Atmospheric Differential Refraction in the Infrared

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1 Introduction

The refraction of light through the atmosphere is a function of wavelength. Although the use of a filter restricts the wavelength range of the light detected from a source, the effective wavelength will vary slightly depending on the shape of the spectrum of the source. The astrometric consequence of this is that the photocentre of a source will be shifted in altitude with respect to its true position as a function of its colour. Corrections to astrometric catalogues are often provided *eg.* Carlsberg Meridian Catalogues (Evans *et al.* 2002), where the effect is comparable to the accuracy of the catalogue.

Most astrometric catalogues are at the visible wavelengths where the effect is significant. The gradient of the angle of refraction as a function of wavelength decreases with increasing wavelength (see Figure 1), so that at infrared wavelengths the gradient is very small and thus this systematic effect is small. However, for completeness, the calculations to estimate the size of this effect for WFCAM and VISTA are presented in this report.

Also investigated is the effect of using a linear plate model to account for the general effect of refraction.

2 Differential astrometry as a function of colour

The equations for calculating this effect can be found in Schubert & Walterscheid 2000 (Section 11.20 of Allen's Astrophysical Quantities, 4th ed.). Another way of presenting these equations can be found in Roe 2002[1], and this is followed here. The refractive index of air, n, is

$$\begin{aligned} n(\lambda, p, T, p_w) &= 1 + \left(64.328 + \frac{29498.1}{146 - \lambda^{-2}} + \frac{255.4}{41 - \lambda^{-2}}\right) \\ &\quad \times \left(\frac{pT_s}{p_sT}\right) 10^{-6} \\ &\quad -43.49 \left(1 - \frac{7.956 \times 10^{-3}}{\lambda^2}\right) \frac{p_w}{p_s} 10^{-6} \end{aligned} \tag{1}$$

where p is the pressure in millibars, T the temperature in kelvin, λ the wavelength in microns. p_s and T_s are 1000 mbar and 288.15 K respectively. p_w is the partial vapour pressure of water.

[1] Note that the units quoted for pressure in Section 2 of this paper are incorrect. They should be in 10^2 Pa, so that p_s is then 1 atmosphere. Alternatively, mbar could be used, but then p_s would be 1000 mbar.

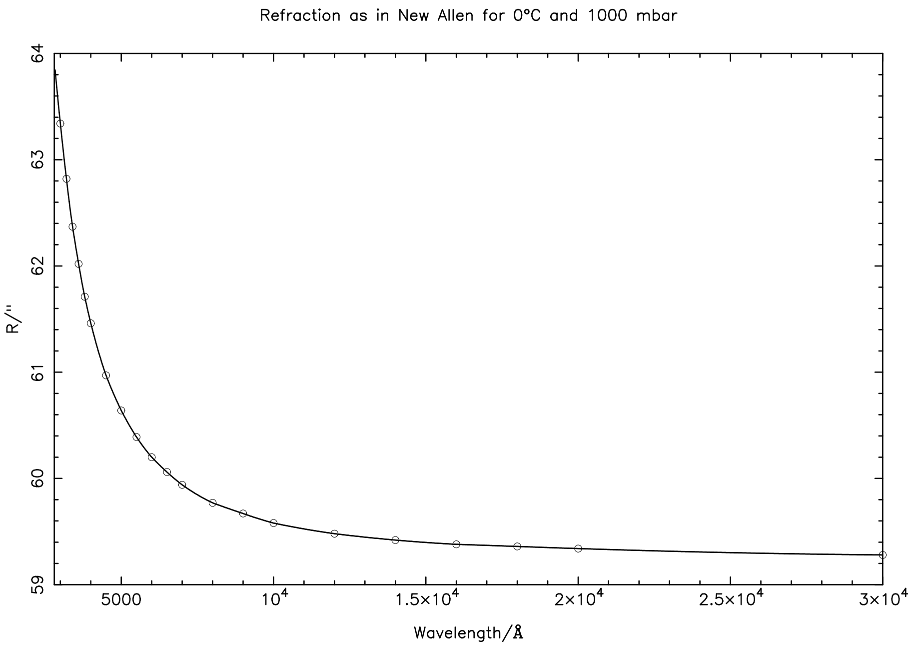

Figure 1: This plot shows the constant of refraction as a function of wavelength for $T = 0°\mathrm{C}$, $p = 1000$ mbar and $p_w = 550$ Pa. These values are from Table 11.24 of Schubert & Walterscheid 2000.

