Numerical Galaxy Formation & Cosmology

Lecture 5: Halo Finders & Semi-analytic Models

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Outline of the lecture course

- Lecture 1: Motivation & Initial conditions
- Lecture 2: Gravity algorithms & parallelization
- Lecture 3: Hydro schemes
- Lecture 4: SPH, Radiative cooling & heating, Subresolution physics
- Lecture 5: Halo and subhalo finders & Semi-analytic models
- Lecture 6: Getting started with Gadget
- Lecture 7: Example galaxy collision
- Lecture 8: Example cosmological box

Outline of this lecture

- Halo and subhalo finders
  - Friends-of-Friends (FOF)
  - Density peak finders
  - Unbinding paricles
  - Examples: BDM, AHF, Subfind, Rockstar
- Semi-analytic models
  - Radiative Cooling
  - Star formation & feedback
  - Mergers & morphology
- Empirical galaxy formation models (abundance matching)

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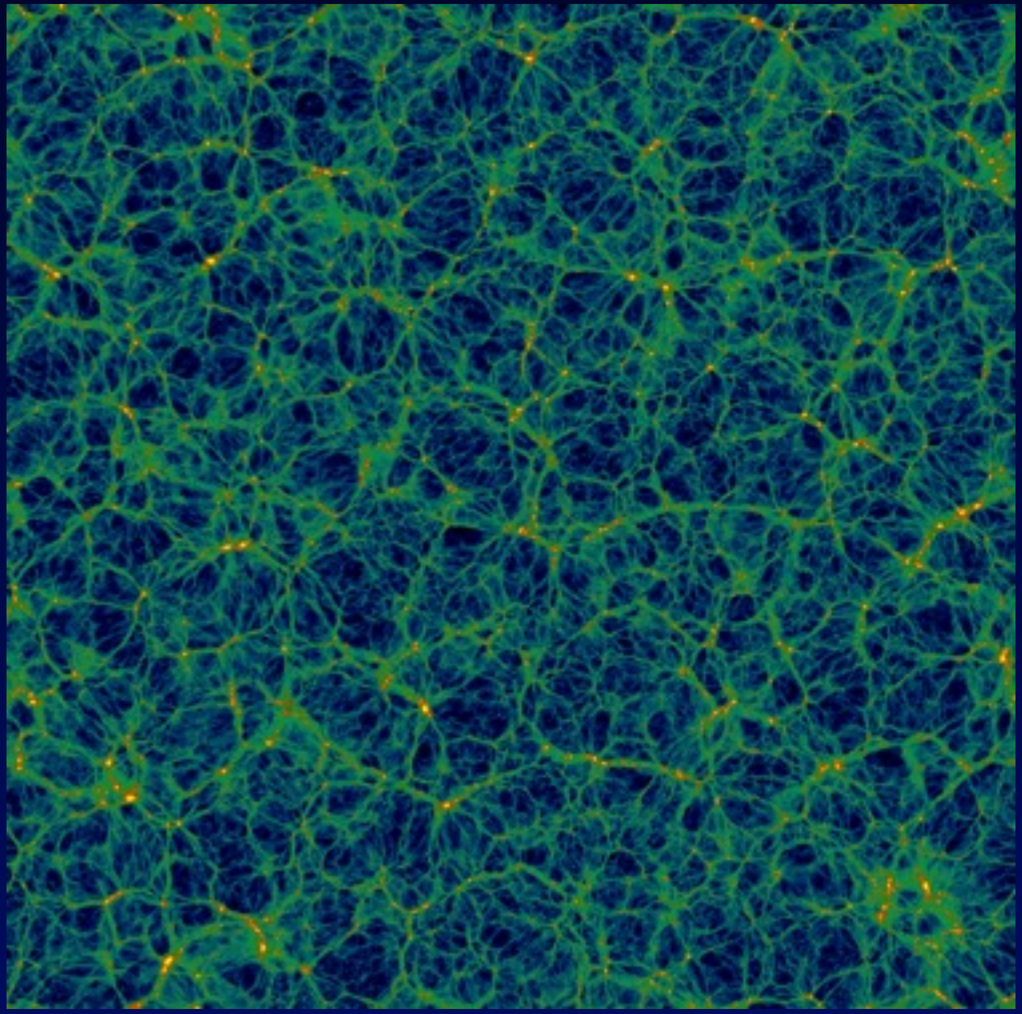
Cosmological Simulations

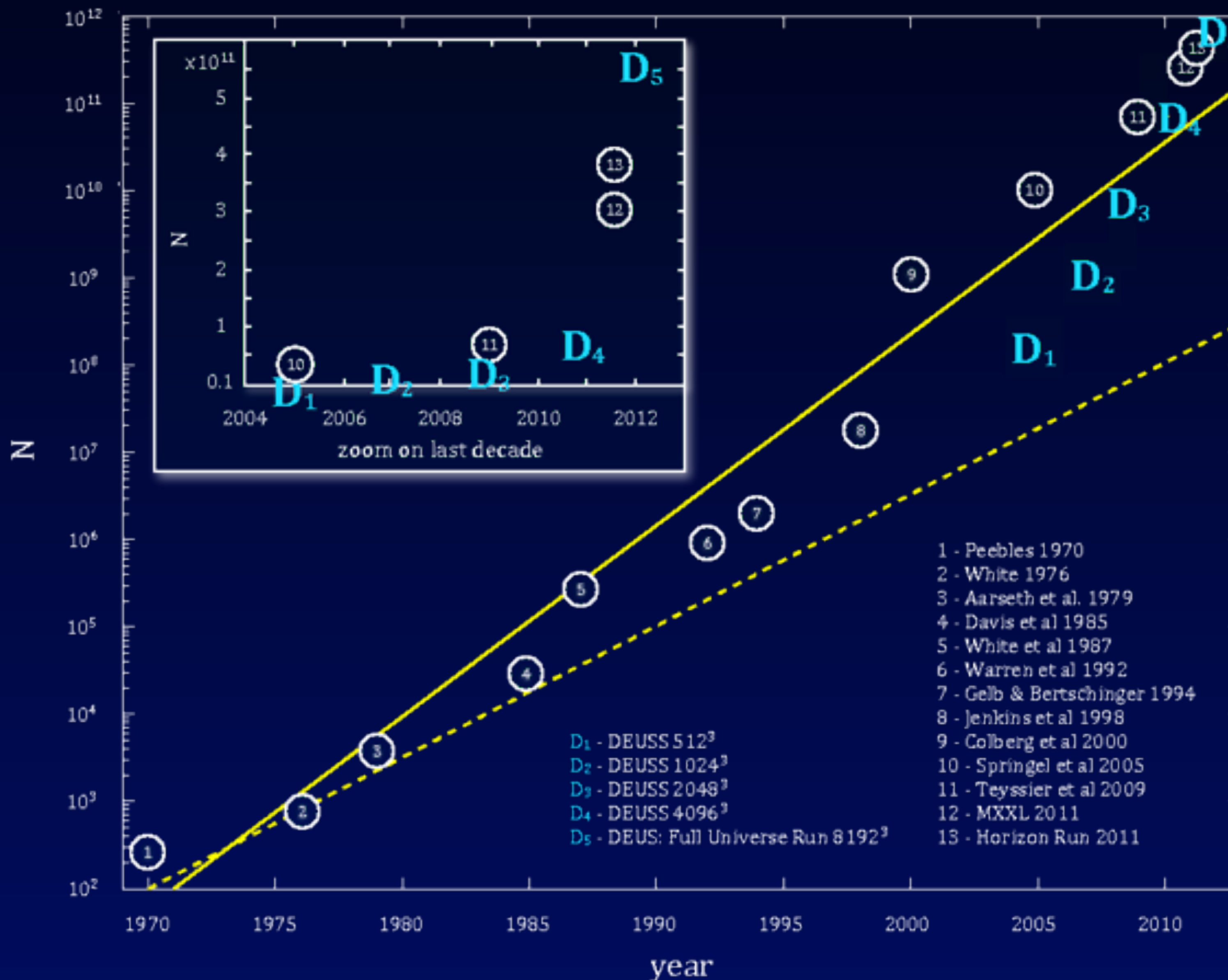
- Up to now: Initial conditions and codes to evolve simulation (Gravity: N-body, Hydro: SPH, AMR, Moving Mesh)
- In the end: Information on particles (and/or cells), NOT haloes/galaxies!
- Need to analyse particle information somehow to get meaningful information about haloes/galaxies



- 1st step: Halo finder ➛ Identifies bound objects in particle data
- Then: analyse particles of given halo (total mass, profile, etc).

