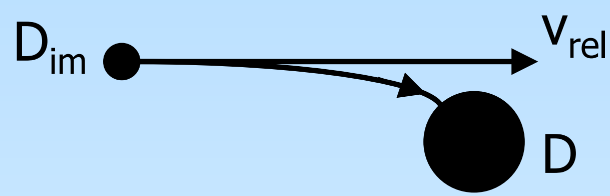


Catastrophic collision rate

The rate of impacts onto a planetesimal of size D from those in the size range D_{im} to $D_{im} + dD_{im}$ is $R_{col}(D,D_{im})dD_{im}$ (Opik 1950) where

$R_{col}(D,D_{im}) = f(D,D_{im})\, \sigma(r,\theta,\phi)\, v_{rel}$

where



$v_{rel} = f(e,I)v_k$ [NB $f(e,I) = (1.25e^2 + I^2)^{0.5}$]

$\sigma(r,\theta,\phi) = \sigma_{tot}/(4\pi r^2 dr I)$

$f(D,D_{im})dD_{im} = [\sigma(D_{im})/\sigma_{tot}][1+(D/D_{im})]^2[1+(v_{esc}(D,D_{im})/v_{rel})^2]$

is the fraction of σ_{tot} that the planetesimal sees

$v_{esc}^2(D,D_{im}) = (2/3)\pi G\rho[D^3+D_{im}^3]/(D_{im}+D)$

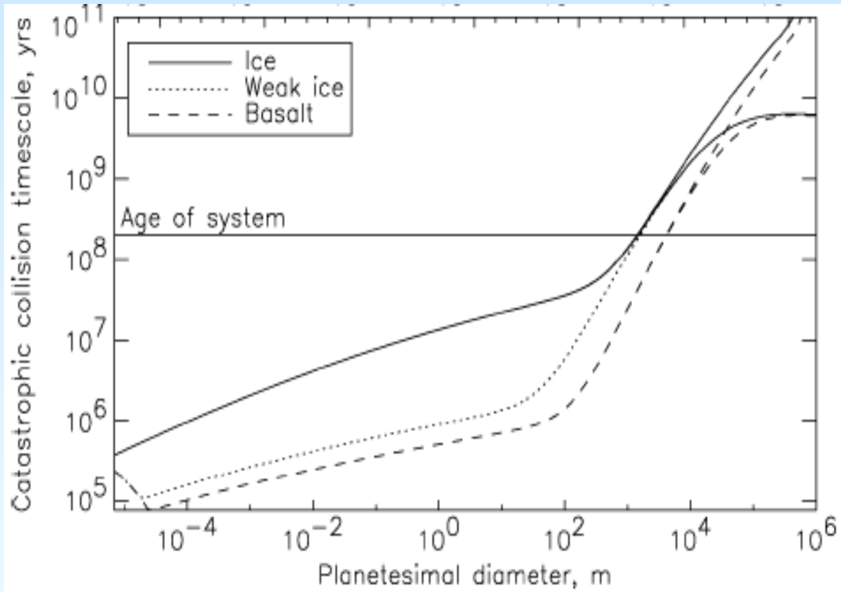
is escape velocity

Mean time between catastrophic collisions

$t_{cc}(D) = t_{per}(r\, dr/\sigma_{tot})[2I/f(e,I)]/f_{cc}(D)$

where

$f_{cc}(D) = \int_{D_{tc}(D)}^{D_{max}} f(D,D_{im})\, dD_{im}$



Simplified collision times

For a disk of same sized particles, $f_{cc}(D) = 4$:

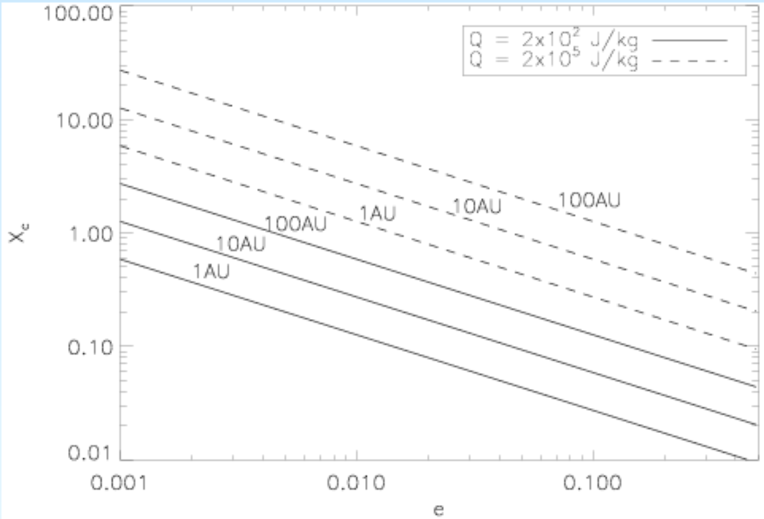
$t_{cc} = t_{per} / [4\pi\tau_{eff} [1+1.25(e/I)^2]^{0.5}] \approx t_{per} / 4\pi\,\tau_{eff}$

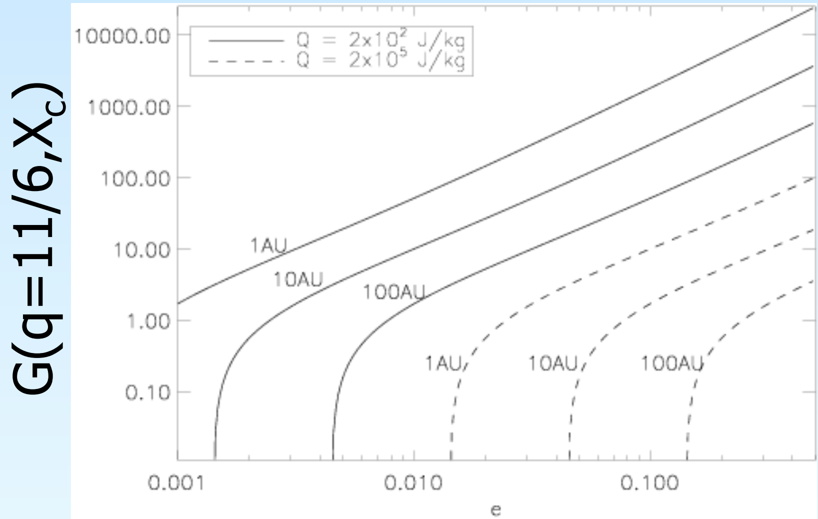
If gravitational focussing can be ignored, then $f_{cc}(D)$ can be solved:

$f_{cc}(D) = (D_{min}/D)^{3q-5}\ G(q,X_c)$

$G(q,X_c) = [(X_c^{5-3q}-1)+(6q-10)(3q-4)^{-1}(X_c^{4-3q}-1)+(3q-5)(3q-3)^{-1}(X_c^{3-3q}-1)]$

$t_{cc}(D) = (D/D_{min})^{3q-5}\ t_{per} / [G(q,X_c)\pi\tau_{eff} [1+1.25(e/I)^2]^{0.5}]$





Actual outcome

Collisions do not either destroy a planetesimal or not

The largest fragment in a collision, $f_{lr} = M_{lr}/M$ is given by

$Q < Q_D^*$ $\quad f_{lr} = 1 - 0.5\,(Q/Q_D^*)$

$Q > Q_D^*$ $\quad f_{lr} = 0.5(Q_D^*/Q)^{1.24}$

The size distribution of the fragments can then be constrained by considering that the total mass of remaining fragments = $M-M_{lr}$

For example, experiments show the fragments to have a size distribution with an exponent

$q_c \approx 1.93$

(although results get 1.83-2.17, and there may be a knee in the size distribution at 1mm)

This means that the second largest fragment must have size:

$D_2/D = [(1-f_{lr})(2/q_c-1)]^{1/3}$

We now know the outcome and frequency of all collisions in a planetesimal disk

Real cascade size distribution

The size distribution is not that of an infinite collisional cascade:

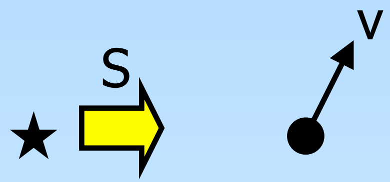
- The largest planetesimals are only so big, D_{max}, so mass is lost from the cascade

- The cascade is not self-similar, since X_{tc} is a function of D

- The smallest dust is removed faster than it is produced in collisions and so its number falls below the q=11/6 value

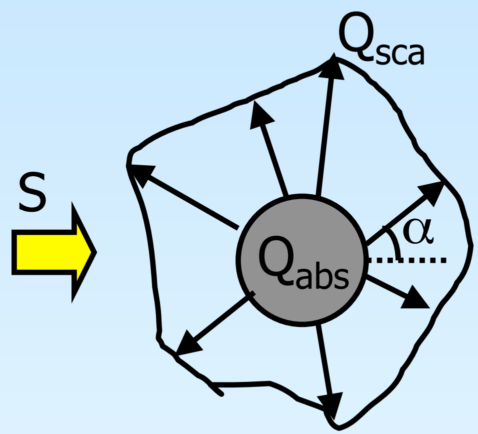
Radiation forces

• Small grains are affected by their interaction with stellar radiation field (Burns et al. 1979)



• This is caused by the fact that grains remove energy from the radiation field by absorption and scattering, and then re-radiate that energy in the frame moving with the particle's velocity:

$$F_{rad} = (SA/c)\, Q_{pr}\, [\, [1 - 2(dr/dt)/c]\mathbf{r} - r(d\theta/dt)\boldsymbol{\theta}\,]$$
$$= \text{radiation pressure } (\mathbf{r}) + \text{Poynting-Robertson drag } (\boldsymbol{\theta})$$



$$Q_{pr} = Q_{abs} + Q_{sca}[1 - \langle \cos(\alpha) \rangle]$$

• The drag forces are defined by the parameter β which is a function of particle size (D):

