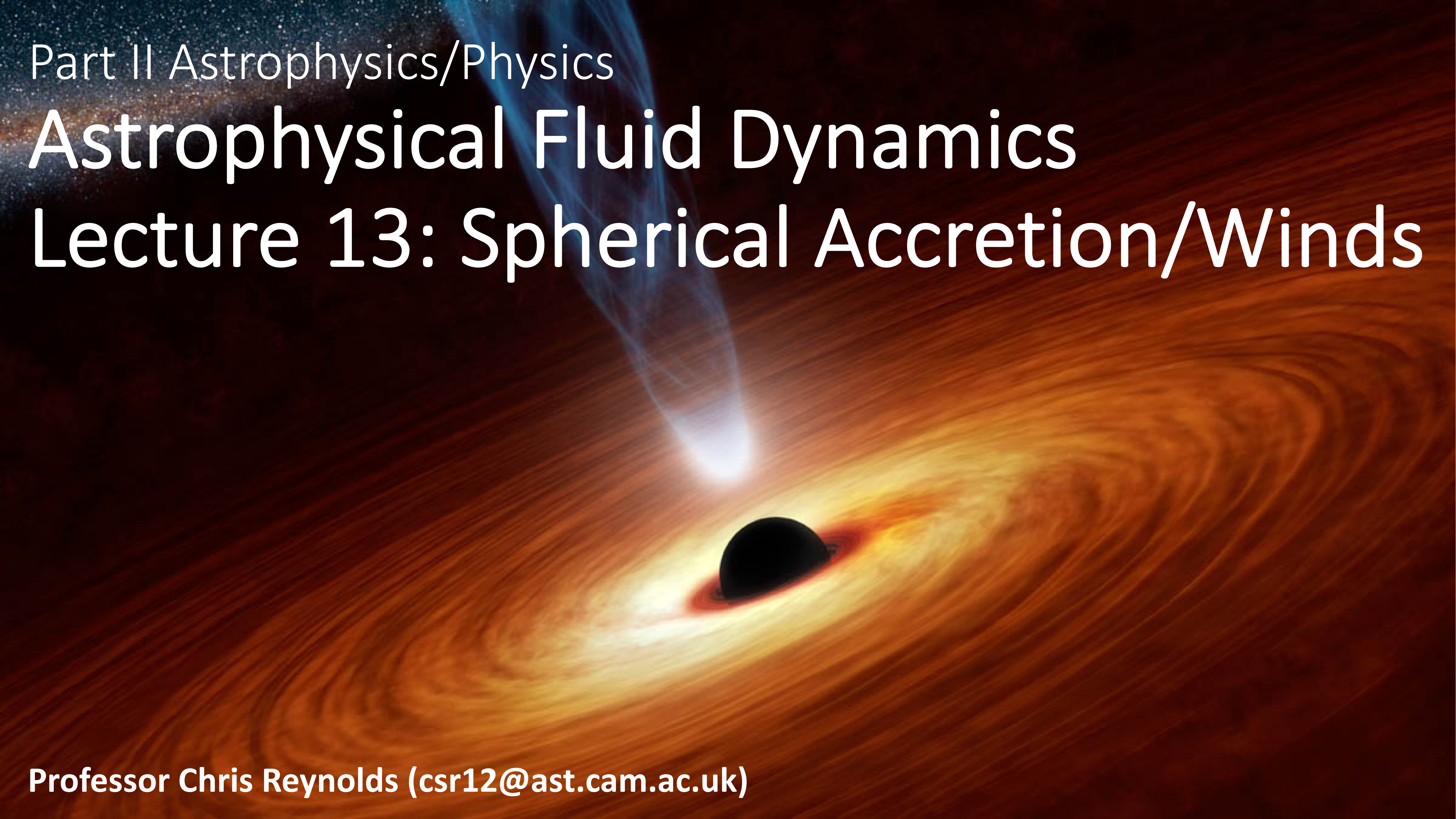
Part II Astrophysics/Physics

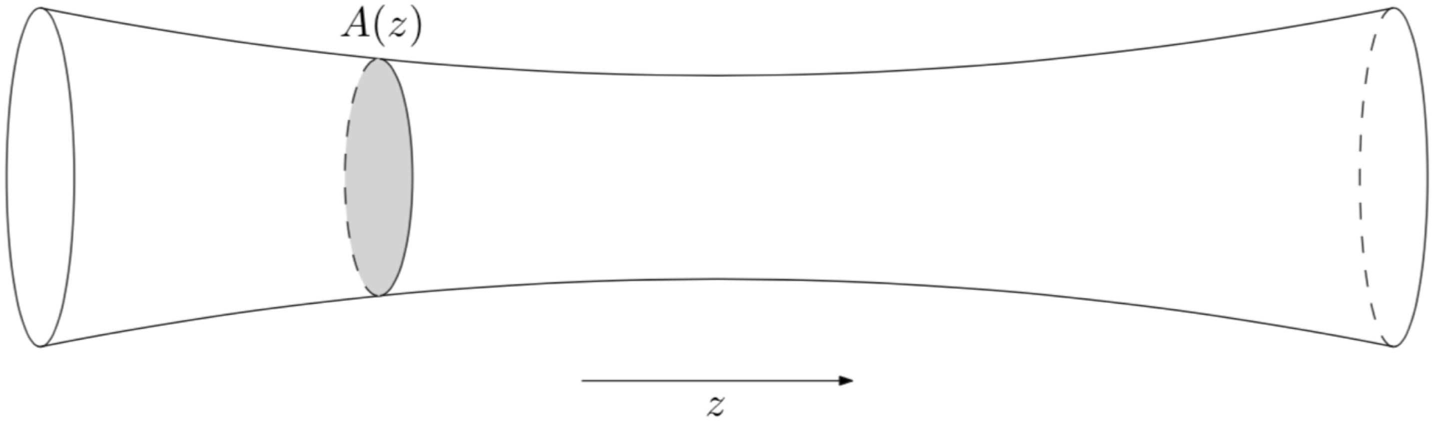
Astrophysical Fluid Dynamics
Lecture 13: Spherical Accretion/Winds



Professor Chris Reynolds (csr12@ast.cam.ac.uk)

Recap - last lecture

- The De Laval Nozzle and transonic flows



$$(u^2 - c_s^2)\boldsymbol{\nabla} \ln u = c_s^2 \boldsymbol{\nabla} \ln A$$

- Can achieve a sonic transition at extremum of A
- Existence of "critical point" allows us to determine structure of flow

This Lecture

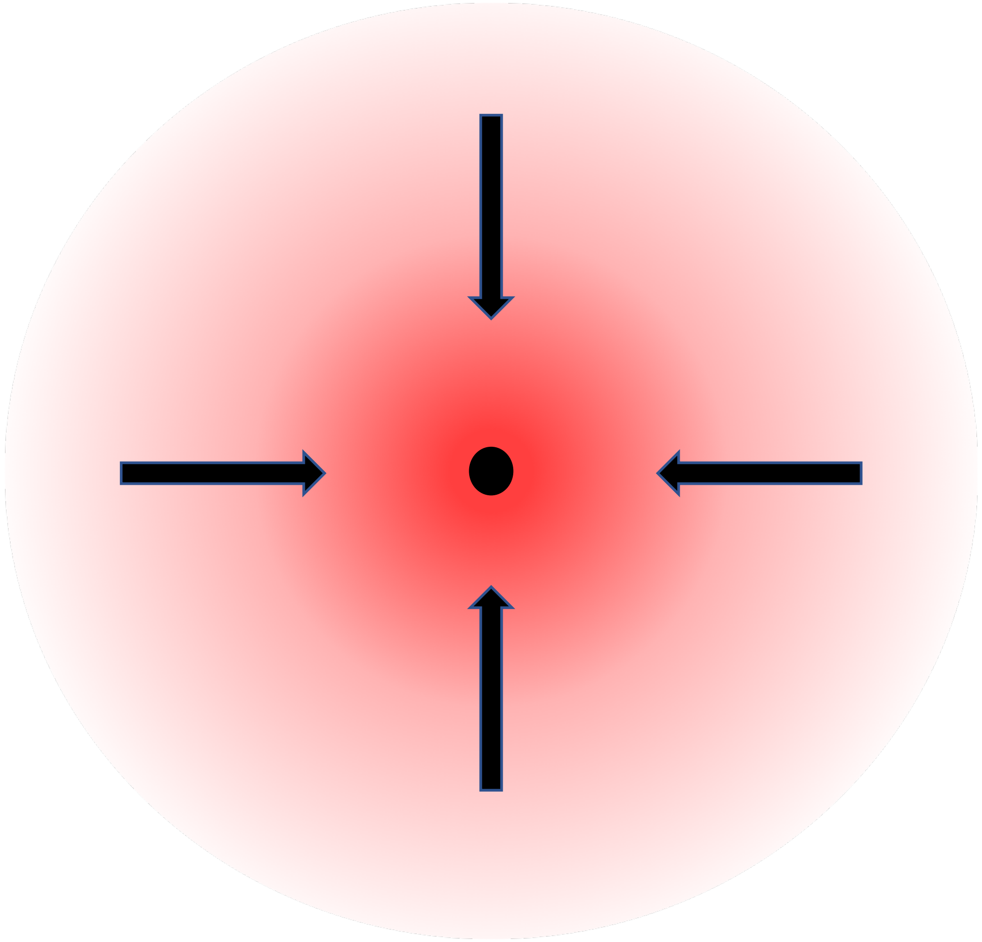
- Other important “critical point” problem…
- Spherical accretion and winds
    - Importance of accretion and winds!
    - Analysis of spherical accretion problem
    - Role of critical (sonic) point
    - Isothermal and polytropic cases
    - Unifying descriptions of accretion flows and winds