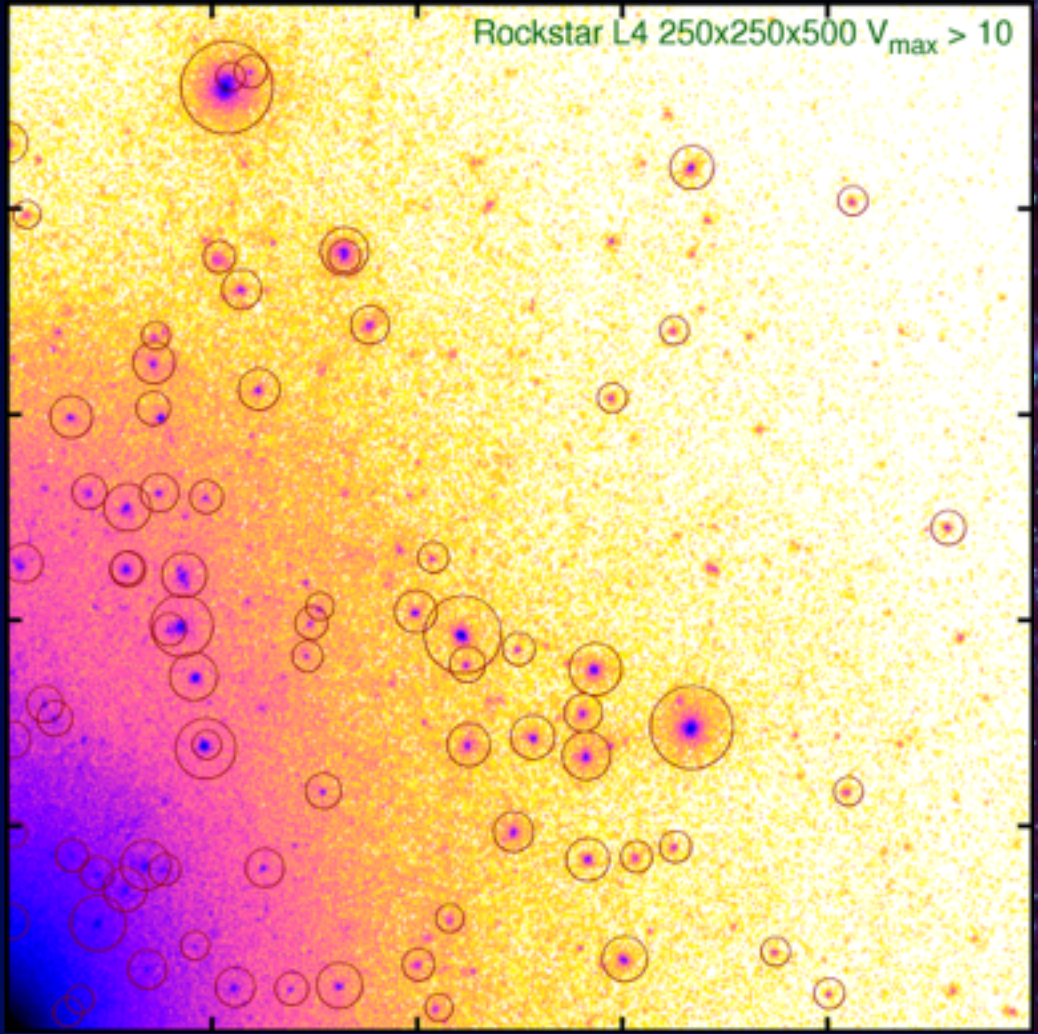
Halo Finders for larger simulations

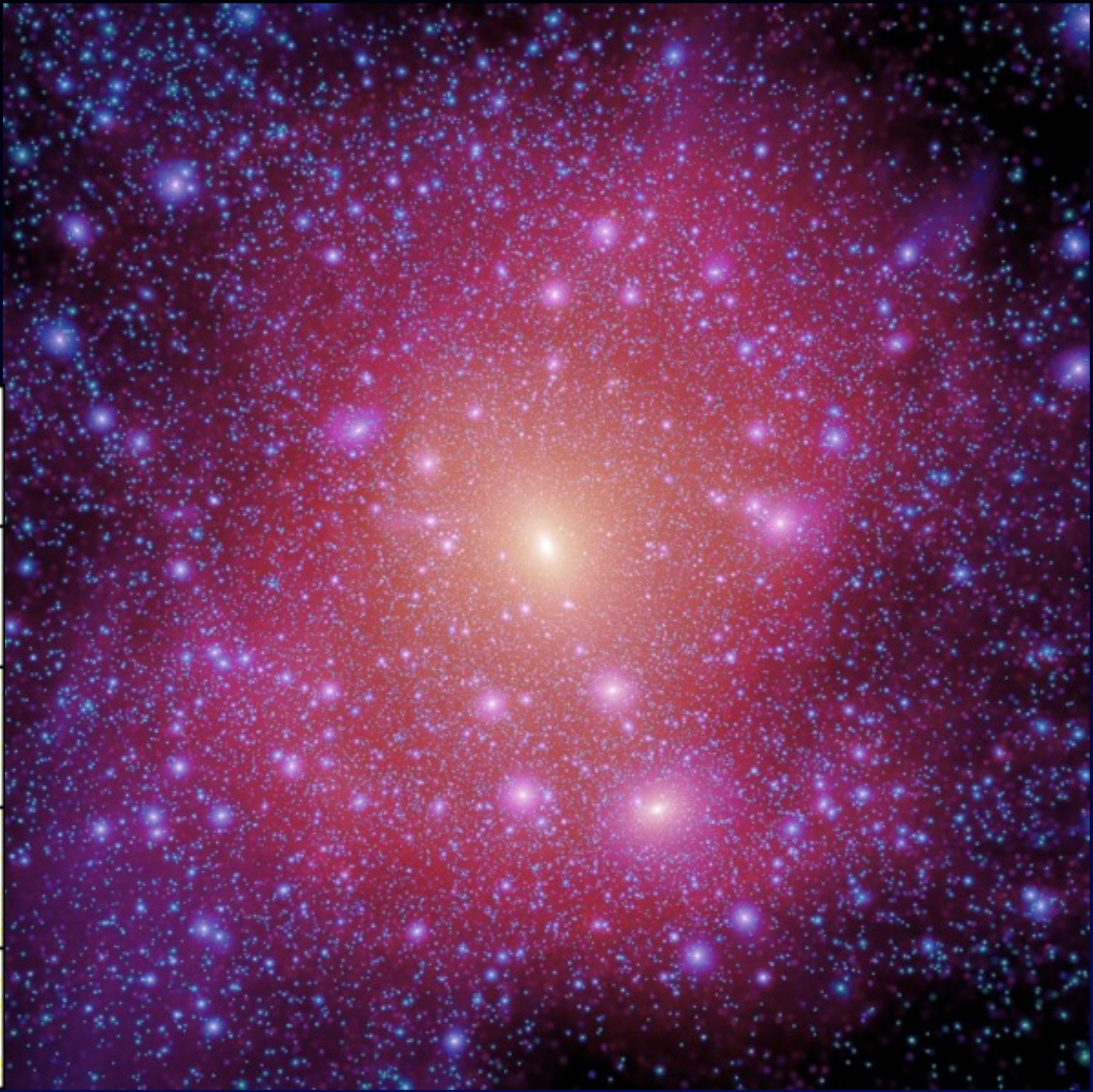
- Larger simulations (Moore's law) require better halo finders
- More and better algorithms have been developed (listing most important ones):
  - 1985: FOF
  - 1991: DENMAX
  - 1995: adaptive FOF
  - 1997: BDM
  - 1988: HOP
  - 1999: hierarchical FOF
  - 2001: SKID
  - 2001: SUBFIND
  - 2006: 6DFOF
  - 2009: AHF
  - 2010: ROCKSTAR



Hierarchical Structure Formation

- In CDM: Structures form hierarchically: identify haloes, subhaloes, subsubhaloes, …





Different Methods

- Can search for haloes using different particle information:
  - Positions (configuration space): 3D
  - Positions & velocities (configuration & velocity space): 3D+3D
  - Positions & velocities (phase space): 6D
  - Positions, velocities & time (phase space & time domain): 7D

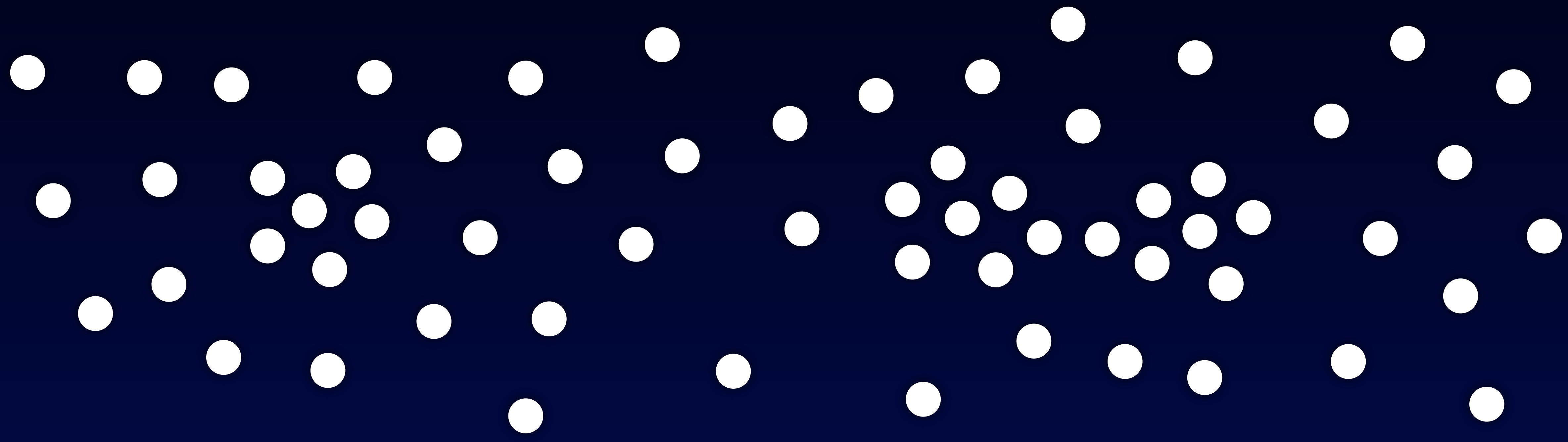
- Easiest methods use just position:
  - Friends-of-Friends (FOF)
  - Density-Peak-Finder

- More advanced methods also use the velocity information to ‘unbind’ particles: adaptive/hierarchical FOF, BDM, SUBFIND, AHF, …

Friends-of-Friends

- Uses only particle positions to group spatially close particles:

  $|\vec{x}_i - \vec{x}_j| \leq b\,\Delta x = b\,B/\sqrt[3]{N}$, B = Boxsize, N = # of particles, b $\approx$ 0.2



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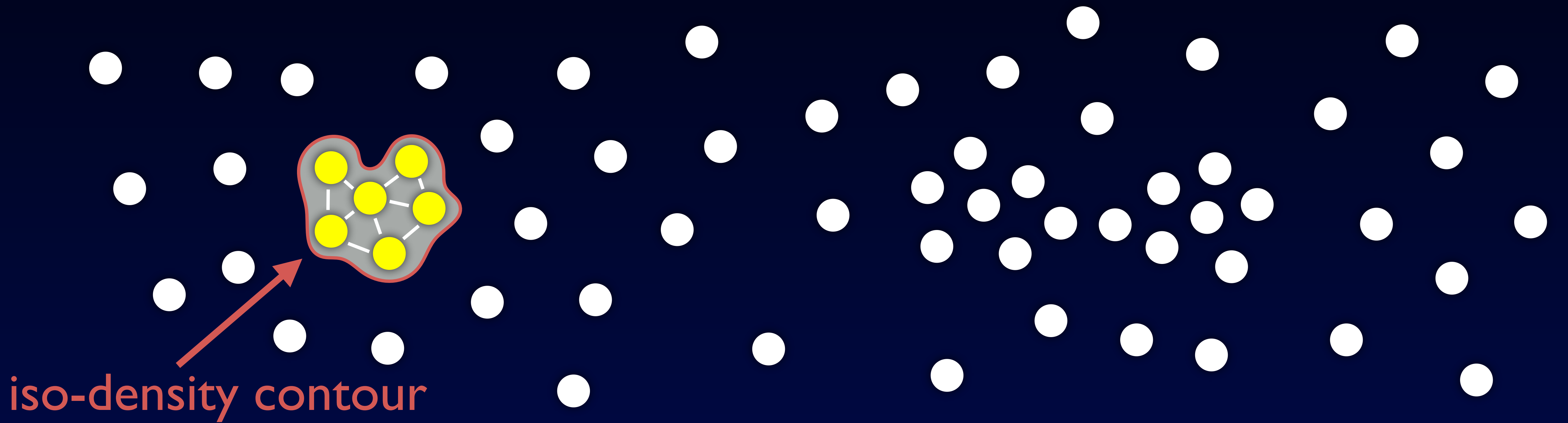

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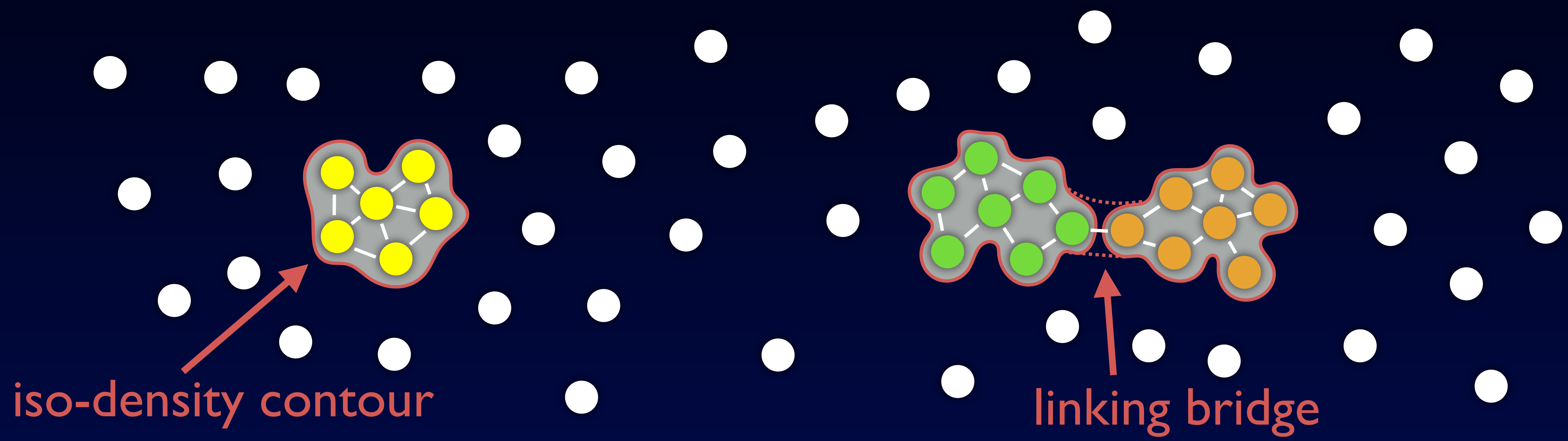
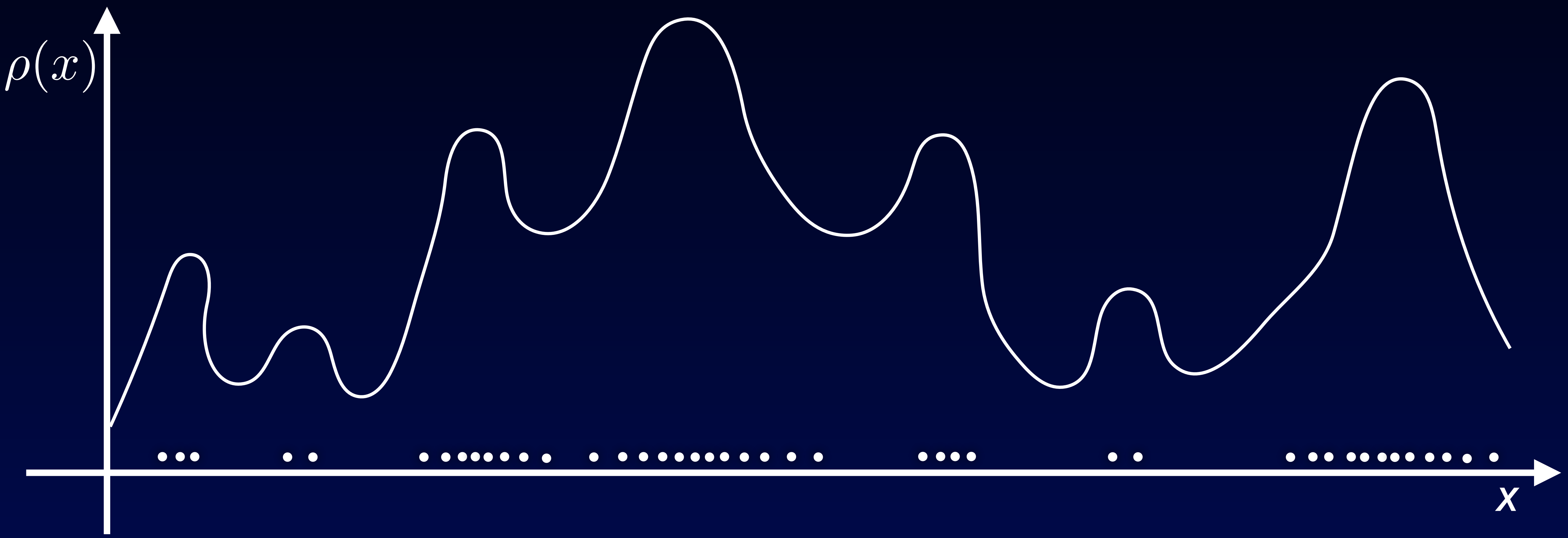


