Introduction to Astrophysics

Michaelmas Term, 2010: Prof Craig Mackay

Module 9: Gravitational Lensing

- Early history
- The lens equation, bending angles, Young diagrams, mass distributions, critical mass surface density.
- Einstein rings, lensing cross-sections, magnification bias, strong lensing by rich clusters.
- Weak lensing, masses of rich clusters.
- Determining H_0, micro-lensing, MACHOs,
- Constraints on baryonic dark matter in the Milky Way Halo.

Gravitational Lensing

- Imagine that photons have mass. When a photon passes near a massive object Newtonian gravity would predict that its path would be deflected by an angle ~ $2GM/(c^2.b)$, where b is the impact parameter. This was realised in 1804.
- Einstein (1915) showed that with General relativity the angular deflection would be twice the above value and predicted that for a star at the solar limb the deflection would be 1.75 arc seconds.
- This was confirmed by Eddington during a solar eclipse in 1919.
- The possibility that stars could be lensed by other stars was proposed by Einstein in 1936 and it was discovered more recently that QSOs can be lensed by galaxies.
- In 1979 the double quasar 0957+561 was discovered (Walsh, Carswell and Weymann, Nature 279, 381). This was a quasar with a redshift z = 1.41 and an angular separation between components of ~ 6 arc seconds. The lensing galaxy (which is in a cluster) has a redshift of z = 0.36.
- The two components of the quasar have identical spectra.
- Note the presence of the galaxy at the top of the lower quasar image.

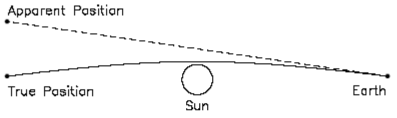
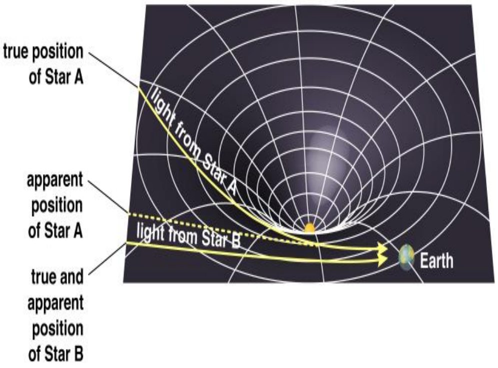
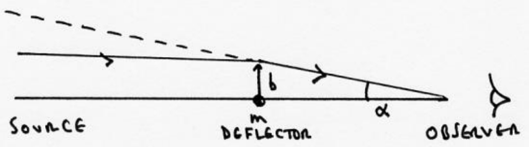


FIG. 1.—Angular deflection of a ray of light passing close to the limb of the Sun. Since the light ray is bent toward the Sun, the apparent positions of stars move away from the Sun.

Gravitational Lensing

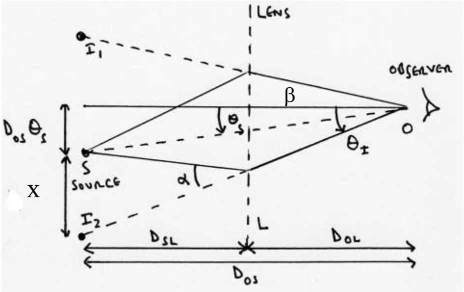
- The origin of the deflection is a distortion of space-time by a mass distribution.
- This means that all photons are equally affected and the lensing is achromatic.
- This picture shows what happens to light from a star passing close to the edge of the sun. Exactly the same thing happens when light from a distant galaxy passes close to another mass concentration on its way towards the observer.
- When gravitational lensing is used for mass determination the assumptions made are generally safer than those for virial masses and x-ray gas masses.





Gravitational Lensing

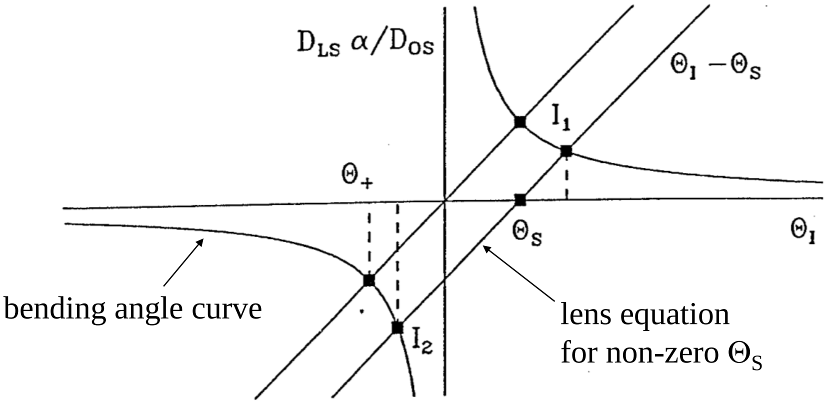
- The basic geometrical layout is shown in the figure opposite.
- The basic geometry is given as follows:
- $\Theta_S D_{OS} + \alpha D_{LS} = \Theta_I D_{OS}$
- α is the bending angle.
- $\Theta_I - \Theta_S = \alpha . D_{LS}/D_{OS}$
- This last equation is known as the “lens equation”.
- Because we know that $\alpha = 4GM/c^2b$ we can predict α versus Θ_I (the bending angle curve) for a given M(r), for example for a point mass, $\alpha \propto 1/b$.
- We can also look at graphical solutions by plotting Θ_I against $\alpha(D_{LS}/D_{OS})$, i.e. plotting the image position in the lens plane versus the bending angle.



$x = \alpha D_{LS}$
$\beta = \Theta_I - \Theta_S$
$b = \Theta_I D_{OL}$
$x = \beta D_{OS}$

Gravitational Lenses: Bending angle or Young Diagram

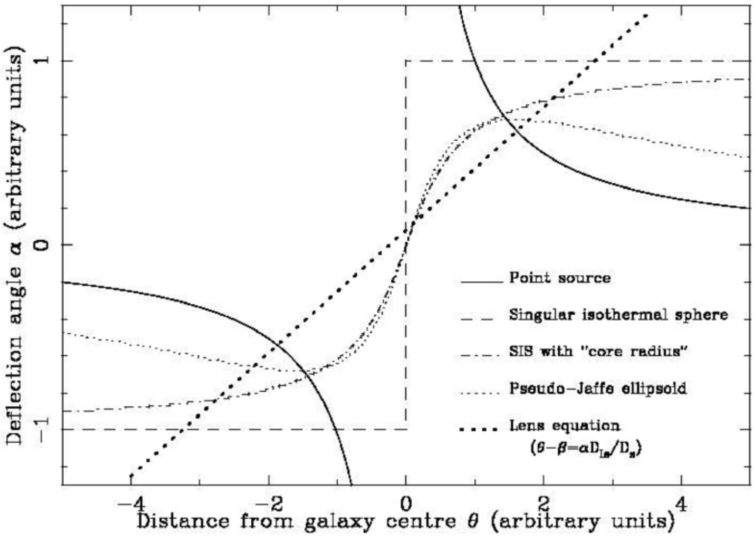
- Solutions (i.e. images) occur when the lens equation line intersects the bending angle curve.
- The number of images will depend on the shape of the bending angle curve.
- For a **point mass** there will be two images.



bending angle curve

lens equation for non-zero Θ_S

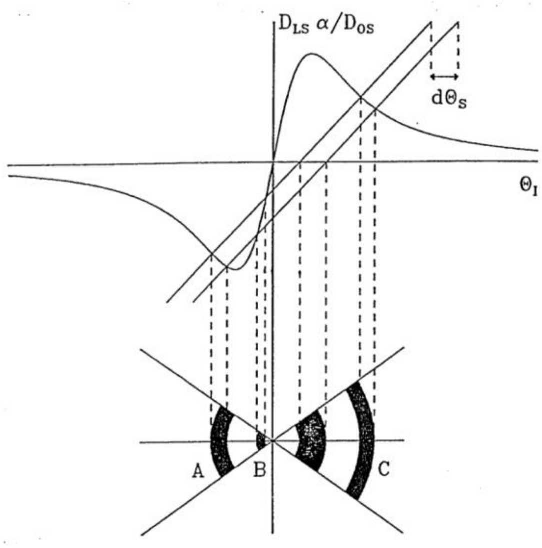
Gravitational Lensing: Bending angle or Young Diagram



This figure shows the bending angle curves for four mass distributions. In the general case the bending angle has to be computed from a vector sum of angles for each element of the three-dimensional mass distribution.