The angle between the true and apparent zenith distance is given by

$$z_t - z_a \simeq R \tan z_t = 206265 \left(\frac{n^2 - 1}{2n^2}\right) \tan z_t \tag{2}$$

in arc seconds. This approximation holds for $z_t < 80°$. The constant of refraction, R, as a function of wavelength is shown in Figure 1.

The correction to allow for water vapour requires a knowledge of the partial vapour pressure of water. Normally, this is not directly available, but can be calculated from the relative humidity. This is defined as the ratio of the actual partial vapour pressure to the saturation vapour pressure at that temperature.

There are many approximations available to describe the variation of the saturation vapour pressure of water with temperature. A reasonably simple formula is

$$p_{sat} = 611 \exp\left(\frac{17.27T}{T + 237.3}\right) \text{ Pa} \tag{3}$$

where T is the temperature in °C (Tetens 1930). This approximation is quoted as being valid for the range 0–100°C.

Tests showed that the effect of water vapour was mainly to cause a shift in the constant of refraction and not to affect the gradient by any significant amount. Therefore all further calculations were carried out for dry air.

In order to calculate the constant of refraction for a particular star you need to calculate its effective wavelength, $\overline{\lambda}$, in that passband and insert this value into the above equations. The effective wavelength is defined by

$$\overline{\lambda} = \frac{\int_0^\infty JDe^{-\kappa\eta}\lambda d\lambda}{\int_0^\infty JDe^{-\kappa\eta}d\lambda} \tag{4}$$

where J is the intensity of the source, D the response of the detector, κ the exponential atmospheric absorption coefficient (all functions of wavelength) and η the air mass (Murray 1983, p175).

Formulating it this way means that you can take into account the air mass. κ is calculated from the transmittance, T_r, ($\kappa = -\log_e T_r$) which is what you normally can find. For Mauna Kea Observatory

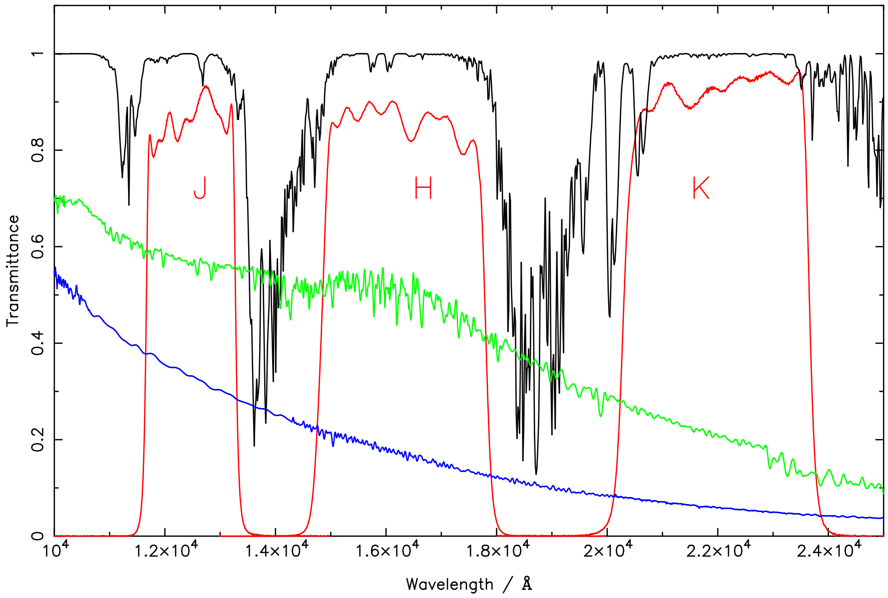

Figure 2: Transmittance as a function of wavelength. Also shown are the passband responses (red) for the J, H, and K filters of the MKO filter set. The spectra of a G2V (blue) and M3III (green) star are superposed.

