Numerical Galaxy Formation & Cosmology

Lecture 5: Halo Finders & Semi-analytic Models

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Outline of the lecture course

• Lecture 1: Motivation & Initial conditions
• Lecture 2: Gravity algorithms & parallelization
• Lecture 3: Hydro schemes
• Lecture 4: SPH, Radiative cooling & heating, Subresolution physics
• Lecture 5: Halo and subhalo finders & Semi-analytic models
• Lecture 6: Getting started with Gadget
• Lecture 7: Example galaxy collision
• Lecture 8: Example cosmological box
Outline of this lecture

• Halo and subhalo finders
  ‣ Friends-of-Friends (FOF)
  ‣ Density peak finders
  ‣ Unbinding particles
  ‣ Examples: BDM, AHF, Subfind, Rockstar

• Semi-analytic models
  ‣ Radiative Cooling
  ‣ Star formation & feedback
  ‣ Mergers & morphology

• Empirical galaxy formation models (abundance matching)
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Cosmological Simulations

- Up to now: Initial conditions and codes to evolve simulation
  (Gravity: N-body, Hydro: SPH, AMR, Moving Mesh)

- In the end: Information on particles (and/or cells), NOT haloes/galaxies!

- Need to analyse particle information somehow to get meaningful
  information about haloes/galaxies

- 1st step: Halo finder ➔ Identifies
  bound objects in particle data

- Then: analyse particles of given halo
  (total mass, profile, etc).
Halo Finders for larger simulations

- Larger simulations (Moore’s law) require better halo finders
- More and better algorithms have been developed (listing most important ones):
  - 1985: FOF
  - 1991: DENMAX
  - 1995: adaptive FOF
  - 1997: BDM
  - 1988: HOP
  - 1999: hierarchical FOF
  - 2001: SKID
  - 2001: SUBFIND
  - 2006: 6DFOF
  - 2009: AHF
  - 2010: ROCKSTAR
Hierarchical Structure Formation

• In CDM: Structures form hierarchically: identify haloes, subhaloes, subsubhaloes, …
Different Methods

- Can search for haloes using different particle information:
  - Positions (configuration space): 3D
  - Positions & velocities (configuration & velocity space): 3D+3D
  - Positions & velocities (phase space): 6D
  - Positions, velocities & time (phase space & time domain): 7D

- Easiest methods use just position:
  - Friends-of-Friends (FOF)
  - Density-Peak-Finder

- More advanced methods also use the velocity information to ‘unbind’ particles: adaptive/hierarchical FOF, BDM, SUBFIND, AHF, …
Friends-of-Friends

- Uses only particle positions to group spatially close particles:

\[ |\vec{x}_i - \vec{x}_j| \leq b \Delta x = b \frac{B}{\sqrt[3]{N}} , \quad B = \text{Boxsize}, \ N = \# \text{ of particles}, \ b \approx 0.2 \]
Friends-of-Friends

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- Advantages: fast, arbitrary halo shapes

- Disadvantages: no subhaloes, danger of linking bridges
Density Peak Finders

- Smooth density field and locate peaks
Density Peak Finders

• Smooth density field and locate peaks

• Collect particles around peaks

• Advantages: haloes clearly separated, can identify subhaloes

• Disadvantages: usually spherical shapes, need to smooth density field
Unbinding particles from haloes

- Use velocity information to remove unbound particles from haloes

- From $\Delta \phi = 4\pi G \rho$ and spherical symmetry:
  
  \[ \frac{d\phi}{dr} = \frac{GM(< r)}{r^2} \]

  \[ \phi(r) = G \int_0^r \frac{M(< r')}{r'^2} \, dr' + \phi(0) \text{ and } \phi(0) \text{ from } \phi(\infty) = 0 \]

- Order particles with respect to distance (from centre):

  \[
  \int_0^r \frac{M(< r')}{r'^2} \, dr' = \int_0^{r_1} \frac{M(< r)}{r^2} \, dr + \int_{r_1}^{r_2} \frac{M(< r)}{r^2} \, dr + \cdots + \int_{r_{N-1}}^{r_N} \frac{M(< r)}{r^2} \, dr
  \]

  \[= \frac{m_1}{r_1^2} r_1 + \frac{m_1 + m_2}{r_2^2} |r_2 - r_1| + \cdots + \sum_{i} \frac{m_i}{r_N^2} |r_N - r_{N-1}| \]