Gravitational Lenses: Bending angle or Young Diagram

- For an elliptical mass distribution there can be three images (labelled A, B and C).
- If the source image is extended then the lens equation can be thought of as a broadened line.
- If the projected mass surface density is below a critical value the slope of the bending angle curve near the origin will be less than that of the lens equation and there will be no lensing (for $\Theta_S = 0$).

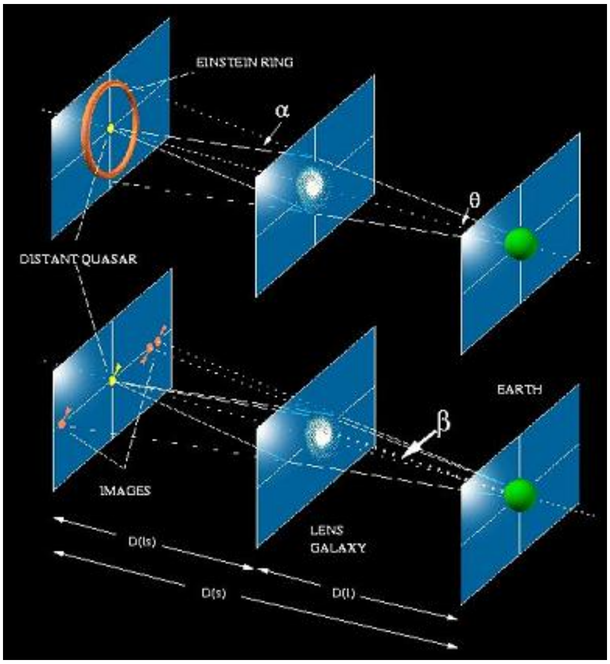


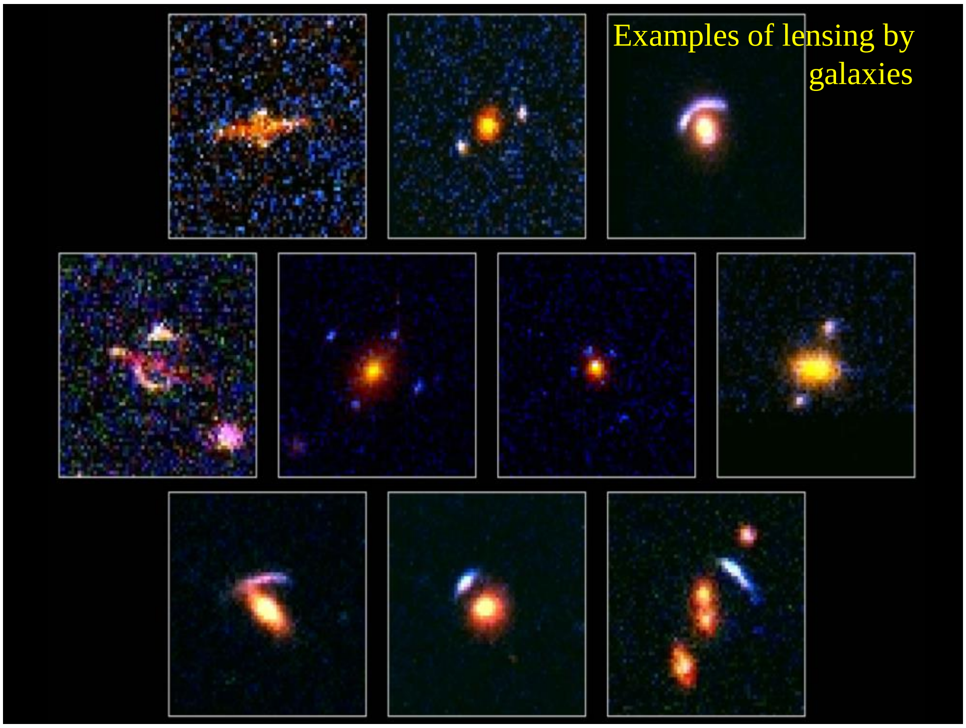
Gravitational Lensing: Critical surface mass density

- There will only be a significant deflection if the lens mass distribution is adequately concentrated and the rays from the source to the observer pass close enough to the centre of this mass distribution.
- There is effectively a critical surface mass density which has to be achieved. This is given by...
- $\Sigma_{crit} = (c^2/4\pi G)\,(D_{OS}/D_{OL}\,D_{LS}) \sim 3.5\,(D_{OL}/1000\mathrm{Mpc})^{-1}$ kg m^{-2}, (exercise: check that 3.5 is right) for $D_{OL} = D_{LS}$, i.e. when the lens is midway between the observer and the source.
- Lensing is most effective when $D_{OL} = D_{LS}$
- Do galaxies and galaxy clusters reached this critical surface mass density, Σ_{crit}? Exercise for the student.

Gravitational Lensing: Einstein Rings

- If the lens has circular symmetry and the source and the lens and the observer lie on a straight line then it should be obvious that the image obtained from this gravitational lens will be a circular ring.
- This ring is called an Einstein Ring. The radius of the ring Θ_E, centred on the lensing object, is called the Einstein Radius.
- In the more general case (source, lens and observer not aligned or asymmetric mass distribution) the distant object is lensed into multiple images.





Examples of lensing by galaxies

Gravitational Lensing: Einstein Rings

- We can work out the angular diameter of an Einstein ring as follows:
- from the lens equation we have [1]:

$$\Theta_S = \Theta_I - \frac{\alpha D_{LS}}{D_{SO}} \qquad \text{-[1]}$$

- the bending angle, α, is then given by [2]:

$$\alpha = \frac{4GM(b)}{c^2 b} = \frac{4GM(\Theta)}{c^2 \Theta D_{OL}} \qquad \text{-[2]}$$

- Setting $\Theta_S = 0$ and eliminating α gives the semi-angular diameter of the ring [3]:

$$>> \Theta_E = \left(\frac{4GM(\Theta_E)}{c^2} . \frac{D_{LS}}{D_{OL} D_{OS}} \right)^{\frac{1}{2}} \qquad \text{-[3]}$$

- For a distant source, and nearby lens:

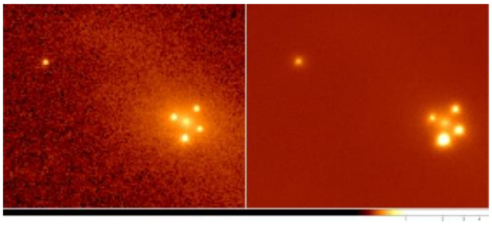
$$D_{OS} \approx D_{LS}$$

$$\Theta_E \propto M^{\frac{1}{2}} D_{OL}^{\frac{-1}{2}}$$

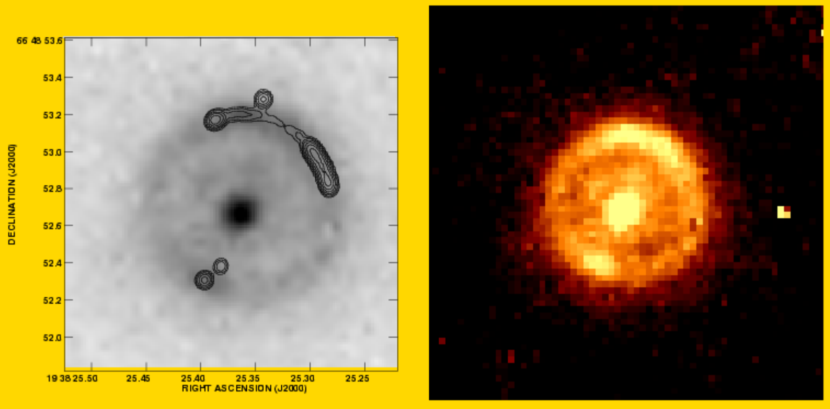
- With mass in solar units and distance in parsecs we get this expression [4] for the radius of the ring in arcsec.

$$\approx \left(\frac{M}{10^{12}} \right)^{\frac{1}{2}} . \left(\frac{D_{OL}}{10^9} \right)^{\frac{-1}{2}} arc\sec \qquad \text{-[4]}$$

- This is approximately one arc second for a lens with the mass of 10^{12} $M_{\odot}$ at a distance of 1000Mpc. i.e. a galaxy.
- This was the first gravitational lens imaged with the Faint Object Camera on HST(right). Quasar z=1.7, galaxy z=0.04



Gravitational Lensing: an Einstein Ring



- This is a nearly complete Einstein ring, a radio AGN B1938+666.
- The left-hand image is the Merlin radio map superimposed on a grey scale copy of the right-hand image which is taken with the Hubble space telescope NICMOS infrared camera.
- It clearly shows the lensing galaxy in the centre and this is found to have a red shift z = 0.88.
- The diameter of the ring is ~ 0.95 arcseconds.
- The source red shift is unknown.