(MKO) you can get the infrared transmittance values from
`http://www.jach.hawaii.edu/JACpublic/UKIRT/astronomy/calib/atmos-index.html`
This is shown in Figure 2. Although the calculations have been carried out for an air mass of 1.15 (a typical zenith distance of 30°), there is not much difference in the results for different values of air mass since the transmittance is fairly flat over the passbands J, H and K.

The passband responses for the J, H and K filters of the MKO filter set can be found at
`http://irtfweb.ifa.hawaii.edu/Facility/nsfcam/hist/newfilters.html`
These are also shown in Figure 2.

The data for the source intensity comes from Pickles 1998.

For each spectrum with IR data within Pickles 1998 the effective wavelength is calculated and the constant of refraction calculated. This is done for each of the IR passbands considered: J, H and K. These values are shown in Figure 3. The pressure and temperature used for these calculations were 620 mbar and 273 K respectively. These are representative conditions for MKO. For Paranal, a typical pressure to use would be 740 mbar.

Figures 4 to 6 shows the detail of the variation with colour. A line has been fitted by least squares to the luminosity class V data points.

It is only for the J passband that there is any significant gradient measured. This is found to be -0.0010 ± 0.0001 arc seconds per unit of B$-$V.

The gradient will only indicate the shifts for typical sources. For an extreme spectrum, one in which the effective wavelength is right at an edge of a passband, what will the size of the shift be? For two extreme stars this would represent the maximum displacement between two stars in a given passband. These values were found to be 40, 28 and 12 mas for J, H and K respectively. The maximum shift for a single star with respect to the average star (which will define the reference frame) will be approximately half these values.

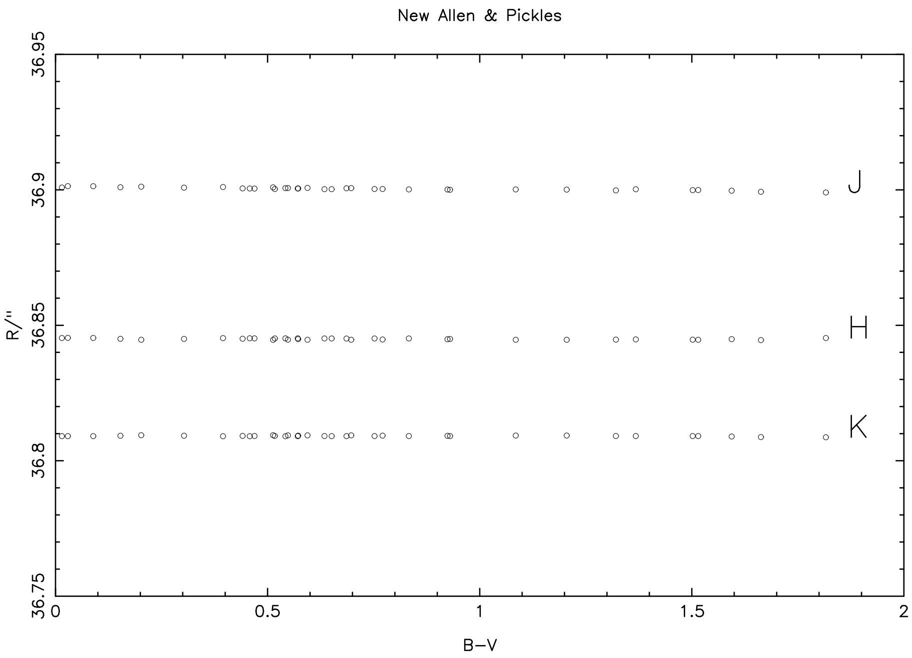

Figure 3: The constant of refraction for J, H and K as a function of colour, B−V. Only luminosity class V stars are plotted. The atmospheric conditions used in these calculations were $p = 620$ mbar and $T = 273.15$ K.

3 Using a linear plate model to account for refraction

When astrometric standards are used to calibrate data from a telescope, after projection effects have been taken into account, it is usual to use a linear plate model. The 6-parameter model usually used,

$$x' = ax + by + c \tag{5}$$
$$y' = dx + ey + f \tag{6}$$

can take into account scale, rotation, shear and offsets. Sometimes it is assumed that the scale is the same in both the x and y directions, however, the effect of the difference in refraction at the top and bottom of the field-of-view is to stretch the scale in the altitude direction. As can be seen from Equation 2, what this model is doing is fitting a straight line to part of the $\tan z$ curve. For a small field-of-view this is a reasonable approximation.