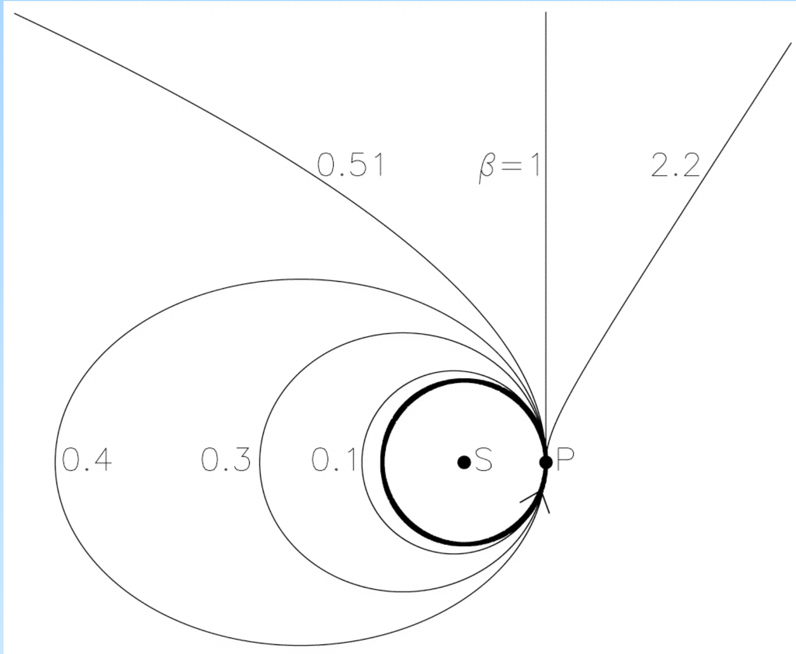
$$\beta = F_{rad}/F_{grav} = C_r(\sigma/m)\langle Q_{pr}\rangle_{T*}(L_*/L_{sun})(M_{sun}/M_*),$$

where $C_r = 7.65\text{x}10^{-4}$ kg/m^2

• For large spherical particles:

$$\beta = (1150/\rho D)(L_*/L_{sun})(M_{sun}/M_*)$$

Radiation pressure



• The radial component is called radiation pressure, and essentially causes a particle to "see" a smaller mass star by a factor $(1-\beta)$, so that particles with $\beta>1$ are not bound and leave the system on hyperbolic trajectories

• This means that a small particle orbiting at "a" has a different orbital period to that of larger objects: $t_{per} = [a^3/M_*(1-\beta)]^{0.5}$ which also moves the locations of resonances etc

Most important consequence is the change in orbital elements for particles released from a large object (can be derived from the 2D orbits from position and velocity at P the same):

$$a_{new}=a(1-\beta)[1-2\beta[1+e\cos(f)][1-e^2]^{-1}]^{-1}$$
$$e_{new}=[e^2+2\beta e\cos(f)+\beta^2]^{0.5}/(1-\beta)$$
$$\varpi_{new}-\varpi=f-f_{new}=\arctan[\beta\sin(f)[\beta\cos(f)+e]^{-1}]$$

which means particles are unbound if $\beta>0.5$

Poynting-Robertson drag

• Poynting-Robertson drag causes dust grains to spiral into the star while at the same time circularising their orbits ($dI_{pr}/dt=d\Omega_{pr}/dt=0$):

$$da_{pr}/dt = -(\alpha/a)(2+3e^2)(1-e^2)^{-1.5} \approx -2\alpha/a$$
$$de_{pr}/dt = -2.5(\alpha/a^2)e(1-e^2)^{-0.5} \approx -2.5e\alpha/a^2$$
where $\alpha = 6.24\times10^{-4}(M_*/M_{sun})\beta$ AU^2/yr

• So time for a particle to migrate in from a_1 to a_2 is

$$t_{pr} = 400(M_{sun}/M_*)[a_1^2 - a_2^2]/\beta \quad \text{years}$$

• On their way in particles can become trapped in resonance with interior planets, or be scattered, or accreted, or pass through secular resonances…

• Large particles move slower, and so suffer no migration before being destroyed in a collision with another large particle ($t_{pr}\propto D$ whereas $t_{cc}\propto D^{0.5}$), with the transition for which P-R drag is important

$$\beta_{pr} = 5000\tau_{eff}(r/M_*)^{0.5}$$

Collisions vs P-R drag

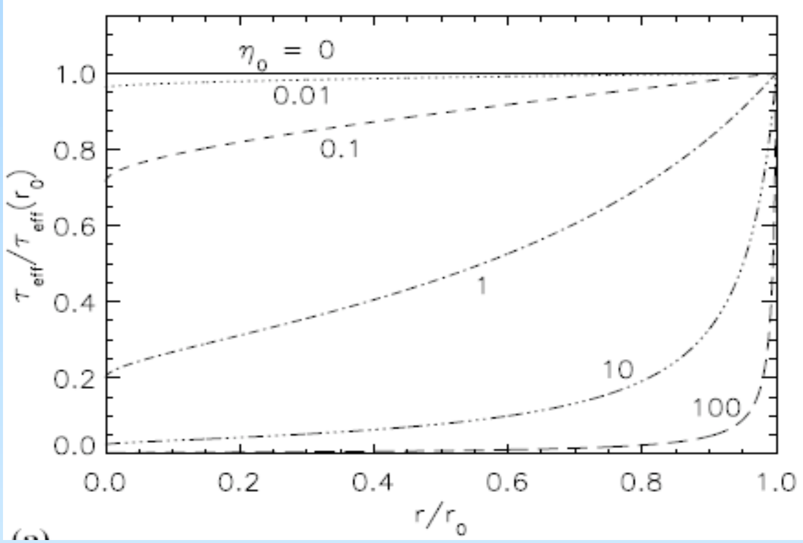
Consider a belt of planetesimals at r_0 which is producing dust of just one size

That dust population then evolves due to
  collisions: $t_{col} = t_{per} / 4\pi\tau_{eff}$
  P-R drag: $dr_{pr}/dt = -2\alpha/r$

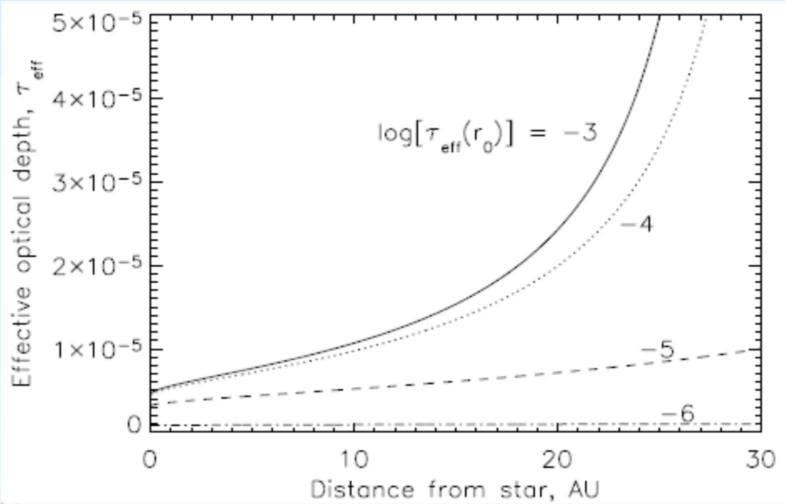
The continuity equation is:
  $d[n(r)dr_{pr}/dt]/dr = -n(r)/t_{col}$
which can be expanded to:
  $dn/dr - n/r = Kn^2r^{-1.5}$
and solved using Bernoulli's equation:
  $\tau_{eff}(r) = \tau_{eff}(r_0)\,[\,1+ 4\eta_0(1-(r/r_0)^{0.5})\,]^{-1}$
where $\eta_0 = 5000\tau_{eff}(r_0)[r_0/M_*]^{0.5}/\beta = t_{pr}/t_{col}$

Note that the same equation implies that particles evolving due to P-R drag have a size distribution $n(D) \propto n_s(D)D$

If $\eta_0 >> 1$ then dust remains confined to the planetesimal belt



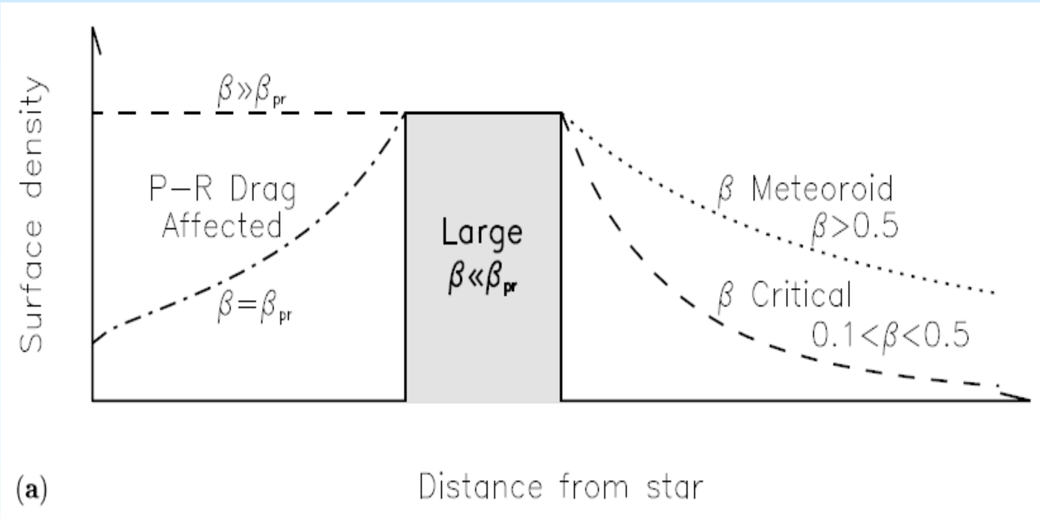
Regardless of $\tau_{eff}(r_0)$, the maximum optical depth at r=0 is $5\text{x}10^{-5}\beta[M_*/r_0]^{0.5}$



Disk particle categories

This motivates a division of disk into particle categories depending on size:

- $\beta << \beta_{pr}$ **(large):** planetesimals confined to belt
- $\beta \approx \beta_{pr}$ **(P-R drag affected):** depleted by collisions before reaching star
- $\beta_{pr} < \beta < 0.5$ **(P-R drag affected):** largely unaffected by collisions (evaporate at star)
- $0.1<\beta<0.5$ (**β critical)**: bound orbits, but extending to larger distances than planetesimals
- $\beta>0.5$ (**β meteoroid)**: blown out on hyperbolic orbits as soon as created