Gravitational Lensing: Cross Sections

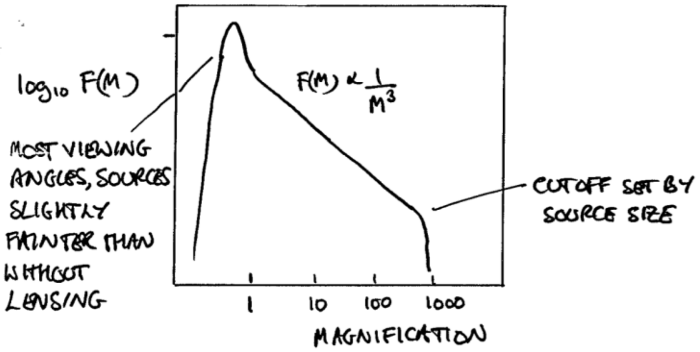
- If we have a massive galaxy ~ 10^{12} $M_\odot$ at 1000 Mpc then Θ_E ~ 1 arc second.
- For a rich cluster of galaxies with M ~ $10^{15} M_\odot$ at the same distance with the surface mass density above the critical level then Θ_E ~ 30 arc seconds.
- A compact very distant source can be magnified considerably by a gravitational lens. The surface brightness is unchanged, (Liouville's Theorem) but the observed solid angle of the object increases greatly.
- If the lensed source has a particular diameter then the thickness of the Einstein ring will be that same diameter. However now it will be multiplied by the circumference of the Einstein ring and therefore we see that small sources can experience very large brightness magnifications.
- These can be as large as $2.\Theta_{E/}\,\Theta_{S.}$
- The converse is that if the diameter of the source is close to that of the Einstein ring diameter then there will be no significant brightening.

Gravitational Lensing: Cross Sections

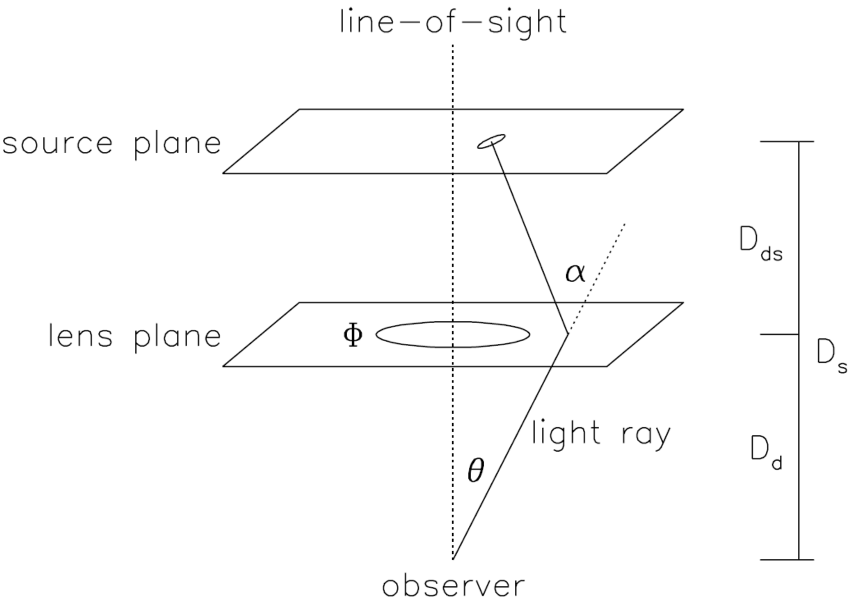
- If the impact parameter, b, is less than the Einstein ring radius Θ_E then we have significant lensing (multiple images).
- Therefore the cross-section for lensing is = $\pi\,\Theta_E^2$.
- $\Theta_E \propto M^{1/2}$ implies the cross-section area is $\propto$ M.
- The total mass cross-section is independent of the individual masses – e.g. lots of small lenses are equivalent to one big one.
- The frequency of lensing is proportional to the fraction of the objects that contain clumps at the critical density or above.
- In the limit of very small masses and a smooth mass distribution we get no gravitational lensing.
- For clusters of galaxies, Mass $\propto v^2 = \sigma^2$, where σ is the velocity dispersion of galaxies.
- Also $L_{xray} \propto n_e^2 \propto M^2 \propto \sigma^4$ and therefore the cross-section for lensing $\pi\,\Theta_E^2 \propto M \propto L_{xray}^{0.5.}$
- The most massive, x-ray luminous clusters will therefore have the largest lensing cross-section.

Gravitational Lensing: Magnification Bias

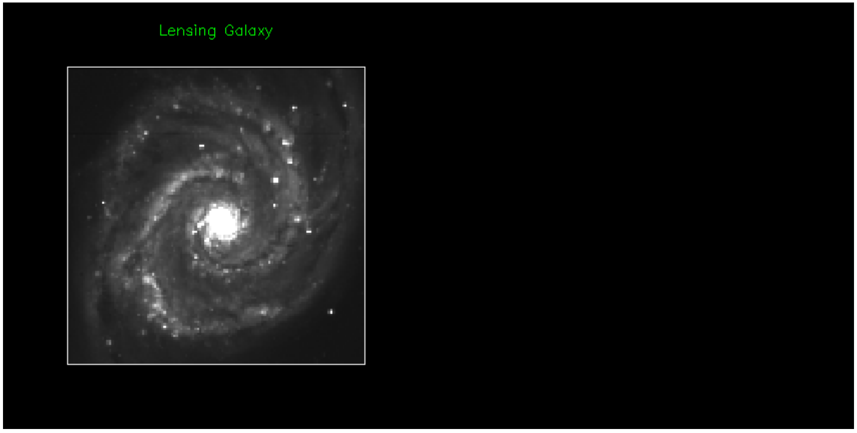
- For a single source, over the whole sky, <magnification> = 1
- Also for a population of sources, over the whole sky, <magnification> = 1



- For a population of sources above a flux limit, <magnification> > 1
- This is called ***magnification bias.***

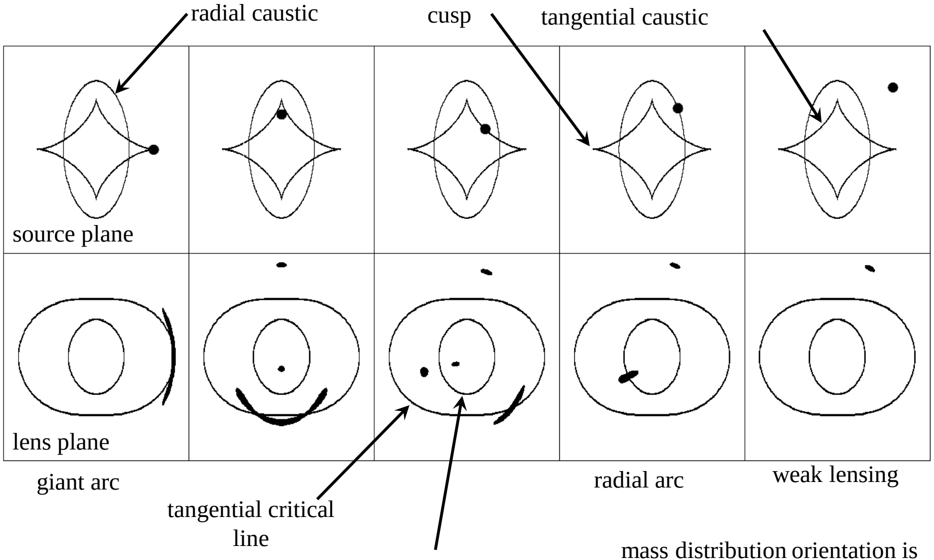


Animation of what happens when the source moves in the source plane with a rich cluster of galaxies acting as a lens.



