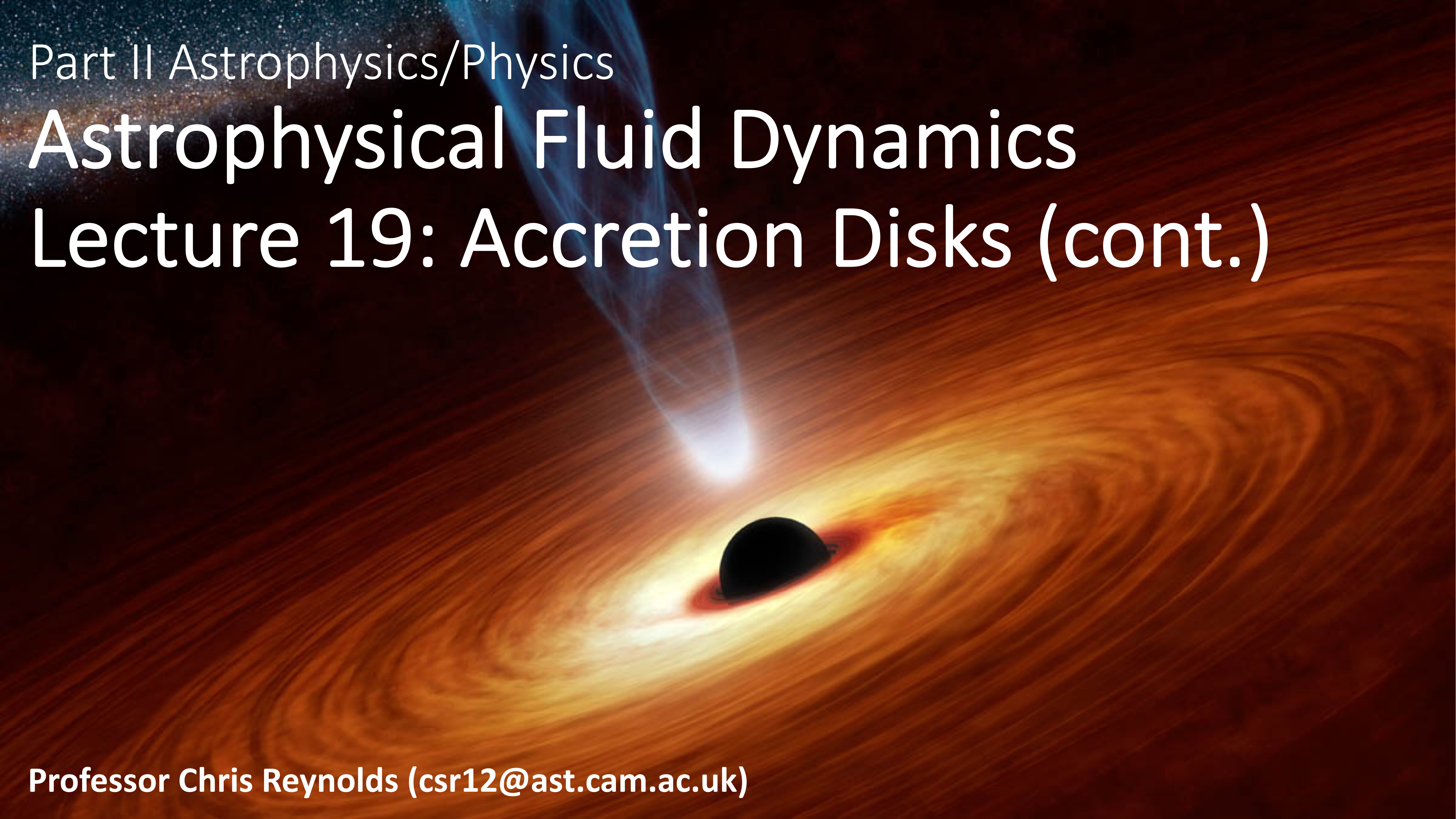


Part II Astrophysics/Physics

Astrophysical Fluid Dynamics
Lecture 19: Accretion Disks (cont.)

Professor Chris Reynolds (csr12@ast.cam.ac.uk)

Recap – last two lecture

- Introduced notion of viscosity
  - Diffusion of momentum through fluid due to the particle nature of fluid
  - Leading order finite-λ correction
- Viscous stress tensor and the Navier-Stokes equations
- Diffusion of vorticity in a viscous flow
- Energy dissipation in a viscous flow (2^{nd} law of TD $\Rightarrow$ positivity of η)
- Viscous flow through pipe
- Accretion disks
  - Importance of angular momentum transport to accretion
  - Basic equations of geometrically thin-disk theory

This Lecture – more on accretion disks

- Revisiting accretion disk formation
- The nature of viscosity – the little white lie of viscous disk models
- Energy dissipation and cooling processes in disks
- Steady-state disks