For WFCAM and VISTA the fields-of-view are 0.93° and 1.65° respectively. Figure 7 shows the effect of using a linear approximation for a zenith distance of 45°. Most of this error comes from the quadratic term in the Taylor expansion of $\tan z$, which goes as the second derivative of $\tan z$. Thus the error plotted in Figure 7 will approximately scale as $\sec^3 z \times \sin z$ for different values of zenith distance. This can be seen in Figure 8, which shows the maximum size of the error as a function of zenith distance. This can be as large as 10 and 40 mas for WFCAM and VISTA respectively.

However, both these instruments are multichip detectors and it is the intention to do an astrometric solution for each chip individually. The effect of this is to restrict the range of zenith distance over which the linear fit is applied thus making it a better approximation. With the z ranges reduced to 0.23° and 0.19° for WFCAM and VISTA respectively, the maximum error is now reduced to less than 1 mas even for zenith distances as large as 60°.

There may be occasions that an astrometric solution for an individual chip may not be possible *eg.* not enough standards lying within the chip boundaries. When this occurs, an astrometric solution covering the whole field-of-view will be necessary and the problems of using a linear solution will be present. The solution in this case is to use a quadratic term in altitude, as can be seen from the form of the plot in Figure 7. Again, the maximum errors drop to below 1 mas for $z = 60°$.

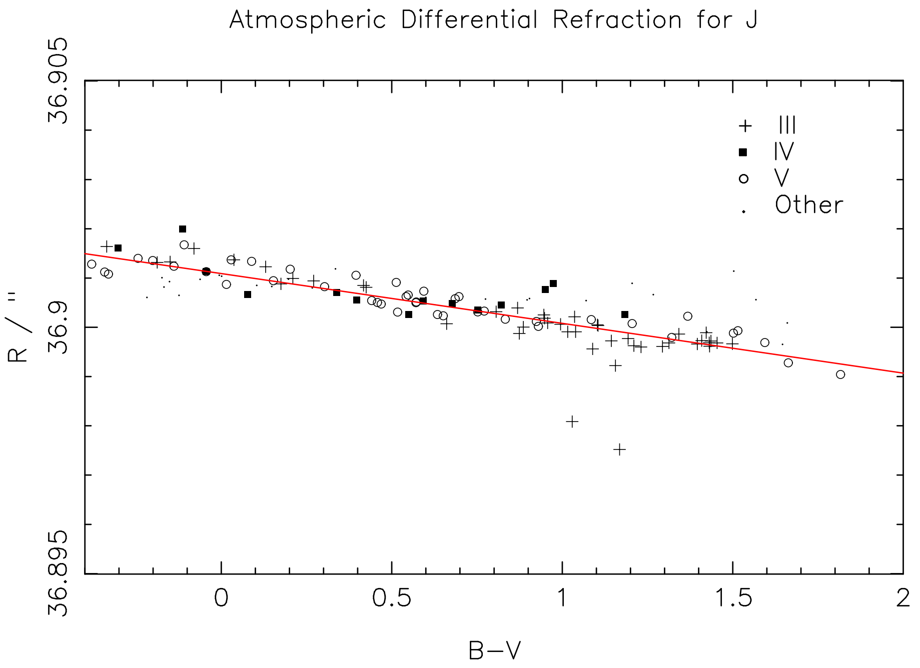

Figure 4: The constant of refraction for J as a function of colour, B−V. The best fit line to the luminosity class V stars is shown.

4 Conclusions

The effect of atmospheric differential refraction in the infrared as a function of colour is shown to be small. The only measurable effect is seen in the J passband and the magnitude of the effect is of order 1 mas. In comparison, the astrometric accuracy for WFCAM at the bright end is expected to be around 20 mas (Evans 2003). This is also below the level required of the astrometry for VISTA.

Using a linear plate model for the astrometric solution has been shown to be suitable to better than 1 mas as long as the solution is applied to each chip individually. If a single solution is required for the whole field of view then a quadratic solution in altitude is necessary. If this is not carried out, then for $z = 60°$, astrometric errors of 10 and 40 mas will occur for WFCAM and VISTA respectively.