The first picture is an optical image of a typical spiral galaxy that acts as the lens in this demonstration. So that the lensing effect is more clearly visible, the galaxy is represented by a yellow circle.
Next, a view of a distant quasar as seen by a radio telescope is shown in the left panel. In this demonstration, the quasar is moved behind the galaxy from the top left of the frame to the lower right. The right panel shows what we would see from Earth as the gravity of the galaxy bends the light from the quasar.
When the quasar is directly behind the galaxy, the light is distorted into an Eintein ring similar to that seen in 1830-211.

Caustics and critical lines for an elliptical mass distribution. (lines of high magnification)



radial caustic

cusp

tangential caustic

source plane

lens plane

giant arc

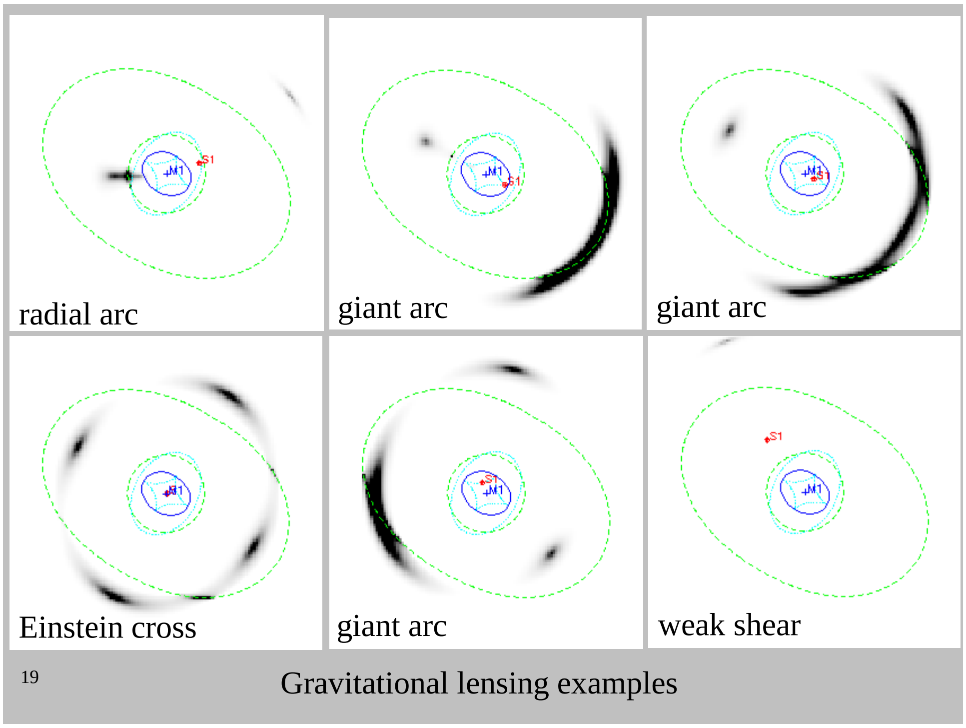
tangential critical line

radial critical line

radial arc

weak lensing

mass distribution orientation is same as tangential critical line



radial arc

giant arc

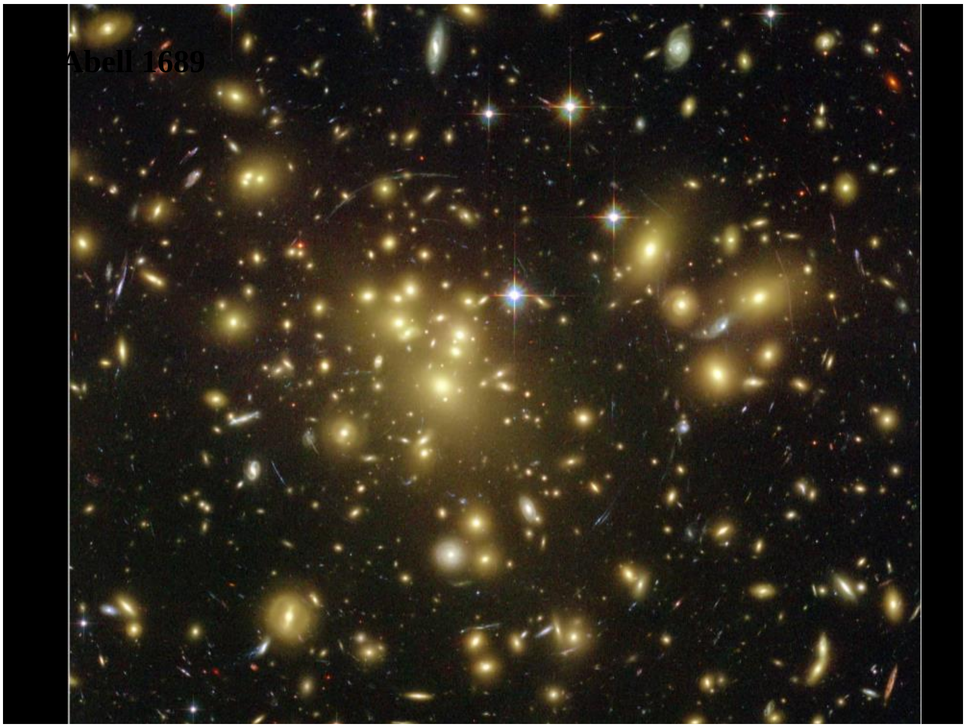
giant arc

Einstein cross

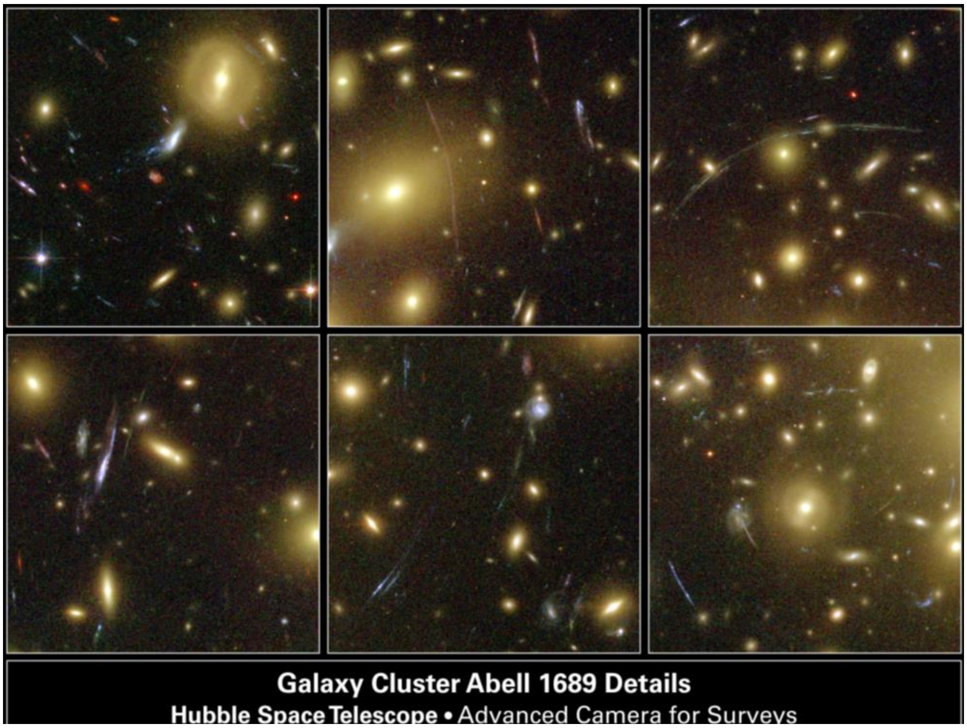
giant arc

weak shear

Gravitational lensing examples



Abell 1689

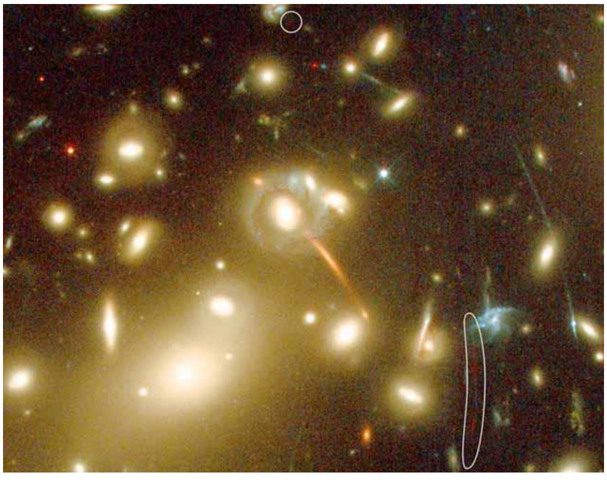


Galaxy Cluster Abell 1689 Details

Hubble Space Telescope • Advanced Camera for Surveys

Gravitational Lensing: Arcs

- Large arc images which comprise a significant fraction of an Einstein ring tell us that the average surface density within the diameter of this arc must be ~ Σ_{crit} , the critical surface density.
- The mass interior to the arc is therefore $\pi\Theta_E^2\Sigma_{crit}$ which is ~ $1.1\text{x}10^{14}$ $(\Theta/30")^2$(D/1000 Mpc) in solar masses.
- It means that we know the mass interior to the arc very precisely but we do not know M(r).

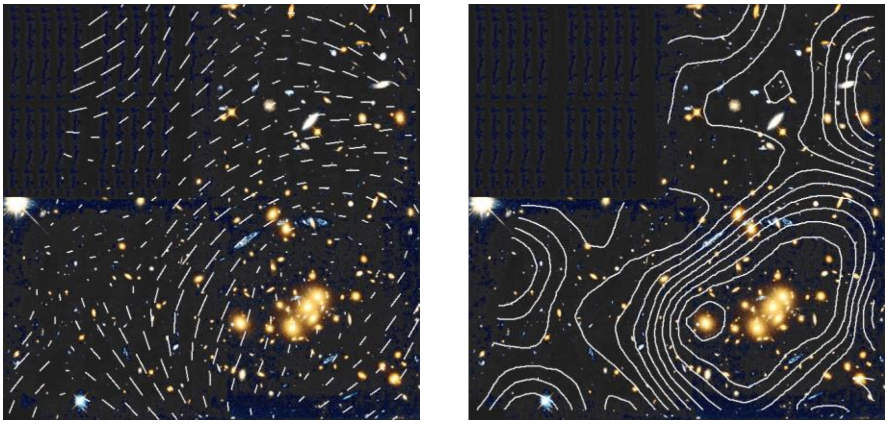


- This cluster, Abell 2218 , has a gravitationally lensed image which may be one of the most distant objects ever detected, a galaxy at a red shift z ~ 7 (not confirmed yet).
- Without magnification by gravitational lensing this galaxy would be much too faint ever to study.
- "Gravitational telescopes" are therefore important for the investigation of the most distant and faintest objects in space.

Weak Gravitational Lensing

- In some of the most impressive pictures from the Hubble space telescope we see great numbers of gravitationally lensed images.
- However we know that over most of the sky that there is little obvious evidence of lensing.
- We would expect, nevertheless, that the general distribution of matter throughout the universe will typically distort the images of distant galaxies and gives us important information on ***the large-scale mass distribution in the universe***.
- Images of typical galaxies in the field (i.e. away from rich clusters) are likely to be extended by only a fraction of one arc second at most.