Revisiting disk formation

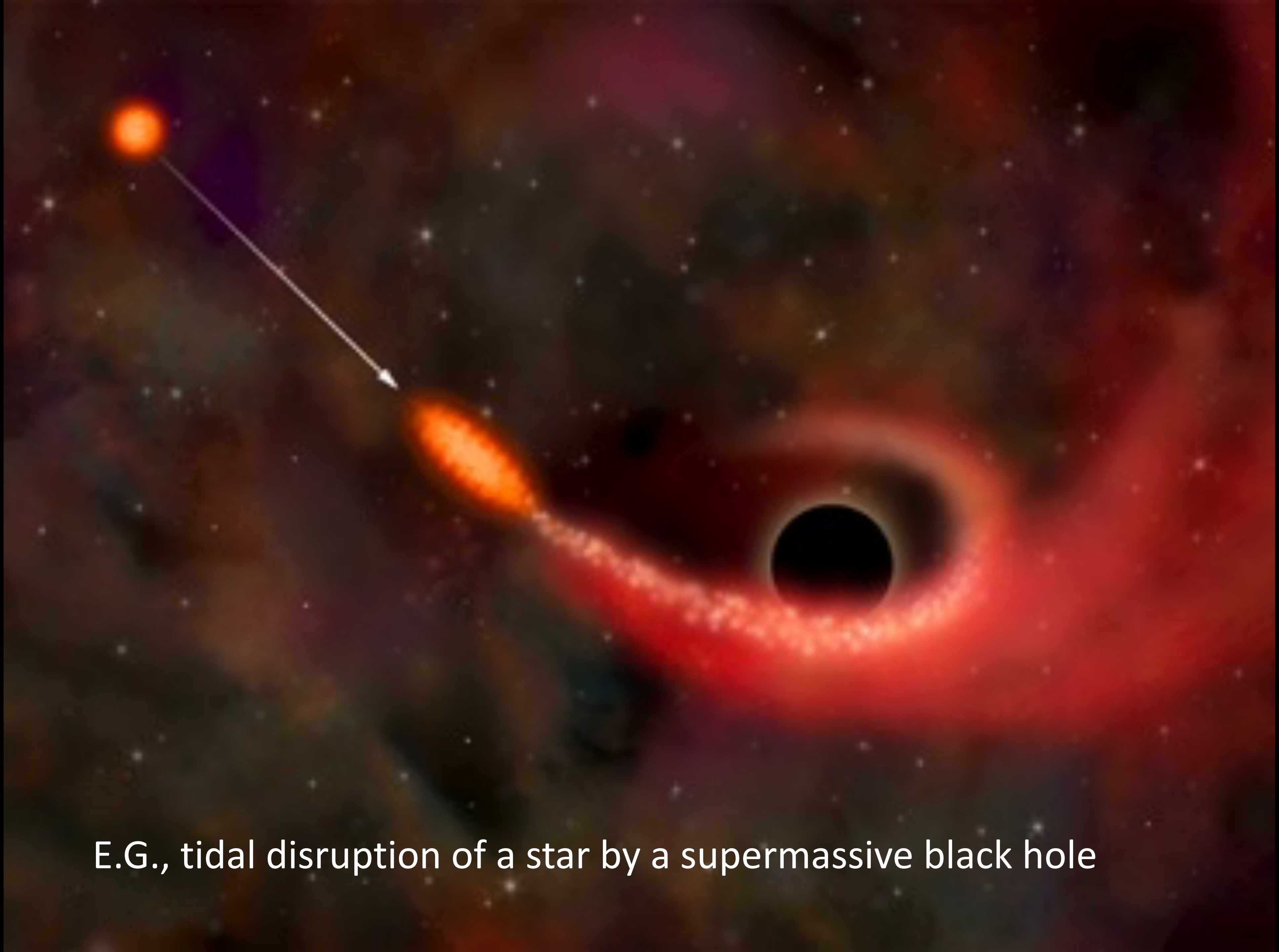
Recall that the surface density of viscous accretion disks evolves according to a diffusion equation:

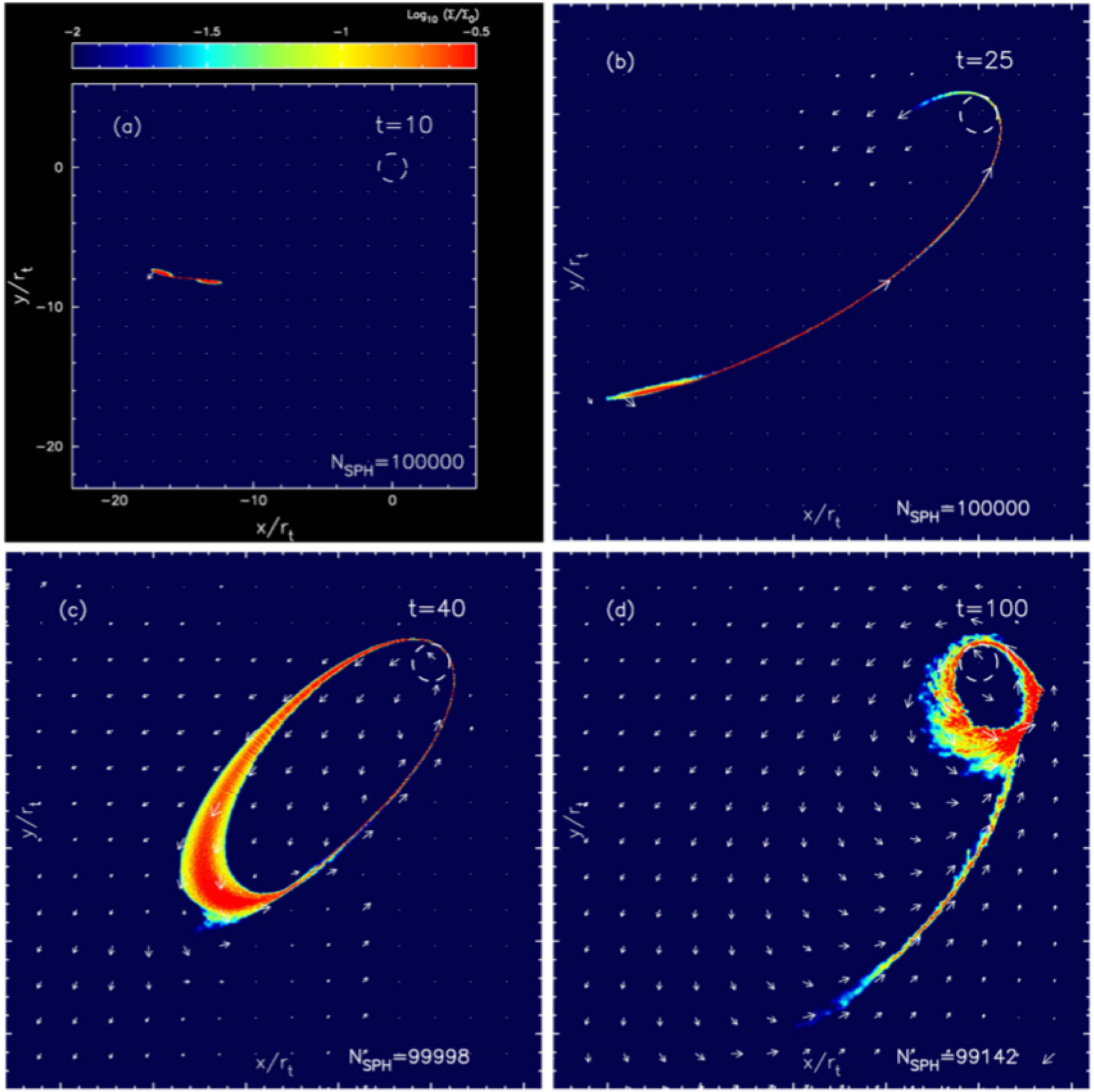
$$\boxed{\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}(\nu \Sigma R^{1/2})\right]}$$

So an initial narrow annulus of matter will spread out to form a full disk.