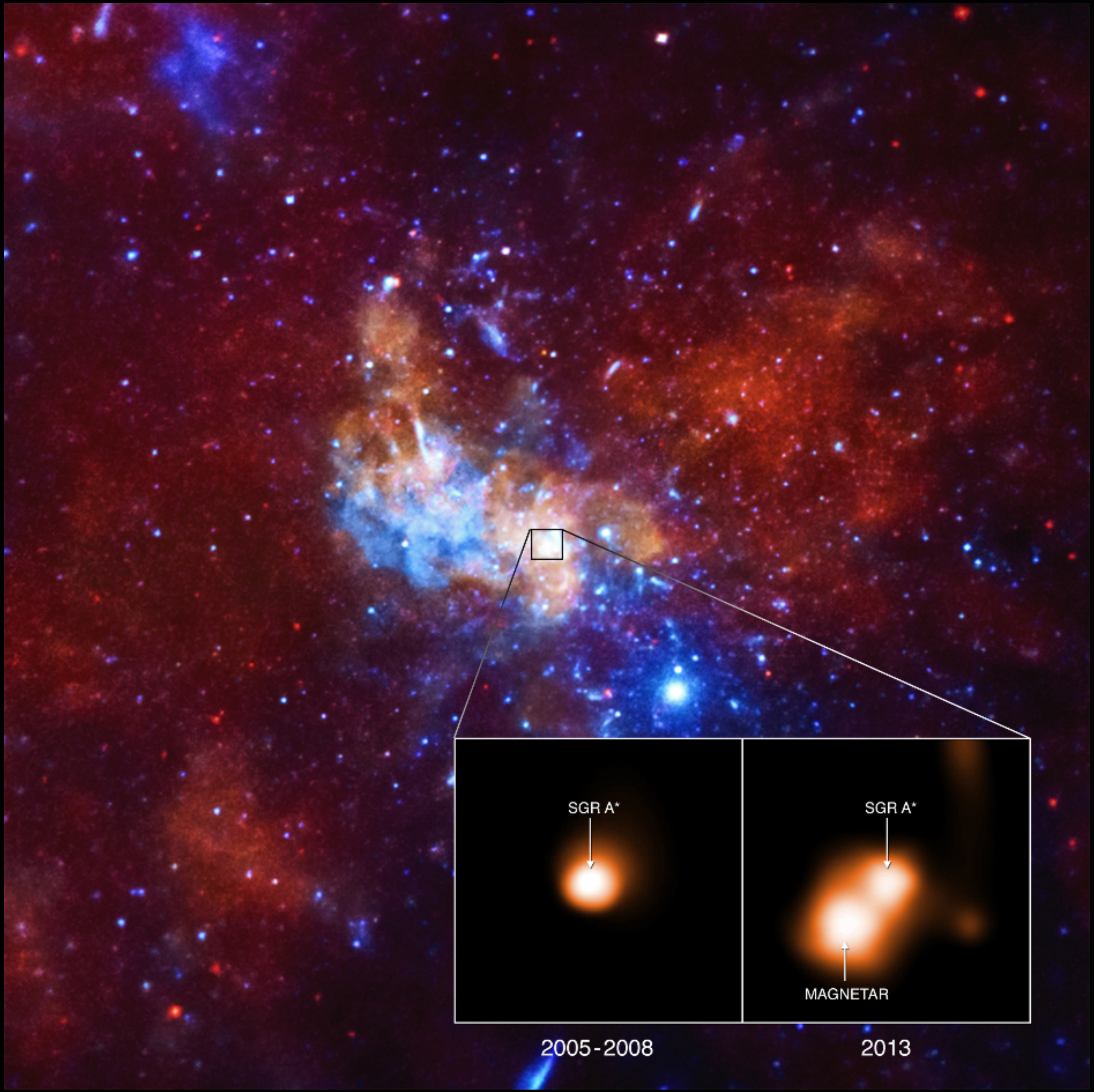
- By end of this lecture, you can do…
    - All of Examples Sheet 1 and 2
    - Example Sheet 3, Q1-4, Q9

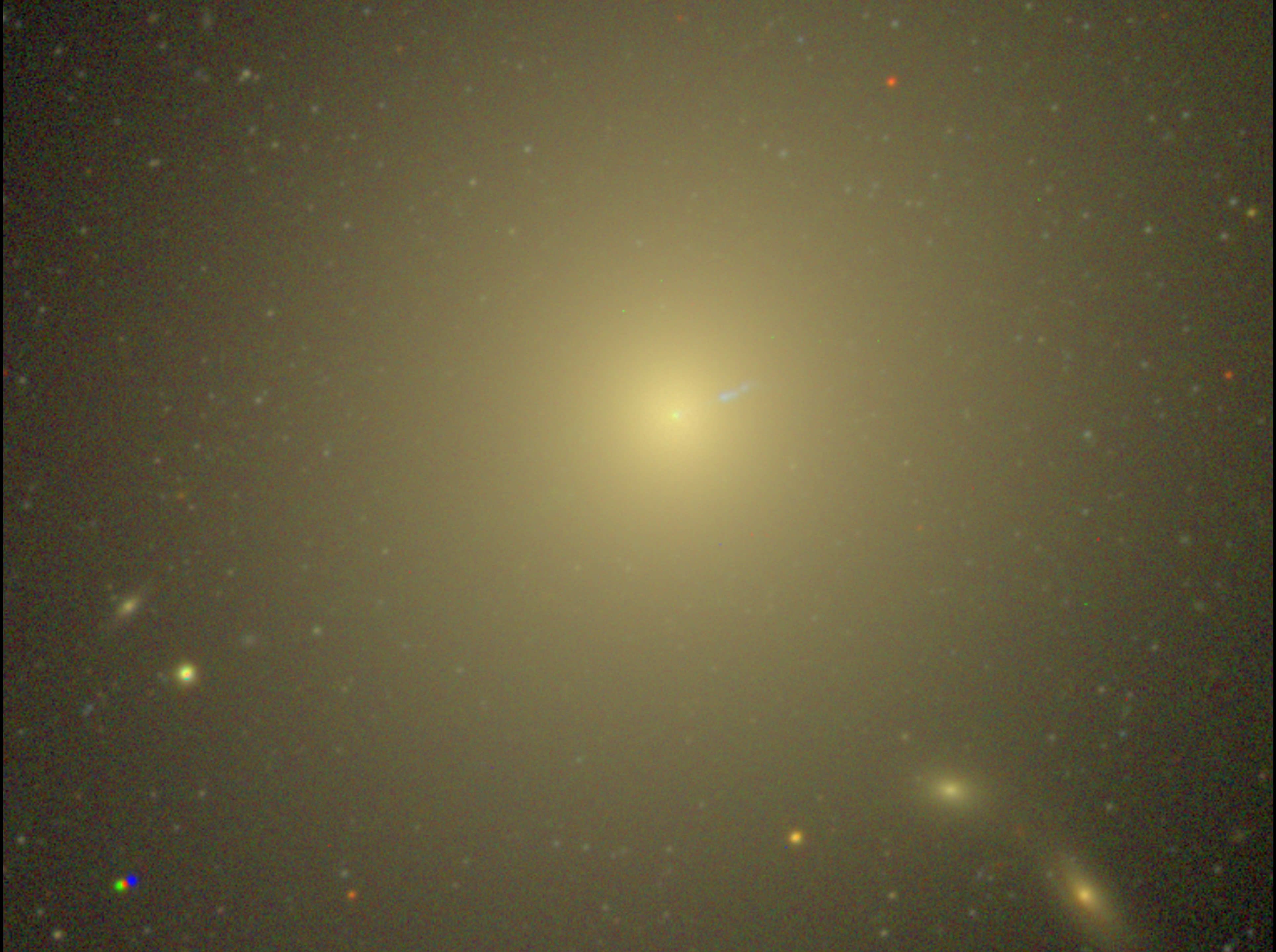
G.4 : Spherical Accretion

Consider spherically symmetric flow of matter in gravitational potential of point-like central body.   Assume:

- Gas at rest at infinity
- Flow is in steady state
- Barotropic equation of state