- Any weak-lensing study therefore has to be able to look over wide areas of sky and look for extremely small distortions which when combined together become statistically significant.
- It turns out this work is extremely difficult and it is very hard to get reliable estimates of the mass distributions from the data that are available from ground based astronomy.
- The technique is probably not going to give significant results until there are telescopes on the ground which are able to provide angular resolutions of about 0.2 arc seconds or better over large fields of view.
- It may be that the surveys would be better done in the near infrared where the atmosphere is less of a problem, now that large area infrared detectors are becoming available.
- Image distortions due to the telescope optics are also a major problem.

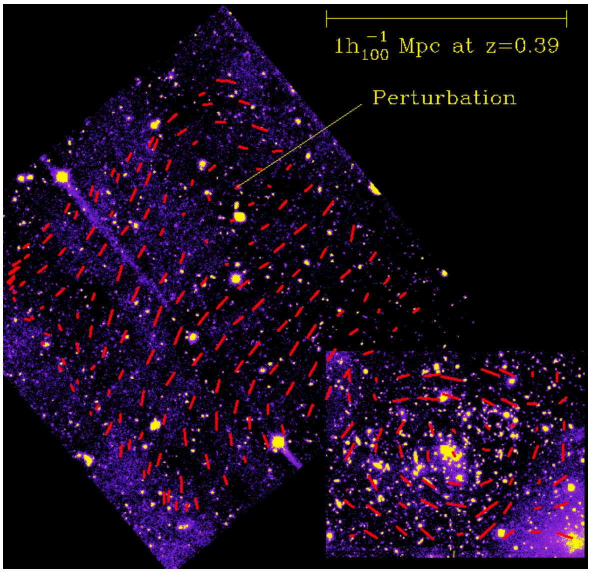
Weak Gravitational Lensing



- These are results from the Hubble space telescope where the elongation of individual objects is measured and local averages provide mean orientation and elongation vectors around a particular galaxy or group of galaxies.
- This is then converted into a projected mass distribution over the cluster and shown in the picture on the right.
- However, we do know that the galaxies in a cluster are preferentially aligned, so small area studies such as these must be somewhat suspect.

Weak Gravitational Lensing

- These images show the weak shear observed by Yannick Mellier and colleagues in the periphery of the rich cluster 0024+16.
- Such measurements offer a promising method of tracing the dark matter profile and constraining the total mass density contributed by clusters of galaxies.
- However they remained technically challenging and the results are so far not as accurate as we would like them to be!



Weak Gravitational Lensing: Survey requirements

- High signal-to-noise
- High spatial definition, better than 0.2 arcsec.
- Surveys need to cover large areas. Rich clusters of galaxies are several Mpcs in diameter, and the surveys need to extend to many times that diameter.
- The image distortions due to the atmosphere and instrumental effects have to be calibrated out with great precision.

Masses of Rich Clusters

- Virial masses give us

$$M_{\mathrm{v}}(\theta) \approx 1.1 \times 10^{14} M_{\odot} \left(\frac{\sigma_v}{10^3\,\mathrm{km\,s^{-1}}}\right)^2 \left(\frac{\theta}{30''}\right) \left(\frac{D}{1\,\mathrm{Gpc}}\right)$$

- Masses from x-ray emitting gas give us

$$M_{\mathrm{X}}(\theta) \approx 0.7 \times 10^{14} M_{\odot} \left(\frac{T}{10^8\,\mathrm{K}}\right) \left(\frac{\theta}{30''}\right) \left(\frac{D}{1\,\mathrm{Gpc}}\right)$$

- Masses from lensing give us

$$M_{\mathrm{gl}}(\theta_{\mathrm{arc}}) \approx 1.1 \times 10^{14} M_{\odot} \left(\frac{\theta_{\mathrm{arc}}}{30''}\right)^2 \left(\frac{D}{1\,\mathrm{Gpc}}\right)$$

- The three agree quite well!