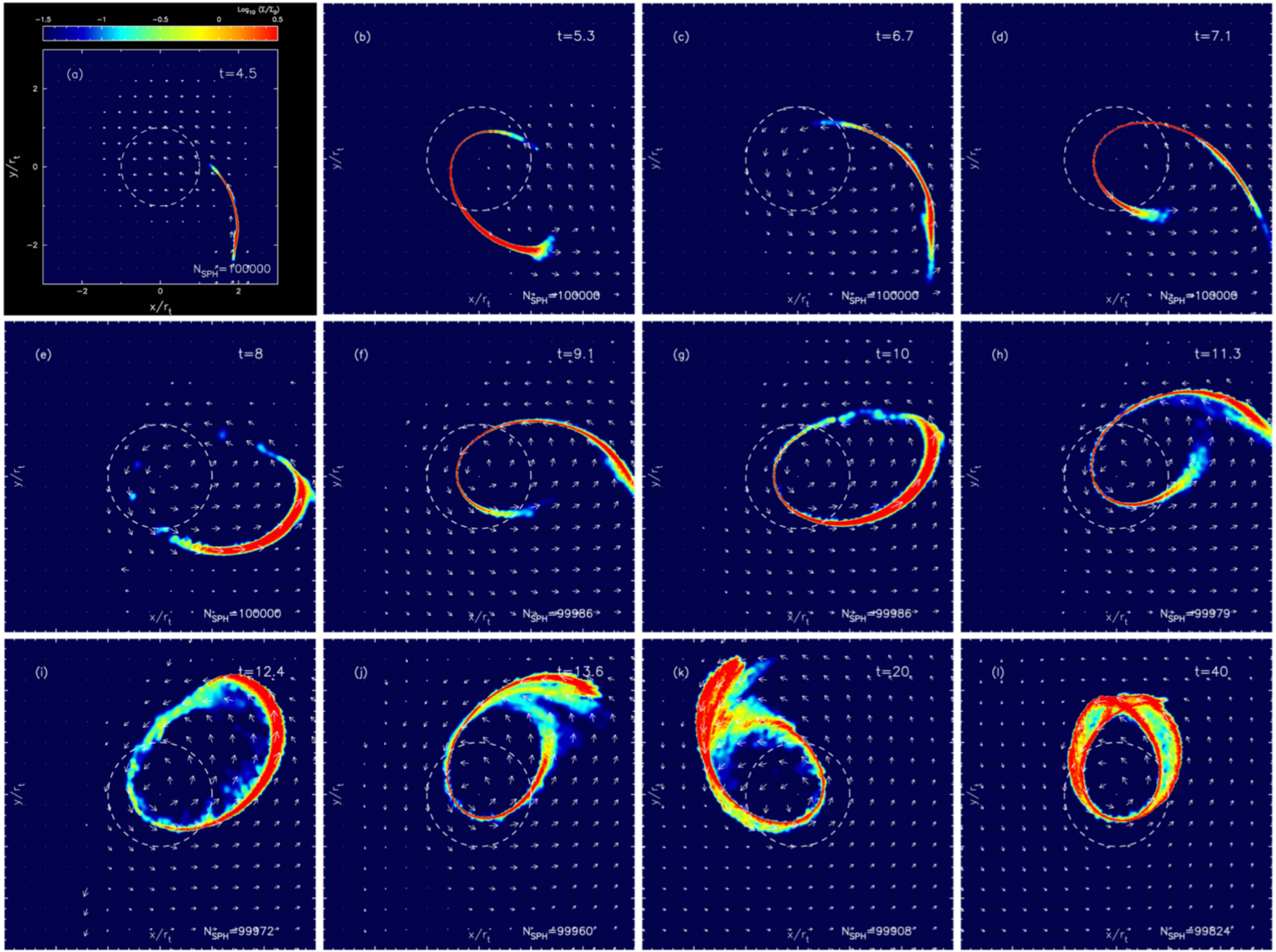
This is a natural way to turn an incoming stream of gas into an accretion disk…

- Stream flows in, swings around central object.
- If orbit is not strictly elliptical, stream self-intersects and shocks
- Form a narrow annulus of gas at location defined by specific angular momentum of matter in stream.