- Advantages: fast, arbitrary halo shapes

- Disadvantages: no subhaloes, danger of linking bridges

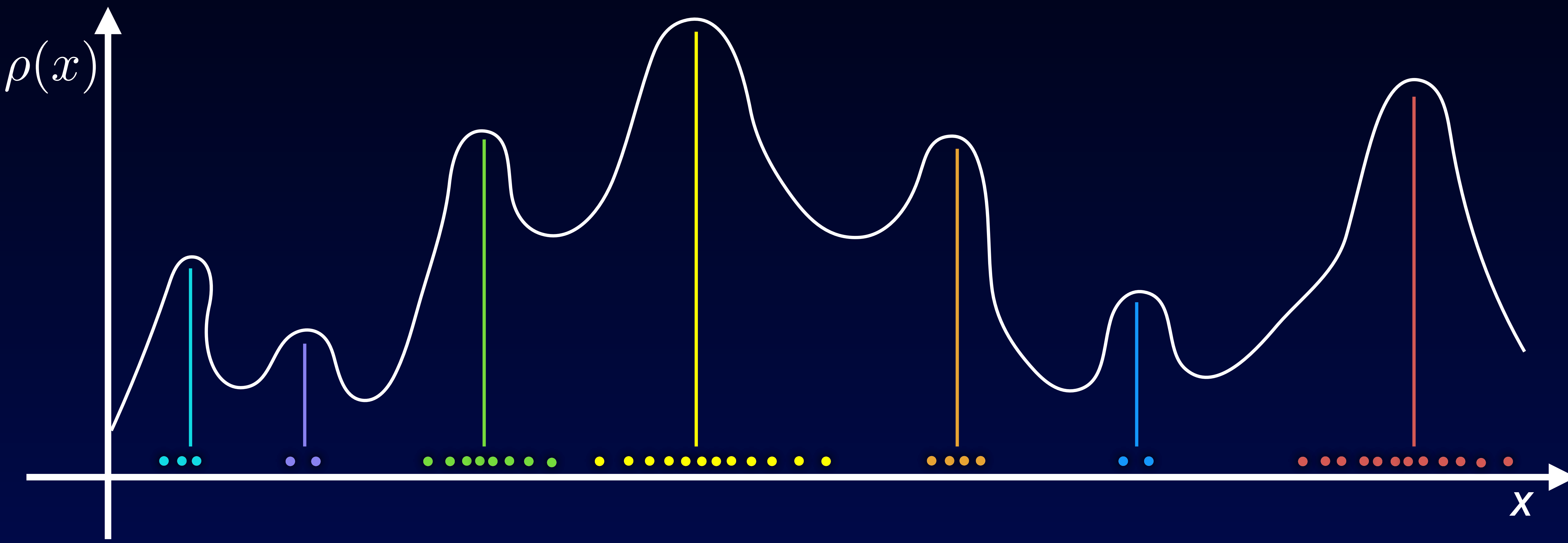
Density Peak Finders

- Smooth density field and locate peaks



Density Peak Finders

- Smooth density field and locate peaks
- Collect particles around peaks



- Advantages: haloes clearly separated, can identify subhaloes
- Disadvantages: usually spherical shapes, need to smooth density field

Unbinding particles from haloes

- Use velocity information to remove unbound particles from haloes
- From $\Delta\phi = 4\pi G\rho$ and spherical symmetry: $\frac{\mathrm{d}\phi}{\mathrm{d}r} = \frac{GM(<r)}{r^2}$

  ➛ $\phi(r) = G\int_0^r \frac{M(<r')}{r'^2}\,\mathrm{d}r' + \phi(0)$ and $\phi(0)$ from $\phi(\infty) = 0$

- Order particles with respect to distance (from centre):

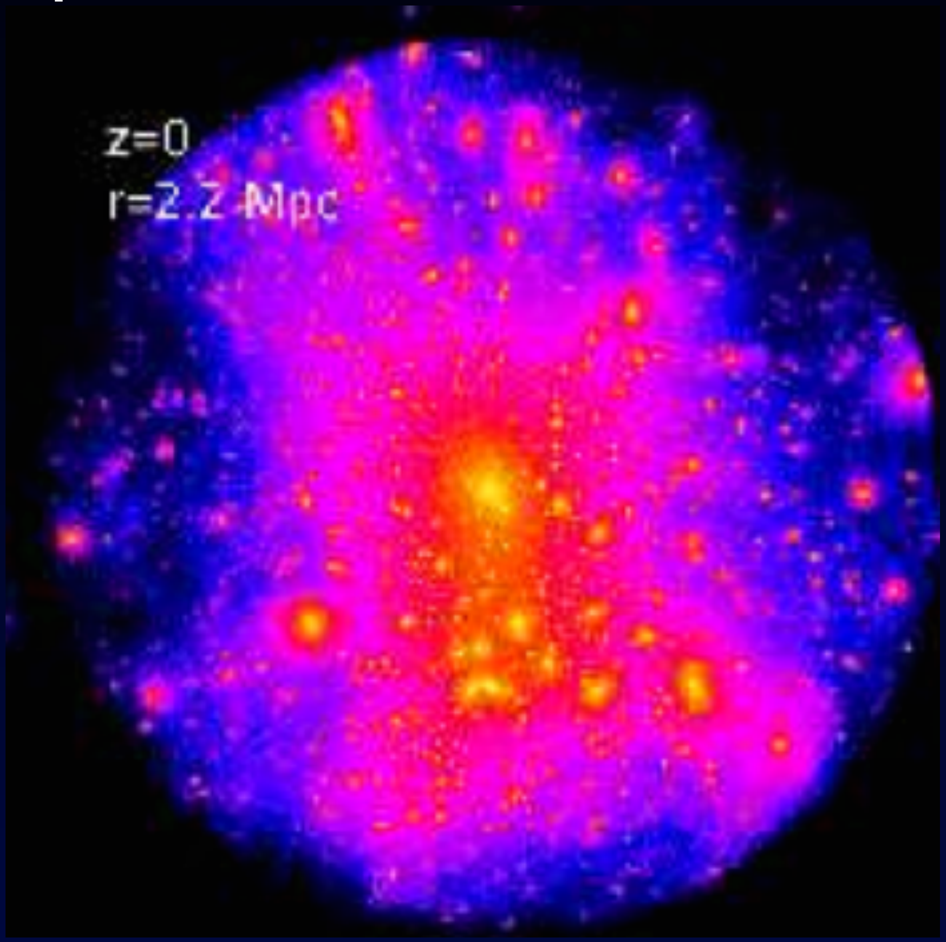
$$\int_0^r \frac{M(<r')}{r'^2}\,\mathrm{d}r' = \int_0^{r_1} \frac{M(<r)}{r^2}\,\mathrm{d}r + \int_{r_1}^{r_2} \frac{M(<r)}{r^2}\,\mathrm{d}r + \cdots + \int_{r_{N-1}}^{r_N} \frac{M(<r)}{r^2}\,\mathrm{d}r$$

$$= \frac{m_1}{r_1^2} r_1 + \frac{m_1 + m_2}{r_2^2} |r_2 - r_1| + \cdots + \frac{\sum_i m_i}{r_N^2} |r_N - r_{N-1}|$$

- Remove particles with $v_i > v_{\mathrm{esc}}(r_i) = \sqrt{2|\phi(r_i)|}$
- Repeat iteratively until no more particles are removed

Phase Space Halo Finders

- Use the complete 6D phase space data to link particles



- 6DFOF: extend standard FOF linking:

$$\frac{(\vec{x}_i - \vec{x}_j)^2}{(b\,\Delta x)^2} + \frac{(\vec{v}_i - \vec{v}_j)^2}{(b_v\,\Delta v)^2} < 1$$

- For $b_v \to \infty$ we recover standard FOF

- Unbinding often not done (as most particles in 6D-linked structures are bound)

- Halo finders can be extended in time-domain
  If halo is lost between N snapshots, add halo with interpolated properties

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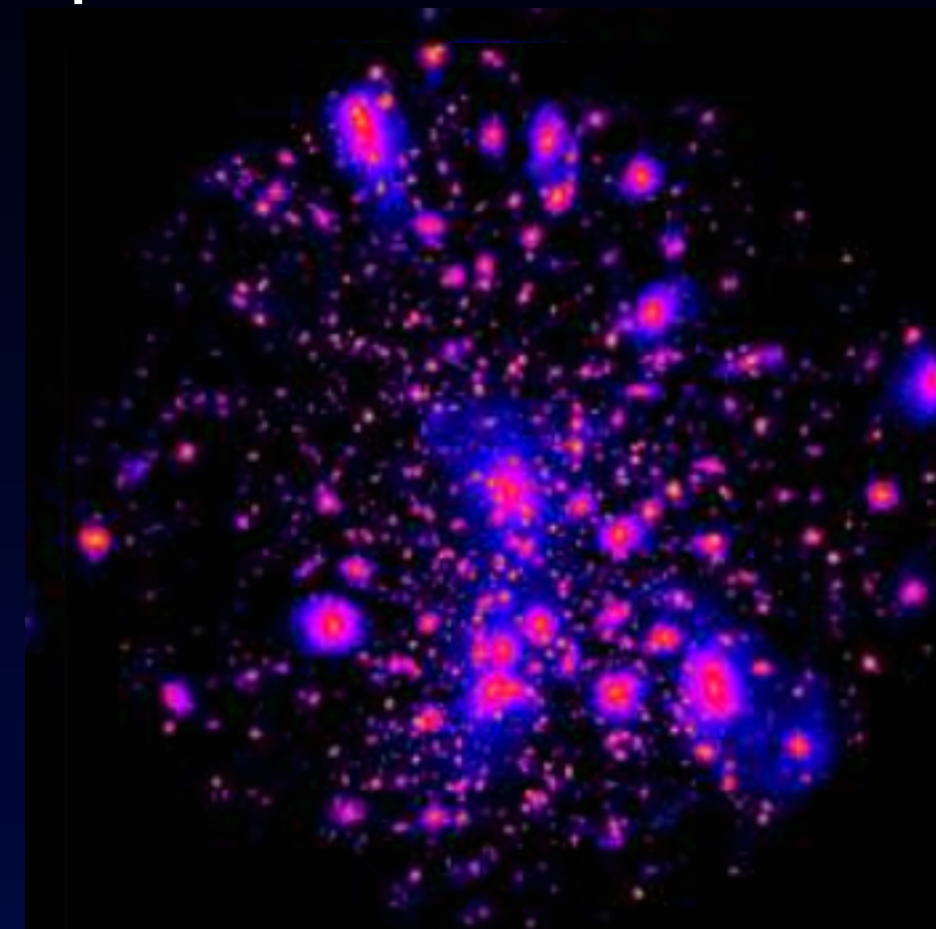