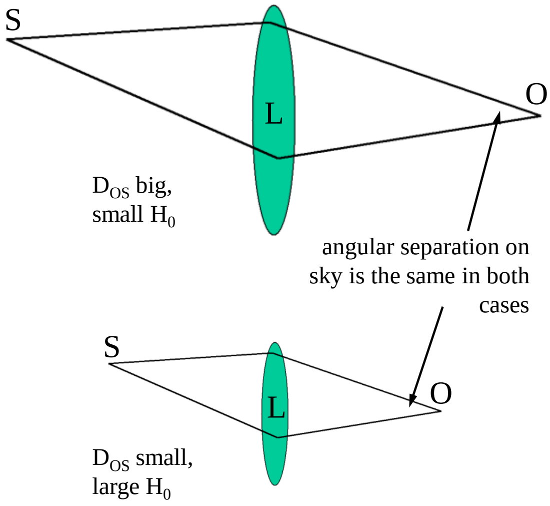
Masses of Rich Clusters

- However.....
- Virial equilibrium is debatable.
- Hydrostatic equilibrium for the x-ray gas is also debatable.
- Simple mass estimates derived from large arcs are systematically too high because of cluster asymmetries and substructure. Only detailed mass models capable of reproducing the observed arcs can yield more reliable cluster masses.
- Weak lensing analyses encompass the whole cluster mass but have to take into account boundary effects due to the finite size of the survey and also a degeneracy known as the "**mass-sheet degeneracy**"
- A uniform mass-sheet can be added to a lens model without altering the observed ellipticities of the weakly lensed images.
- However, a mass-sheet does affect the sizes of the images so including this information in the analysis breaks the degeneracy. This is a second order effect so is not so easy to detect when the first-order effect is so hard to see in the first place.

Gravitational lensing: Determining H_0

- Measuring the redshifts (i.e. radial velocities) of the lens and the source combined with an adopted cosmology (i.e. H_0) defines exactly the geometry.
- This means we can determine all the D parameters (i.e. the observer-lens distance, the observer-source distance and the lens-source distance).
- A model for the lens mass distribution can be constructed that accurately predicts the observed lens images.
- We can also estimate the difference in path length from the source to the observer that corresponds to each lensed image.
- However, all the distances are dependent on H_0.
- For example, both cases shown on the right are consistent with the measured redshifts and imaging data (measured angles).
- We need to measure one of the distances in the model independently so we can set the scale and hence get H_0.

Radial velocity = $H_0 \times$distance



D_{OS} big, small H_0

angular separation on sky is the same in both cases

D_{OS} small, large H_0

Gravitational Lensing: Measuring H_o

- For a variable source (and many lensed QSOs are variable) we can measure the time delay if we have two or more image paths.
- dt = Δ(distance)/c, and because $H_o \propto 1/\Delta$(distance) we are able to measure H_o directly.
- After many years of observation we know the time delay (417 days) for the two images from the gravitationally lensed quasar 0957+561 to about 3%.
- The time delay directly gives the path difference which is essentially a standard ruler that allows us to determine H_0.
- The largest uncertainty is the lens model.

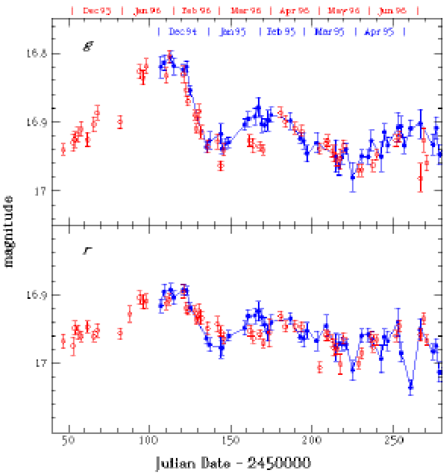
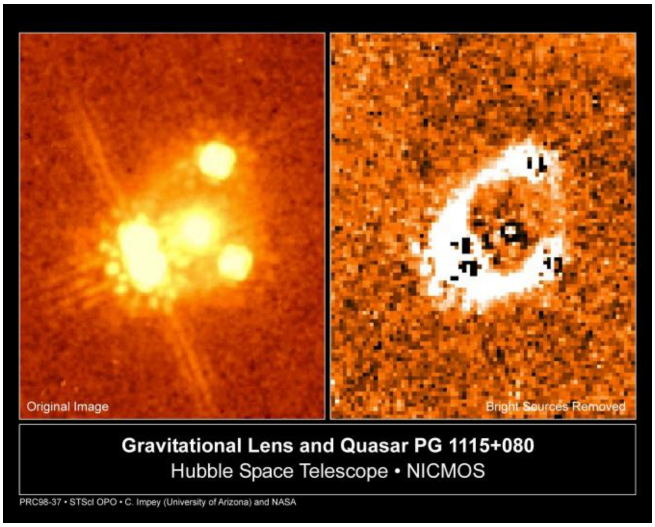


Figure 8: *Optical Lightcurves of images Q0957+561 A and B (top panel: g-band; bottom panel: r-band). The blue curve is the one of leading image A, the red one the trailing image B. Note the steep drop that occured in December 1994 in image A and was seen in February 1996 in image B. The light curves are shifted in time by about 417 days relative to each other. (Credits: Tomislav Kundić; see also [79])*

Gravitational Lensing: Measuring H_o

- On the left of this image is the lensing galaxy and four images of the lensed quasar.
- After subtraction of the five bright images we can see most of the Einstein ring.
- Such a system of four images plus a ring constrains the model for the galaxy mass distribution very well and so allows an improved estimate of H_0 using the observed time delays.
- This should be contrasted with 0957+561 which had a galaxy plus cluster and only two quasar images



Gravitational Lens and Quasar PG 1115+080
Hubble Space Telescope • NICMOS

PRC98-37 • STScI OPO • C. Impey (University of Arizona) and NASA

Gravitational Lensing: Microlensing and MACHOs

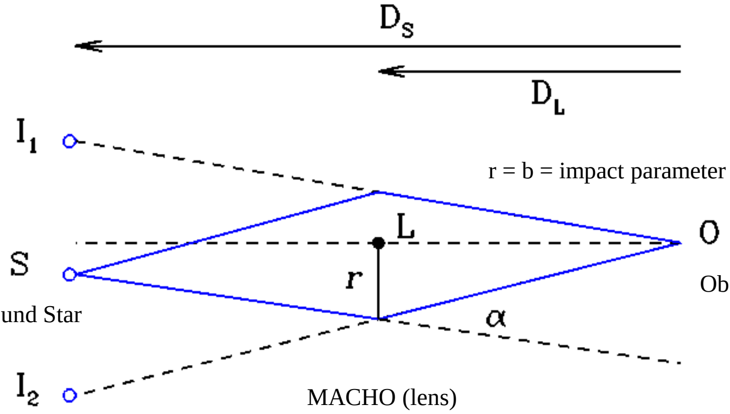
- What is the nature of the dark matter that is essential to hold our galaxy together?
- From observations of the rotation of the disk of our galaxy and the motions of globular clusters and satellite galaxies the mass of the Milky Way halo is ~2×10^{12} solar masses.
- One possibility is that there is a halo of dark, massive, baryonic, compact objects around our galaxy. These are known as **MACHO**s (**MA**ssive **C**ompact **H**alo **O**bjects).
- The combined motion of one of these MACHOs , the Earth and a background star can give rise to an observed brightening of the background star due to gravitational lensing. This is called ***micro-lensing.***
- We have seen that the radius of the Einstein ring is given by: $$\Theta_E = \left(\frac{4GM(\Theta_E)}{c^2}\cdot\frac{D_{LS}}{D_{OL}D_{OS}}\right)^{\frac{1}{2}}$$
- If we had a lens of 0.1 solar masses at a distance of ~ 25 kpc (half the distance of the Large Magellanic Cloud (LMC), and the source was in the LMC at ~ 50 kpc then Θ_E ~ 3AU.
- At 25 kpc this angle is 0.0001 arc seconds, so not even close to being resolvable from earth.
- The amplification factor of the source is given by:
- (u is dimensionless: Θ_E and b in same units).