SGR A*
SGR A*
MAGNETAR
2005-2008
2013



Mass conservation:

$$\rho u A = \text{constant } \dot{M}$$

$$\Rightarrow \quad 4\pi r^2 \rho u = \dot{M},$$

$$\Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}r}\left(\ln \dot{M}\right) = 0$$

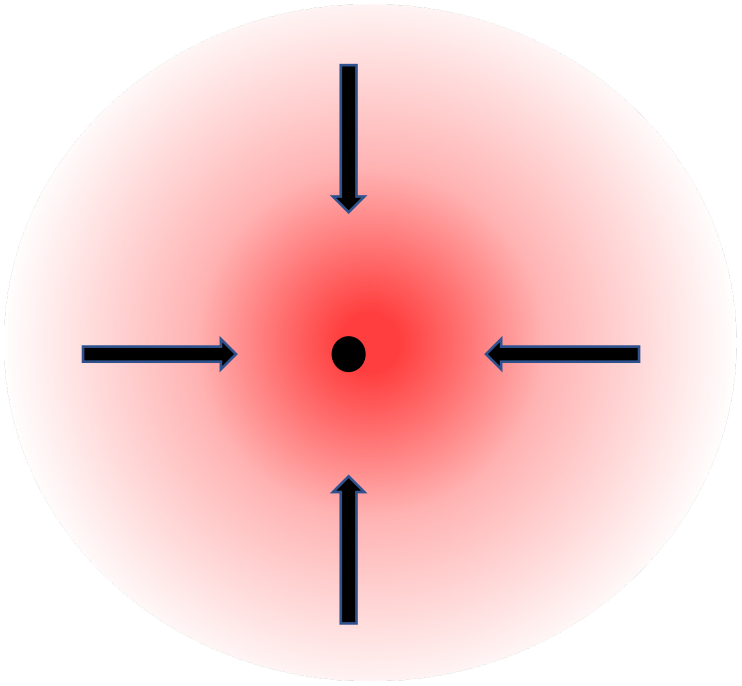
$$\Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}r}\ln\rho + \frac{\mathrm{d}}{\mathrm{d}r}\ln u + \frac{\mathrm{d}}{\mathrm{d}r}\ln r^2 = 0$$

$$\Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}r}\ln\rho = -\frac{\mathrm{d}}{\mathrm{d}r}\ln u - \frac{2}{r}.$$

Momentum eq (steady state):

$$u\frac{\mathrm{d}u}{\mathrm{d}r} = -\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{GM}{r^2}$$

$$\Rightarrow \quad u^2\frac{\mathrm{d}\ln u}{\mathrm{d}r} = -c_s^2\frac{\mathrm{d}\ln\rho}{\mathrm{d}r} - \frac{GM}{r^2}$$



So

$$u^2 \frac{\mathrm{d}}{\mathrm{d}r} \ln u = c_s^2 \left(\frac{\mathrm{d}}{\mathrm{d}r} \ln u + \frac{2}{r} \right) - \frac{GM}{r^2}$$

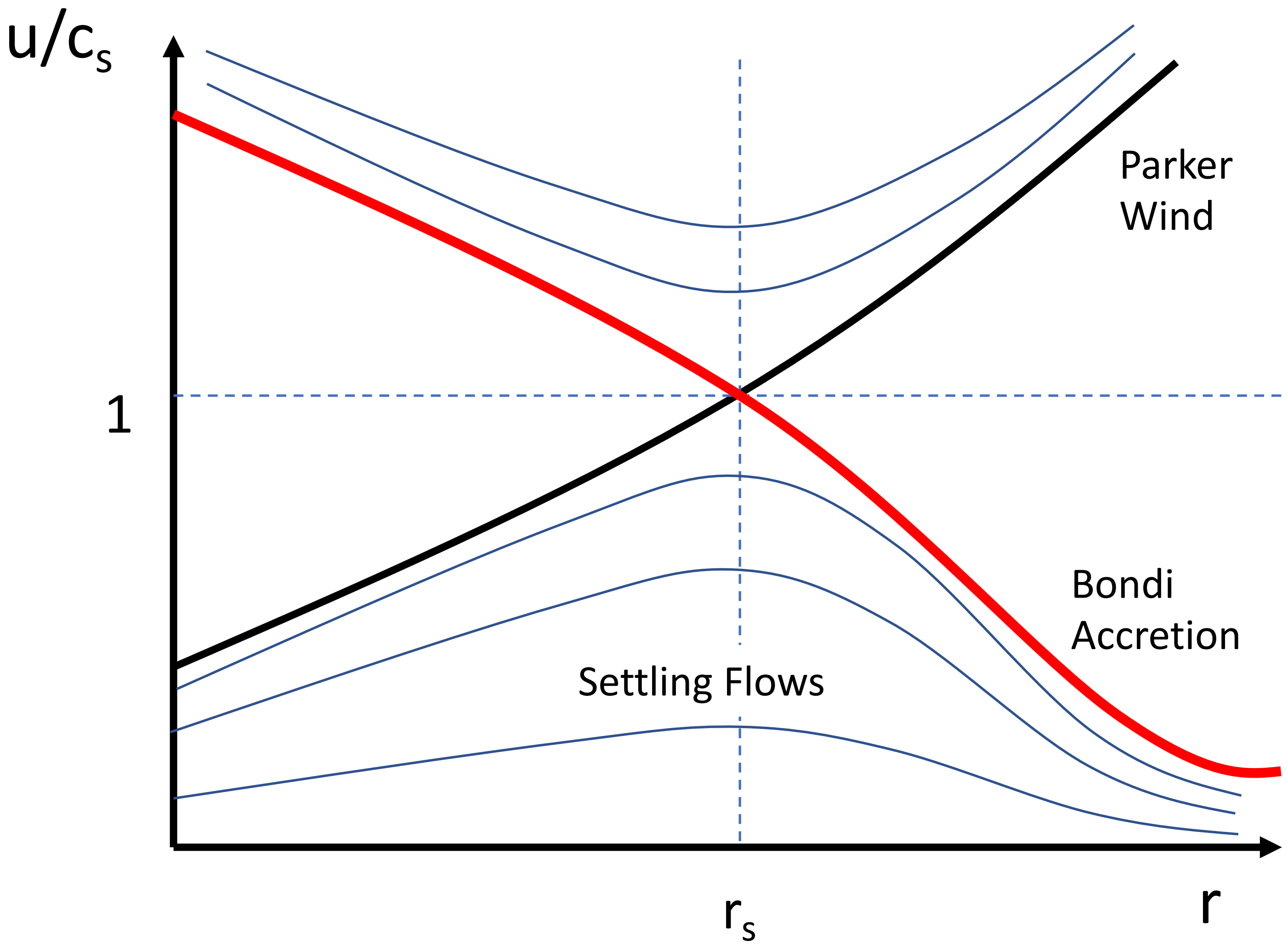
$$\Rightarrow \quad \boxed{(u^2 - c_s^2) \frac{\mathrm{d}}{\mathrm{d}r} \ln u = \frac{2c_s^2}{r} \left(1 - \frac{GM}{2c_s^2 r} \right)}.$$

Thus there is a critical point where

$$r = r_s = \frac{GM}{2c_s^2} \qquad \textsc{Sonic Point}$$

and either a) *u* has an extremum, or

b) $u = c_s$


u/c_s
1
Parker
Wind
Bondi
Accretion
Settling Flows
r_s
r

Example : Isothermal accretion case

$$c_s = \sqrt{\frac{\mathcal{R}_* T}{\mu}} = \text{const.}$$

So the flow passes through a sonic point $u = c_s$ at location $r = r_s = GM/2c_s^2$.

Use Bernoulli to compare flow at sonic point with a general point:

$$H = \frac{1}{2}u^2 + \underbrace{\int \frac{\mathrm{d}p}{\rho}}_{c_s^2 \ln \rho} + \Psi = \text{const.}$$

$$\Rightarrow \qquad \frac{1}{2}u^2 + c_s^2 \ln \rho - \frac{GM}{r} = \frac{1}{2}c_s^2 + c_s^2 \ln \rho_s - \frac{GM}{r_s}$$

$$\Rightarrow \qquad \frac{1}{2}u^2 + c_s^2 \ln \rho - \frac{GM}{r} = c_s^2 \left(\ln \rho_s - \frac{3}{2} \right)$$

$$\Rightarrow \qquad u^2 = 2c_s^2 \left[\ln \left(\frac{\rho_s}{\rho} \right) - \frac{3}{2} \right] + \frac{2GM}{r}$$

$$u^2 = 2c_s^2 \left[\ln\left(\frac{\rho_s}{\rho}\right) - \frac{3}{2} \right] + \frac{2GM}{r}$$