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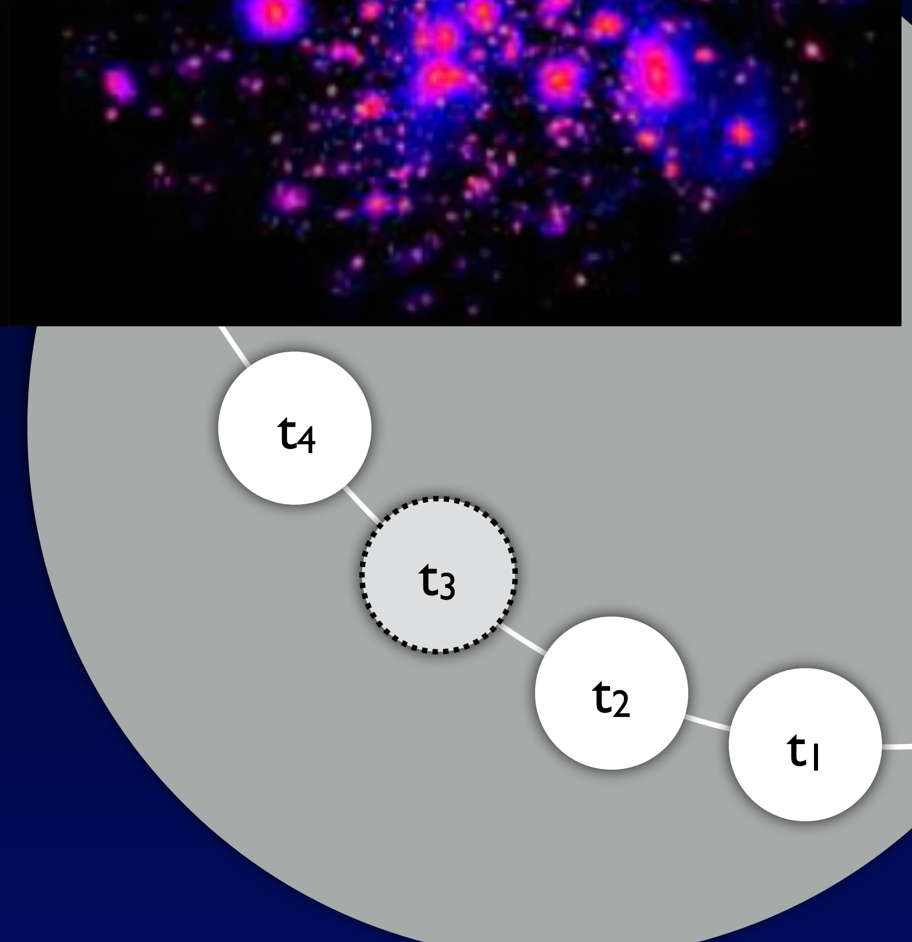

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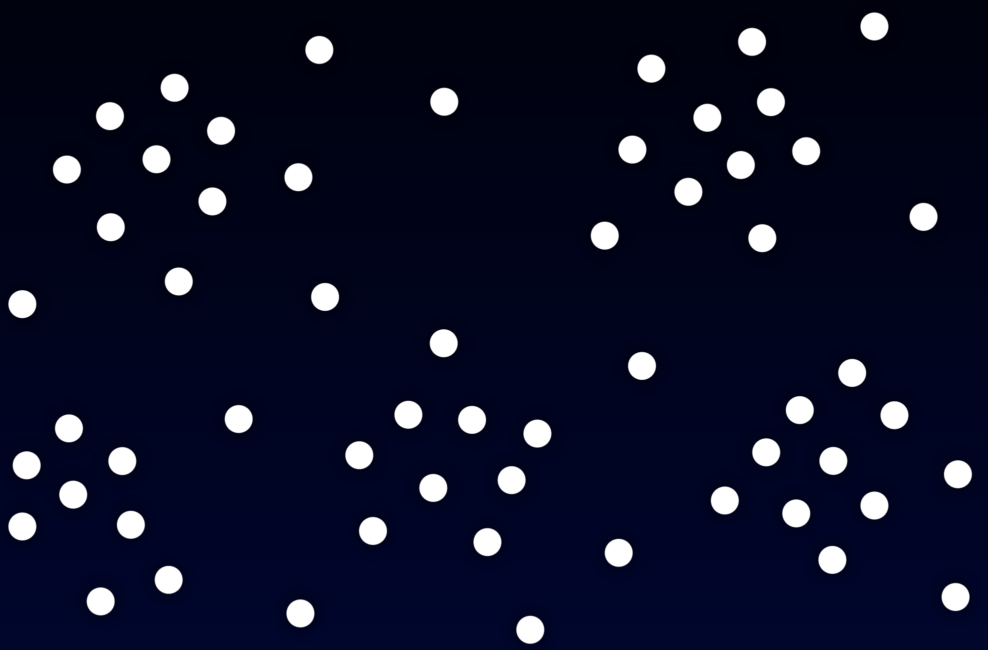
Bound Density Maxima (BDM)

- BDM first identifies peaks (potential halo centres) via sphere jittering
- Randomly place N spheres and iteratively move to center of mass



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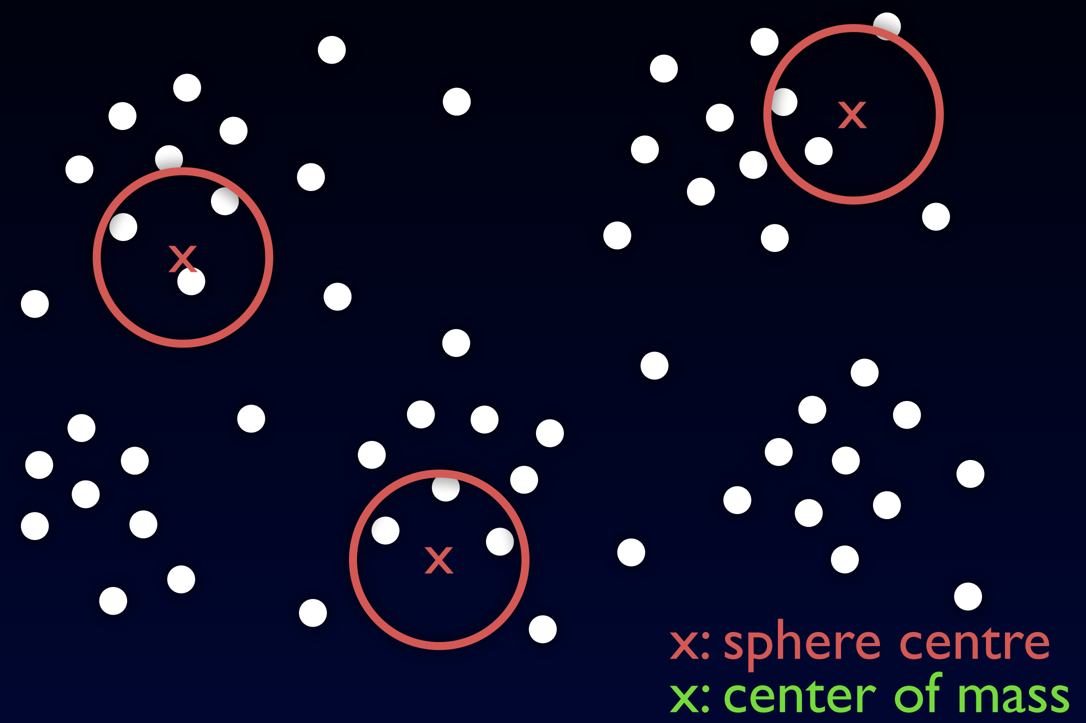

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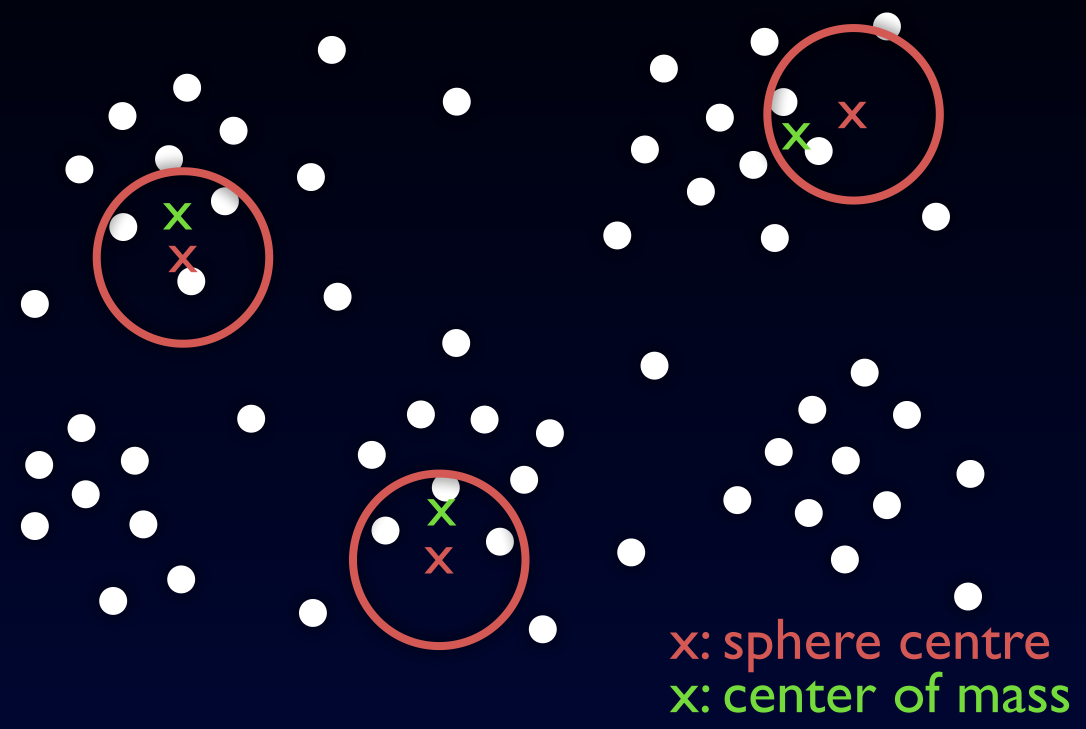