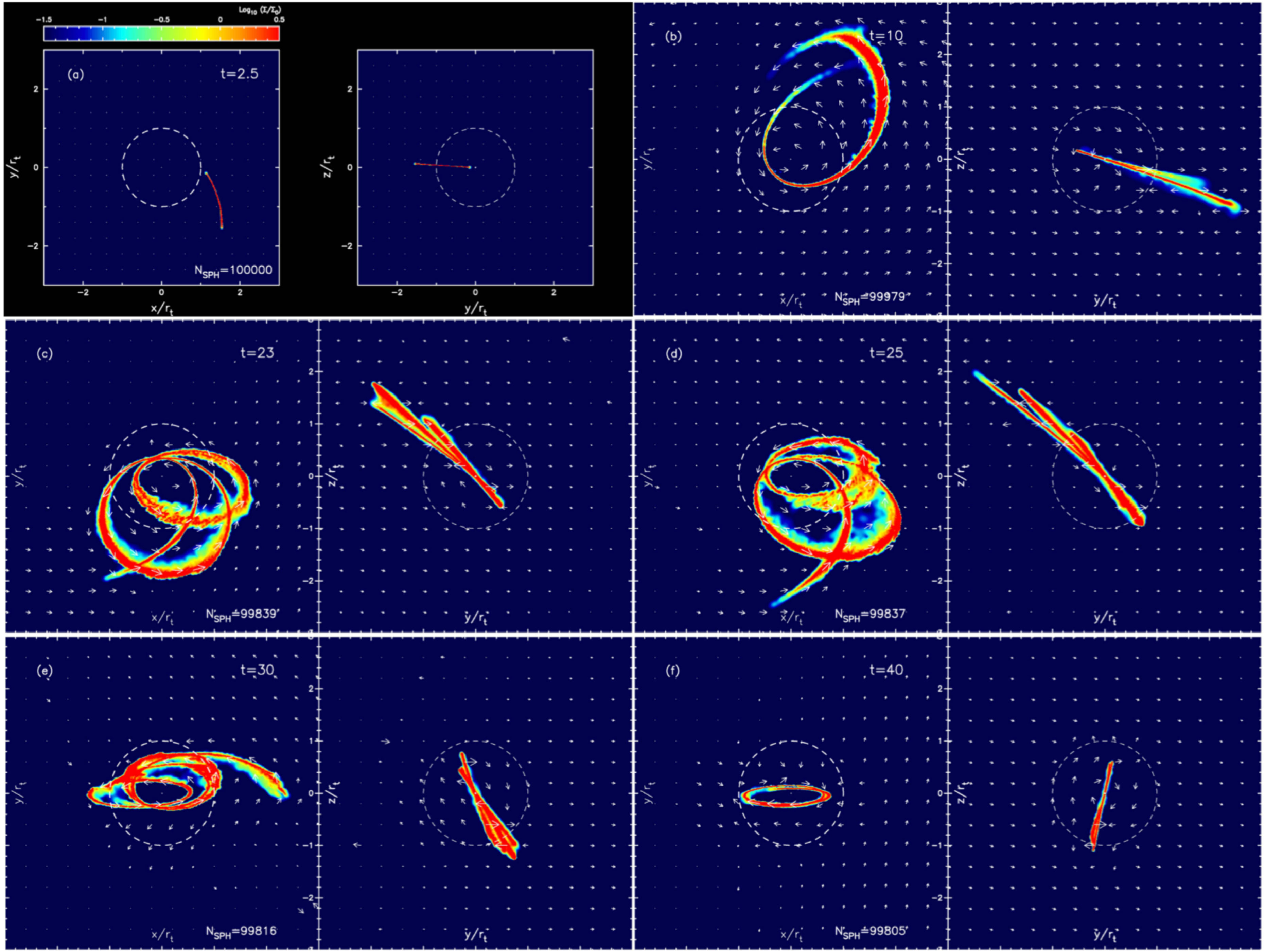

E.G., tidal disruption of a star by a supermassive black hole



Hayakasi, Stone & Loeb (2016)