Look at some limits…

$$r \to 0, u^2 \to 2GM/r$$

(free fall speed once $r \ll r_s$)

$$r \to \infty \text{ and } u \to 0, \rho = \rho_s e^{-3/2}$$

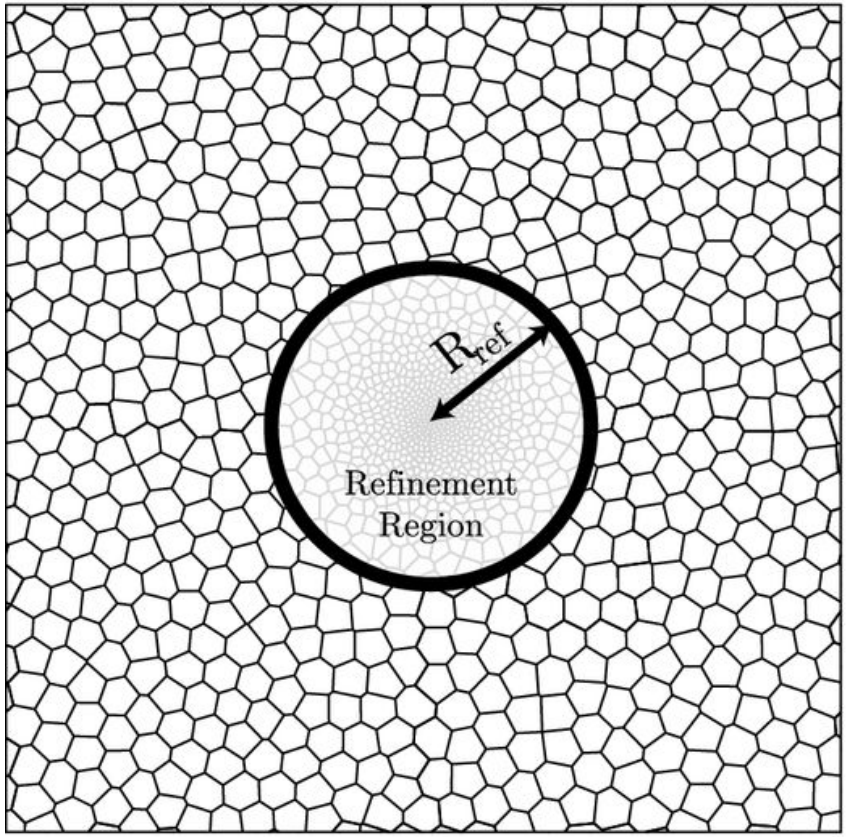
Since we can relate ρ_s to ρ_∞, we can use properties at sonic radius to determine the mass accretion rate

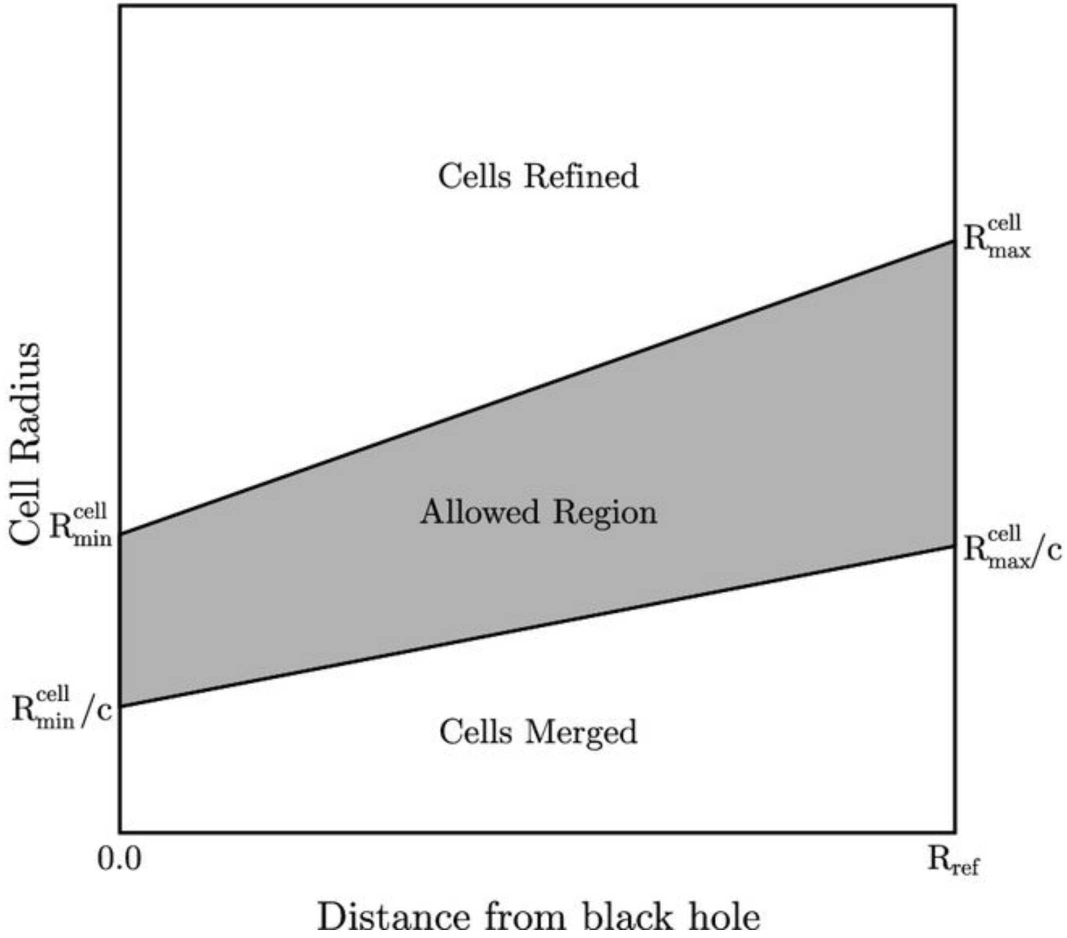
$$\dot{M} = 4\pi r_s^2 \rho_s c_s$$

$$\Rightarrow \quad \dot{M} = \frac{\pi G^2 M^2 e^{3/2} \rho_\infty}{c_s^3}$$

(uniquely determined by conditions in the medium at infinity)

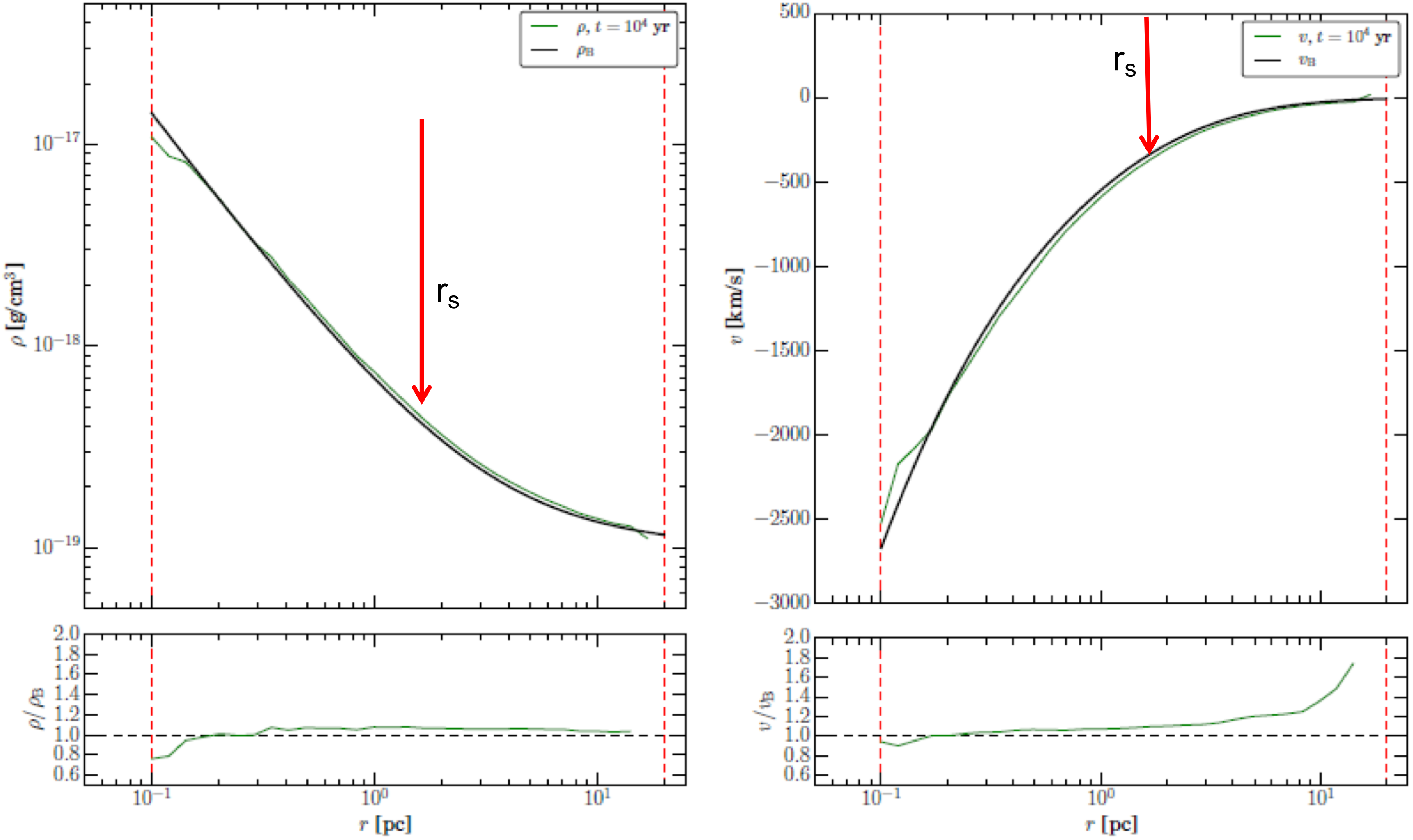
Simulating Bondi accretion





Curtis & Sijacki, 2015

Simulating Bondi accretion



Curtis & Sijacki, 2015

Example : Polytropic accretion case

$$p = K\rho^{1+1/n}; \qquad \int \frac{\mathrm{d}p}{\rho} = K(n+1)\rho^{1/n} = nc_s^2 \qquad c_s^2 = \frac{n+1}{n}K\rho_s^{1/n}$$

