Measures and Eternal Inflation

Steve Gratton

25 July 2009

Inflation

- Successes
  - Homogeneity
    - A fraction more time before the big bang
  - Inhomogeneity
    - Small quantum fluctutations
- Problems
  - Initial conditions
    - How did inflation start?

Stochastic Eternal Inflation

- Successes
  - Homogeneity
    - Lots more time before the big bang
  - Inhomogeneity
    - Large quantum fluctutations
- Problems
  - Initial conditions
    - Who cares?

Volume-Weighting

- Basically “rewards” histories that inflate a lot
- Perhaps related to “spatial averaging”
- Perhaps related to the anthropic principle: more inflation might lead to more final volume and so to more observers

The Youngness Paradox (1)

- “Normal” version:
  - Universes that have evolved less will have formed later, so there should be more of them
    - A “typical” low-energy observer should be the first one in a pocket, and so say see as hot a CMB as possible

The Youngness Paradox (2)

- “Extreme” version:
  - Should crazy things have happened in our past, with the skewed statistics dominating over normal dynamics?
    - A “typical” low-energy observer should see Planck-scale inflation just to the past!
    - So no nice density perturbations…

Stochastic Inflation

- We start with the slow-roll equation…

$$\dot{\phi} + \frac{V_{,\phi}}{3H} = 0 \qquad \left(H^2 = V(\phi)/3\right)$$

- …and add noise:

$$\dot{\phi} + \frac{V_{,\phi}}{3H} = \frac{H^{3/2}}{2\pi} n(t)$$

- Trying to solve this directly is the *Langevin* approach
  - Cf. Heisenberg approach to QM
- Can also convert this into a PDE for the probability density; the *Fokker-Planck* approach
  - Cf. Schrodinger approach to QM

New Approach: Path Integral, cf Feynman

- Consider a general path $\phi(t)$.
- After a short time the field will be at $\phi + \dot{\phi}\Delta t$.
- According to the Langevin equation, the field "should" change by

$$-\frac{V_{,\phi}}{3H}\Delta t,$$

with a spread of $H^3\Delta t / 4\pi^2$.

- So, the probability of this segment of history occurring is:

$$\sqrt{\frac{2\pi}{H^3\Delta t}}e^{-2\pi^2\left(\dot{\phi}+V_{,\phi}/3H\right)^2\Delta t/H^3}d\phi.$$

- Multiplying for all the segments and taking the limit $\Delta t \to 0,$ we obtain:

Probability of a history…

$$P[\phi(t)] \propto e^{-S[\phi(t)]}$$

with $\quad S[\phi(t)] \equiv \int_0^T dt L(t),$

where $\quad L = \dfrac{2\pi^2(\dot{\phi} + V_{,\phi}/3H)^2}{H^3},$

Addition of the Volume-Weighting Term

- Just need to multiply each trajectory by its volume, namely:

$$a^3(T) = e^{3\int H(\phi(t))dt}$$

- So, putting it together we obtain…

...The Measure for Volume-Weighted Stochastic Inflation

$$P_V[\phi(t)] \propto e^{-S_V[\phi(t)]}$$

with $S_V[\phi(t)] \equiv \int_0^T dt L_V(t),$

where $L_V = \frac{2\pi^2(\dot{\phi} + V_{,\phi}/3H)^2}{H^3} - 3H.$

Now we can treat vol. weighting seriously!