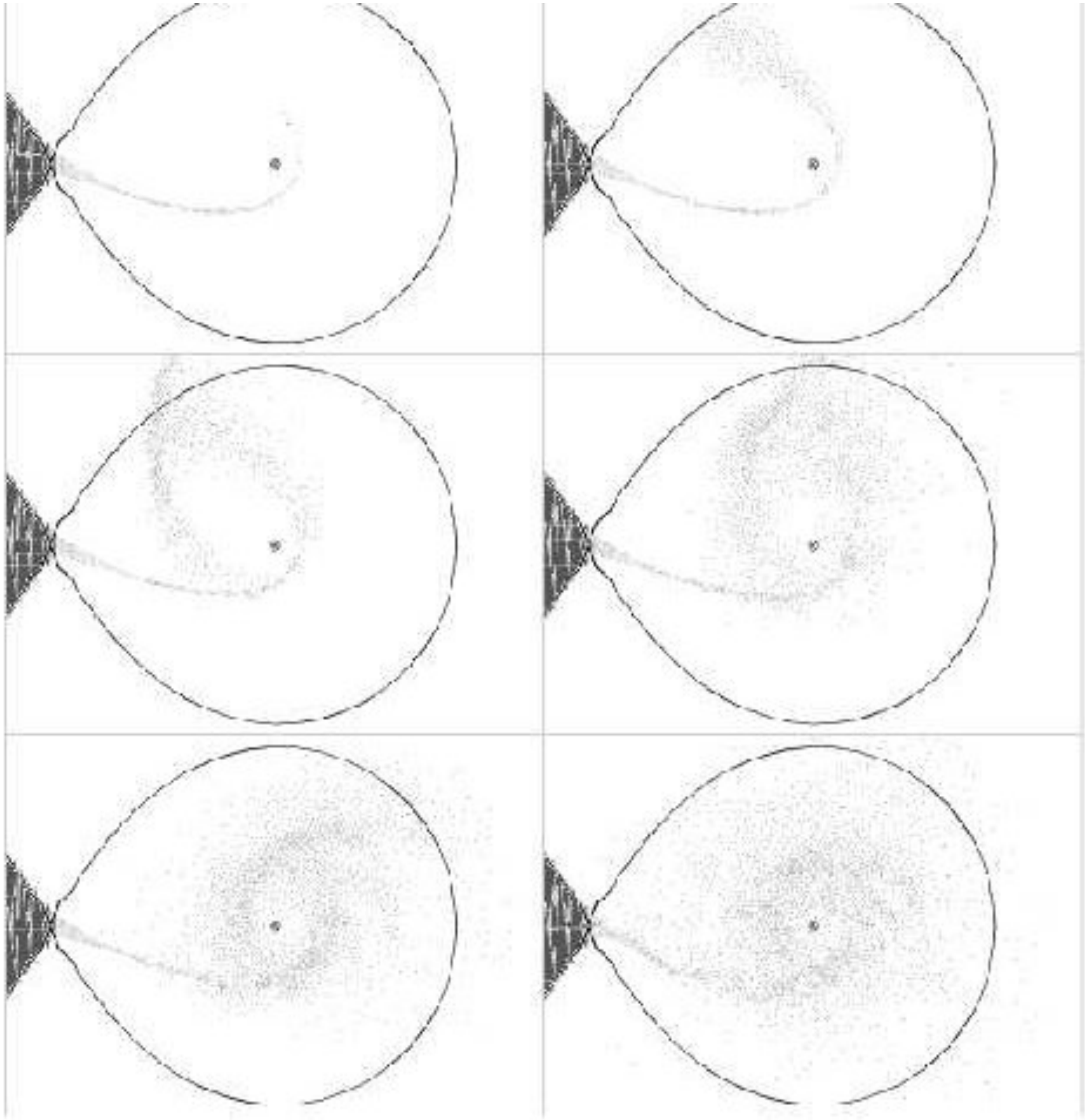


$\mathrm{Log}_{10}\,(\Sigma/\Sigma_0)$
-1.5 -1 -0.5 0 0.5
(a) t=4.5
(b) t=5.3
(c) t=6.7
(d) t=7.1
(e) t=8
(f) t=9.1
(g) t=10
(h) t=11.3
(i) t=12.4
(j) t=13.6
(k) t=20
(l) t=40
y/r_t
x/r_t
N_{SPH}=100000
N_{SPH}=99986
N_{SPH}=99979
N_{SPH}=99972
N_{SPH}=99960
N_{SPH}=99908
N_{SPH}=99824



Log10 (Σ/Σ0)
(a)
t=2.5
NSPH=100000
x/rt
y/rt
z/rt
(b)
t=10
NSPH=99979
(c)
t=23
NSPH=99839
(d)
t=25
NSPH=99837
(e)
t=30
NSPH=99816
(f)
t=40
NSPH=99805

Roche Lobe overflow in a compact binary star system



Boffin et al. (2008)

The nature of viscosity in accretion disks

We can make a simple order-of-magnitude estimate for the timescale on which matter will accrete…

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}(\nu \Sigma R^{1/2})\right]$$

$$\frac{\Sigma}{t_\nu} \sim \frac{1}{R}\frac{1}{R}R^{1/2}\frac{1}{R}\nu\Sigma R^{1/2} \sim \frac{\nu\Sigma}{R^2}$$

$$\Rightarrow \qquad t_\nu \sim \frac{R^2}{\nu} = \frac{R}{u_\phi}\frac{R u_\phi}{\nu} = \Omega^{-1}\,\mathrm{Re}$$

Suppose we compute ν from our usual kinetic theory approach… for typical parameters, we will find small values giving very large Reynolds (Re~10^{14}) and hence accretion timescales exceeding the age of the Universe.

So, the "viscosity" driving accretion cannot be the real microphysical viscosity.

Need an additional source of anomalous viscosity.

Believed to be due to the action of turbulence.

Dimensional analysis… $[\nu] = [L]^2[T]^{-1}$

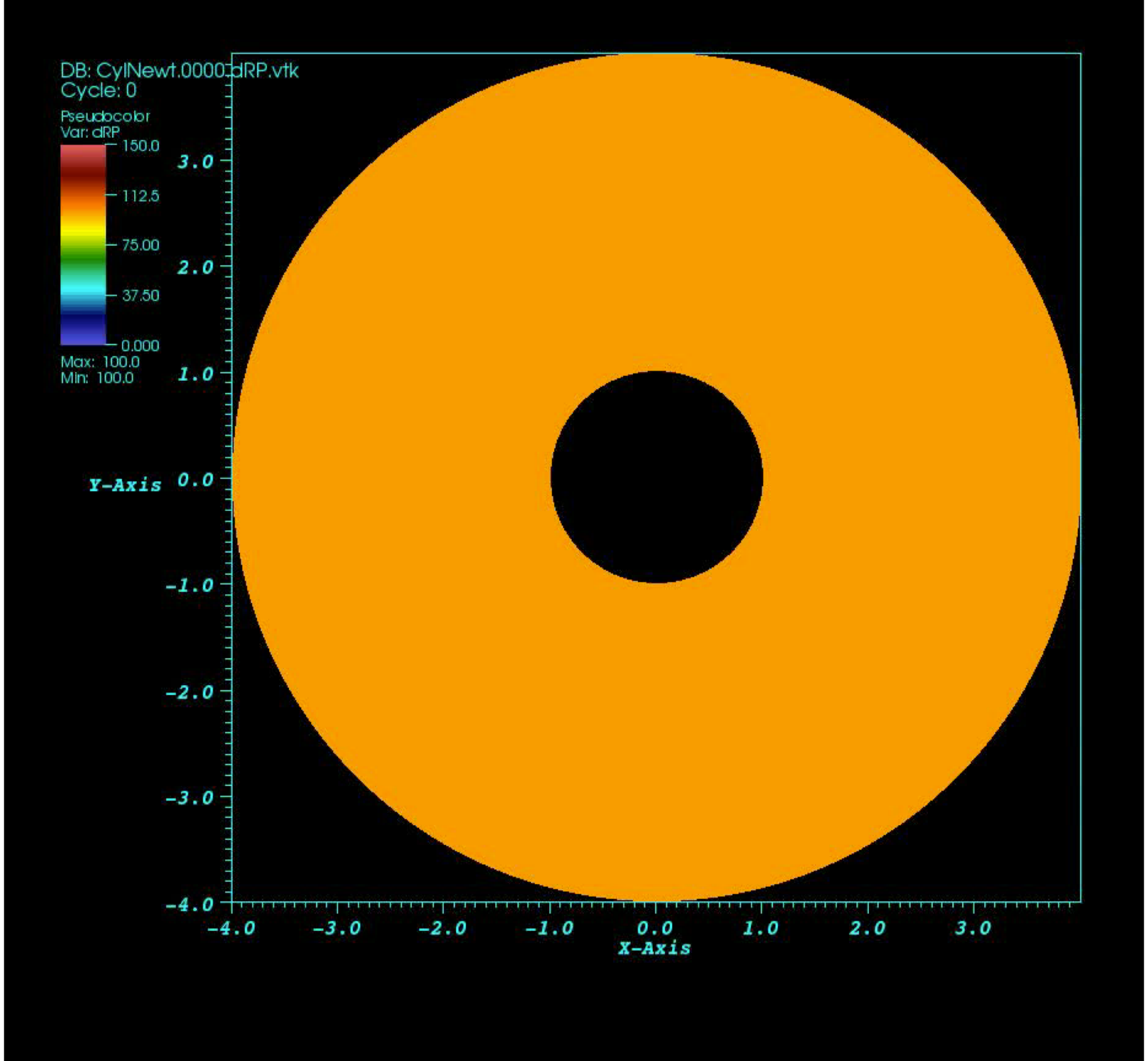
Turbulence gives effective viscosity $\nu_{eff} = u\, l$ where l and u is the size and velocity of a typical eddy in the turbulence.