Comparing sonic radius to general point, Bernoulli gives:

$$\frac{1}{2}u^2 + (n+1)K\rho^{1/n} - \frac{GM}{r} = \frac{1}{2}c_s^2 + nc_s^2 - \frac{GM}{r_s}$$

$$\Rightarrow \quad \frac{1}{2}u^2 + (n+1)K\rho^{1/n} - \frac{GM}{r} = c_s^2\left(n - \frac{3}{2}\right)$$ using $r_s = \frac{GM}{2c_s^2}$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{\dot{M}}{4\pi r^2\rho}\right)^2 + (n+1)K\rho^{1/n} = c_s^2\left(n - \frac{3}{2}\right) + \frac{GM}{r}$$ using $\dot{M} = 4\pi r_s^2 \rho_s c_s$

So taking this to infinity, we can relate sound speed at sonic point at infinity…

$$\rho_\infty = \left[\frac{c_s^2\left(n - \frac{3}{2}\right)}{(n+1)K}\right]^n, \quad c_{s,\infty}^2 = \frac{n+1}{n}K\rho_\infty^{1/n} \quad \Rightarrow \quad c_s^2 = \left(\frac{n}{n-\frac{3}{2}}\right)c_{s,\infty}^2, \quad \rho_s = \left(\frac{n}{n-\frac{3}{2}}\right)^n \rho_\infty$$

Can now derive accretion rate…

$$
\begin{aligned}
\dot{M} &= 4\pi r_s^2 \rho_s c_s \\
&= \frac{4\pi G^2 M^2}{4c_s^4} \cdot c_s \rho_\infty \left(\frac{n}{n - \frac{3}{2}} \right)^n \\
&= \frac{\pi G^2 M^2}{c_{s,\infty}^3} \rho_\infty \left(\frac{n}{n - \frac{3}{2}} \right)^{n-3/2}
\end{aligned}
$$

$$
\boxed{\dot{M} = \frac{\pi (GM)^2 \rho_\infty}{c_{s,\infty}^3} \left(\frac{n}{n - \frac{3}{2}} \right)^{n-3/2}}
$$

BONDI ACCRETION

Same form as isothermal case (just different pre-factor).

Recover the isothermal case by taking limit $n \rightarrow \infty$.

$$\dot{M} = \frac{\pi (GM)^2 \rho_\infty}{c_{s,\infty}^3} \left(\frac{n}{n - \frac{3}{2}} \right)^{n-3/2}$$

Notes:

1. Dependence of accretion rate on mass of object

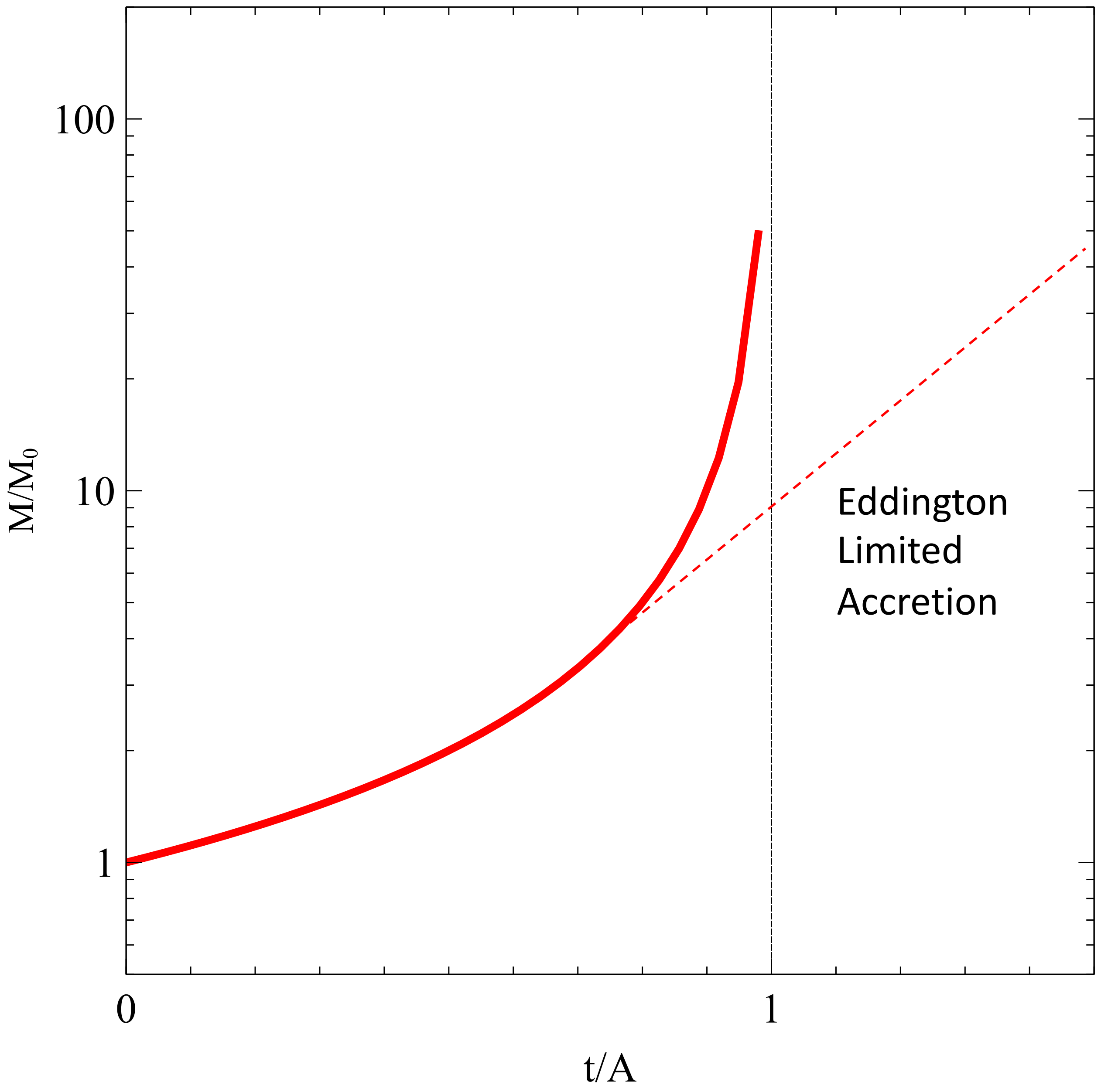
$$\dot{M} = AM^2$$

- If initial mass is M_0 and it accretes like this for time t, then we can integrate to get

$$\int_{M_0}^{M} \frac{\mathrm{d}M}{M^2} = At \qquad \Rightarrow \qquad M = \frac{M_0}{1 - AM_0 t}$$

so $\quad M \to \infty \text{ as } t \to 1/AM_0$

- In reality, the accretion rate will become limited by fuel supply and/or the Eddington limit (which has $\dot{M} \propto M$, so exponential growth).


100
10
1
M/M$_0$
0
1
t/A
Eddington
Limited
Accretion

2. Dependence of accretion rate on reservoir properties:

$$\dot{M} \propto \frac{\rho_\infty}{c_\infty^3} \propto \frac{p_\infty}{c_\infty^5}$$

- Much higher accretion rates from colder material.

E.g. old model for "Ultra Luminous X-ray Sources" (ULXs)… rouge intermediate-mass black hole entering into a cold molecular cloud and undergoing Bondi accretion.

(Now know that at least some of these sources are neutron stars that are accreting at super-Eddington rates from a companion star.)

