i)

Conversion of $L_{orbit} \Rightarrow L_{rotation}$

1. Before collapse all L in orbit, L_{orb}

After collapse L in orbit + in rotation of planet : $L'_{orb} + L_{rot}$

$$\therefore L_{orb} = L'_{orb} + L_{rot}$$

$$L_{rot} = L_{orb} - L'_{orb} \qquad (1)$$

2. $L'_{orb} = M_p \Omega_k a^2 \qquad (2)$

3. $L_{rot} = \frac{2}{5} M_p R_p^2 \, \omega \qquad (3)$

angular frequency

assuming rigid body rotation

assuming uniform density

4. Integrate to find orbital angular momentum contained in collapsing patch of disk, L_{orb}

$$dL_{orb} = dm \, \Omega_k(r) \, r^2$$

$$dm = \Sigma \, dA$$

$$dA = 2\pi r \, dr$$

$$\therefore dm = \Sigma 2\pi r \, dr$$

And $\Omega_k = \left(\frac{G M_*}{r^3}\right)^{\frac{1}{2}} = \mu^{1/2} r^{-3/2}$

So, $dL_{disk} = \mu^{\frac{1}{2}} r^{-3/2} \cdot r^2 \cdot \Sigma\, 2\pi r\, dr$

$$= \mu^{1/2}\, 2\pi \Sigma\, r^{3/2}\, dr$$

$$L_{disk} = \mu^{\frac{1}{2}}\, 2\pi \Sigma \int_{a-R_R}^{a+R_R} r^{3/2}\, dr$$

$$= \frac{2}{5}\mu^{1/2}\, 2\pi \Sigma \left[(a+R_R)^{5/2} - (a-R_R)^{5/2}\right] \quad (4)$$

sub (2), (3), (4) $\rightarrow$ (1)

$$\frac{2}{5} M_p R_p^2 \omega = \frac{4}{5}\mu^{\frac{1}{2}} M_p \left(\frac{(a+R_R)^{5/2} - (a-R_R)^{5/2}}{4aR_R}\right) - M_p \mu^{\frac{1}{2}} a^{\frac{1}{2}}$$

$$R_p^2 \omega = 2\mu^{\frac{1}{2}} \left[\frac{a^{3/2}}{4R_a}\left[\left(1+R_R/a\right)^{5/2} - \left(1-R_R/a\right)^{5/2}\right]\right] - \mu^{\frac{1}{2}} a^{\frac{1}{2}}$$

using: $\Sigma = \frac{M_p}{\pi}\frac{1}{(a+R_R)^2-(a-R_R)^2} = \frac{M_p}{\pi}\frac{1}{4aR_R}$

Now, $\left(1+R_R/a\right)^{5/2} \rightarrow 1 + \frac{5}{2}\frac{R_R}{a} \ldots$ $\qquad \frac{f^n(a)(x-a)^n}{n!}$

$\left(1-R_R/a\right)^{5/2} \rightarrow 1 - \frac{5}{2}\frac{R_R}{a} \ldots$

$$\therefore \; R_p^2 \omega \approx \frac{\mu^{\frac{1}{2}} a^{3/2}}{2 R_R} \left(1 + \frac{5}{2} \frac{R_R}{a} - 1 + \frac{5}{2} \frac{R_R}{a} \right) - \mu^{\frac{1}{2}} a^{\frac{1}{2}}$$

$$\approx \frac{5}{2} \mu^{\frac{1}{2}} a^{\frac{1}{2}} - \mu a^{\frac{1}{2}} \qquad (5)$$

$$\approx \frac{3}{2} \mu^{\frac{1}{2}} a^{\frac{1}{2}}$$

So $\omega \approx \frac{3}{2} \frac{\mu^{\frac{1}{2}} a^{\frac{1}{2}}}{R_p^2}$ //

only depend on M_*, orbital radius & final radius of planet

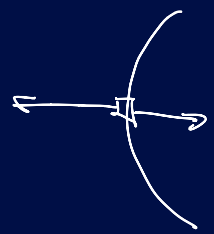
ii) $$P_{day} = \omega^{-1} \sim \left[\frac{3}{2} \frac{\left(2\pi \times 10^{-11} \times 10^{30} \; \frac{\pi}{2} \times 10^{11}\right)^{\frac{1}{2}}}{\left(2\pi \times 10^{7}\right)^2} \right]^{-1}$$

$$\approx \left[\frac{10^{15}}{10^{15}} \right]^{-1}$$

$$\approx 1 \text{ s}$$ //

iii) Compare with self gravity

$$F_c = \frac{m v^2}{r} = \frac{m \omega^2 R_p^2}{R_p}$$



$$F_g = \frac{G M_p m}{R_p^2}$$

$$= m \omega^2 R_p$$

$$\frac{F_g}{F_c} = \frac{G M_p m}{R_p^2} \cdot \frac{1}{m \omega^2 R_p}$$

$$= \frac{G M_p}{\omega^2 R_p^3} \sim \frac{2\pi \times 10^{-11} \cdot 10^{-3} \cdot 10^{30}}{1 \cdot (2\pi \cdot 10^{7})^3}$$

$$\sim \frac{10^{18}}{10^{23}}$$

$$\sim 10^{-5}$$

$\therefore$ planet would tear apart under its own rotation.

iv) Return to (5)

$$R_p^2 \omega \sim \underbrace{\frac{5}{2}\mu^{\frac{1}{2}} a^{\frac{1}{2}}}_{L \text{ disk}} - \mu^{\frac{1}{2}} a^{\frac{1}{2}}$$

$$\omega \sim \frac{\mu^{\frac{1}{2}} a^{\frac{1}{2}} \left(\frac{5}{2} f - 1\right)}{R_p^2}$$ // introduce L efficiency factor

Now, $\frac{F_g}{F_c} \geq 1$ $\therefore$ $\frac{GM_p m}{R_p^2} \cdot \frac{1}{m \omega_{max}^2 R_p} \simeq 1$

$$\frac{GM_p}{R_p^3} \cdot \frac{R_p^4}{GM_* a \left(\frac{5}{2} f_{max} - 1\right)} \simeq 1$$

so $\frac{5}{2} f_{max} = \frac{M_p}{M_*} \cdot \frac{R_p}{a} + 1$

$$f_{max} = \frac{2}{5}\left(\frac{M_p}{M_*} \cdot \frac{R_p}{a} + 1\right)$$

$$\sim \frac{2}{5}\left(10^{-3} \cdot \frac{10^7}{10^{11}} + 1\right)$$

$$\sim \frac{2}{5}$$ //

$\therefore$ at most 40% of angular momentum of disk can be kept in planet and this is almost entirely in orbit, not in planet's rotation. //

$\Rightarrow$ jets, winds, viscous spreading of planet's subdisk, and magnetic torques on disk from planet must combine to allow accretion. //