Thinking about physics in comoving frame of orbital disk, only characteristic velocity is c_s and only characteristic distance is thickness H. These set the upper bounds on the velocity and size of turbulent eddies.

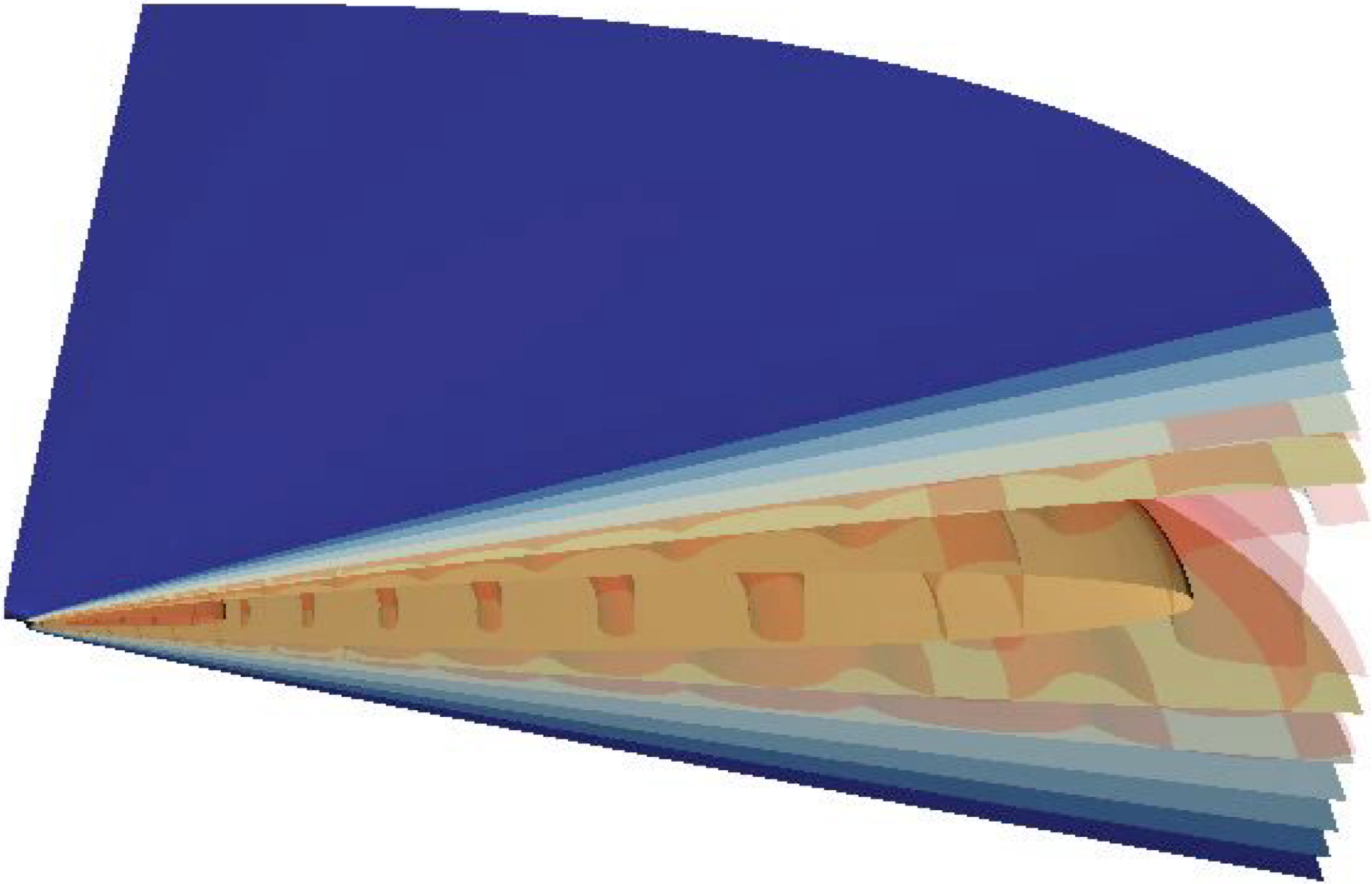
So, we can say

$$\nu = \alpha c_s H \quad (\alpha < 1)$$

Disk models using this prescription are known as α-disks (gives ν dependence upon disk temperature and vertical structure. (Shakura & Sunyaev 1973)



Sorathia et al. (2012)



Energy Dissipation in an Accretion Disk

Viscosity gives energy dissipation. Let's compute this…

The dissipation per unit area of a geometrically-thin disk is

$$
\begin{aligned}
F_{\mathrm{diss}} &= -\int \sigma'_{ij} \partial_j u_i \frac{\mathrm{d}V}{2\pi R \, \mathrm{d}R \, \mathrm{d}\phi} \\
&= \frac{1}{2} \int \eta (\partial_j u_i + \partial_i u_j)^2 \, \mathrm{d}z \\
&= \int \eta R^2 \left(\frac{\mathrm{d}\Omega}{\mathrm{d}R} \right)^2 \mathrm{d}z \\
&= \nu \Sigma R^2 \left(\frac{\mathrm{d}\Omega}{\mathrm{d}R} \right)^2
\end{aligned}
$$

I.7 : Steady-State Accretion Disks

We now look at disks that have reaches a stationary state.