- Remove particles with $v_i > v_{\text{esc}}(r_i) = \sqrt{2|\phi(r_i)|}$

- Repeat iteratively until no more particles are removed
Phase Space Halo Finders

- Use the complete 6D phase space data to link particles
- 6DFOF: extend standard FOF linking:
  \[
  \frac{(\vec{x}_i - \vec{x}_j)^2}{(b \Delta x)^2} + \frac{(\vec{v}_i - \vec{v}_j)^2}{(b_v \Delta v)^2} < 1
  \]
- For \( b_v \to \infty \) we recover standard FOF
- Unbinding often not done (as most particles in 6D-linked structures are bound)
- Halo finders can be extended in time-domain
  If halo is lost between N snapshots, add halo with interpolated properties
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Bound Density Maxima (BDM)

- BDM first identifies peaks (potential halo centres) via sphere jittering
- Randomly place N spheres and iteratively move to center of mass
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\[ x: \text{center of mass} \]
\[ x: \text{sphere centre} \]
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- Problems: sphere radius (smoothing scale)
  - too big → too much smoothing, too small → spheres get stuck
  - Too many spheres → duplicates (can be removed)
  - Too few spheres → missing haloes
- Use spherical overdensity criterion to compute $R_{200}$: $\rho(R_{200}) = 200\rho_c$
  Compute $M_{200} = \frac{4\pi}{3} R_{200}^3 200\rho_c$ and unbind particles iteratively
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AMIGA HALO FINDER (AHF)

- Uses adaptive mesh to locate possible halo centres
- Refinement levels define isodensity contours
- The AMR hierarchy is organized into a tree structure
- Identify Halo, sub-halo, sub-sub-halo, etc
- Unbind particles from haloes and compute properties
SUBFIND (integrated into Gadget)

• First step is to define parent haloes with 3D FOF

• Local (smoothed) density is computed for particles with SPH kernel

• Identify saddle points: locations where neighboring densities are larger

• Compute isodensity contours around saddle points

• Subhaloes = regions enclosed by isodensity surfaces

• Remove unbound particles
ROCKSTAR

- Selects particle groups with 3D FOF and large $b = 0.28$
- For each group: divide positions and velocity by dispersion:
  - Choose phase-space linking length adaptively to link 70% of group’s particles in sub-groups
- Repeat in each sub-group and define sub-subgroups until minimum number of particles is reached (10).
- Place seed haloes at lowest substructure level and assign particles according to their phase-space proximity
- Remove unbound particles and compute halo properties
Merger trees from N-body simulations

- Link haloes through time
- Identify descendants in later snapshots (Or progenitors in earlier snapshots)
- In hierarchical structure formation: several progenitors but one descendant
- When subhaloes merge their particles move to the ‘main’ halo
- Use particle IDs: for given halo, identify all haloes in later snapshot that contain its particles (possibly weigh by binding energy)
- Descendant is the halo with most matching particles (or highest weight)
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Semi-analytic models

- Semi-analytic models populate dark matter halo merger trees with galaxies using simple, physically motivated recipes for baryonic physics.

- As physics behind galaxy formation is not very well understood (c.f. sub-grid models in simulations), each recipe contains a number of free parameters.

- Recipes are either motivated by observations or detailed simulations.

- Free parameters are ‘tuned’ such that observed statistical galaxy properties are reproduced.
Gas cooling

- Gas is initially distributed as dark matter
- Falls into dark matter haloes and shock heats

**Virial temperature:** \( T_{\text{vir}} = \mu m_p V_{\text{vir}}^2 / 2k = 35.9\left[V_{\text{vir}}/(\text{km s}^{-1})\right]^2 K \)

- Gas cools from hot gaseous halo, loses pressure support, falls to centre and forms disc

\[
t_c = \frac{(3/2)n_gkT}{n_g^2 \Lambda(T_{\text{vir}}, Z_g)} = \frac{3\mu m_p kT}{2\rho_g \Lambda(T_{\text{vir}}, Z_g)}
\]

- For given \( \rho(r) \) compute cooling radius \( r_c \)
- After a time \( t = t_c \) all gas in \( r_c \) had time to cool
Star formation