$$A = \frac{u^2+2}{u\sqrt{(u^2+4)}}, where\ u = \frac{b}{\Theta_E}$$

$$\text{So } A = 1.34 \text{ for } b = \Theta_E \quad (u = 1)$$

- We can also work out the duration of one microlensing event using estimates of the velocity of a MACHO as being approximately equal to the velocity dispersion of stars in our galaxy.
- This gives us duration estimates of: $\Delta t \sim 3\text{days}\ \sqrt{M/M_J}$ and $97\text{days}\ \sqrt{M/M_{SUN}}$

Geometry of a MACHO micro-lens



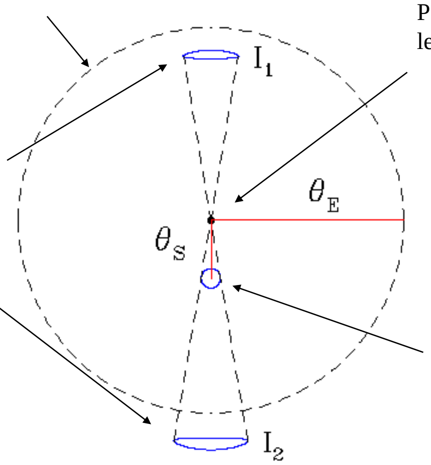
r = b = impact parameter

Observer

Background Star

MACHO (lens)

What the observer sees (with perfect vision)



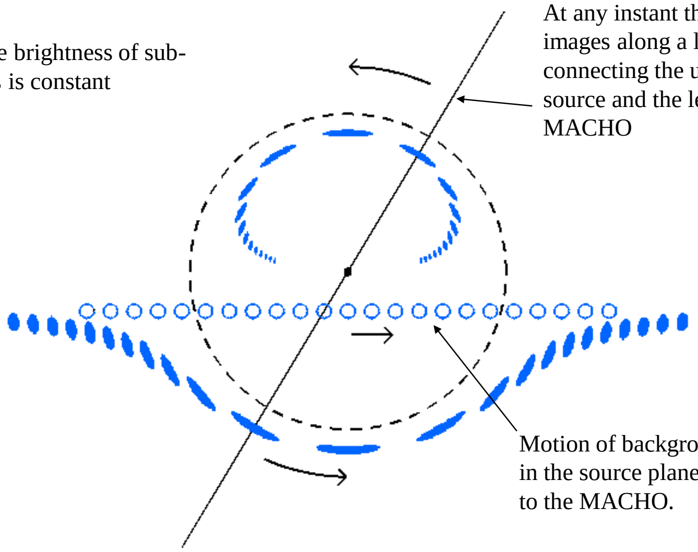
Einstein ring

Position of the lensing star

2 images of the background star

Where the background star would be if it wasn’t lensed

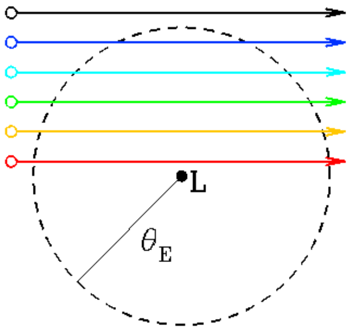
What the observer sees as the stars move



Surface brightness of sub-images is constant

At any instant there are two images along a line connecting the unlensed source and the lensing MACHO

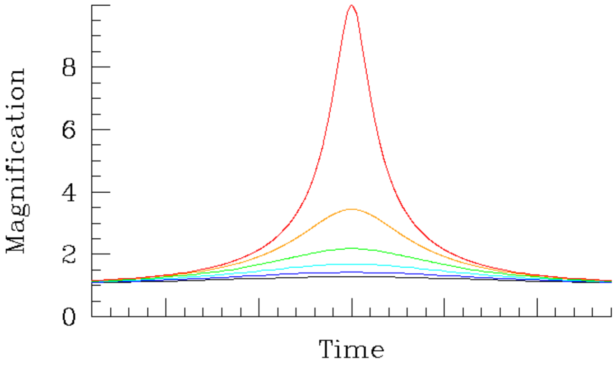
Motion of background star in the source plane relative to the MACHO.

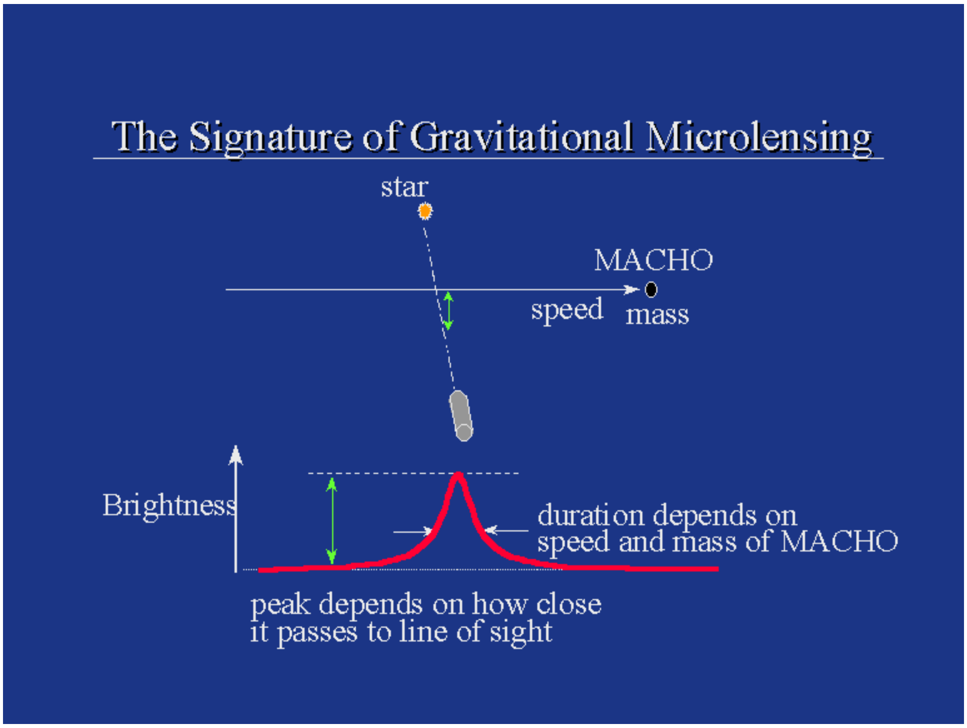


The closer they line up the brighter the lensed image becomes

$$A = \frac{u^2+2}{u\sqrt{u^2+4}}, \quad u = \frac{b}{\Theta_E}$$

Signal is proportional to the total area of the images →



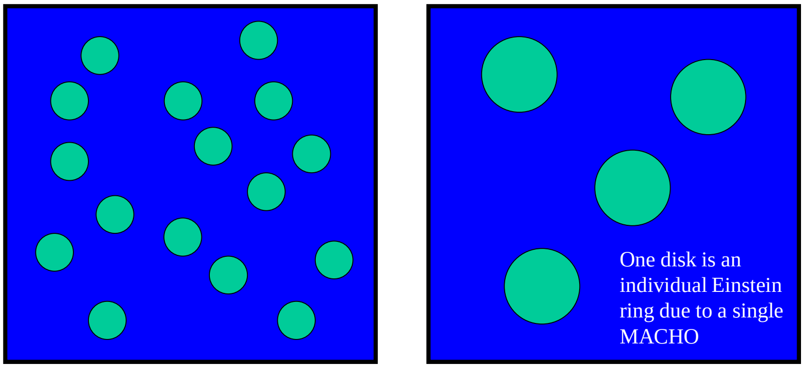


The Signature of Gravitational Microlensing

duration depends on speed and mass of MACHO

peak depends on how close it passes to line of sight

Micro-lensing optical depths and cross-sections

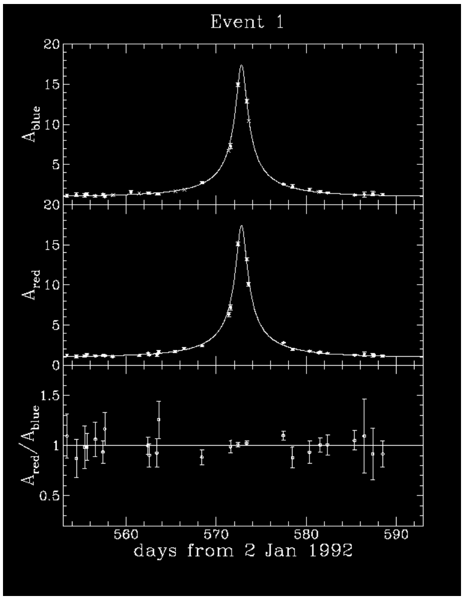


One disk is an individual Einstein ring due to a single MACHO

- Total mass $\propto$ Total cross-section.
- Total cross-section depends on total mass but is independent of the mass distribution. Total mass and cross-section are the same in the two diagrams.
- FOV=field of view (solid angle, steradians or square arcmins etc.)
- fill-factor = [Total cross-section in the FOV]/FOV
- Optical depth = fill-factor