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(R\Sigma u_R) = 0$$

$$\Rightarrow \qquad R\Sigma u_R = C_1 = -\frac{\dot{m}}{2\pi}$$

Mass accretion rate

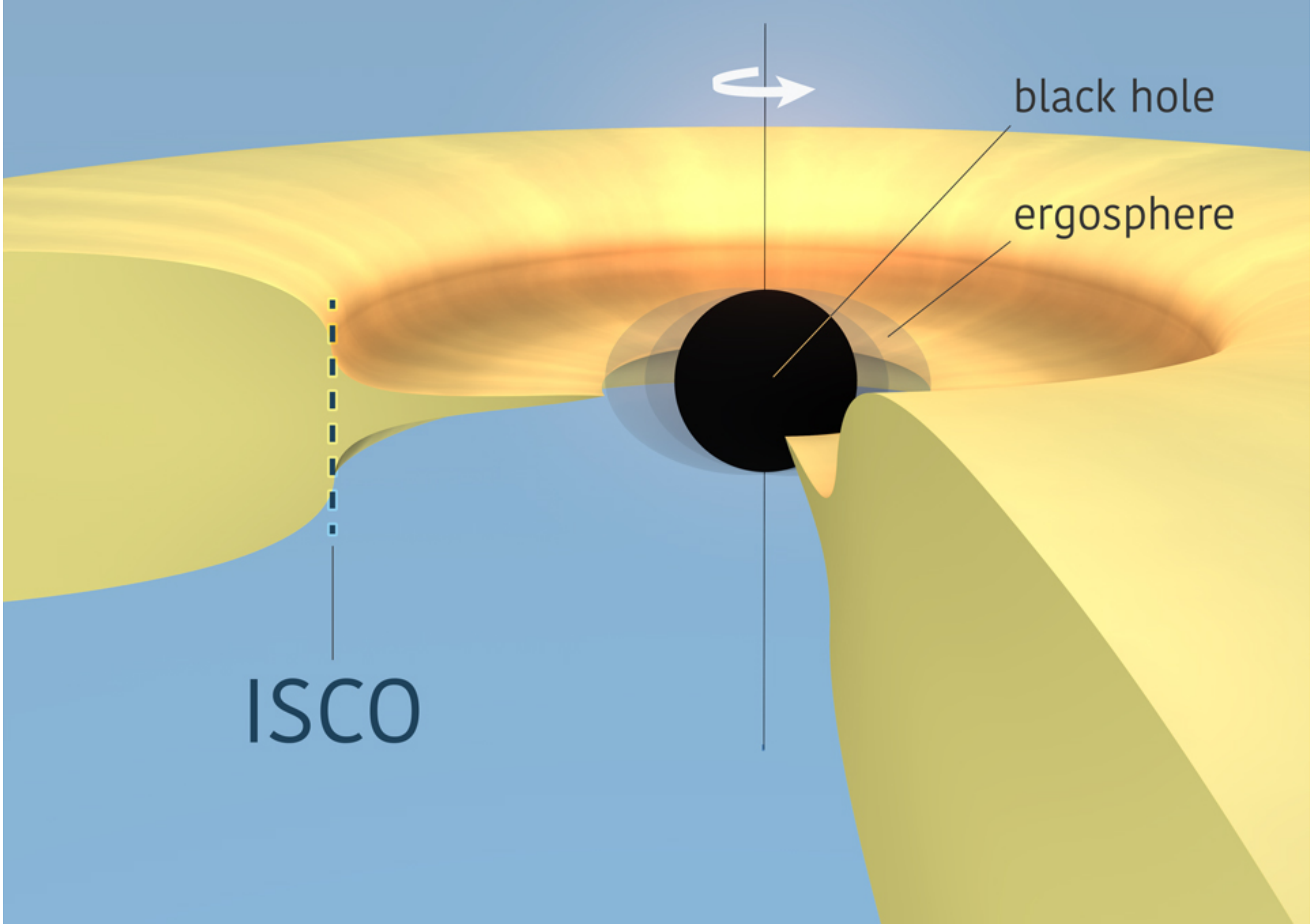
Recall:

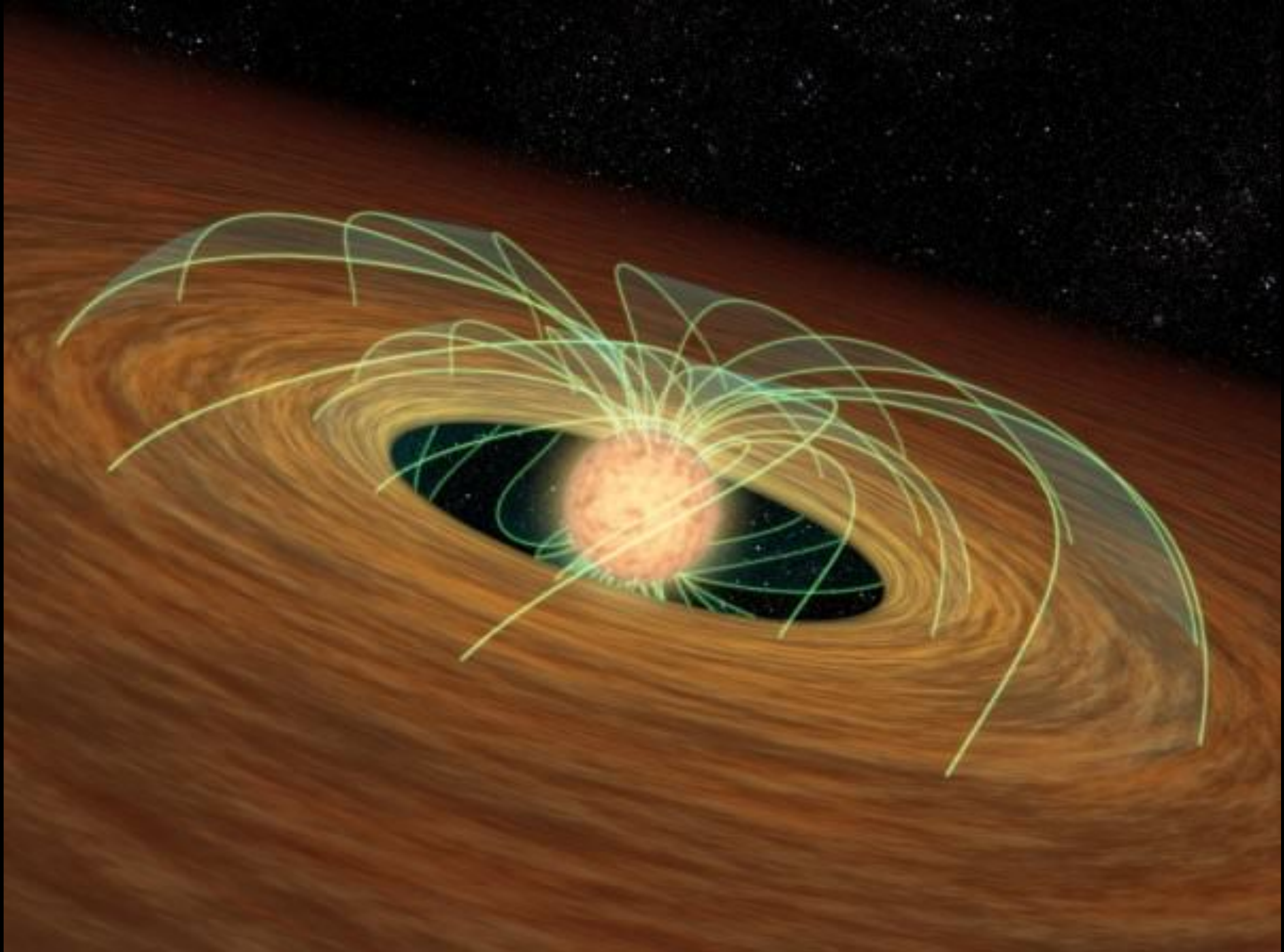
$$u_R = \frac{\frac{\partial}{\partial R}\left(\nu\Sigma R^3 \frac{\mathrm{d}\Omega}{\mathrm{d}R}\right)}{R\Sigma \frac{\partial}{\partial R}(R\Omega^2)}$$