Which categories exist in a disk depends on the disk density

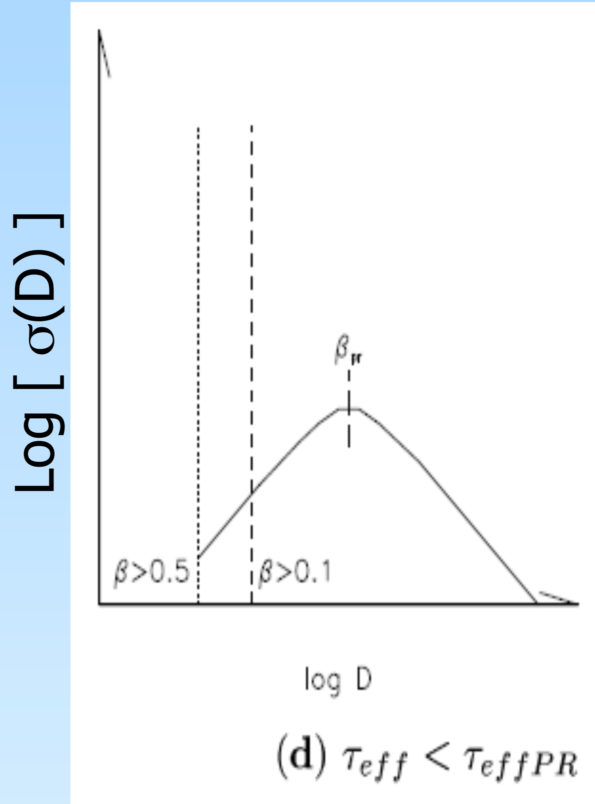
P-R drag dominated disks

A significant P-R drag affected grain population is only expected in tenuous disks for which

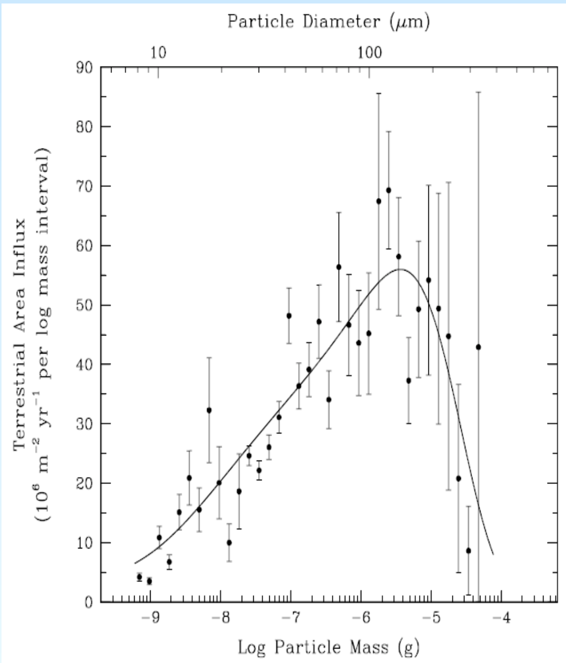
$\tau_{eff} < \tau_{effPR} = 10^{-4}\,[M_*/r]^{0.5}$

since then $\beta_{pr}<0.5$

Such disks have a size distribution with area dominated by grains $\sim\beta_{pr}$ in size



(d) $\tau_{eff} < \tau_{effPR}$



The asteroid belt and zodiacal cloud are examples of this regime, since $\tau_{eff} \approx 10^{-7}$ meaning the material in the asteroid belt should be concentrated in particles D_{pr}~500μm with smaller particles dominating closer to the Sun (100-200μm dominate accretion by Earth, (Love & Brownlee 1993)

Collision dominated disks

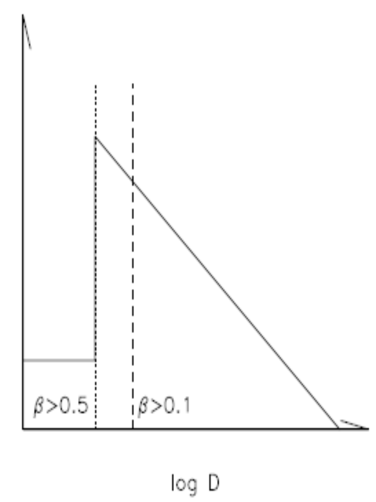
The majority of debris disks have

$\tau_{effPR} < \tau_{eff} < 0.1$

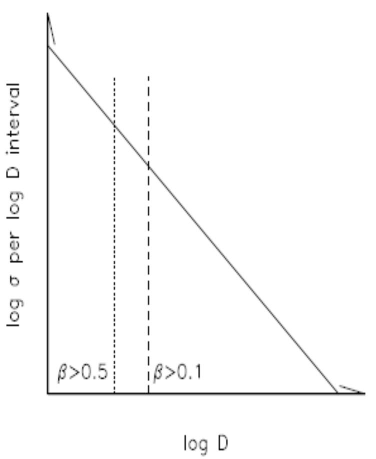
meaning that P-R drag is insignificant, but that grains getting blown out by radiation pressure are not created quickly enough for them to contribute much to σ_{tot}

Such disks have a size distribution with area dominated by grains β~0.1-0.5 in size and so may have a large β critical component

Since grains with β>0.5 are removed on orbital timescales (e.g., consider that when β=1 velocity is constant so one orbital time moves grains from r to 6.4r), they become important when $t_{col} < t_{per}$ and so $\tau_{eff} > 0.1$ (note that such disks are becoming optically thick)



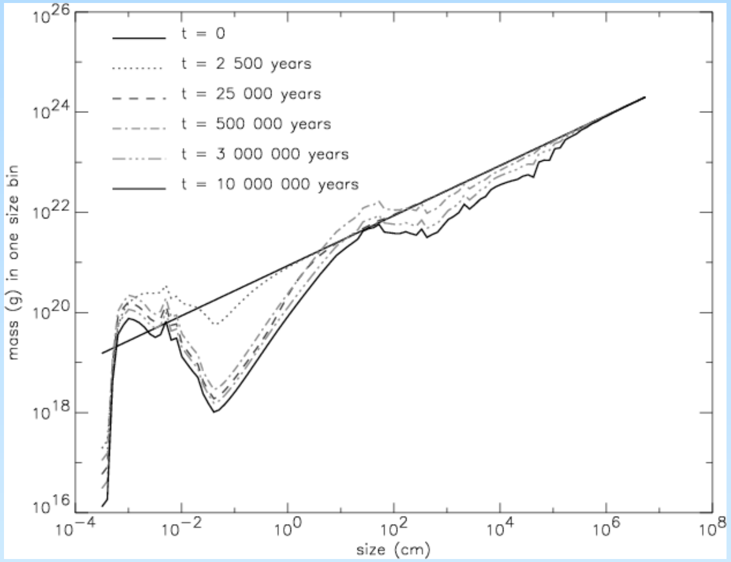
(c) $\tau_{effPR} < \tau_{eff} < 0.1$



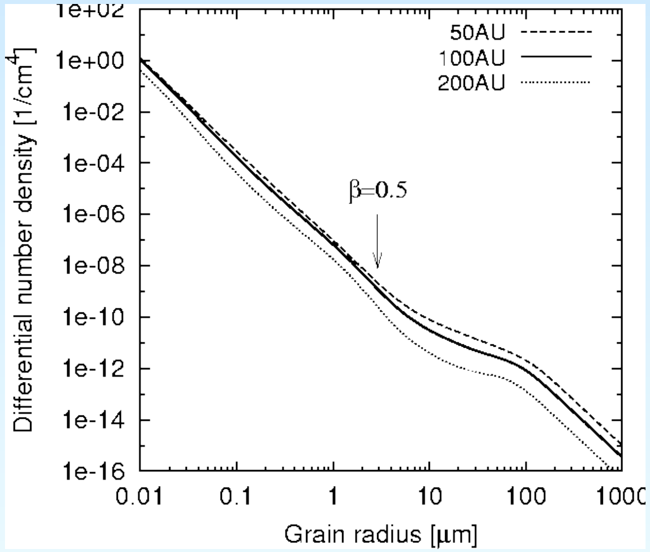
(b) $\tau_{eff} > 0.1$

Wavy size distribution: bottom end

We expect the size distribution to differ from q=11/6 for small sizes because of their removal by radiation forces



• a sharp cut-off causes a wave, since β critical grains should be destroyed by β>0.5 grains (Thebault, Augereau & Beust 2003) [the period of the wave is indicative of X_{tc}]



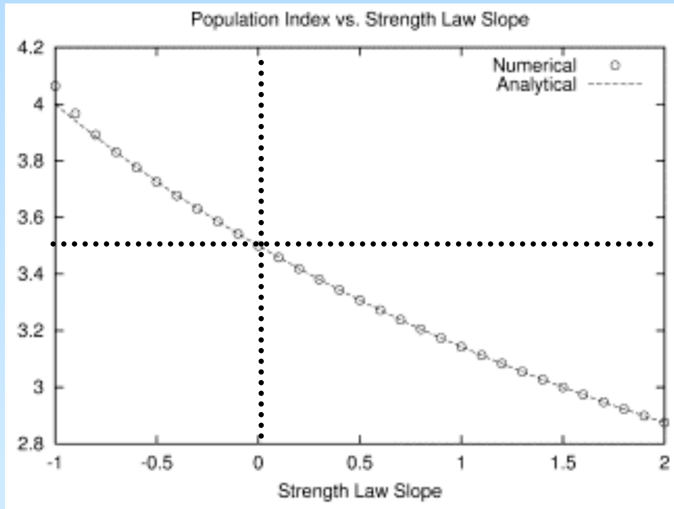
• if a large number of blow-out grains do exist, however, their large velocities can significantly erode the β critical population (Krivov, Mann & Krivova 2000)

