Appendix A

Three models of the Galaxy

Here we provide the equations that describe the three models of the Galaxy exploited in Chapter 4.

Let x, y and z be the coordinates defined by the principal axes of the galactic bulge/bar, with x-axis directed along the long bar axis, which is at angle ϕ_0 from the Sun-centre line. Then the following equations describe the distribution of the mass density ρ (ρ_b and ρ_d refer to the density contributions of the bar and the disk correspondingly) in the Milky Way galaxy in the models of Binney et al., Freudenreich and Dwek et al.

In the model of Binney et al.:

$$\rho = \Upsilon_B j_0 (f_b + f_d) \tag{A.1}$$

$$f_b = f_0 \frac{e^{-a^2/a_m^2}}{(1 + a/a_0)^{1.8}} \tag{A.2}$$

$$f_d = \left(\frac{e^{-|z|/z_0}}{z_0} + \alpha \frac{e^{-|z|/z_1}}{z_1} \right) R_d e^{-R/R_d} \tag{A.3}$$

$$a = \left(x^2 + \frac{y^2}{\eta^2} + \frac{z^2}{\zeta^2} \right)^{1/2} \tag{A.4}$$

Binney et al.		Freudenreich		Dwek et al. (E2)	
ϕ_0	$20°$	ϕ_0	$14°$	ϕ_0	$24°$
Υ_B	$3.57 \times 10^8 \frac{M_\odot}{COBE\text{unit}}$	Υ_F	$0.84 \frac{M_\odot}{L_\odot}$	ρ_0	$4.46 \frac{M_\odot}{\text{pc}^3}$
j_0	$0.456 \frac{COBEunits}{\text{kpc}^3}$	I_b	$1300 \frac{L_\odot}{\text{pc}^2}$	Σ	$50 \frac{M_\odot}{\text{pc}^2}$
f_0	624	(a_x, a_y, a_z)	(1.69, 0.63, 0.42) kpc	H	0.325 kpc
a_m	3.82 kpc	$(c_\perp, c_\parallel)$	(1.55, 3.43)	R_d	3.5 kpc
a_0	100 pc	(R_{bar}, h_{bar})	(2.97, 044) kpc	x_0	900 pc
z_0	210 pc	I_d	$867 \frac{L_\odot}{\text{pc}^2}$	y_0	385 pc
z_1	42 pc	R_d	2.5 kpc	z_0	250 pc
R_d	2.5 kpc	z_0	167 pc		
(η, ζ)	(0.3,0.3)	R_h	2.94 kpc		
α	0.27	(q_h, β)	(0.87, 1.72)		

Table A.1: This lists the values of the parameters in the Galactic models of Binney et al. (first column), Freudenreich (second column) and Dwek et al. (third column).

In the model of Freudenreich:

$$\rho = \Upsilon_F(j_b + j_d) \tag{A.5}$$

$$j_b = \frac{I_b}{a_z} sech^2(m_b)T(R) \tag{A.6}$$

$$j_d = \frac{I_d}{2z_0} sech^2(z/2z_0)e^{-R/R_d}(1 - e^{-m_h^\beta}) \tag{A.7}$$

$$m_h \equiv \sqrt{(x^2 + q_h^2 y^2)/R_h^2} \tag{A.8}$$

$$m_b \equiv \left\{ \left[\left(\frac{|x|}{a_x}\right)^{c_\perp} + \left(\frac{|y|}{a_y}\right)^{c_\perp} \right]^{c_\parallel/c_\perp} + \left(\frac{|z|}{a_z}\right)^{c_\parallel} \right\}^{1/c_\parallel} \tag{A.9}$$

$$T(R) \equiv \begin{cases} 1, & \text{for } R \leq R_{bar} \\ e^{-(R-R_{bar})^2/h_{bar}^2}, & \text{otherwise} \end{cases} \tag{A.10}$$

In the model of Dwek et al.:

$$\rho = \rho_b + \rho_d \tag{A.11}$$

$$\rho_b = \rho_0 e^{-r_D} \tag{A.12}$$

$$\rho_d = \frac{\Sigma}{2H} e^{-(R-R_\odot)/R_d} e^{-|z|/H} \tag{A.13}$$

$$r_D = \left[\left(\frac{x}{x_0}\right)^2 + \left(\frac{y}{y_0}\right)^2 + \left(\frac{z}{z_0}\right)^2 \right]^{1/2} \tag{A.14}$$