Q2

i) $P_e = 2.6 \times 10^7$ years

$\therefore$ Nemesis must pass close to Oort cloud with period P_e

From K3: $\frac{a^3}{P^2} \simeq \frac{GM}{4\pi^2}$

So, $a_{nem} = \left(\frac{G M_\odot P_e^2}{4\pi^2}\right)^{\frac{1}{3}}$

$= P_e^{2/3}$ (au)

$= 9 \times 10^4$ au

To calculate eccentricity, note that Nemesis must be approaching 10^4 au to disturb Oort cloud.

as, $r_p = a(1-e)$

$\therefore -\frac{r_p - a}{a} = e$

$e = -\frac{10^4 - 9\times10^4}{9\times10^4}$

$\simeq 0.9$

ii) For orbit just grazing that of the Earth assume

$$r_p = 1 \text{ au}$$

$$r_a = 10^4 \text{ au}$$

From conservation of energy, $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$

From conservation of angular momentum, $L = mrv$

$\mathcal{E}_a = \mathcal{E}_p \qquad L_a = L_p$

$$\frac{V_a^2}{2} - \frac{\mu}{r_a} = \frac{V_p^2}{2} - \frac{\mu}{r_p}, \text{ where } \mu = GM_\odot$$

$$V_a = \left(\frac{V_p^2}{2} - \frac{\mu}{r_p} + \frac{\mu}{r_a}\right)^{\frac{1}{2}} 2^{\frac{1}{2}} \qquad (1)$$

Now conserve L

$$m r_p V_p = m r_a V_a$$

$$V_a = \frac{r_p V_p}{r_a} \qquad (2)$$

Combining (1) + (2)

$$\frac{r_p^2 V_p^2}{r_a^2} = V_p^2 - \frac{2\mu}{r_p} + \frac{2\mu}{r_a}$$

$$V_p^2 \left(r_p^2/r_a^2 - 1\right) = \frac{2\mu}{r_a} - \frac{2\mu}{r_p}$$

$$v_p^2 = 2\mu\left(\frac{1}{r_a} - \frac{1}{r_p}\right)\left(r_p^2/r_a^2 - 1\right)^{-1}$$

Now, $r_a \gg r_p$

$$\therefore v_p^2 \simeq \frac{2\mu}{r_p} \quad //$$

iii) $$v_p \simeq \frac{2 \times 2\pi \times 10^{-11} \times 2 \times 10^{30}}{\frac{\pi}{2} \times 10^4}$$

$$= \frac{8\pi\ 10^{19}}{\pi/2\ 10^4}$$

$$= 16 \times 10^8$$

$$\therefore v_p \sim 4 \times 10^4$$

$$= 40\ \text{kms}^{-1}$$

Now, $v_i = 40\ \text{kms}^{-1} + v_{esc}$

$$\sim 50\ \text{kms}^{-1} \quad //$$

iv) Estimate atm loss by energy conservation:

Energy of comet: $K = \frac{1}{2} m v_{imp}^2$

Energy to move material to ∞

$$U_g(\infty) = \int_{R_p}^{\infty} \frac{G M_p m_e}{r^2} \, dr$$

$$= \frac{G M_p m_e}{R_p}$$

Equate to find m_e, mass of ejected material

$$\frac{1}{2} m_c V_{imp}^2 = \frac{G M_p m_e}{R_p}$$

$$m_e = \frac{\frac{1}{2} m_c R_p V_{imp}^2}{G M_p}$$

$$\simeq \frac{\frac{1}{2} \cdot 5 \times 10^{14} \cdot 6 \times 10^{6} \cdot 25 \times 10^{8}}{2\pi \times 2\pi \times 10^{24}}$$

$$\simeq \frac{\pi \times 10^{28}}{4\pi^2 \times 10^{24}}$$

$\simeq 10^3$ kg In this case, net gain of volatiles

But, this will be an underestimate. Only need to remove mass from Earth's Hill Sphere.

$$r_H \sim a\left(\frac{M_p}{3M_\odot}\right)^{\frac{1}{3}}$$

$$\sim 1.5\times10^{11}\cdot\left(\frac{2\pi\times10^{24}}{3\cdot2\times10^{30}}\right)^{\frac{1}{3}}$$

$$\approx 1.5\times10^{11}\cdot\left(10^{-6}\right)^{\frac{1}{3}}$$

$$= 1.5\times10^{9}\ m$$

Returning to U_g

$$U_g(r_H) = \int_{R_p}^{r_H} \frac{GMm_e}{r^2}\,dr$$

$$= GM_p m_e\left[-\frac{1}{r_H}+\frac{1}{R_p}\right]$$

$$= 2\pi\cdot10^{-11}\cdot2\pi\cdot10^{24}\cdot\left[\frac{1}{2\pi\times10^{6}}-\frac{1}{\frac{\pi}{2}\times10^{9}}\right]m_e$$

$$= 4\pi\,10^{13}\left[5\times10^{-7}-2\times10^{-9}\right]m_e$$

$$= 2\pi\times10^{6}\ m_e$$

equaling to K

$$m_e = \frac{\frac{1}{2}\,5\times10^{14}\cdot25\times10^{8}}{2\pi\times10^{6}}$$

$\sim 10^{17}$ kg //

In the case where atmosphere just needs to get beyond the Hill radius there is a net loss of atmosphere.

v) $$V_{orb} = \left(\frac{G M_\odot}{10^5 \, au}\right)^{\frac{1}{2}} \sim \left(\frac{4\pi \times 10^{19}}{10^5 \cdot \frac{\pi}{2} 10^{11}}\right)^{\frac{1}{2}}$$

$$\sim 8 \text{ km s}^{-1}$$

$\therefore$ $V_{disp} \gg V_{orb}$ and there will not be significant gravitational focusing

Geometric cross section:

$$\text{Apocenter} = (1+e)a$$
$$= 1.9 \times 9 \times 10^4$$
$$r_a = 1.7 \times 10^5 \text{ au}$$

$$A_x \sim \pi (2 \times r_a)^2$$
$$\sim \pi (2 \times 0.8 \text{ pc})^2$$
$$\sim 8 \text{ pc}^2$$

Rate of encounter :

$$8\,pc^2 \times 0.1\,pc^{3} \times V$$

$$V \sim \underbrace{20 \times 10^3}_{ms^{-1}} \times \underbrace{10^6}_{Myr} \times \underbrace{\pi \times 10^7}_{s/yr} \Big/ 3 \times 10^{16}$$

$$\sim \frac{2 \times 10^{17}}{10^{16}}\ pc\ Myr^{-1}$$

$$\sim 20\ pc\ Myr^{-1}$$

$$\therefore \text{rate} = 16\ Myr^{-1}$$

vi) Unlikely : Nemesis - Star interactions are sufficiently frequent that Nemesis's orbit would be perturbed & the period of the extinctions would change.