Wavy size distributions: middle/top

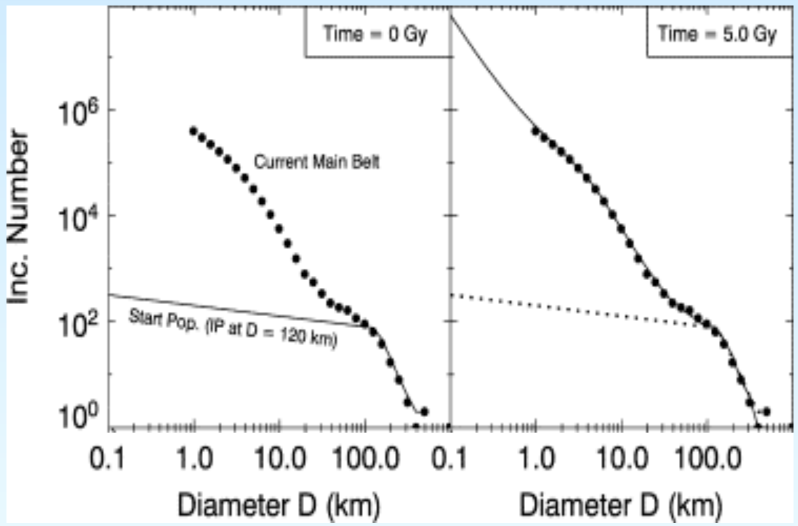
The transition from strength to gravity scaling also causes a wave in the size distribution

- If $Q_D^* \propto D^s$ then equilibrium size distribution has (O'Brien & Richardson 2003):
    - $q>11/6$ if $s<0$ (strength regime)
    - $q<11/6$ if $s>0$ (gravity regime)

p=3q-2



- The transition between the two size distributions causes a wave in the distribution (Durda et al. 1998), and asteroid belt size distribution well fitted thus constraining Q_D^* vs D and concluding that D>120km are primordial (Bottke et al. 2005)



Simple evolution model

The cut-off in the size distribution at D_{max} means no mass input at the top end of the cascade resulting in a net decrease of mass with time:

$$dM_{tot}/dt = - M_{tot}/t_{col}$$

where M_{tot} is dominated by grains of size D_{max} which, assuming a size distribution described by q, have a lifetime of

$$t_{col} = r^{1.5}\, M_*^{-0.5}\, (\rho r dr D_{max}/M_{tot})\, (12q-20)(18-9q)^{-1}[1+1.25(e/I)^2]^{-0.5}/G(q,X_c)$$

This can be solved to give:

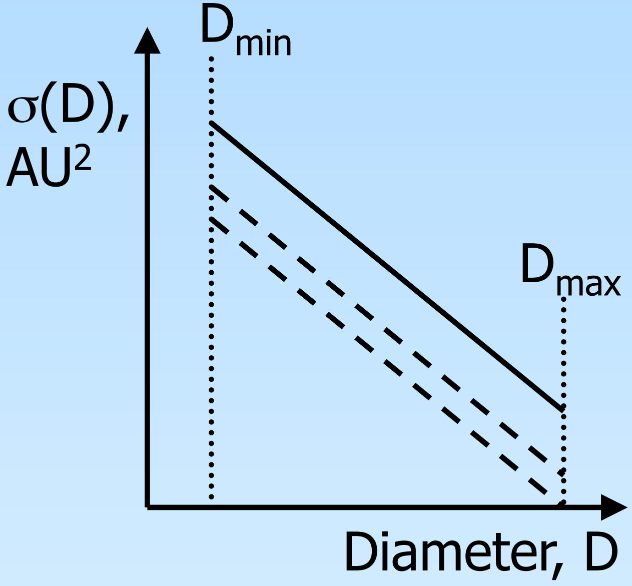
$$M_{tot}(t) = M_{tot}(0)\, [\ 1 + t/t_{col}(0)\ ]^{-1}$$

In other words, mass is constant until a significant fraction of the planetesimals of size D_{max} have been catastrophically destroyed at which point it falls of $\propto 1/t$

Dust evolution: collision dominated

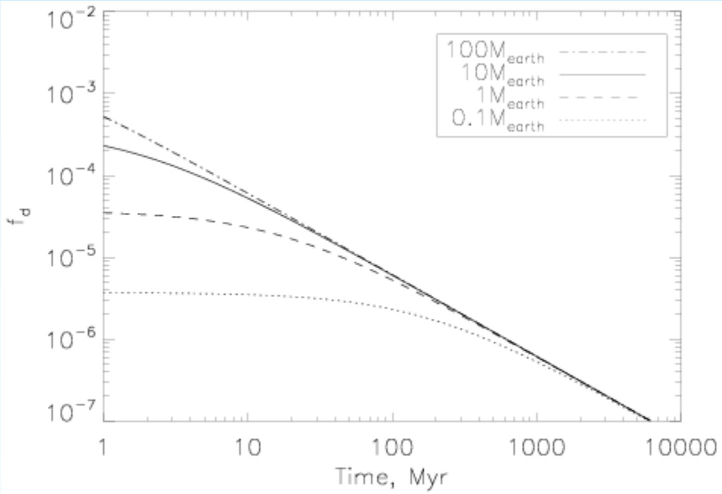
Forgetting waviness, the size distribution of dust in a collisionally dominated disk is proportional to the number of large planetesimals and so is the area of the dust (which is what is seen):

$$\sigma_{tot} = M_{tot}\,(18\text{-}9q)(6q\text{-}10)^{-1}\rho^{-1}(D_{min}/D_{max})^{3q-6}D_{min}^{-1}$$



A common way of expressing this observationally is the fractional luminosity of the dust, which if you assume the black body grains:

$$f = L_{ir}/L_* = \sigma_{tot} / 4\pi r^2$$

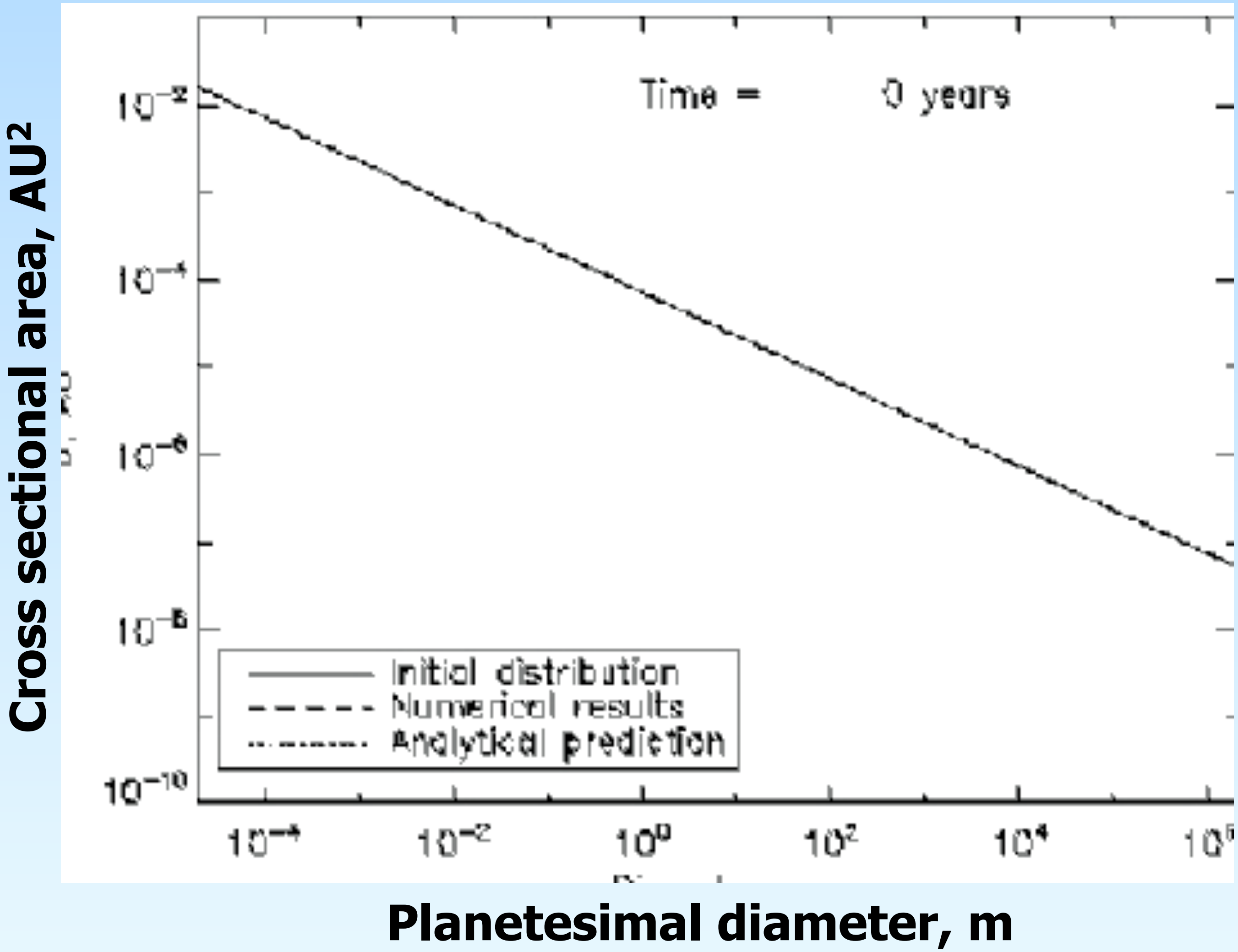


The mass (or f) of a disk at late times is independent of the initial disk mass; i.e., there is a maximum possible disk luminosity at a given age:

$$f_{max} = r^{1.5}M_*^{-0.5}(dr/4\pi r t_{age})(D_{min}/D_{max})^{5-3q} * 2[1+1.25(e/I)^2]^{-0.5}/G(q,X_c)$$