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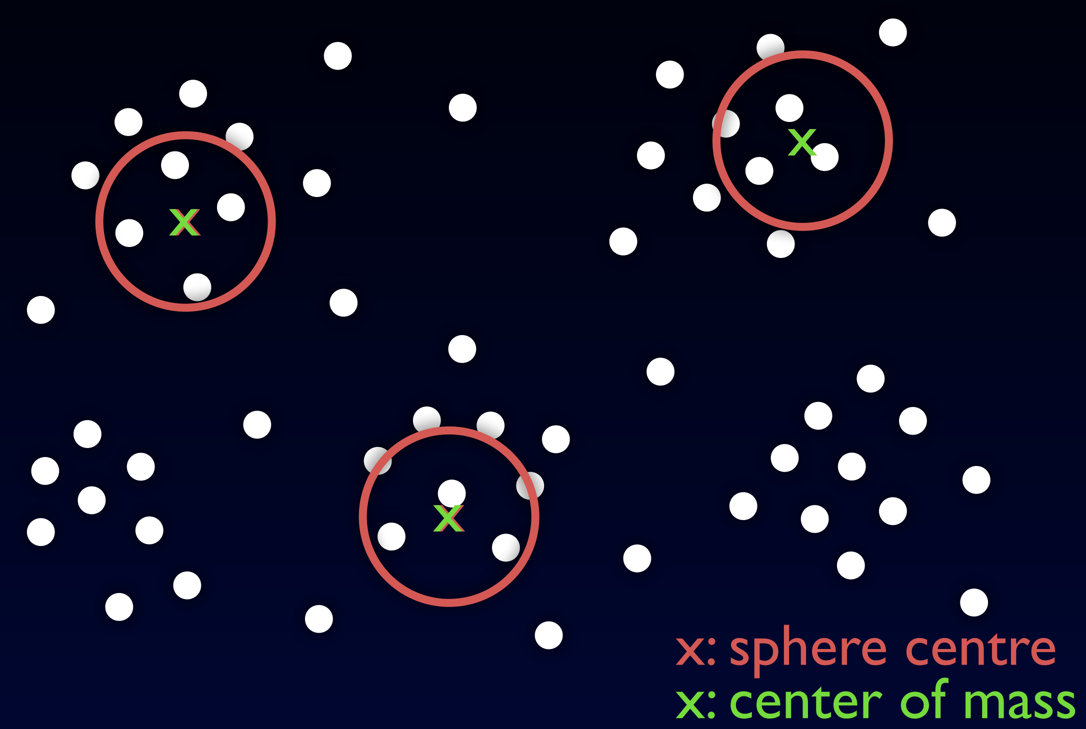

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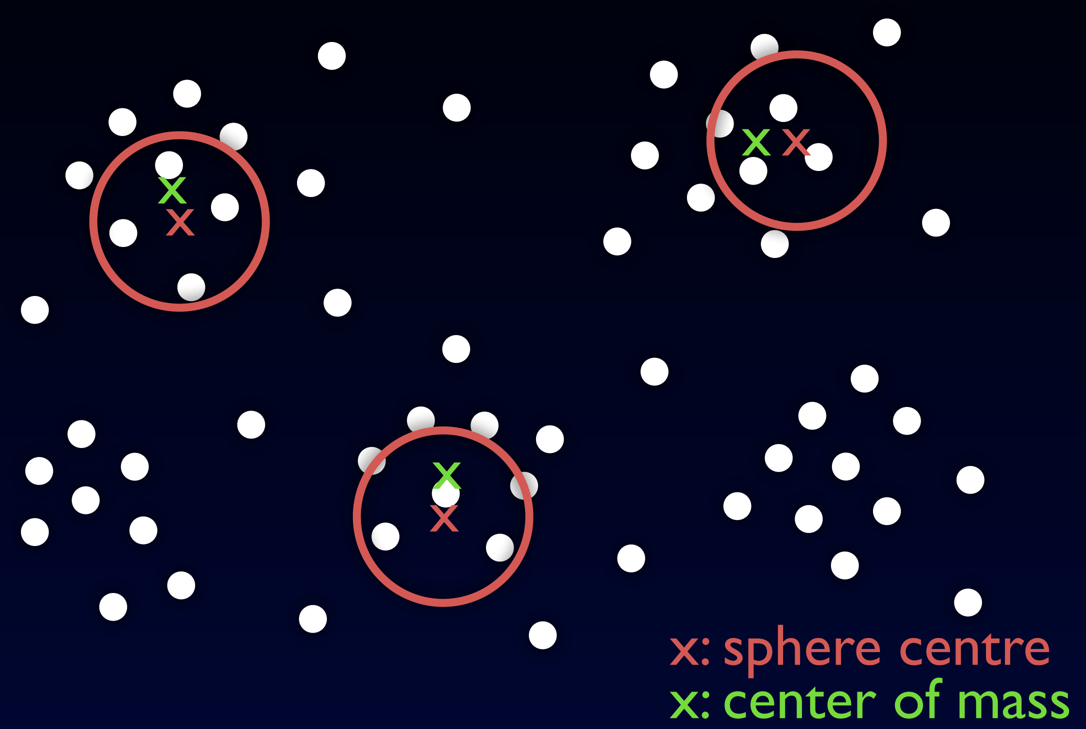

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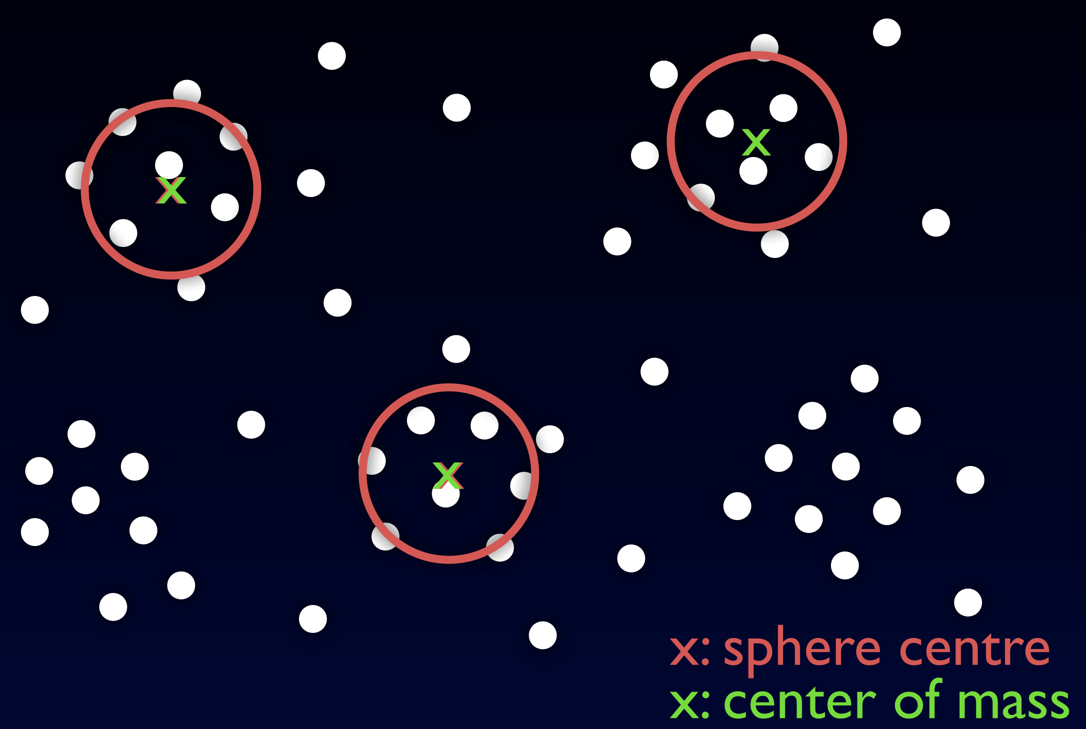

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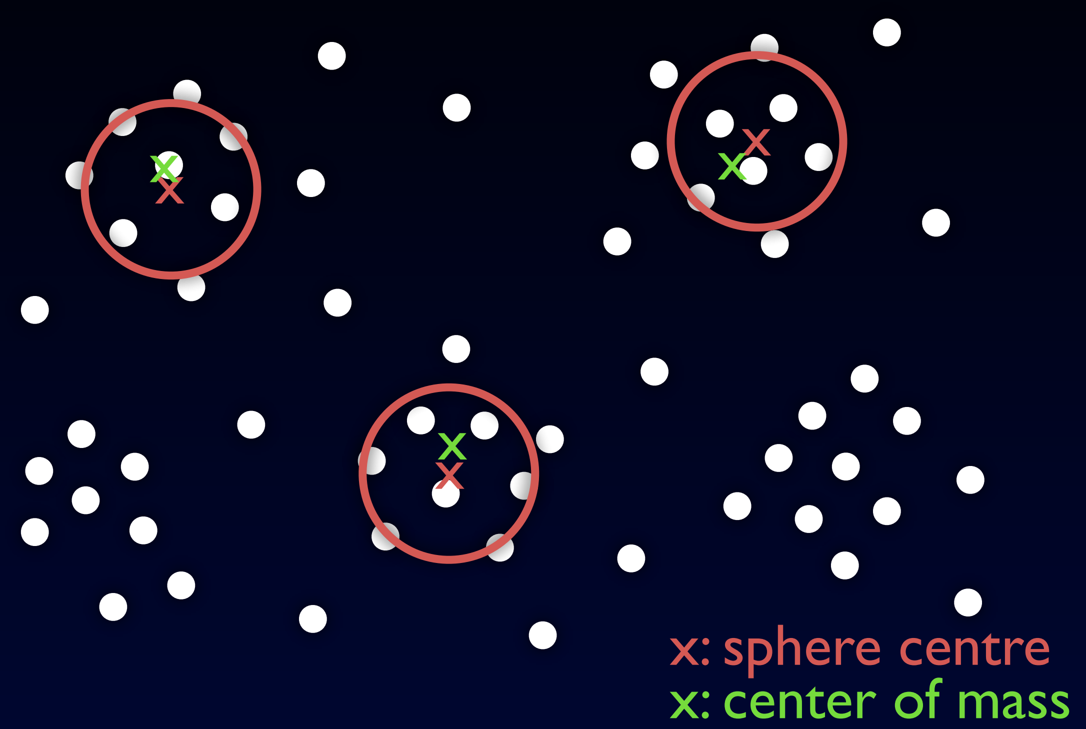

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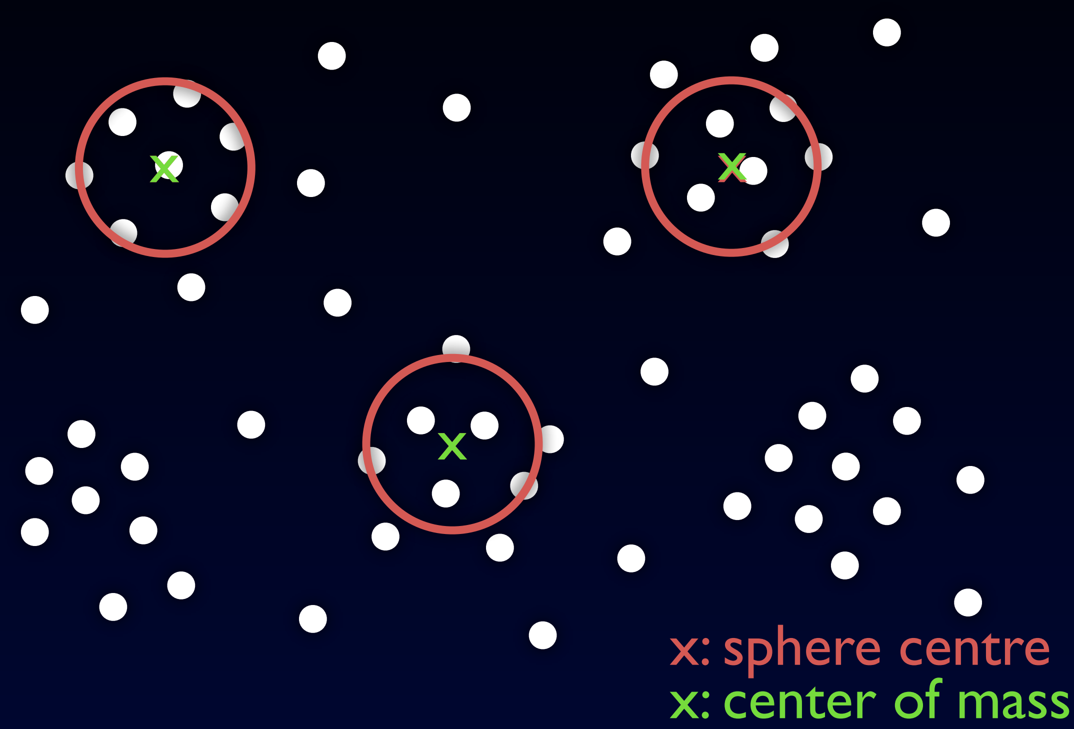