3. The n=3/2 is a singular case:

- Sonic point goes to origin, with infinite sound speed and density

$$c_s^2 = \left(\frac{n}{n - \frac{3}{2}}\right) c_{s,\infty}^2 \quad , \quad \rho_s = \left(\frac{n}{n - \frac{3}{2}}\right)^n \rho_\infty$$

$$r_s = \frac{GM}{2c_s^2} \to 0 \text{ as } n \to \frac{3}{2}$$

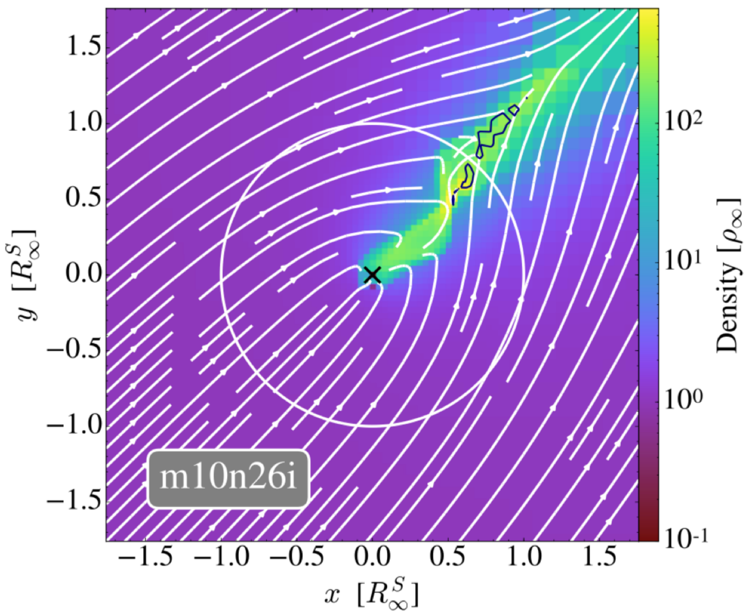
- But accretion rate remains finite

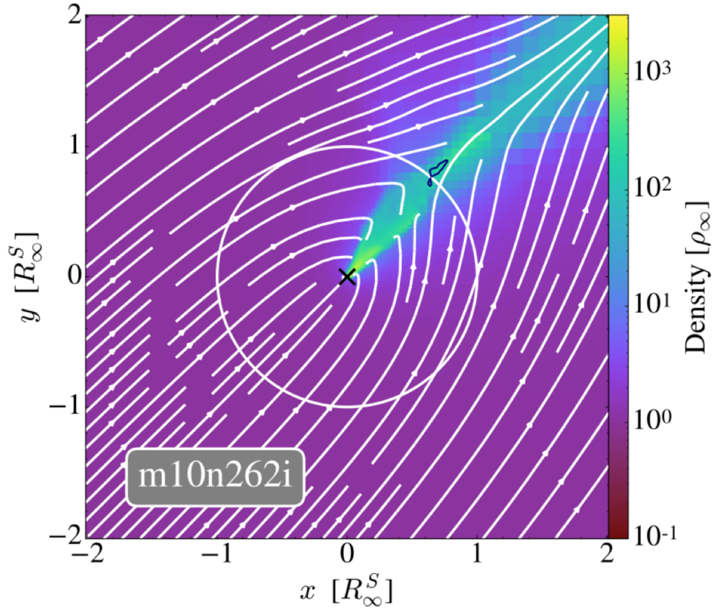
$$\dot{M} = \frac{\pi (GM)^2 \rho_\infty}{c_{s,\infty}^3} \left(\frac{n}{n - \frac{3}{2}}\right)^{n-3/2} \to \frac{\pi (GM)^2 \rho_\infty}{c_{s,\infty}^3} \qquad \text{as } n \to \frac{3}{2}$$

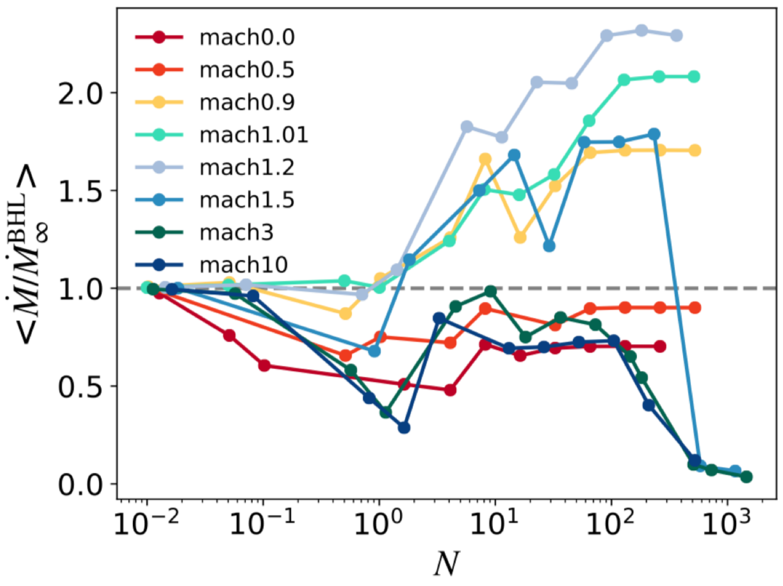
4. Can extend (with less rigor) to the case of the mass moving at speed v_∞ through a uniform medium. Accretion rate

$$\dot{M} \sim \frac{(GM)^2 \rho_\infty}{(c_\infty^2 + v_\infty^2)^{3/2}}$$

BONDI-HOYLE ACCRETION



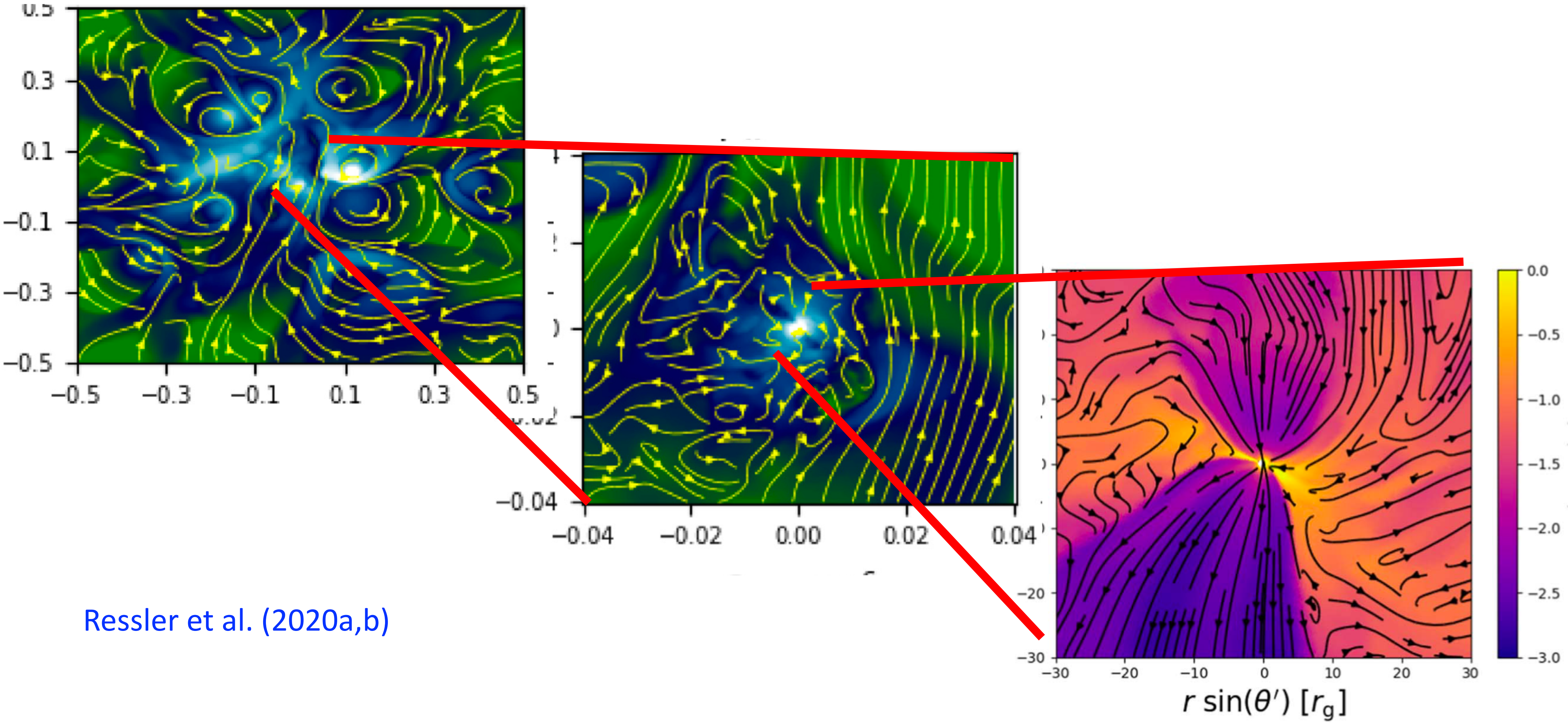




Beckmann et al. (2018)

5. Due to importance of critical point, the nature of the accretion flow can be strongly affected by even modest departures from assumptions, e.g.
   - Non-zero angular momentum of the incoming gas
   - Magnetic fields

Sgr A*, from stellar winds to the black hole



Ressler et al. (2020a,b)

Mon. Not. R. Astron. Soc. **283**, L111–L116 (1996)