Evolution of size distribution

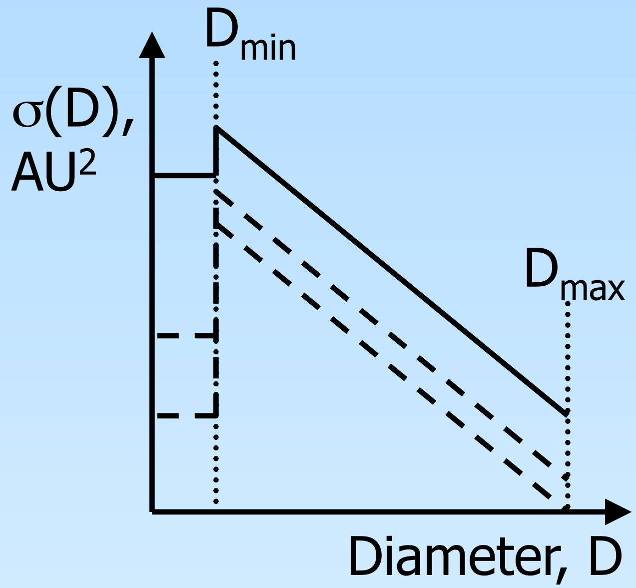
The evolution of the size distribution can be followed using schemes where, in each timestep, mass is lost from each size bin due to destructive collisions with other planetesimals, and mass is gained due to the fragmentation of larger particles



Dust evolution: blow-out grains

The exact number of grains below the radiation pressure blow-out limit depends on how many are created in different collisions:

- planetesimals with dusty regoliths may release large quantities in collisions
- tiny grains may condense after massive collision
- small grains have higher velocities and so preferentially escape in gravity regime

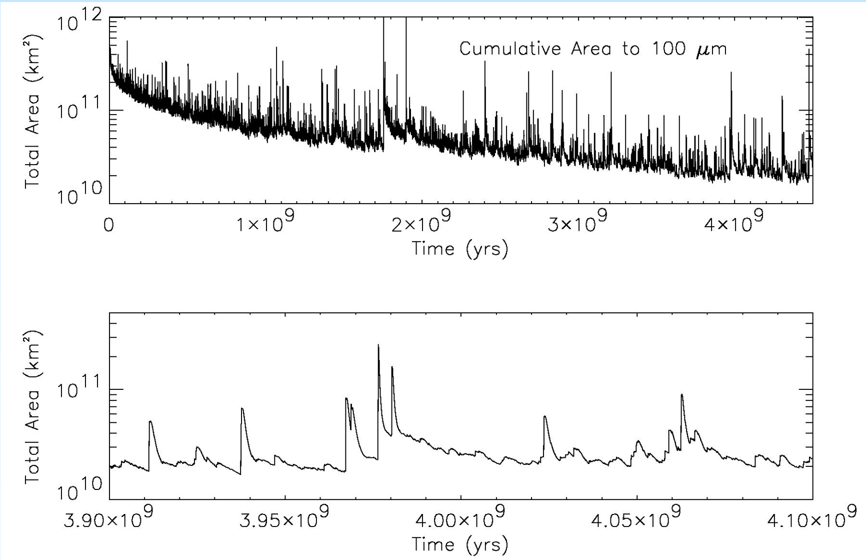


Regardless, since their production rate is $\propto M_{tot}^2$ and their loss rate is constant, their number will fall $\propto M_{tot}^{-2}$ and so $\propto t^{-2}$

Massive collisions

The collision rate, R_{col}, gives a mean time between collisions, t_{col}, which the steady state model can be used to work out the number of collisions that occur between objects of size D to D+dD and D_{im} and $D_{im}+dD_{im}$

However, the actual number of collisions in the timestep is a random number and should be chosen by Poisson statistics (Durda et al. 1997)



Important for collisions between largest objects which happen infrequently, but have large impact on disk

Models show asteroid belt evolution is punctuated by increases in dust when large asteroids are destroyed

This is not usually the case for debris disks for which single asteroid collisions do not produce a detectable signature

Steady state vs stochastic evolution

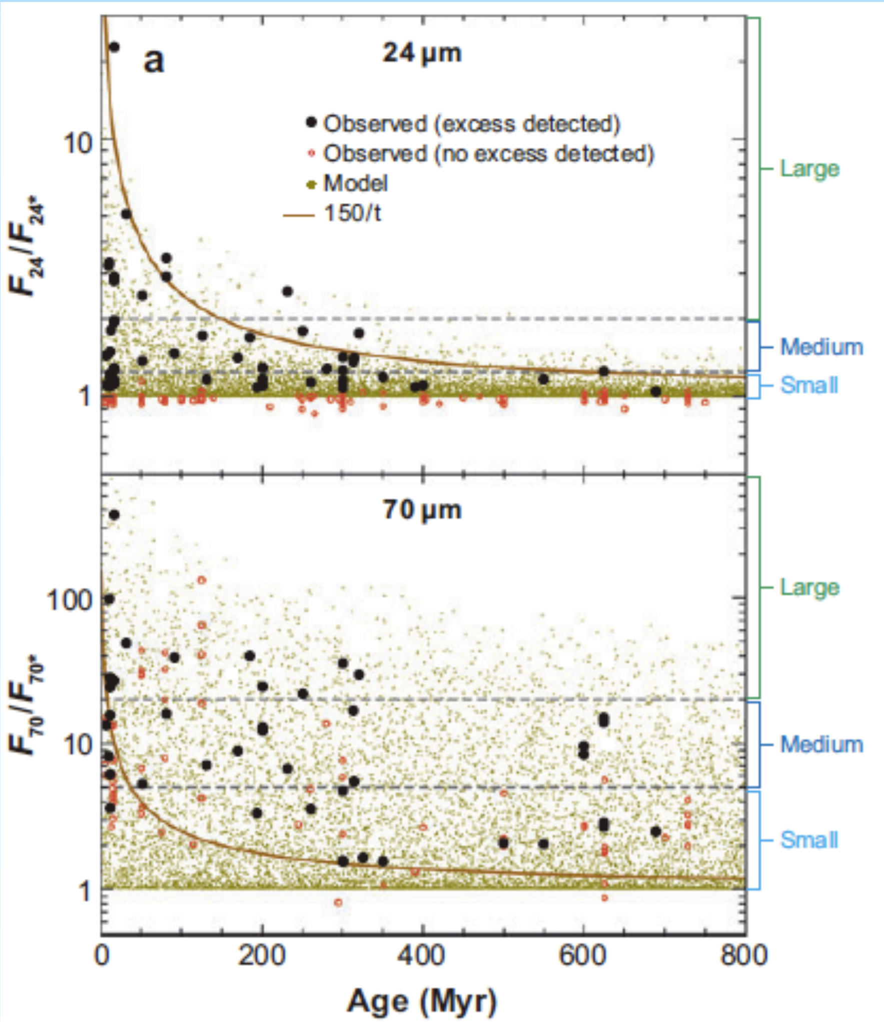
That is the steady state model for planetesimal belt evolution, and explains the observed $\propto t^{-1}$ evolution

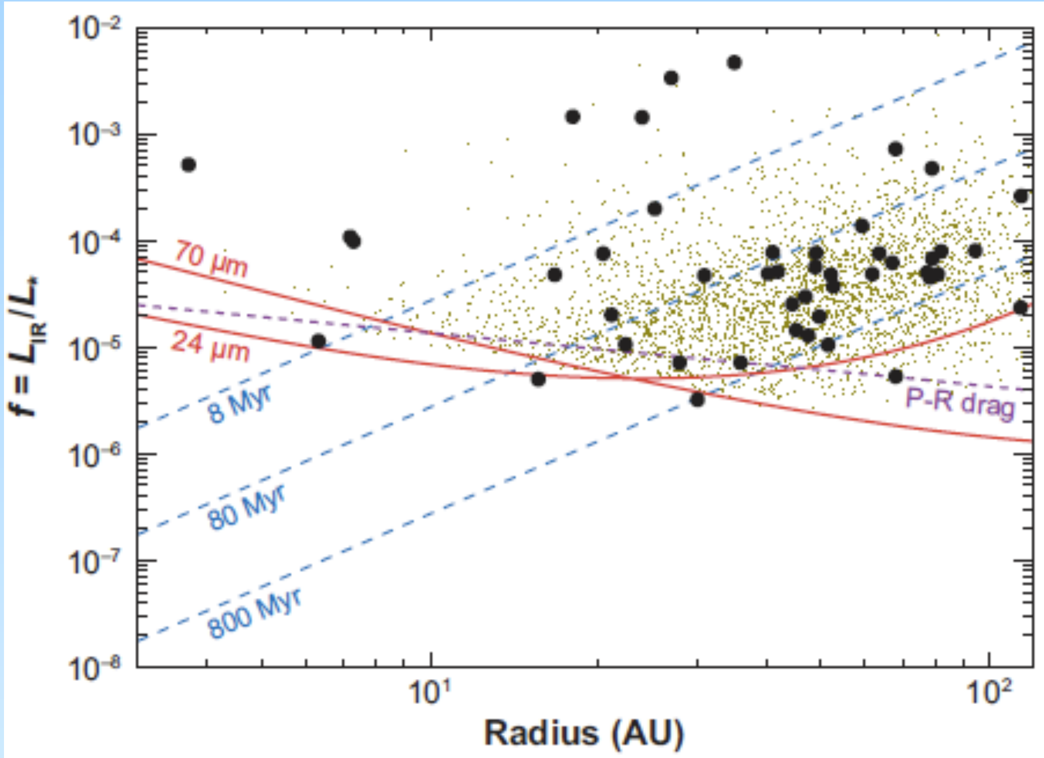
Several mechanisms have been proposed to cause non-equilibrium evolution, including:

- close passage of nearby star (Kenyon & Bromley 2002)
- formation of Pluto-sized object in the disk (Kenyon & Bromley 2004)
- passge through dense patch of ISM (Arytmowicz & Lubow 1997)
- dynamical instability in the disk (e.g., LHB type event; Gomes et al. 2005)
- sublimation of supercomet (Beichman et al. 2005)
- massive collision between two asteroids (Wyatt & Dent 2002)

All of these models can be interpreted in terms of the steady state model: a collisional cascade is rapidly set up in the system and the same physics applies

Population synthesis





To explain the observed distributions of excesses, assume stars all have a planetesimal belt, chosen from a distribution of initial masses and from a distribution of radii, then choose their spectral types and ages from appropriate ranges to get the observed populations
(Wyatt et al. 2007b)