$$\Rightarrow \qquad -\frac{\dot{m}}{2\pi R\Sigma} = -\frac{3}{\Sigma R^{1/2}}\frac{\partial}{\partial R}(\nu\Sigma R^{1/2}) \qquad \text{for } \Omega^2 = GM/R^3$$

$$\Rightarrow \qquad \nu\Sigma = \frac{\dot{m}}{3\pi}\left(1 - \sqrt{\frac{R_*}{R}}\right)$$

Applying boundary condition of $\nu\Sigma = 0$ at $R = R_*$
(zero torque boundary condition)


black hole
ergosphere
ISCO



So,

$$\nu\Sigma = \frac{\dot{m}}{3\pi}\left(1 - \sqrt{\frac{R_*}{R}}\right)$$

and

$$F_{\mathrm{diss}} = \nu\Sigma R^2 \left(\frac{d\Omega}{dR}\right)^2$$

Thus,

$$\boxed{F_{\mathrm{diss}} = \frac{3GM\dot{m}}{4\pi R^3}\left(1 - \sqrt{\frac{R_*}{R}}\right)}$$

Notes on dissipation from steady-state accretion disks:

1. Total energy emitted

$$L = \int_{R_*}^{\infty} F_{\mathrm{diss}} 2\pi R \,\mathrm{d}R = \frac{GM\dot{m}}{2R_*}$$

So the disk **radiates half of the binding energy** of the accreting matter. The remaining half is carried through the inner edge of the disk as kinetic energy.

In case of black hole, remaining energy is carried into the black holes in kinetic form.

In other cases, energy dumped into boundary layer and eventually radiated.

$$\boxed{F_{\mathrm{diss}} = \frac{3GM\dot{m}}{4\pi R^3}\left(1 - \sqrt{\frac{R_*}{R}}\right)}$$

2. Far from inner boundary, we have

$$F_{\mathrm{diss}} \approx \frac{3GM\dot{m}}{4\pi R^3}$$

Compare with the local rate of loss of binding energy…

$$F_{\mathrm{diss,est}} = \underbrace{\frac{1}{2\pi R\,\mathrm{d}R}}_{\substack{\text{area of}\\ \text{annulus}}} \cdot \underbrace{\left|\frac{\partial}{\partial R}\left(\frac{GM\dot{m}}{R}\right)\right|}_{\substack{\text{change in grav.}\\ \text{potential of } \dot{m}\\ \text{over annulus}}} \mathrm{dR} \quad \underbrace{\frac{1}{2}}_{\substack{\text{half converts}\\ \text{to radiation,}\\ \text{rest to kinetic}}} = \frac{GM\dot{m}}{4\pi R^3}$$

So there is three times more energy dissipated locally than is associate with the local accretion. This is due to the viscous transport of energy from the inner disk into the outer disk.

$$\boxed{F_{\rm diss} = \frac{3GM\dot{m}}{4\pi R^3}\left(1 - \sqrt{\frac{R_*}{R}}\right)}$$

3. Can now estimate the temperature of the accretion disk... assuming that the locally dissipated energy is radiated as a black body, we have

$$\underbrace{2}_{\substack{\text{top and} \\ \text{bottom of disk}}} \cdot \sigma_{\rm SB} T_{\rm eff}^4 = \frac{3GM\dot{m}}{4\pi R^3}\left(1 - \sqrt{\frac{R_*}{R}}\right)$$

$$\Rightarrow \qquad T_{\rm eff} = \left[\frac{3GM\dot{m}}{8\pi\sigma R^3}\left(1 - \sqrt{\frac{R_*}{R}}\right)\right]^{1/4}$$

At large distance, temperature declines as $R^{-3/4}$.

For (non-rotating) black hole, scale radii to gravitational radius $R = x(GM/c^2)$ so

$$T_{\rm eff} \propto \dot{m}^{-1/4} M^{-1/2} x^{-3/4}$$

$$T_{\rm eff,max} \propto M^{-1/4} \quad \text{if } \dot{m} \propto M$$

Black hole X-ray binaries (10Msun), T~10^7K
Active Galactic Nuclei (10^9Msun), T~10^5K