- No theory of star formation that gives SFR as function of ISM
- More pragmatic approach based on dimensional argument: \( \dot{m}_* \propto \frac{m_c}{\tau} \)
- All stars form in exponential disc with scale length \( \lambda \dot{r}_d R_{200} \)
- Gas disc is also in exponential profile with \( r_g = \chi r_d \) and \( \chi \approx 1.5 \)
- Star formation rate given by Kennicutt-Schmidt-Relation: \( \dot{\Sigma}_* = A_K \Sigma_g^N \)
  \[
  \dot{m}_* = \int_0^{r_c} \dot{\Sigma}_* 2\pi r dr = \int_0^{r_c} A_K \Sigma_g^N 2\pi r dr
  \]
- Fraction of short-lived stars \( \beta \) leads to modified SFR: \( \dot{m}'_* = (1 - \beta)\dot{m}_* \)
- In timestep \( dt \) the mass \( dm = \dot{m}'_* dt \) is formed
Supernova feedback

- Cold gas is ejected from the central disc by SN-driven winds
- Rate of reheating by winds: \( \dot{m}_{rh} = \epsilon_0^{SN} \left( \frac{V_d}{200 \text{ km s}^{-1}} \right)^{\alpha_{rh}} \dot{m}_* \)
  with \( \epsilon_0^{SN} \approx 1.3 \) and \( \alpha_{rh} \approx 2 \)
- Heated gas either trapped in halo, or ejected into diffuse IGM
- Fraction of ejected gas is \( f_e(V_{\text{vir}}) = \left[ 1 + \left( \frac{V_{\text{vir}}}{V_e} \right)^{\alpha_e} \right]^{-1} \)
  with \( \alpha_e \approx 6 \) and \( V_e \approx 100 - 150 \text{ km s}^{-1} \)
- Ejected gas can recollapse into halo at later times and become available for cooling: \( \dot{m}_{re} = \chi_{re} \left( \frac{m_e}{t_{\text{dyn}}} \right) \)
  with \( \chi_{re} \approx 0.1 \)
AGN feedback

• Every top-level halo contains a seed black hole (usually $M_{\text{seed}} = 100M_\odot$)

• BHs grow during galaxy mergers: $\Delta m_{\text{BH}} = \frac{f_{\text{BH}} (m_s/m_c) m_{\text{cg}}}{1 + (280 \text{ km s}^{-1}/V_{\text{vir}})^2}$

• Quasar mode: energy released during rapid BH growth drives wind.
  Outflow rate: $p_{\text{rad}} = p_{\text{wind}}$ or $\epsilon_w \eta r m_{\text{acc}} c = m_{\text{out}} V_e$
  yields $m_{\text{out}} = \epsilon_w \eta r (c/V_e) m_{\text{acc}}$

• Radio mode: hot gas accretes onto BH and releases energy to hot halo
  Accretion rate: $m_{\text{radio}} = \kappa_r \left( \frac{kT}{\Lambda(T, Z)} \right) \left( \frac{m_{\text{BH}}}{10^8 M_\odot} \right)$
  Energy that suppresses cooling: $L_{\text{BH}} = \eta r m_{\text{radio}} c^2$
  Modified cooling rate: $m'_{\text{cool}} = m_{\text{cool}} - 4L_{\text{BH}}/3V_{\text{vir}}^2$
  Cooling never allowed to fall below 0
Starbursts during mergers

• Main haloes host central galaxies, subhaloes host satellite galaxies

• Stars can be either in the disc, or in a spherical bulge component

• When satellites merge with central: merger triggered starburst

  Fraction of cold gas turned into stars: \( e_{sb} = e_0 \mu \gamma_{sb} \) \( \mu \): mass ratio

• Slope \( \gamma_{sb} \) depends on bulge-to-total ratio B/T

  (more massive bulges suppress starburst efficiency)

• Normalisation \( e_{sb} \) depends on halo mass and gas fraction

• Stars created in a starburst are added to the bulge
Morphological transformations

- All stars formed from the cold gas disc are added to the stellar disc
- All stars formed in a starburst are added to the bulge
- In a galaxy merger: a fraction of the disc is moved to the bulge:
  - Major merger: all stars are moved to the bulge
  - Minor merger: Stars from satellite are added to the bulge
    
    \[
    \text{Fraction of central stars added to bulge: } f(\mu)
    \]

- Secular evolution: if stellar disc becomes too massive for its size, a disc instability develops (bar forms and gets destroyed \(\rightarrow\) bulge)
  