References

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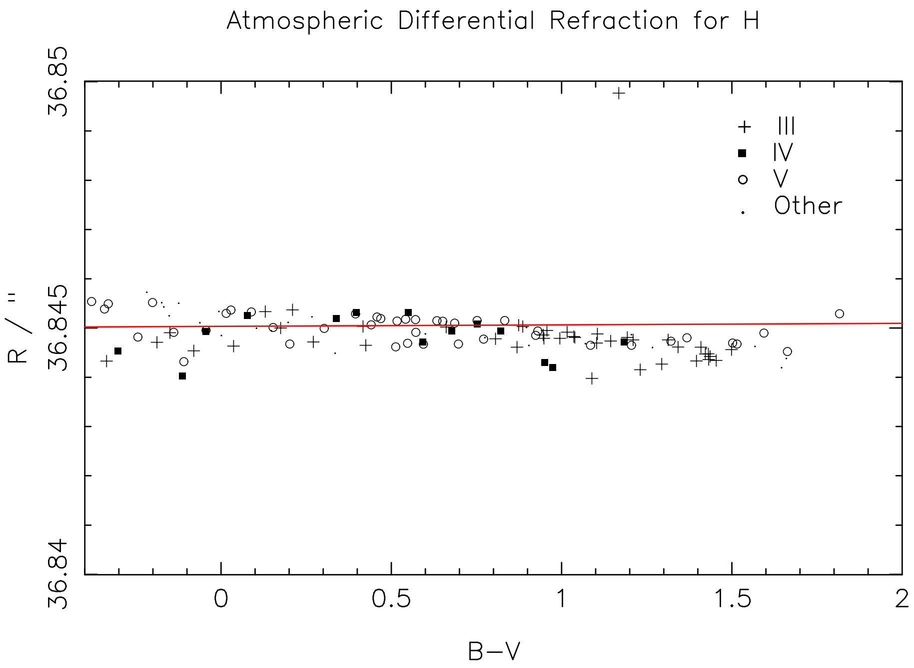

Figure 5: The same as Figure 4, but for the H passband.

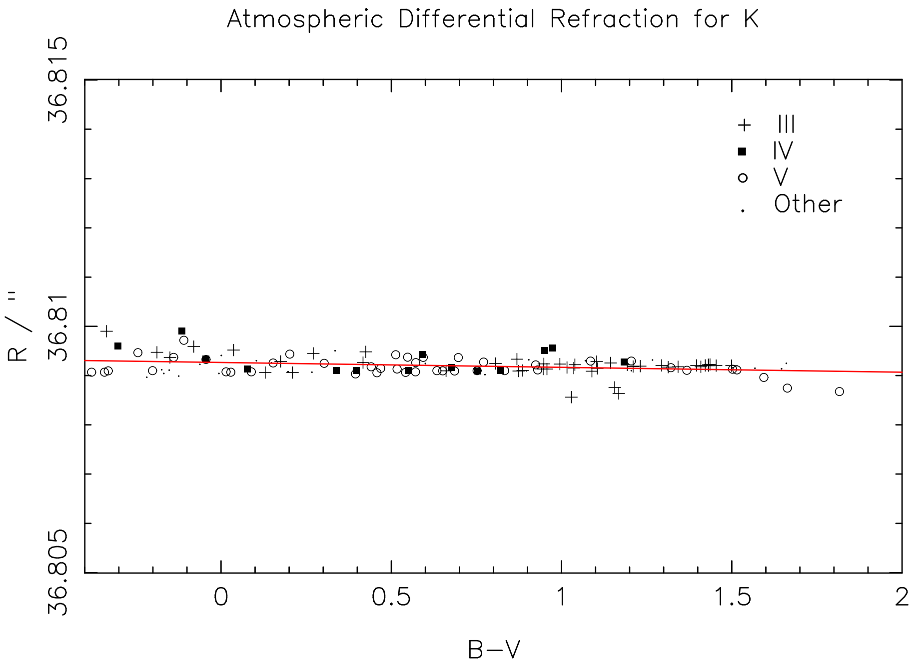

Figure 6: The same as Figure 4, but for the K passband.