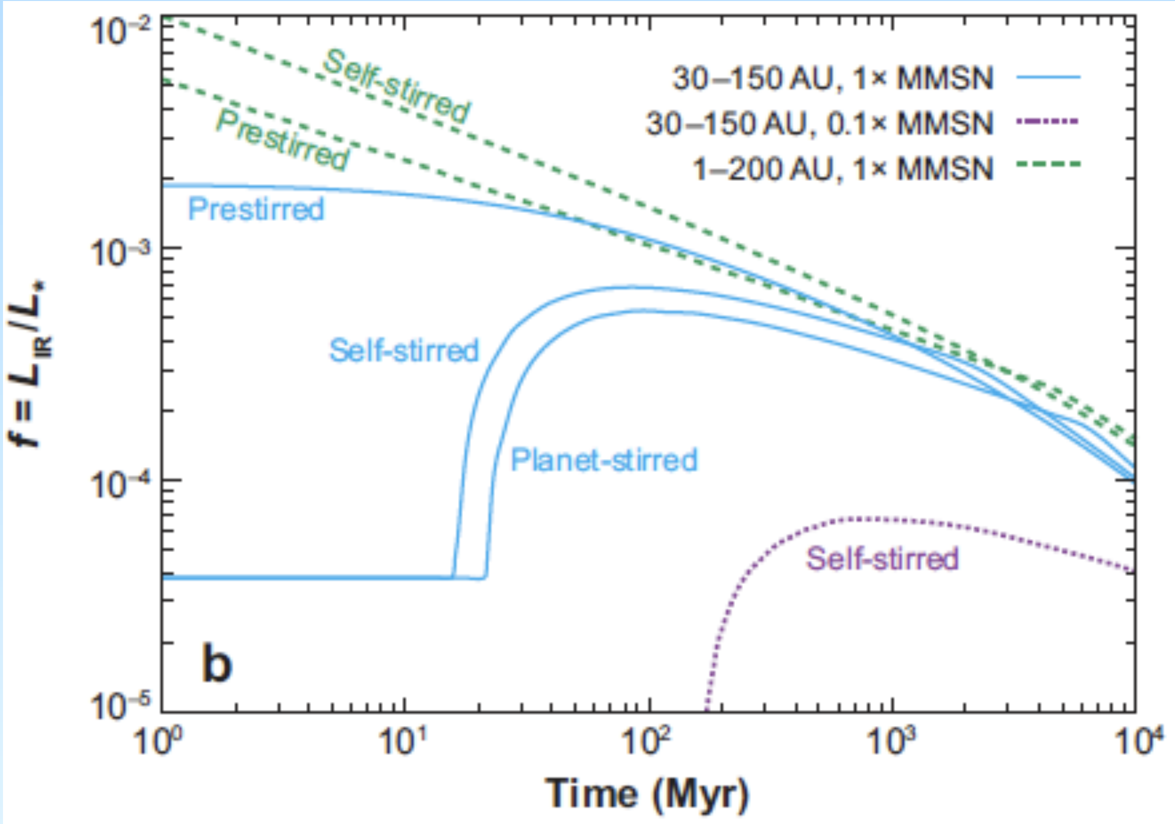
Self-stirring

Assume that:

• debris disks are stirred by the formation of Pluto-sized objects within them, a process which takes longer further from the star

• before the disk is "self-stirred", the dust/disk mass ratio is lower by 50x that of a collisional cascade

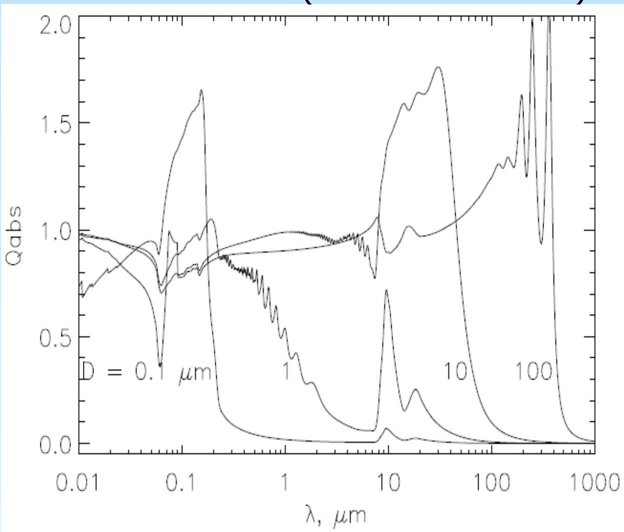
• each annulus is evolved separately



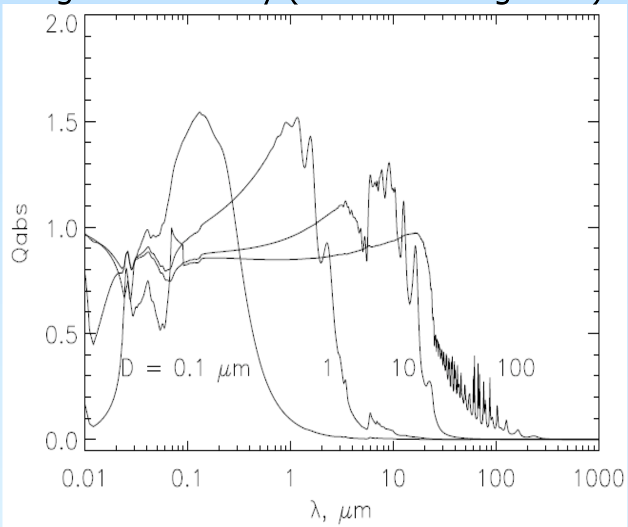
Optical properties

Optical constants can be used to work out the bulk properties of the grains (Q_{abs}, Q_{sca} and Q_{pr}) using **Mie theory** for compact spherical grains, and **geometric optics** and **Rayleigh-Gans** theory in appropriate limits (Laor & Draine 1993)

Astronomical silicate (Draine & Lee 1984)



Organic refractory (Li & Greenberg 1997)

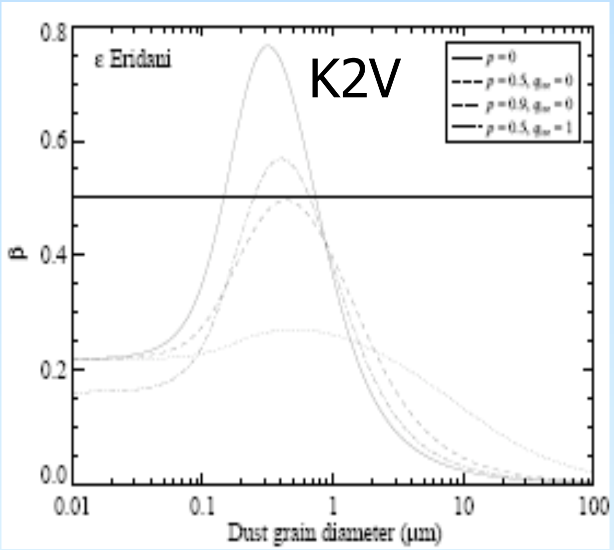


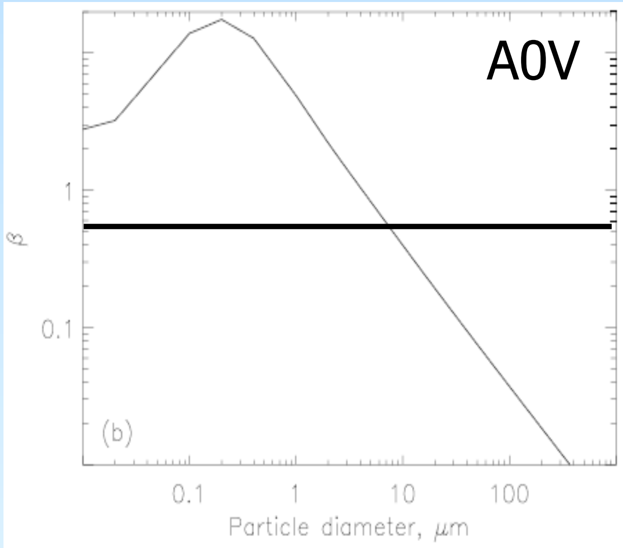
- Emission efficiency $Q_{em}=Q_{abs}$ ~1 for $\lambda<D$ and $\sim(\lambda/D)^n$ for $\lambda>D$ (although there are emission features, e.g., 10 and 20 μm features of silicates from Si-O stretching and O-Si-O bending modes)
- Albedo = $Q_{sca}/(Q_{abs}+Q_{sca})$

Radiation pressure coefficient

Remember $Q_{pr}=Q_{abs}+Q_{sca}[1-\langle\cos(\alpha)\rangle]$ where $\langle\cos(\alpha)\rangle$ is the asymmetry parameter (asymmetry in light scattered in forward/backward direction)

But we're interested in $\beta=F_{rad}/F_{grav}=(1150/\rho D)(L_*/M_*)\langle Q_{pr}\rangle_{T*}$
where $\langle Q_{pr}\rangle_{T*} = \int Q_{pr}F_* d\lambda \;/\; \int F_* d\lambda$ is Q_{pr} averaged over stellar spectrum





• higher mass stars remove larger grains by radiation pressure (~1μm for K2V and 10μm for A0V)
• porous grains are removed for larger sizes
• turnover at low D means small grains still bound to K and M stars

Equilibrium dust temperature

The equilibrium temperature of a dust grain is determined by the balance between energy absorbed from the star and that re-emited as thermal radiation:

$$[g/(4\pi r^2)] \int Q_{abs}(\lambda,D)\, L_*(\lambda)\, d\lambda = G \int Q_{abs}(\lambda,D)\, B_\nu(\lambda,T(D,r))\, d\lambda$$