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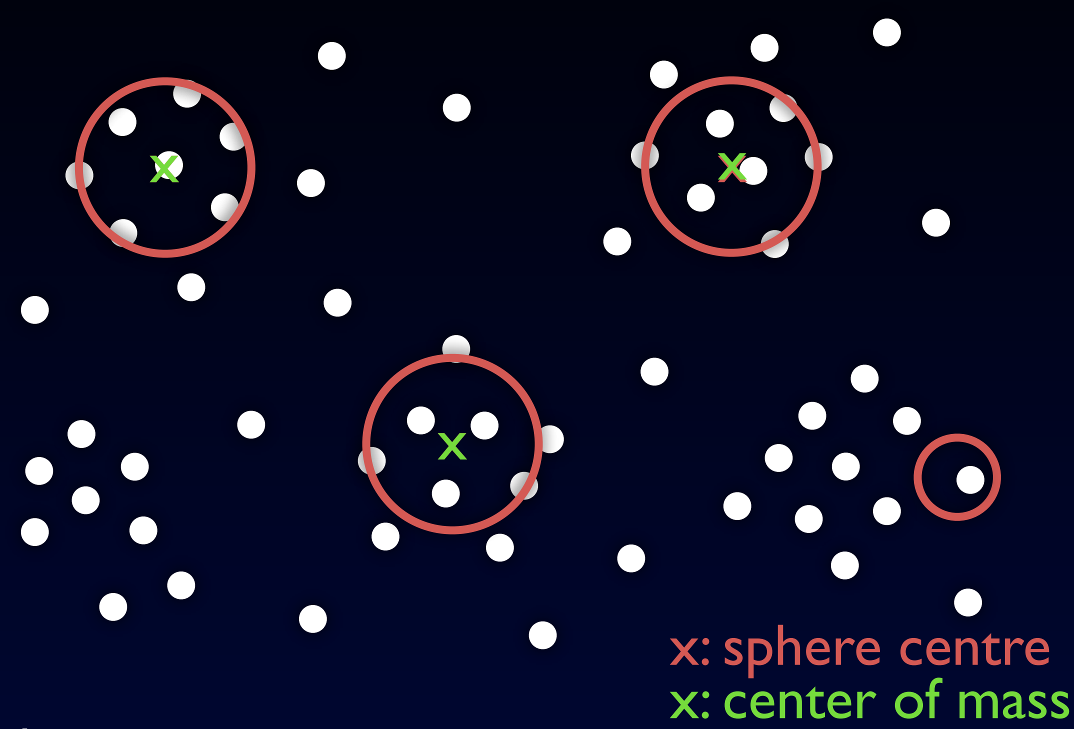

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  too big ➛ too much smoothing, too small ➛ spheres get stuck
  Too many spheres ➛ duplicates (can be removed)
  To few spheres ➛ missing haloes
- Use spherical overdensity criterion to compute R_{200}: $\rho(R_{200}) = 200\rho_c$
  Compute $M_{200} = \frac{4\pi}{3} R_{200}^3 \, 200\rho_c$ and unbind particles iteratively

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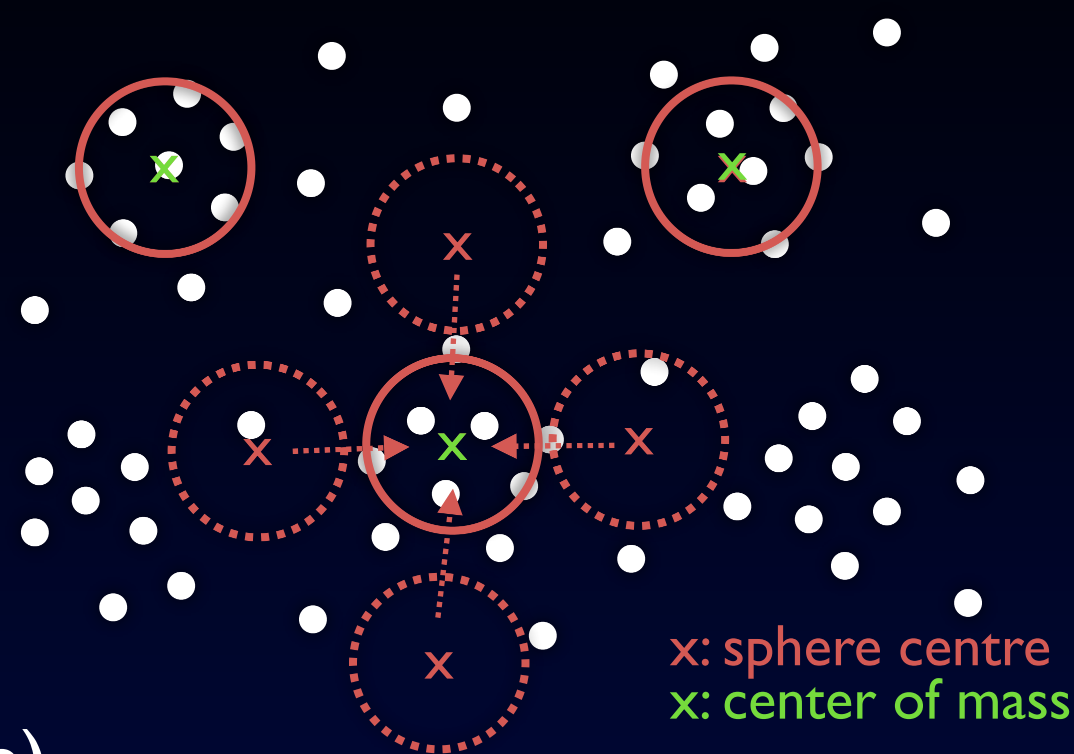

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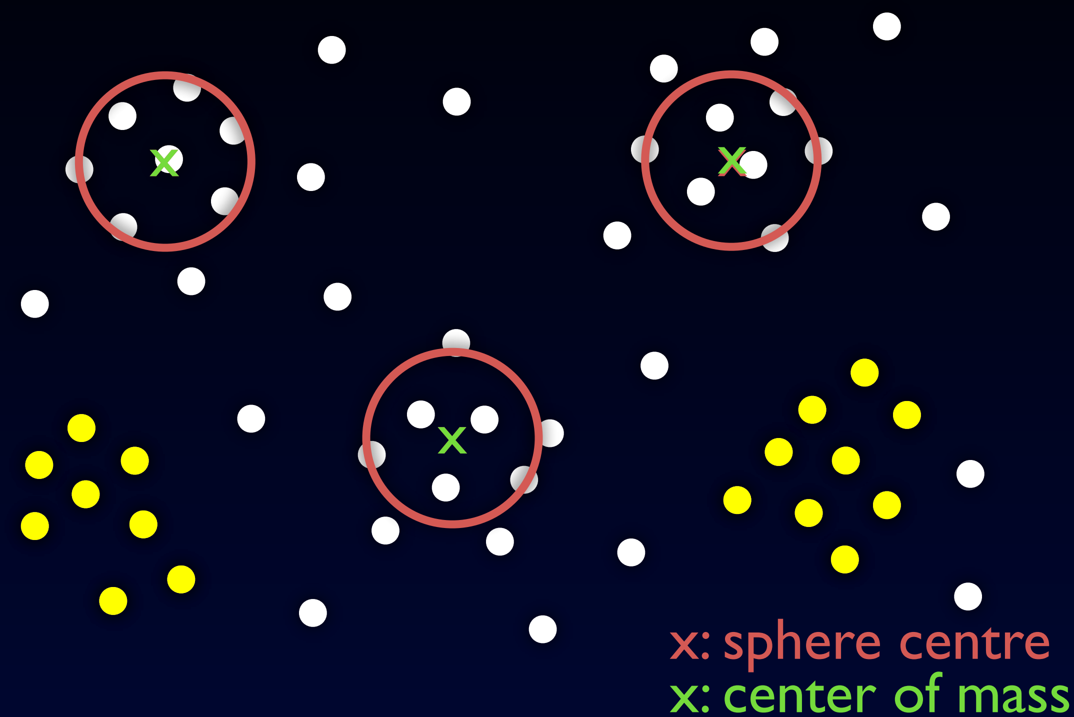

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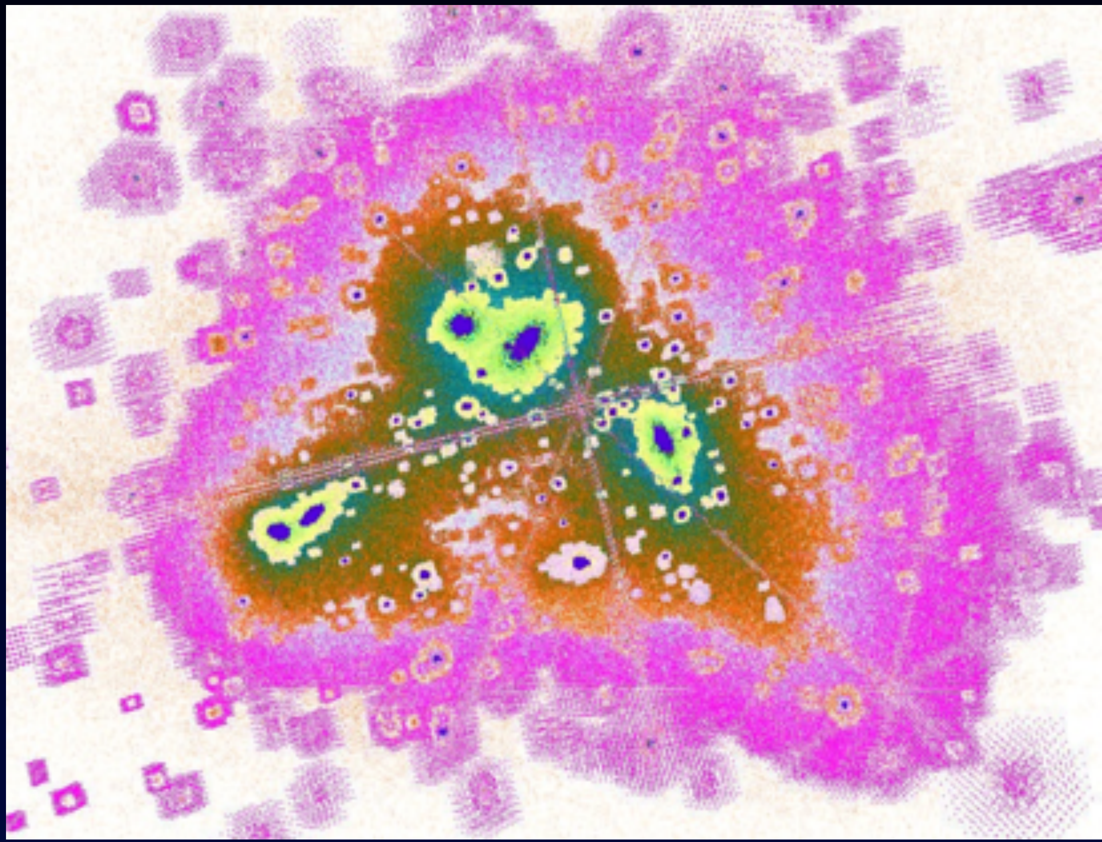
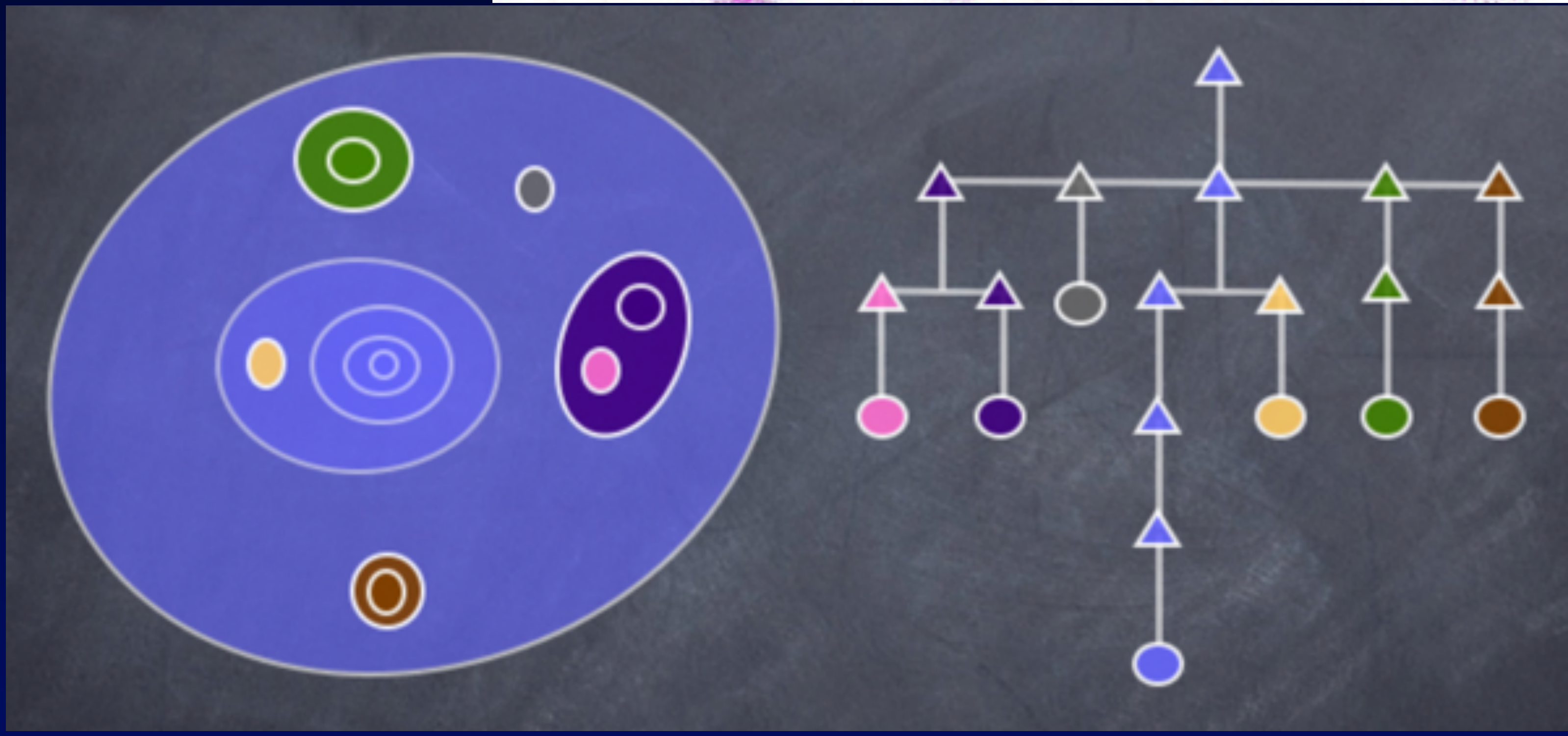
AMIGA HALO FINDER (AHF)