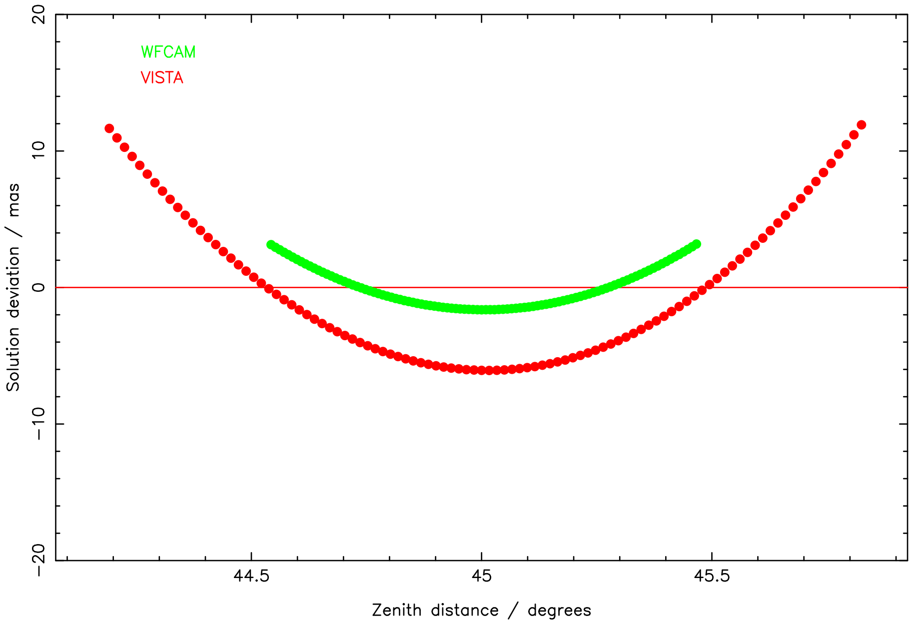

Figure 7: The astrometric error caused by using a linear plate solution to fit $R\tan z$ for a telescope pointing of $z = 45°$. Appropriate parameters for WFCAM and VISTA have been used.

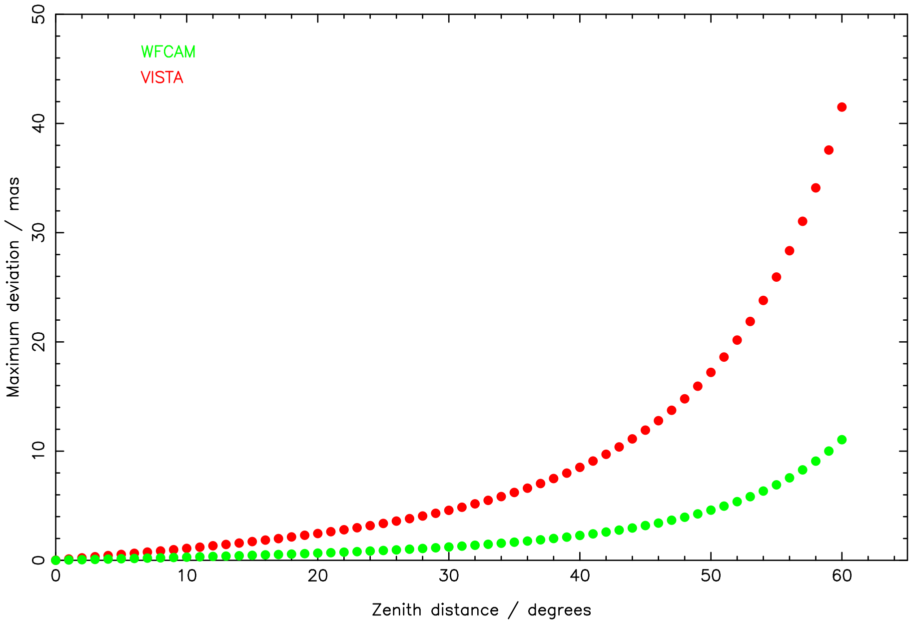

Figure 8: The maximum astrometric error caused by using a linear model to account for refraction as a function of zenith distance, z.