The 'quiescent' black hole in M87

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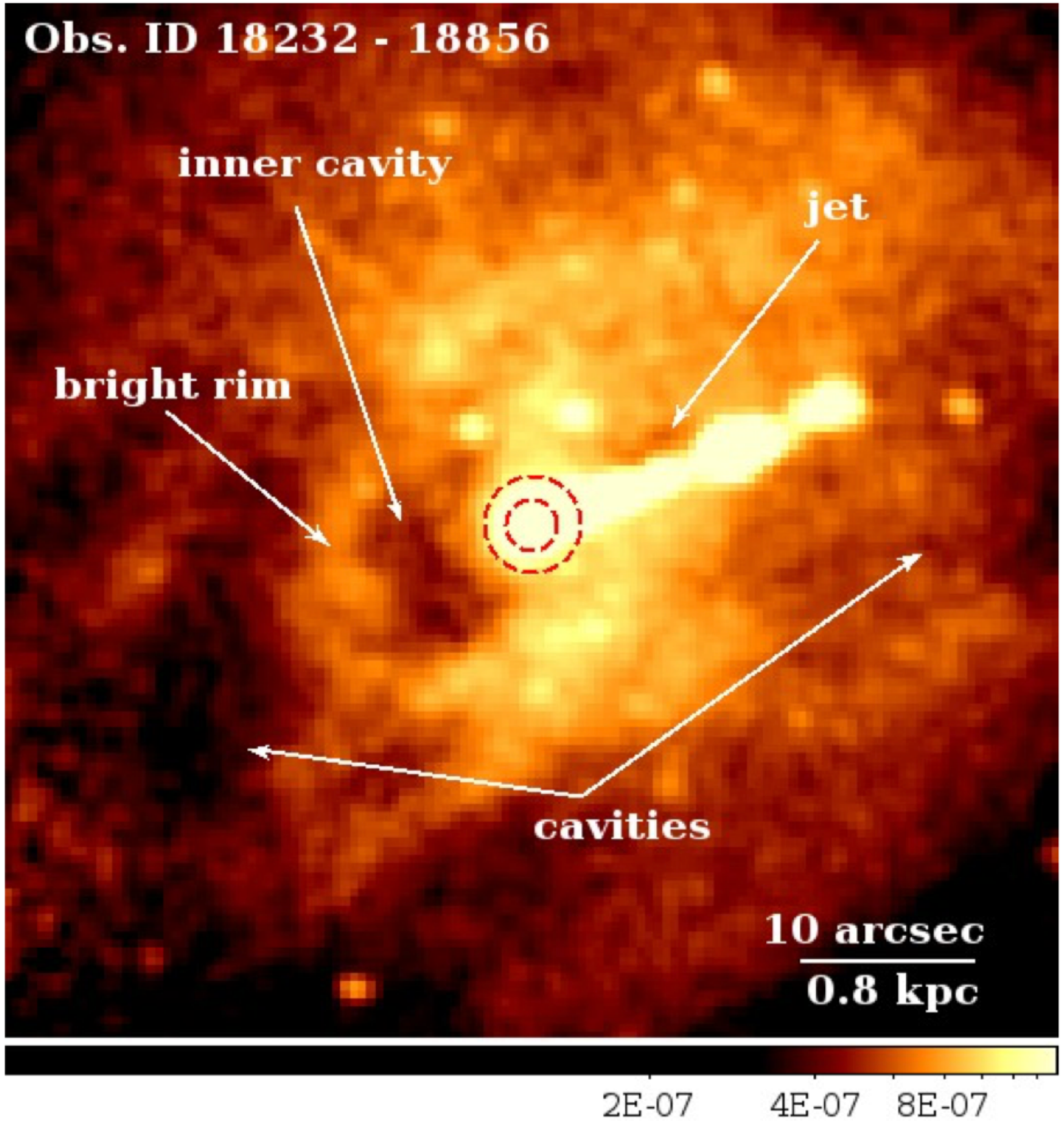
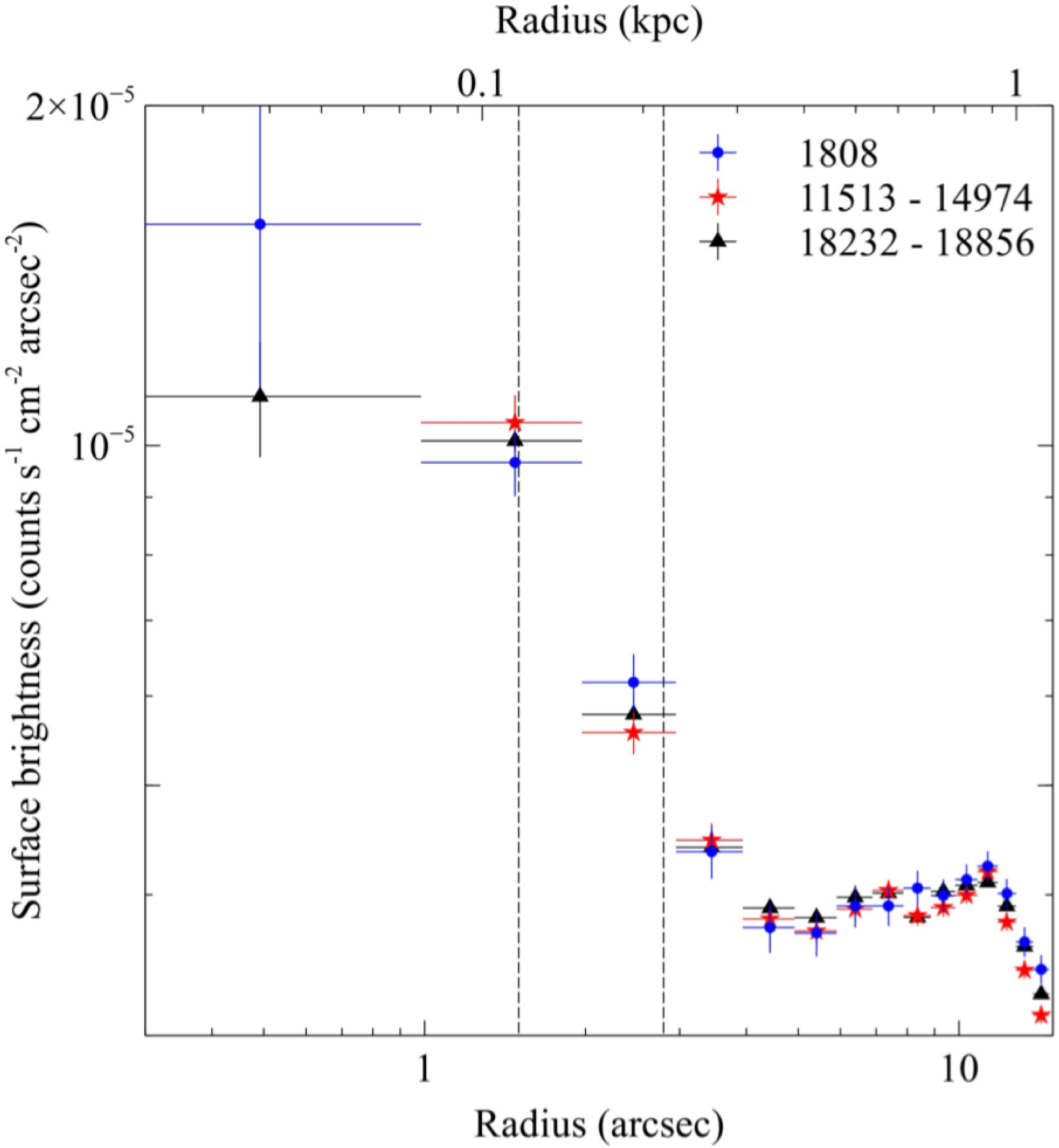
ABSTRACT

It is believed that most giant elliptical galaxies possess nuclear black holes with masses in excess of 10^8 $M_\odot$. Bondi accretion from the interstellar medium might then be expected to produce quasar-like luminosities from the nuclei of even quiescent elliptical galaxies. It is a puzzle that such luminosities are *not observed*. Motivated by this problem, Fabian & Rees have recently suggested that the final stages of accretion in these objects occurs in an advection-dominated mode with a correspondingly small radiative efficiency. Despite possessing a long-known active nucleus and dynamical evidence for a black hole, the low radiative and kinetic luminosities of the core of M87 provide the best illustration of this problem. We examine an advection-dominated model for the nucleus of M87, and show that accretion at the Bondi rate is compatible with the best-known estimates for the core flux from radio through to X-ray wavelengths. The success of this model prompts us to propose that Fanaroff–Riley (FR)I radio galaxies and quiescent elliptical galaxies accrete in an advection-dominated mode whereas FRII-type radio-loud nuclei





Russell et al. (2018)

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The imprints of AGN feedback within a supermassive black hole's sphere of influence