- Uses adaptive mesh to locate possible halo centres

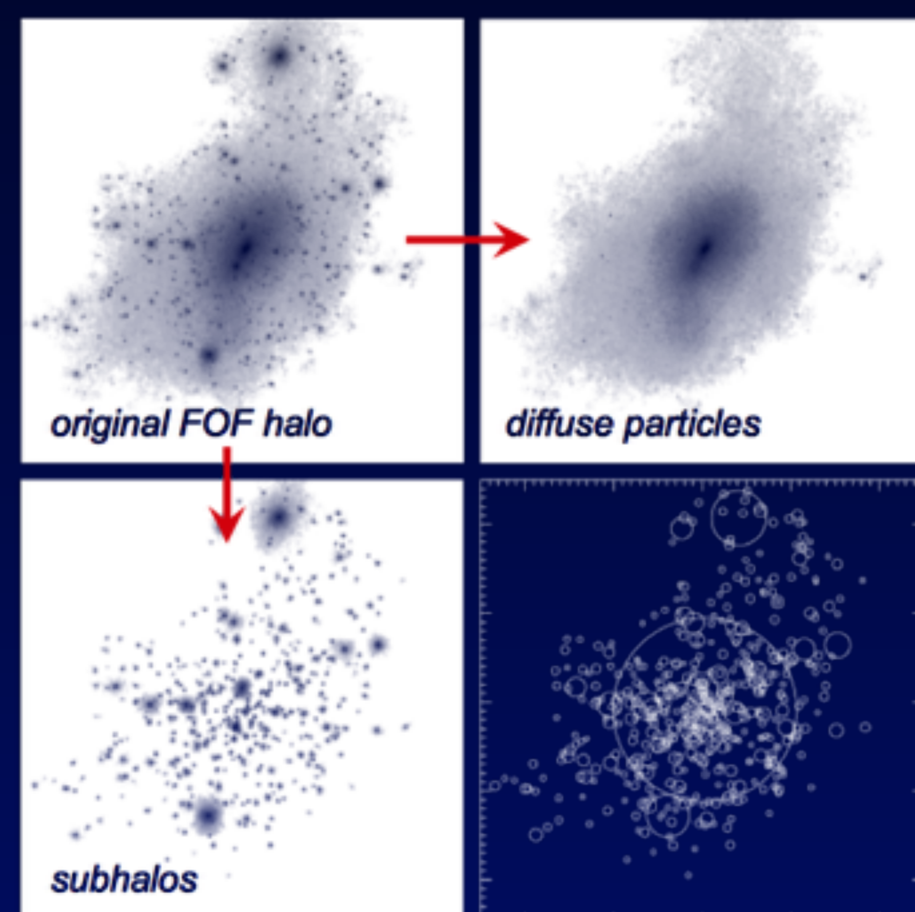


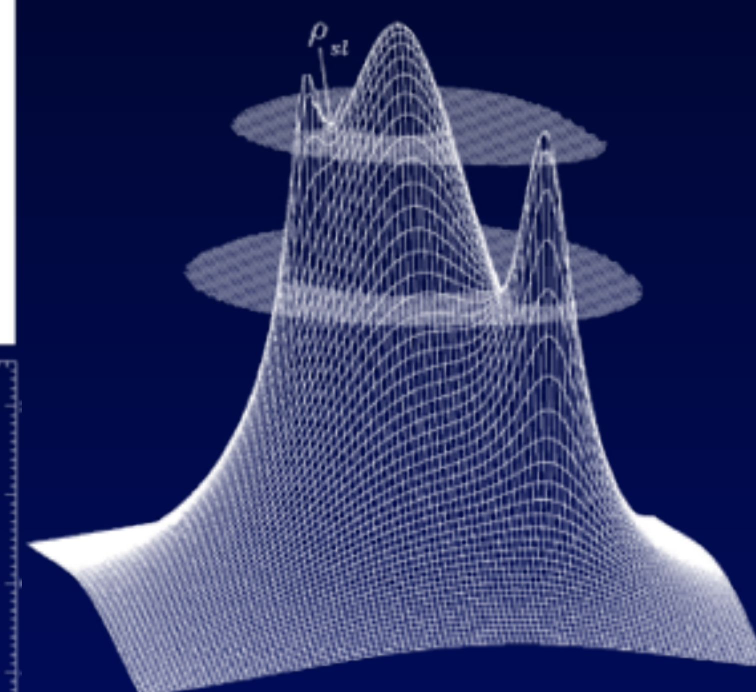
- Refinement levels define isodensity contours
- The AMR hierarchy is organized into a tree structure
- Identify Halo, sub-halo, sub-sub-halo, etc
- Unbind particles from haloes and compute properties



SUBFIND (integrated into Gadget)

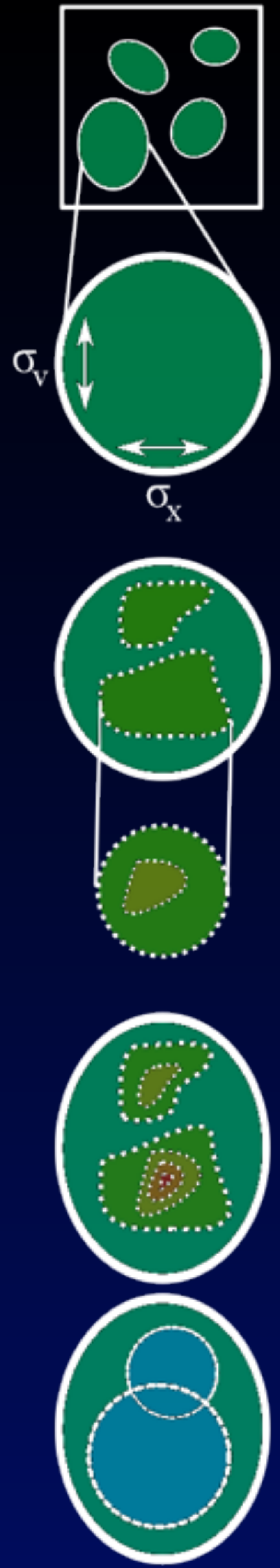
- First step is to define parent haloes with 3D FOF
- Local (smoothed) density is computed for particles with SPH kernel
- Identify saddle points: locations where neighboring densities are larger
- Compute isodensity contours around saddle points
- Subhaloes = regions enclosed by isodensity surfaces
- Remove unbound particles





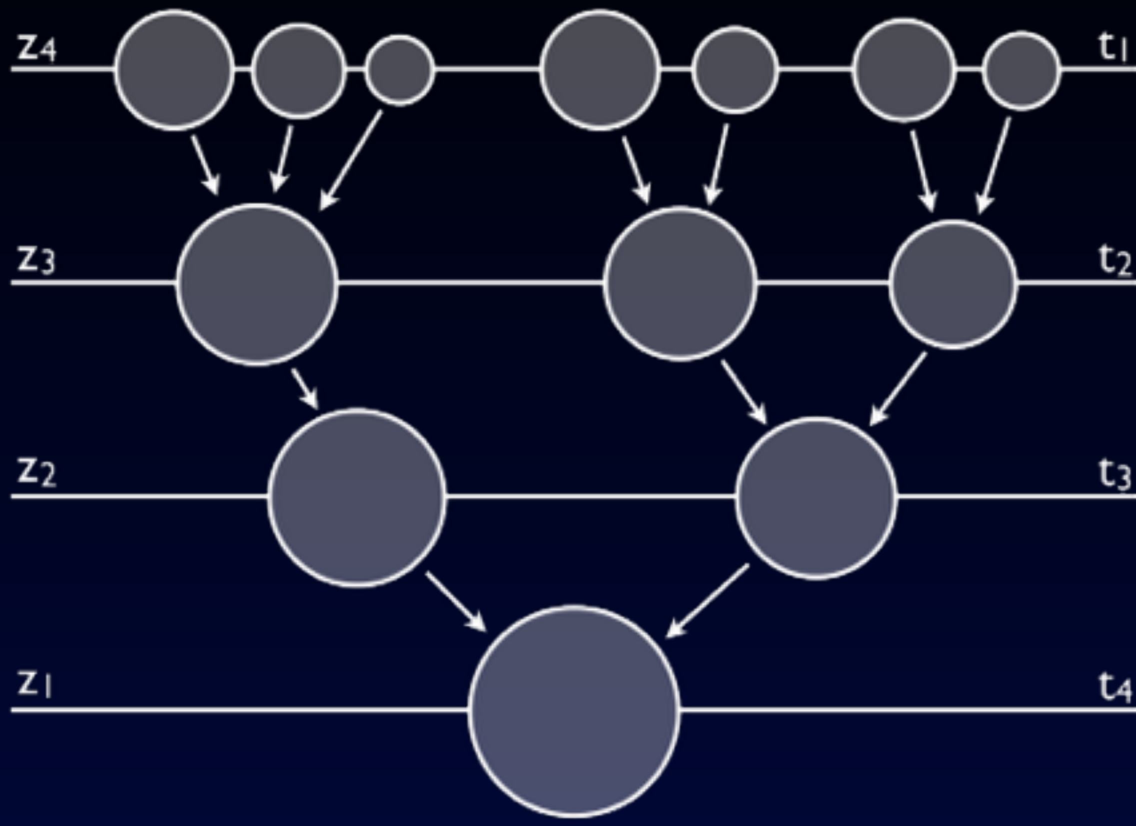
ROCKSTAR

- Selects particle groups with 3D FOF and large b = 0.28
- For each group: divide positions and velocity by dispersion:
- Choose phase-space linking length adaptively to link 70% of group's particles in sub-groups
- Repeat in each sub-group and define sub-subgroups until minimum number of particles is reached (10).
- Place seed haloes at lowest substructure level and assign particles according to their phase-space proximity
- Remove unbound particles and compute halo properties