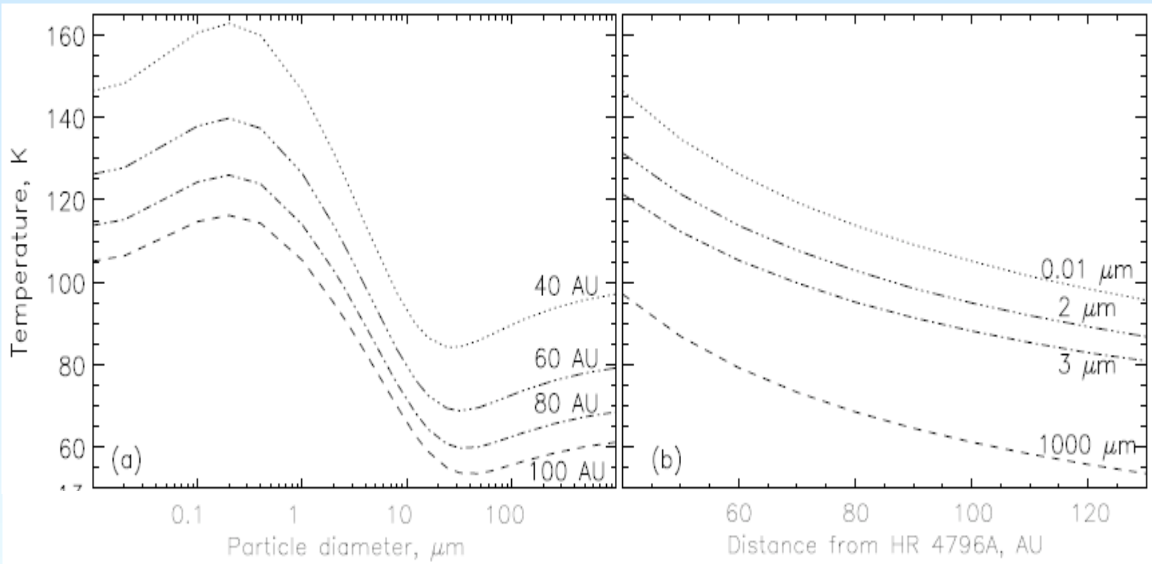
where dust temperature is a function of D and r, $g=0.25\pi D^2$, $G=\pi D^2$

Since $\int L_*(\lambda)\, d\lambda = L_*$ and $\int B_\nu(\lambda,T)\, d\lambda = \sigma T^4$, then

$$T(D,r) = [\, \langle Q_{abs}\rangle_{T*} / \langle Q_{abs}\rangle_{T(D,r)}\,]^{0.25}\, T_{bb}$$

where $T_{bb} = 278.3\, L_*^{0.25}\, r^{-0.5}$ and $\langle Q_{abs}\rangle_{T*}$ is average over stellar spectrum

Small particles are hotter than black body because they absorb starlight efficiently, but reemit inefficiently



Emission spectrum

The emission from a single grain is given by

$F_\nu(\lambda,D,r) = Q_{abs}(\lambda,D)\ B_\nu(\lambda,T(D,r))\ \Omega(D)$

where $\Omega = 0.25\pi D^2/d^2$ is the solid angle subtended by the particle at the Earth

If the disk is axisymmetric then define the distribution of cross-sectional area such that $\sigma(D,r)dDdr$ is the area in the range D to D+dD and r to r+dr and so $\int\int\sigma(D,r)dDdr = \sigma_{tot}$

Thus the total flux in Jy from the disk is

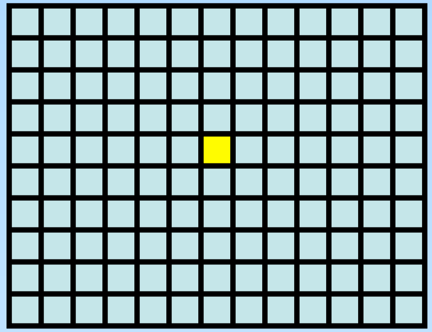
$$F_\nu = 2.35\times10^{-11} \int_{r_{min}}^{r_{max}}\int_{D_{min}}^{D_{max}} Q_{abs}(\lambda,D)\ B_\nu(\lambda,T(D,r))\ \sigma(D,r)\ d^{-2}\ dD\ dr$$

where area is in AU^2 and distance d is in pc

This equation can be simplified by setting $\sigma(D,r)=\sigma(D)\sigma(r)$ or just $=\sigma(D)$

Even more simply the grains can be assumed to be black bodies $Q_{abs}=1$ at the same distance giving $\mathbf{F_\nu = 2.35\times10^{-11}\ B_\nu(\lambda,T_{bb})\ \sigma_{tot}\ d^{-2}}$

Modelling images

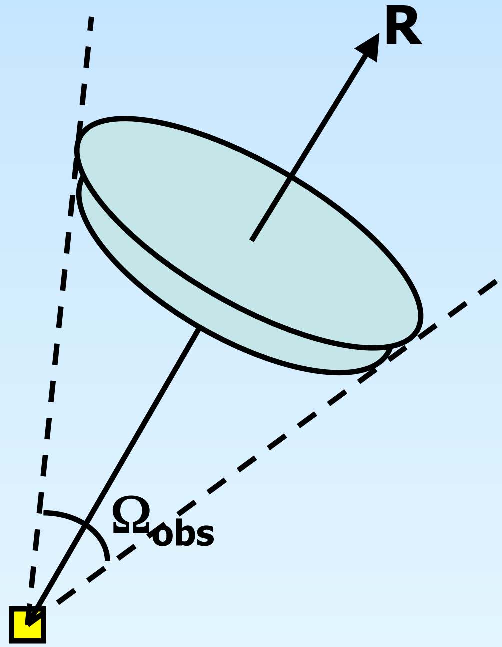


An image is made up of many pixels, each of which has a different line-of-sight through the disk

The surface brightness of emission in each pixel in Jy/sr is worked out using a line-of-sight integrator:

$$F_\nu/\Omega_{obs} = 2.35\mathrm{x}10^{-11} \int_{R_{min}}^{R_{max}} \int_{D_{min}}^{D_{max}} Q_{abs}(\lambda,D) B_\nu(\lambda,T(D,r)) \sigma(D,r,\theta,\phi) dD d\mathbf{R}$$

where $\sigma(D,r,\theta,\phi)$ is volume density of cross-sectional area in AU^2/AU^3 per diameter, and **R** is the line of sight vector



This equation can be simplified by setting
$\sigma(D,r,\theta,\phi)=\sigma(D)\sigma(r,\theta,\phi)$

$$F_\nu/\Omega_{obs} = P(\lambda,r)\ \sigma(r,\theta,\phi)\ d\mathbf{R}$$

$$P(\lambda,r) = 2.35\mathrm{x}10^{-11} \int_{D_{min}}^{D_{max}} Q_{abs}(\lambda,D) B_\nu(\lambda,T(D,r)) [\sigma(D)/\sigma_{tot}] dD$$

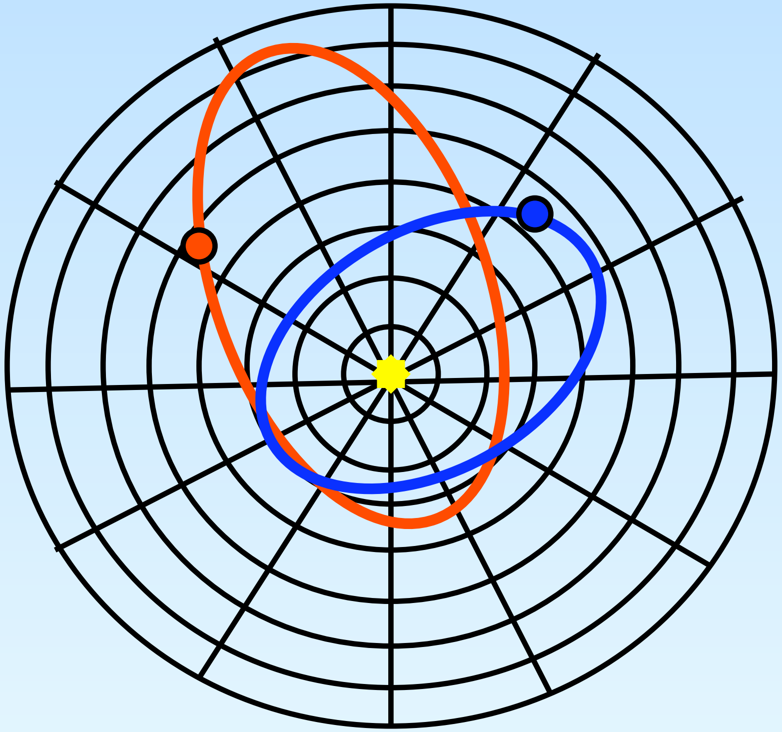
where $\sigma(r,\theta,\phi)$ depends on dynamics, and $P(\lambda,r)$ depends on composition/size distribution

Modelling structure

A model for the spatial distribution of material, $\sigma(r,\theta,\phi)$, can be derived from 2 body dynamics and assuming distributions of orbital elements.

For example, in 2D:

(1) Make a grid in r and θ
(2) Choose N particles on orbits with
- Semimajor axis, a
  e.g., between a_1 and a_2
- Eccentricity, e
  e.g., between 0 and e_{max}
- Pericentre orientation, ϖ
  e.g., between 0^0 and 360^0
- Mean longitude, λ
  e.g., between 0^0 and 360^0



(3) Convert particle location into r and θ
(4) Add up number of particles in each grid cell

Real disk images

The line-of-sight integrator will give a perfect image of the disk, the one that arrives at the Earth’s atmosphere

