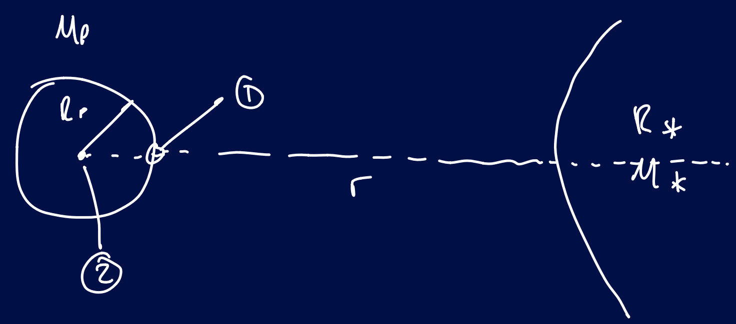
i) Derive tidal force approximation



$$F_1 = \frac{G M_* m}{r^2}$$

$$F_2 = \frac{G M_* m}{(r + R_p)^2}$$

$$F_T = F_1 - F_2 = G M_* m \left(\frac{1}{r^2} - \frac{1}{(r + R_p)^2} \right)$$

$$= G M_* m \left(\frac{1}{r^2} - \frac{1}{r^2 + R_p^2 + 2 r R_p} \right)$$

$$= G M_* m \left(\frac{1 + R_p^2/r^2 + 2 R_p/r}{r^2 + R_p^2 + 2 r R_p} - \frac{1}{r^2 + R_p^2 + 2 r R_p} \right)$$

$$= G M_* m \left(\underbrace{\frac{R_p^2/r^2}{r^2 + R_p^2 + 2 r R_p}}_{a} + \underbrace{\frac{2 R_p/r}{r^2 + R_p^2 + 2 r R_p}}_{b} \right)$$

Now if $r >> R_p$:

$a \to 0$

denominator of $b \to r^3$

$$\therefore F_T \sim \frac{2GM_* m R_p}{r^3}$$

So, specific force, $\frac{F_T}{m} \sim \frac{GM_* R_p}{r^3}$ //

or note that $F = -\frac{GMm}{r^2}$

$$dF = \left(\frac{dF}{dr}\right) dr$$

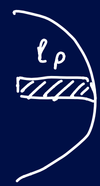
$$\frac{dF}{dr} = \frac{2GMm}{r^3}$$

and so $dF = \frac{2GMm}{r^3} dr$

for $r >> R_p$

$$\Delta F = F_T \sim \frac{2GMm R_p}{r^3}$$ //

ii) $m = R_p \rho$



$m = R_p \rho$

$$\Delta F = 100 \times 10^6 = \frac{2GM\,\ell_{max}\,\rho\,R_{max}}{r^3}$$

$$\ell_{max}^2 = \frac{10^8\,r^3}{2GM\rho}$$

$$= \frac{10^8 \cdot (0.2 \times 10^8)^3}{2 \times 2\pi \times 10^{-11} \times 10^{30} \times 3000}$$

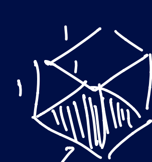
$$= \frac{10^{30}}{3 \times 10^{23}}$$

$$\approx 3 \times 10^6$$

$$\sim 10^3 \text{ m}$$

iii) $F_g = \frac{GMm}{\ell_p^2}$

$$\frac{F_g}{F_T} = \frac{G\,\frac{4}{3}\pi\,\ell_{max}^3\,\rho\,m}{\ell_{max}^2} \bigg/ 10^6$$