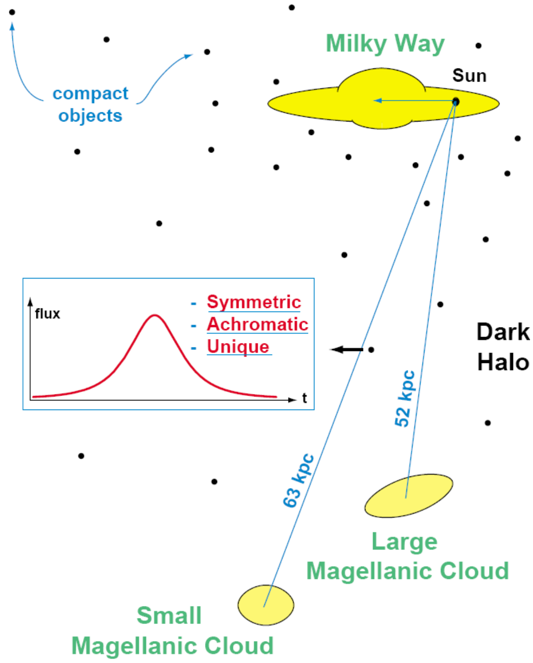
Gravitational Lensing: Microlensing

- We can calculate the optical depth to microlensing in the halo of our galaxy. For example we can assume that the entire missing mass of the halo is due to MACHOs. This gives an optical depth of about 5×10^{-7}
- Many stars vary intrinsically and therefore can produce light curves that cause confusion in microlensing surveys.
- A microlensing event should be distinguishable from other events because…
- The brightening should be **achromatic** (no wavelength dependence).
- The brightening should be **unique** (i.e not repeat).
- The brightening should be **symmetrical**, because the lens should move in front of the source star and then move away at the same rate.
- Such events have indeed been found.



The MACHO project:

- Survey of the bar of the LMC taken in Australia. About 13-17 lensing events seen in 5.7 years.
- EROS 1 and EROS 2: surveys taken from Chile of the LMC and the SMC. Four events seen towards the SMC and 5 seen towards the LMC.
- Expect one part in 2 million of the surface area of the sky to be lensed at any one time (2 million is the reciprocal of the optical depth).



surveys : EROS 2, MACHO, OGLE 2
follow-up : PLANET, GMAN

Gravitational Lensing: Microlensing

- The EROS and MACHO data constrain how many MACHOs contribute to the dark matter content of our galaxy.
- We can draw contours on a plot of the fraction of the halo mass that might be in such objects as a function of the mass of the deflectors.
- There was no evidence from either project for short microlensing toward the LMC with event durations between a few hours and 20 days. This rules out a large mass in the form of planets and brown dwarfs.
- EROS tells us that there could be a moderate fraction of the total Halo mass in deflectors with mass between 10^{-7} and 1 solar mass – but not enough to account for the missing mass in our galaxy.
- On the other hand the MACHO experiment says that a substantial fraction of the Halo could be in the form of objects between 100% and 10% of one solar mass (between 8% and 50% of the Halo in the form of 0.5 solar-mass objects).
- Why the differences?
- Variable star contamination? MACHO more crowded.
- EROS bigger survey area – less self-lensing.
- EROS both SMC and LMC, not just one.

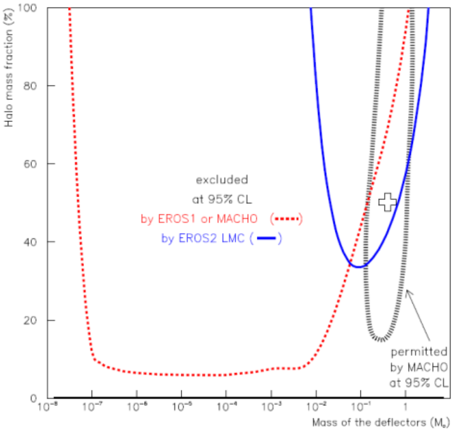
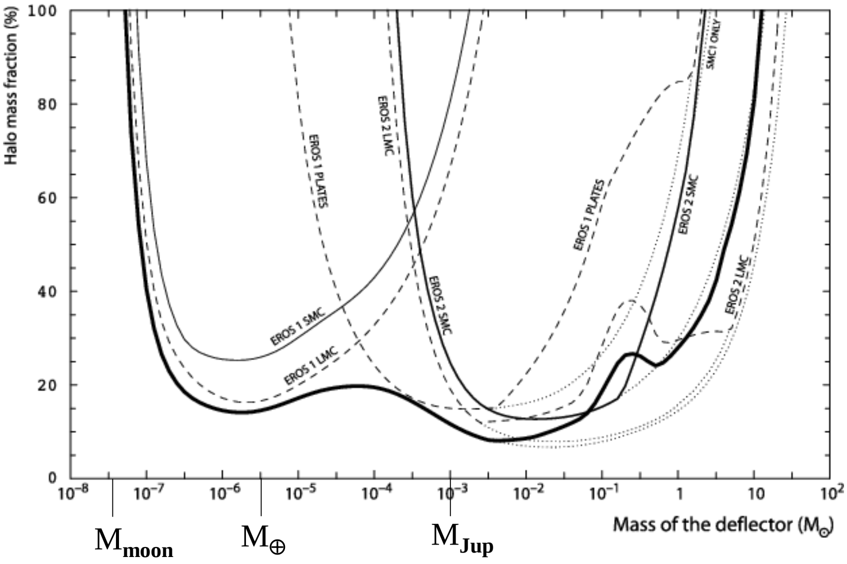


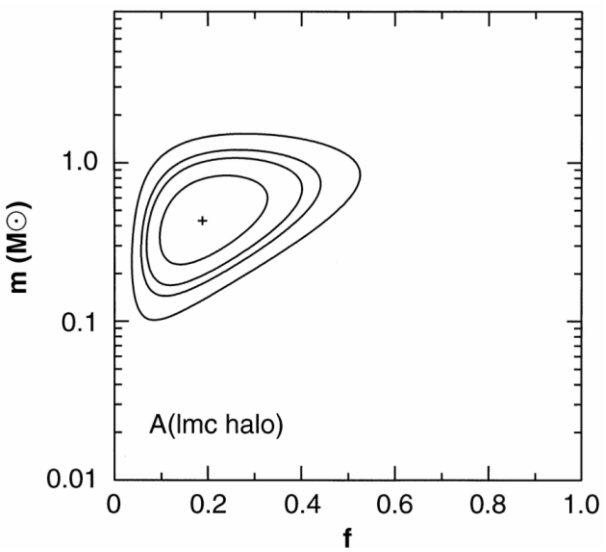
Figure 1. 95% CL exclusion diagram on the halo mass fraction in compact objects for the standard halo model. The dashed line indicates the merging of the MACHO results (at low mass) and the EROS1 (photographic plates+CCD) experiments. The cross is centered on the area allowed at 95% CL obtained by the MACHO LMC two year analysis (Alcock et al. 1997a) (thick dashed line). The full line shows the preliminary exclusion limit derived from our LMC analysis.

Halo Mass fraction:
95% confidence limit (CL) constraints from EROS



Constraints on Halo objects from the MACHO project

Most likely result:
20% of the Halo mass fraction, f, is in the form of 0.5 solar mass stars.



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