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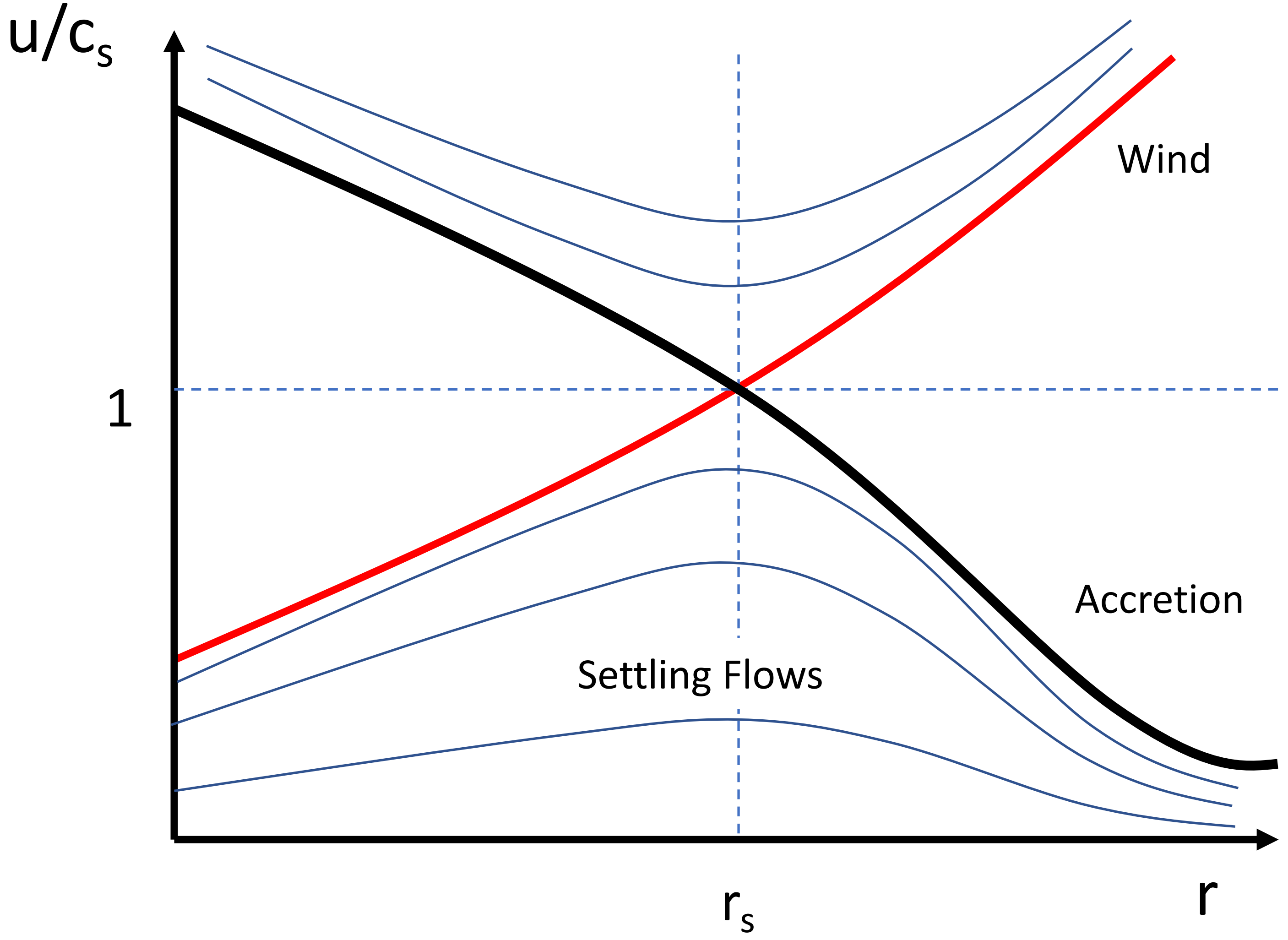
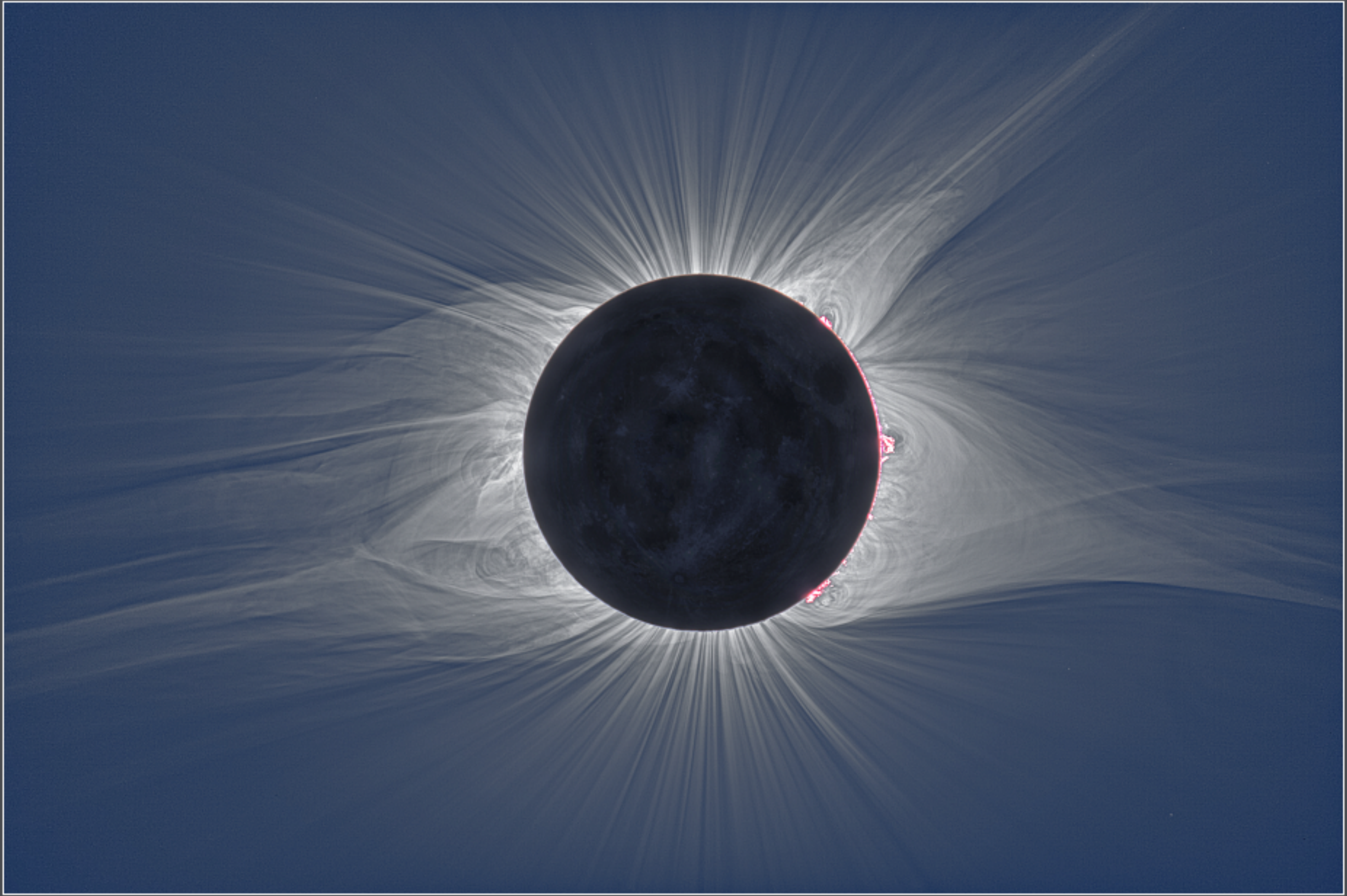
ABSTRACT

We present a new 300 ks *Chandra* observation of M87 that limits pileup to only a few per cent of photon events and maps the hot gas properties closer to the nucleus than has previously been possible. Within the supermassive black hole's gravitational sphere of influence, the hot gas is multiphase and spans temperatures from 0.2 to 1 keV. The radiative cooling time of the lowest temperature gas drops to only 0.1–0.5 Myr, which is comparable to its free fall time. Whilst the temperature structure is remarkably symmetric about the nucleus, the density gradient is steep in sectors to the N and S, with $\rho \propto r^{-1.5\pm0.1}$, and significantly shallower along the jet axis to the E, where $\rho \propto r^{-0.93\pm0.07}$. The density structure within the Bondi radius is therefore consistent with steady inflows perpendicular to the jet axis and an outflow directed E along the jet axis. By putting limits on the radial flow speed, we rule out Bondi accretion on the scale resolved at the Bondi radius. We show that deprojected spectra extracted within the Bondi radius can be equivalently fitted with only a single cooling flow model, where gas cools from 1.5 keV down below 0.1 keV at a rate of 0.03 $M_{\odot}$ yr^{-1}. For the alternative multitemperature spectral fits, the emission measures for each temperature component are also consistent with a

6. Same basic theory can be used to describe thermally driven wind; called the **Parker Wind solution.**





Total Solar Eclipse 2017