  If \( \epsilon = \frac{v_{\text{max}}}{\sqrt{GM_{\text{disc}}/R_{\text{disc}}}} \) a fraction of disc stars are moved to the bulge
  
  (either total stellar disc, or just the ‘unstable mass’)

\[\text{Benjamin Moster} \quad \text{Numerical Galaxy Formation & Cosmology V} \quad \text{IoA, 10.02.2016}\]
Morphological transformations

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Fraction of central stars added to bulge: \( f(\mu) \)

Secular evolution: if stellar disc becomes too massive for its size, a disc instability develops (bar forms and gets destroyed)
  - A fraction of disc stars are moved to the bulge (either total stellar disc, or just the 'unstable mass').

\[
24 \quad v_{\text{max}} = \frac{v_G M_{\text{disc}}}{R_{\text{disc}}}
\]
Semi-analytic models in action

- The gaseous & stellar structure are not modelled in detail
  But: profiles & positions of galaxies are given and can be visualised:

\[ \text{GALFORM galaxies} \]
\[ \text{Dark matter density} \]
\[ \text{Redshift} = 0.00 \]
Global properties

- Semi-analytic models are able to reproduce a large number of observed statistical galaxy properties
- Advantage: flexibility, fast, large number of systems, statistics
- Disadvantage: little spatial information, simplified approximations, many free parameters
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Subhalo abundance matching

- In CDM paradigm: galaxies form in dark matter haloes
- Galaxy properties (luminosity, stellar mass) are coupled to depth of potential well and depend on halo mass
- Don’t model baryonic physics \( \rightarrow \) link \( m^* \) and \( M_{\text{halo}} \) statistically

Subhalo abundance matching: extract positions and masses of haloes and subhaloes from N-body simulations

- Link galaxies and haloes using simple assumption:
  Most massive galaxies live in most massive haloes
- Connection can be done in 2 ways:
  a) assuming a non-parametric monotonic relation
  b) assuming parameterized functions
Abundance matching & parameterized linking

- Produce galaxy catalogue from observed SMF in same volume as halo catalogue
- Match galaxies-haloes by mass
- Optional: Use fitting-function to get $m^*(M_h)$

$$m^*(M_h) = 2 R M_h \left[ \left( \frac{M_h}{M_1} \right)^{-\beta} + \left( \frac{M_h}{M_1} \right)^\gamma \right]$$
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$$m^*(M_h) = 2 R M_h \left[ \left( \frac{M_h}{M_1} \right)^{-\beta} + \left( \frac{M_h}{M_1} \right)^{\gamma} \right]$$

- Assume function for $m^*(M_h)$
- Populate haloes with galaxies
- Compute model SMF
- Fit parameters to observed SMF
- Derive $m^*(M_h)$ individually for a set of redshifts
Abundance matching & parameterized linking

- Produce galaxy catalogue from observed SMF in same volume as halo catalogue
- Match galaxies-haloes by mass
- Optional: Use fitting-function to get $m^*$ ($M_h$)

Abundance matching & parameterized linking

- Assume function for $m^*$ ($M_h$)
- Populate haloes with galaxies
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- Derive $m^*$ ($M_h$) individually for a set of redshifts

$ m^*(M_h) = \frac{2}{3} M_h^{0.6} + \frac{5}{3} M_h^{0.2} \times \ldots $
Abundance matching & parameterized linking

- Produce galaxy catalogue from observed SMF in same volume
- Match galaxies-haloes by mass
- Optional: Use fitting-function to get $m_*(M_h)$

- Assume function for $m_*(M_h)$
- Populate haloes with galaxies
- Compute model SMF
- Fit parameters to observed SMF

- Derive $m_*(M_h)$ individually for a set of redshifts

\[ m_*(M_h) = \begin{cases} \frac{2}{3} M_h - M_1 & \text{if } M_h > M_1 \\ \frac{1}{2} M_h - M_1 & \text{otherwise} \end{cases} \]
Final notes

- **Text Books:**
  - Cosmology: Galaxy Formation and Evolution (Mo, vdBosch, White)

- **Reviews:**

- **Papers:**
  - Knebe et al. (2011), MNRAS, 415, 2293
  - Guo et al. (2011), MNRAS, 435, 897

- **Gadget and N-GenIC website:**
  
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