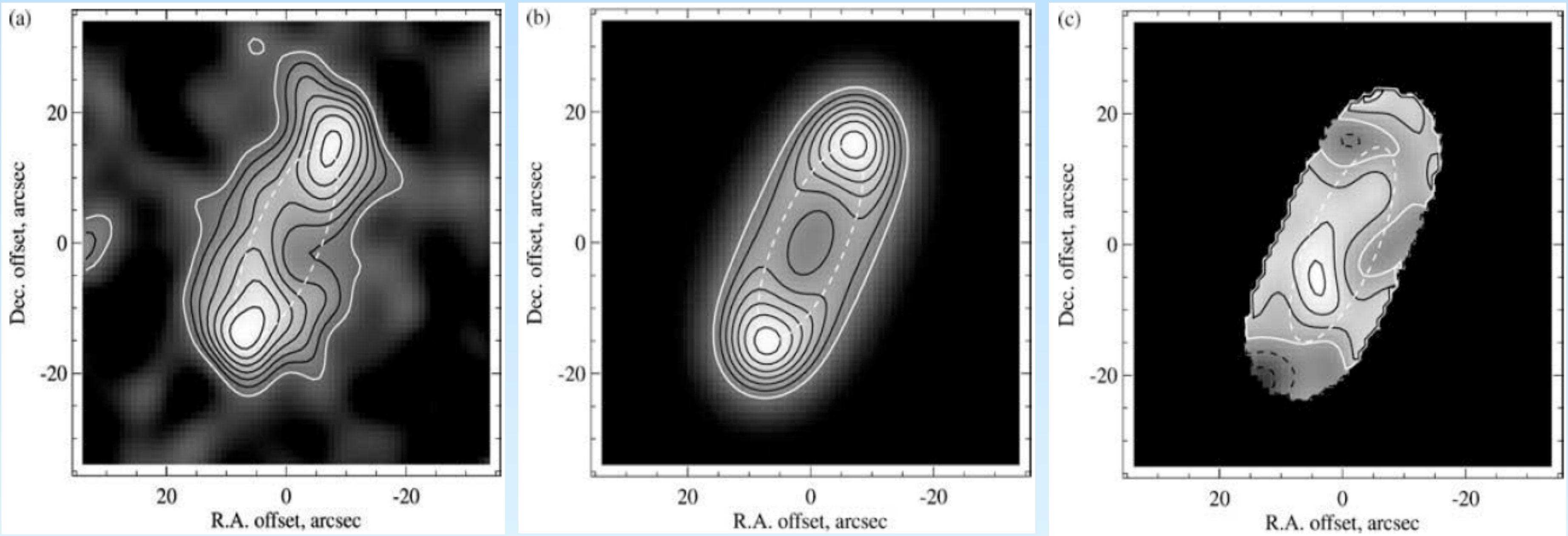
The image is blurred by the point spread function of the telescope
• ideally there will be a psf image to convolve the perfect image with
• if not, can assume Gaussian smoothing with FWHM=λ/Diameter telescope
• this is what you compare to the observation

The images are noisy
• often assume each pixel has additional uncertainty defined by gaussian statistics with given 1σ
• **Monte-Carlo:** to ascertain effect on image, create many noisy model images (each with random noise component) and see how diagnostics of model are affected

Application to Fomalhaut images

The disk modelling process is evident through the example of the Fomalhaut disk (Holland et al. 2003; Wyatt & Dent 2002):

450μm observation | model | residuals



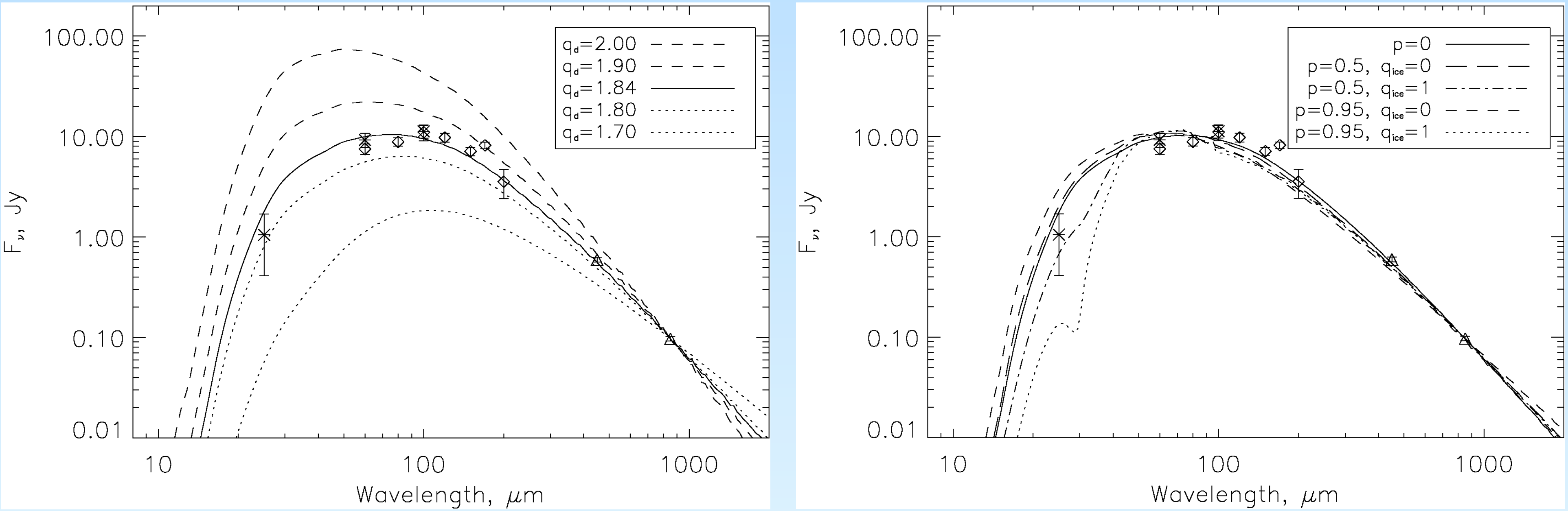
- The observation has three observables: mean peak brightness of the lobes, mean radial offset, mean vertical half maximum width
- The model had three free parameters: total area, radius, and inclination (although slightly more information on radial and vertical structure)

Fomalhaut SED

Once the radial distribution was constrained using the image, the full SED could be used to constrain the size distribution

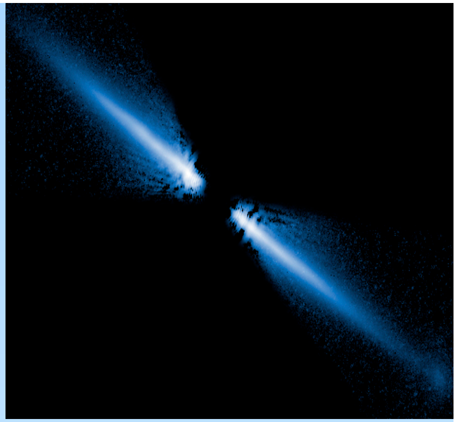
model changing q

model changing composition

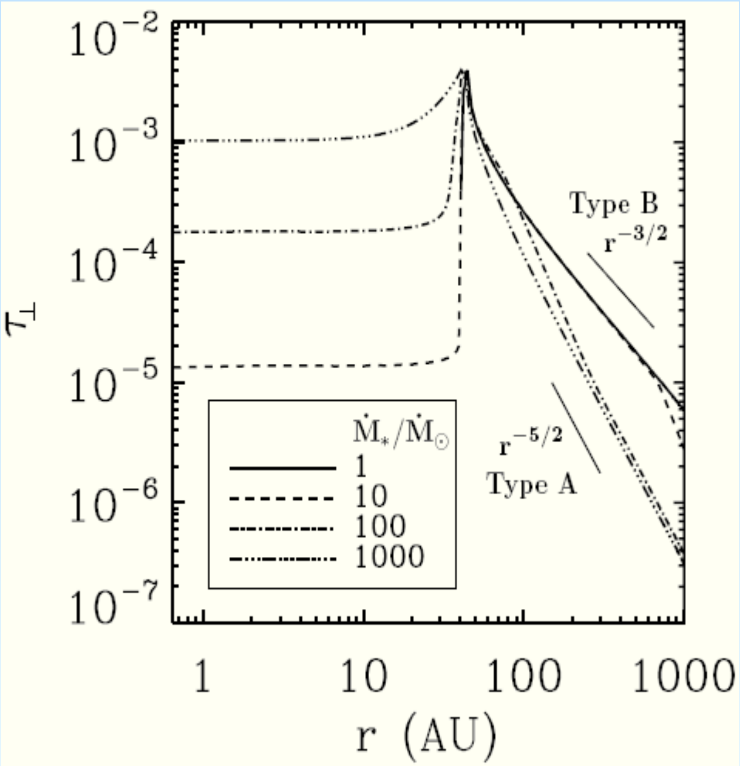


The slope of the size distribution could be well constrained as different dust sizes (at the same distance) have different temperatures, but the composition could not

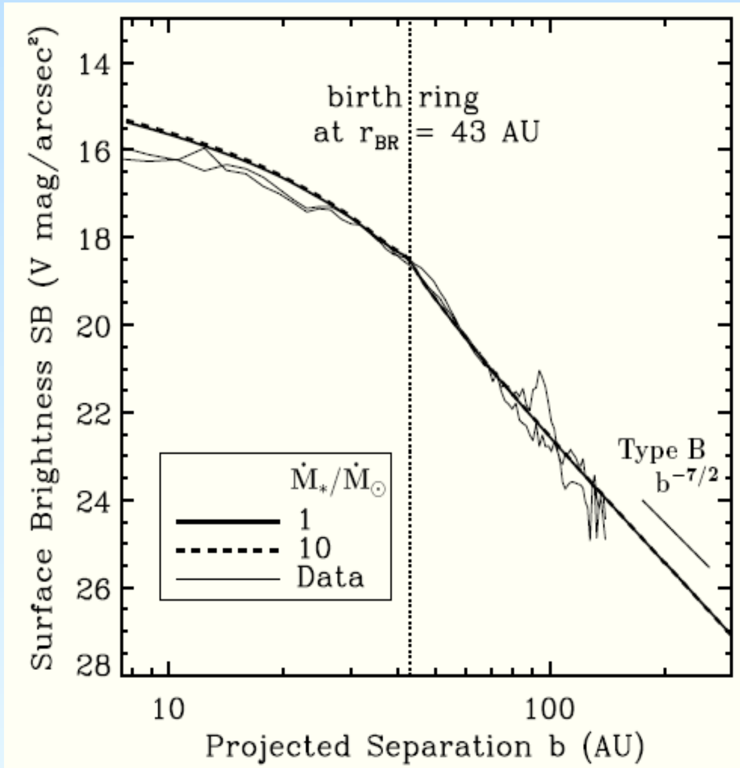
Extended dust distributions



The extended structure of AU Mic can be explained by dust created in a narrow belt at ~40AU (Augereau & Beust 2006; Strubbe & Chiang 2006)



Short wavelengths probe smallest grains and so are dominated by dust on orbits affected by pressure and drag forces



β Pictoris dust distribution can be explained in the same way (Augereau et al. 2001)