Simplification to Quartic Case

- If the potential is $\lambda\phi^4$,

a change of variables simplifies the lagrangian to

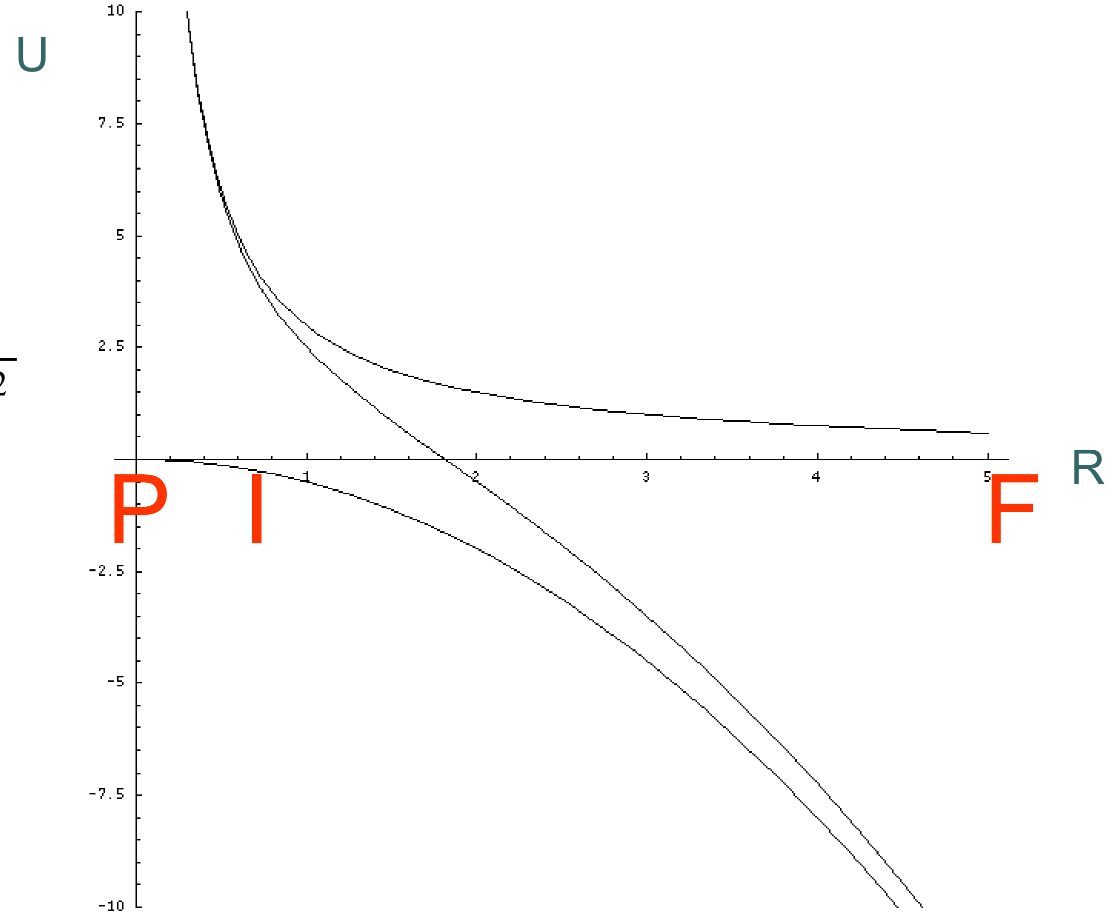
$$L_V \propto \frac{(\dot{R}-R)^2}{2} - \frac{3}{R},$$

where $R \propto 1/\phi^2$.

Get Approximate Results via Saddle Point Histories

$$U = -\frac{R^2}{2} + \frac{3}{R},$$

$$-\frac{dU}{dR} = R + \frac{3}{R^2}$$



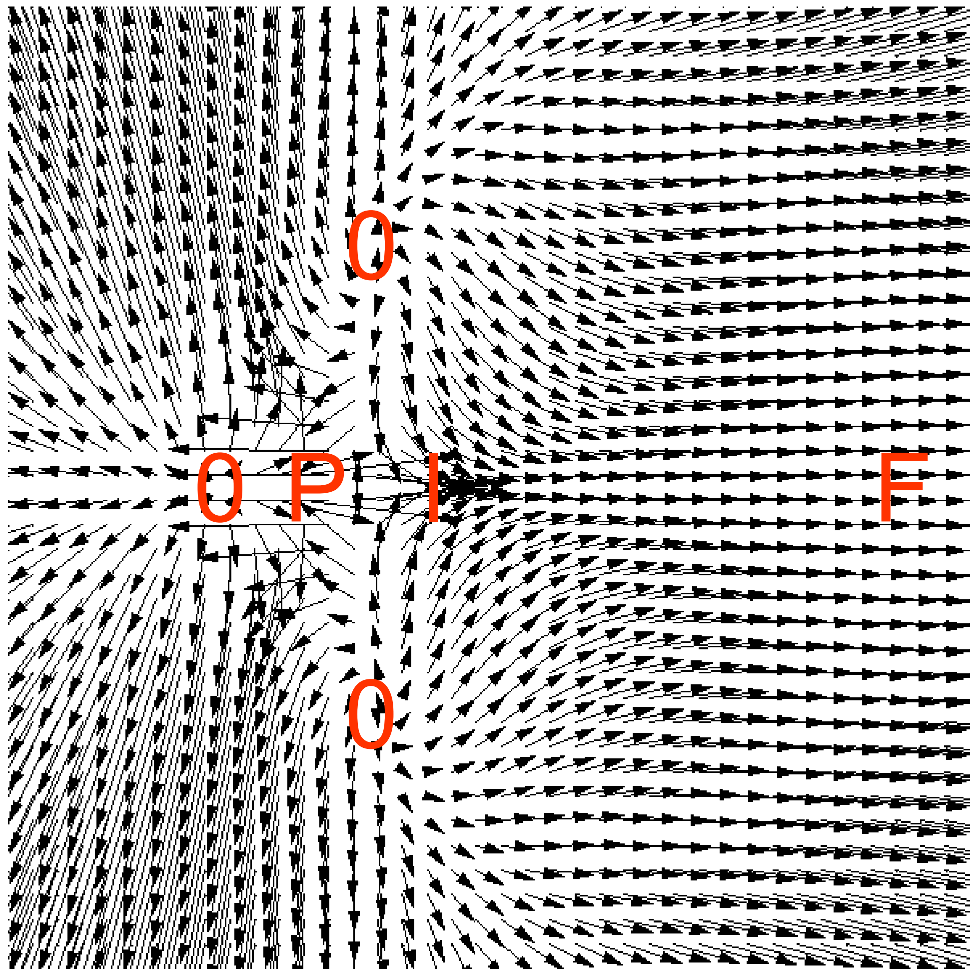
As the time interval increases, such solutions stop existing!