$m = V\rho = 1 \cdot \rho$

$= 3000 \text{ kg m}^{-3}$

$$\frac{F_g}{F_r} = \frac{G\,4\pi\, l_{max}\,\rho^2}{10^6}$$

$$\sim \frac{2\pi\times10^{-11}\cdot 4\pi\cdot 10^3\cdot 10^7}{10^6}$$

$$\sim 10^{-6}$$

Self gravity much weaker than tensile strength of rock

iv) $F = \sigma T^4 \cdot 2\pi\, l_{max}^2$

$$T = \left(\frac{F}{2\pi\sigma\, l_{max}^2}\right)^{\frac{1}{4}}$$

$$F = \frac{30 L_\odot \times \pi\, l_{max}^2}{4\pi a^2}$$

$$\therefore\ T = \left(\frac{30 L_\odot}{8\pi\sigma\, a^2}\right)^{\frac{1}{4}}$$

$$= \left(\frac{10^{28}}{8\pi\cdot 2\pi\times10^{-8}\cdot 0.2\times(2\pi\times10^8)^2}\right)^{\frac{1}{4}}$$

$$= \left(\frac{10^{28}}{3\pi^2\times10^{-8}\cdot 4\pi^2\times10^{16}}\right)^{\frac{1}{4}}$$

$$= \left(\frac{10^{28}}{12\pi^4\times10^8}\right)^{\frac{1}{4}}$$

$\sim 10^4\,K$ $>$ T_{sub} $(\sim 10^3\,K)$

v) $$t_{sub} = \frac{\overbrace{\tfrac{4}{3}\pi R_{rock}^3 \cdot \rho \cdot \Delta H_v}}{F}$$ energy to sublimate entire body

from above $F = \dfrac{30 L_\odot \pi R_{rock}^2}{4\pi a^2}$

$$\therefore\ t_{sub} = \frac{4\pi R_{rock}^3\, \rho\, \Delta H_v \cdot 4\pi a^2}{30 L_\odot \pi R_{rock}^2}$$

$$= \frac{16\pi}{30}\, \frac{R_{rock}\, a^2\, \rho\, \Delta H_v}{L_\odot}$$

vi) If $t_{sub} > \Omega_k^{-1}$ then tidal disruption will break the object up. Some material will move onto shorter period orbits, and some longer periods, as there is a velocity dispersion of broken fragments of the body

$\therefore$ Some material $> 1\,au$

vii)

Magnitude of velocity dispersion:

$$v = \left(\frac{GM}{a}\right)^{\frac{1}{2}}$$

$$v_o = \left(\frac{GM}{a+R}\right)^{\frac{1}{2}}$$

$$v_i = \left(\frac{GM}{a-R}\right)^{\frac{1}{2}}$$

$$v_o - v_i = (GM)^{\frac{1}{2}}\left(\frac{1}{(a+R)^{\frac{1}{2}}} - \frac{1}{(a-R)^{\frac{1}{2}}}\right)$$

$$= (GM)^{\frac{1}{2}}\left(\frac{(a-R)^{\frac{1}{2}}}{(a^2-R^2)^{\frac{1}{2}}} - \frac{(a+R)^{\frac{1}{2}}}{(a^2-R^2)^{\frac{1}{2}}}\right)$$

$$\approx (GM)^{\frac{1}{2}} a^{\frac{1}{2}}\left(\frac{1-\frac{R}{2a} - 1+\frac{R}{2a}}{a}\right)$$

$$\approx -\left(\frac{GM}{a^3}\right)^{\frac{1}{2}} R$$

viii) These fragments will have size R_{max}

ix) In the opposite limit $t_{sub} < \Omega^{-1}$

$\Rightarrow$ Sublimation of the body, gas will then be subject to radiation pressure from the star, being blown from system.

x) $t_{sub} \sim \dfrac{\frac{1}{2}\pi \cdot 10^3 \cdot (0.2 \cdot 2\pi \times 10^8)^2 \cdot 3000 \cdot 3\times 10^6}{\pi 10^{26} \times 30}$

$$\sim \frac{10^{29}}{10^{28}} \sim 10 \text{ s}$$

Breakup will occur on orbital timescale, Ω_k^{-1}

$$\Omega_k^{-1} = \left(\frac{a^3}{GM_*}\right)^{\frac{1}{2}} = \left(\frac{(10^8)^3}{2\pi \times 10^{-11} \cdot 10^{30}}\right)^{\frac{1}{2}} = \left(\frac{10^{24}}{10^{20}}\right)^{\frac{1}{2}} = 100 \text{ s}$$

$\therefore$ timescales comparable, breakup + sublimation happening concurrently.