Where do we have to go? Into the complex plane!

$$\propto \frac{1}{\phi^2} \quad \angle R$$

$$U = -\frac{R^2}{2} + \frac{3}{R},$$

$$-\frac{dU}{dR} = R + \frac{3}{R^2}$$

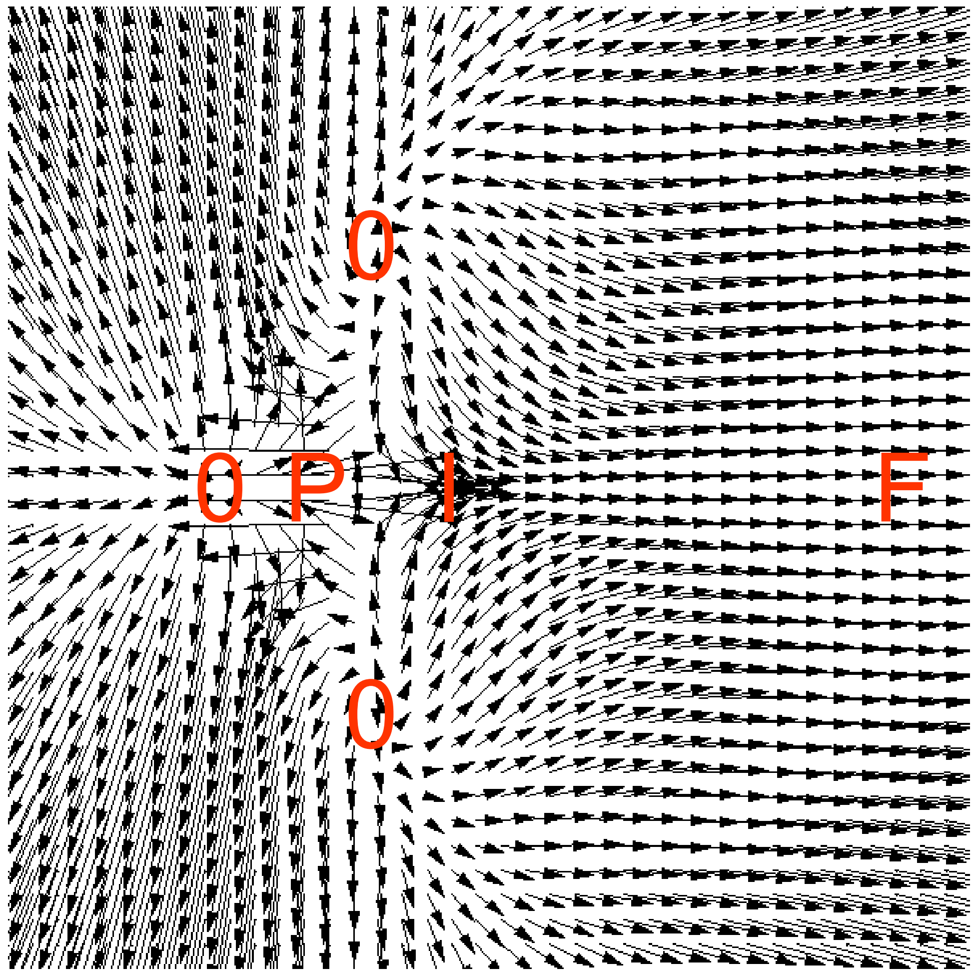


Loitering and Attractors, or Why Eternal Inflation Forgets

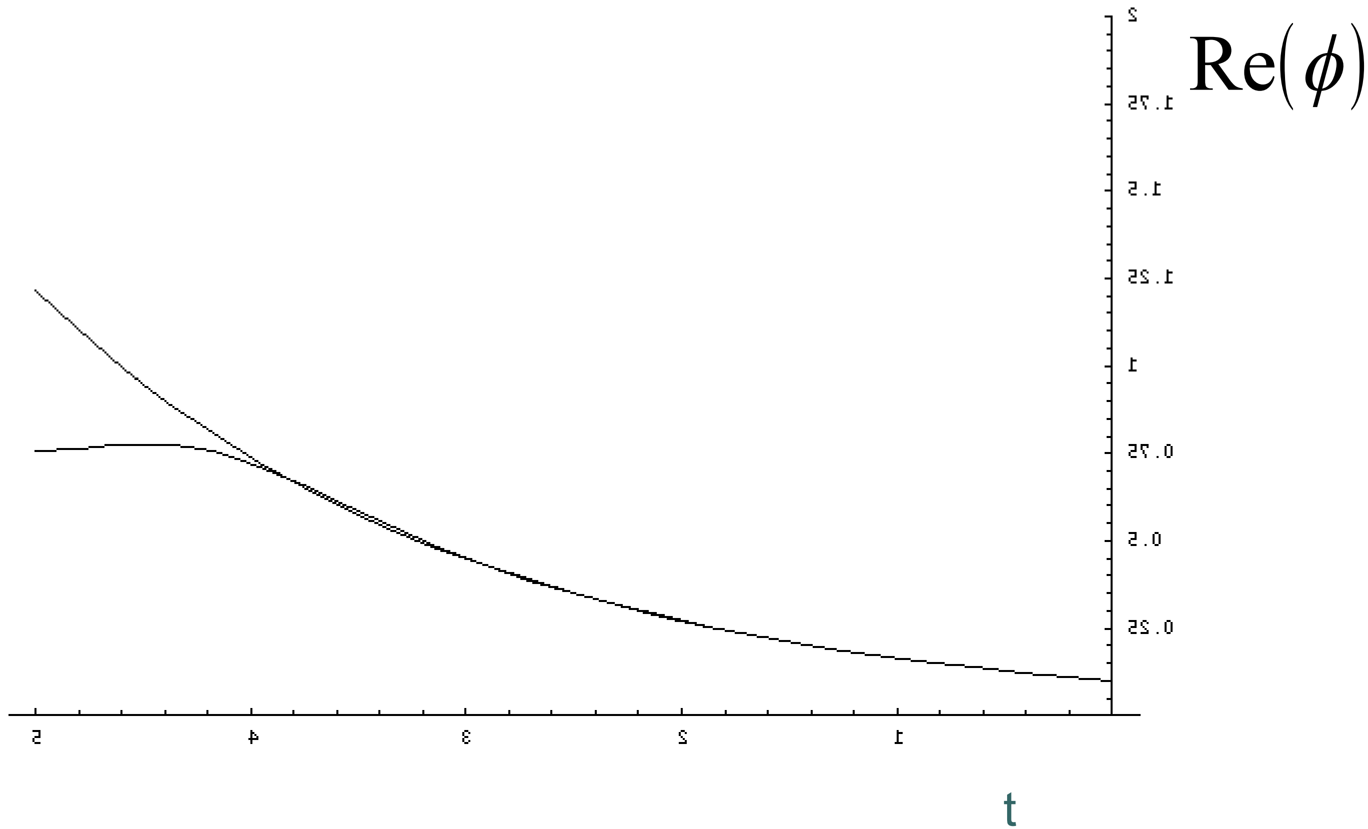
$$\propto \frac{1}{\phi^2}$$ R

$$U = -\frac{R^2}{2} + \frac{3}{R},$$

$$-\frac{dU}{dR} = R + \frac{3}{R^2}$$



And Inflation Ends Classically!



Conclusions

- There exists a powerful path integral formulation of stochastic eternal inflation,
  - in which volume-weighting can naturally be incorporated and taken seriously.
- Complex saddle points emerge, that are vital in allowing one to see
  - eternal inflation depends only weakly on initial conditions
  - that the "extreme" youngness paradox doesn't occur

(…very hard with Langevin/Fokker-Planck)

Other measures…

- Scale Factor cutoff

Reflection:
A democracy of measures?

- As inflation ends in the same way basically independently of final time, we’ve seen that the much-maligned proper time volume weighting can be used to answer at least this question unambiguously…
- Perhaps the “measure problem” is as much a problem with specific questions as for the weighting schemes themselves!