Merger trees from N-body simulations

- Link haloes through time
- Identify descendants in later snapshots (Or progenitors in earlier snapshots)
- In hierarchical structure formation: several progenitors but one descendant



- When subhaloes merge their particles move to the 'main' halo
- Use particle IDs: for given halo, identify all haloes in later snapshot that contain its particles (possibly weigh by binding energy)
- Descendant is the halo with most matching particles (or highest weight)

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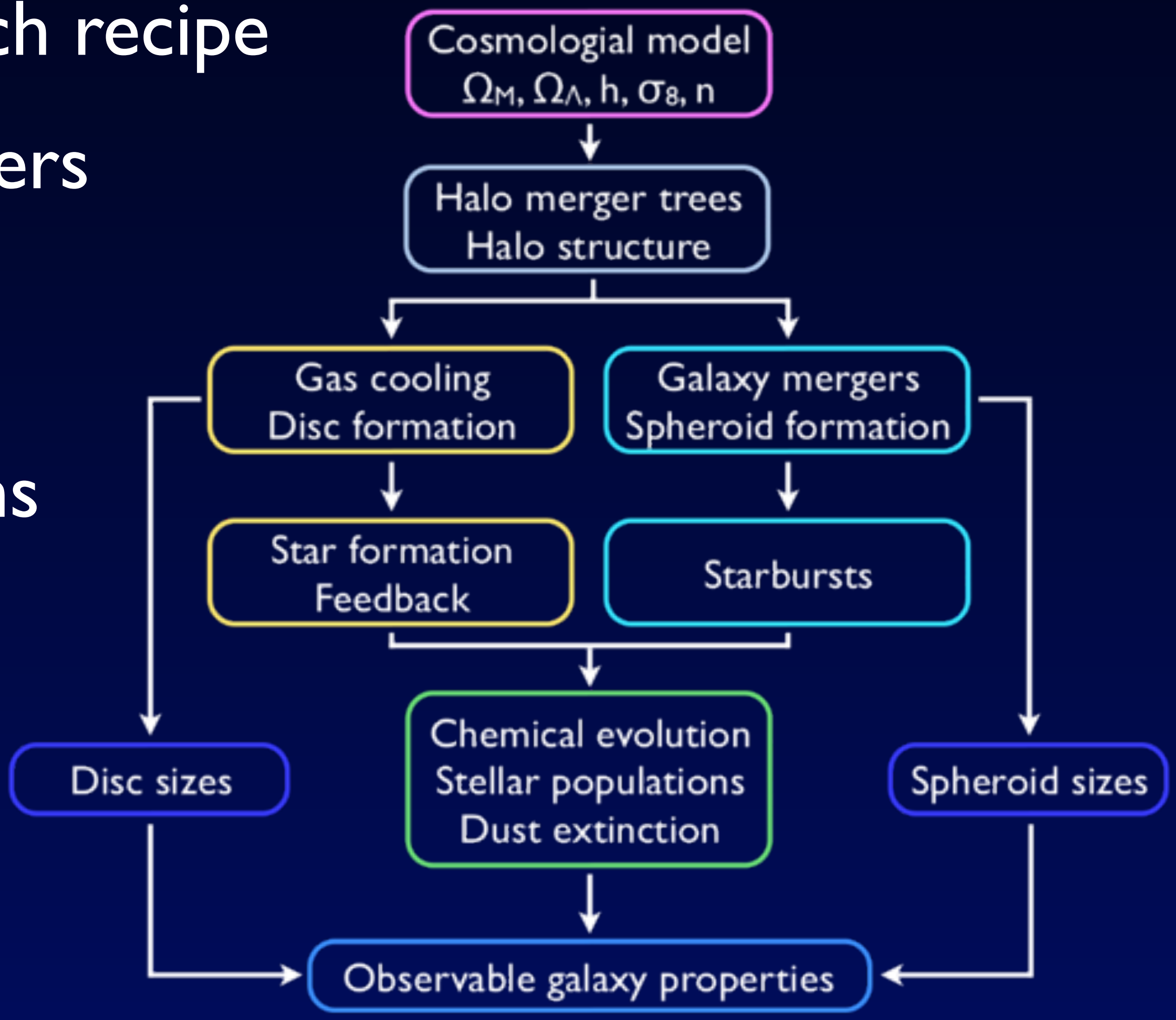
Semi-analytic models

- Semi-analytic models populate dark matter halo merger trees with galaxies using simple, physically motivated recipes for baryonic physics
- As physics behind galaxy formation is not very well understood (c.f. sub-grid models in simulations), each recipe contains a number of free parameters
- Recipes are either motivated by observations or detailed simulations
- Free parameters are 'tuned' such that observed statistical galaxy properties are reproduced



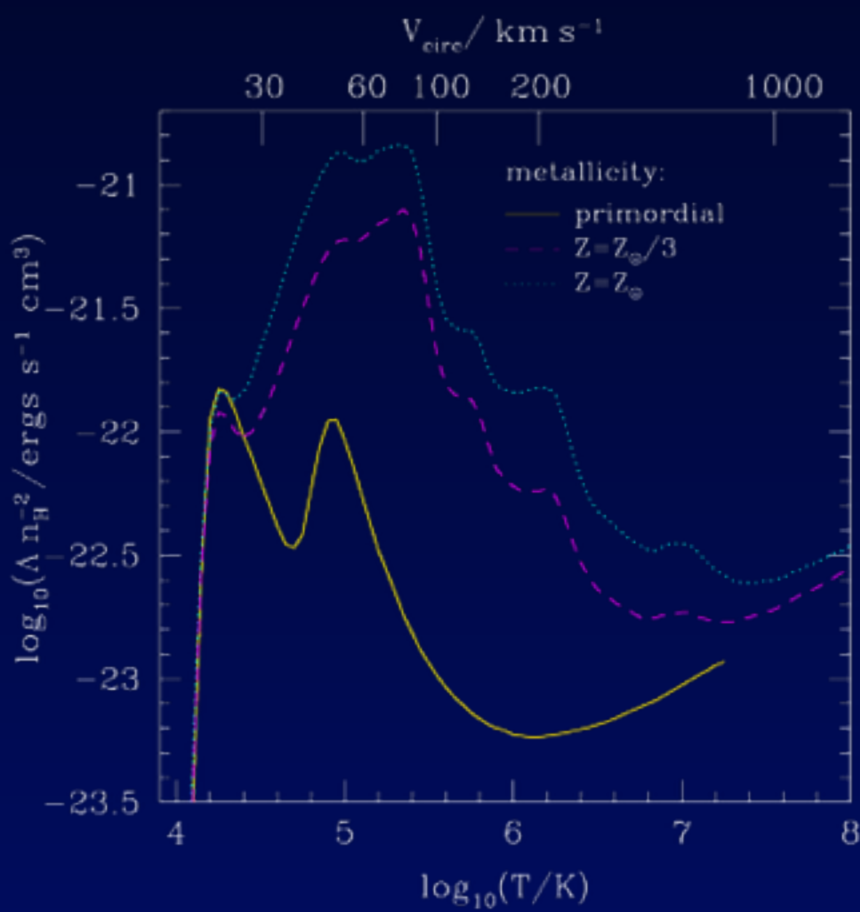
Gas cooling

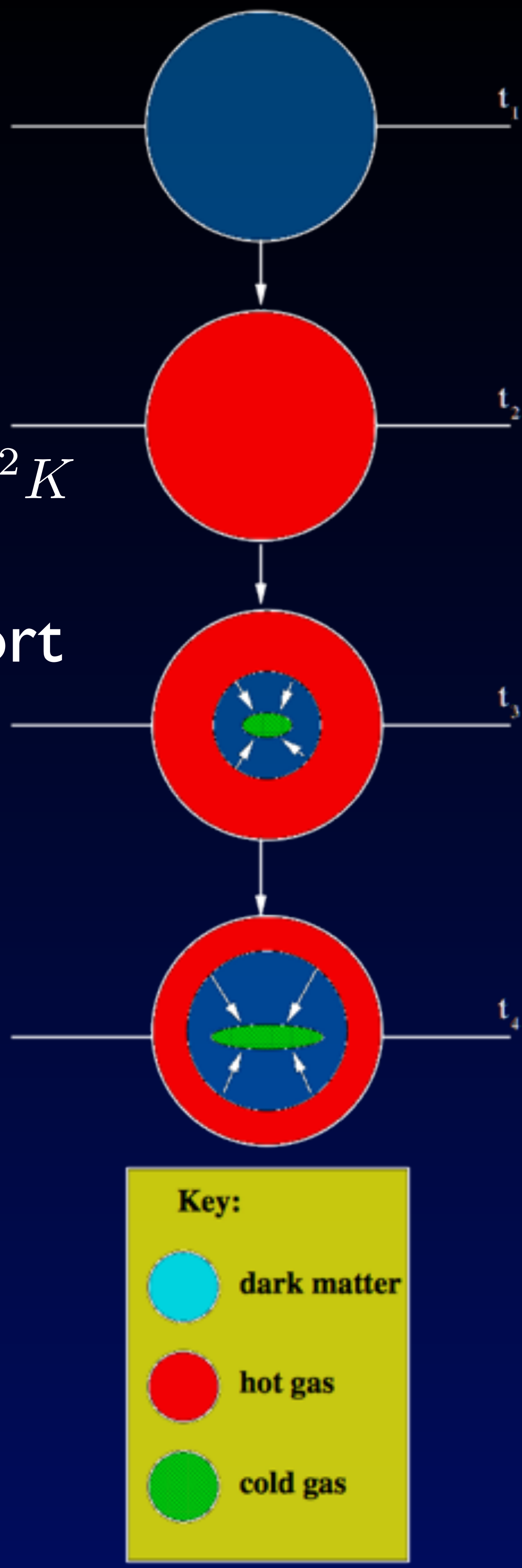
- Gas is initially distributed as dark matter
- Falls into dark matter haloes and shock heats

  Virial temperature: $T_{\rm vir} = \mu m_p V_{\rm vir}^2/2k = 35.9[V_{\rm vir}/(km\,s^{-1})]^2 K$
- Gas cools from hot gaseous halo, loses pressure support falls to centre and forms disc

$$t_c = \frac{(3/2)n_g kT}{n_g^2\Lambda(T_{\rm vir}, Z_g)} = \frac{3\mu m_p kT}{2\rho_g \Lambda(T_{\rm vir}, Z_g)}$$

- For given $\rho(r)$ compute cooling radius r_c
- After a time $t = t_c$ all gas in r_c had time to cool





Star formation

- No theory of star formation that gives SFR as function of ISM
- More pragmatic approach based on dimensional argument: $\dot{m}_* \propto \frac{m_c}{\tau}$
- All stars form in exponential disc with scale length $r_d = \frac{\lambda}{\sqrt{2}} \frac{j_d}{j_h} R_{200}$
- Gas disc is also in exponential profile with $r_g = \chi r_d$ and $\chi \approx 1.5$
- Star formation rate given by Kennicutt-Schmidt-Relation: $\dot{\Sigma}_* = A_K \Sigma_g^N$

$$\dot{m}_* = \int_0^{r_c} \dot{\Sigma}_* 2\pi r \mathrm{d}r = \int_0^{r_c} A_K \Sigma_g^N 2\pi r \mathrm{d}r$$

- Fraction of short-lived stars β leads to modified SFR: $\dot{m}'_* = (1-\beta)\dot{m}_*$
- In timestep dt the mass $\mathrm{d}m = \dot{m}'_* \, \mathrm{d}t$ is formed

Supernova feedback

- Cold gas is ejected from the central disc by SN-driven winds
- Rate of reheating by winds: $\dot{m}_{\rm rh} = \epsilon_0^{\rm SN} \left(\frac{V_d}{200\,{\rm km\,s^{-1}}} \right)^{\alpha_{\rm rh}} \dot{m}_*$
  with $\epsilon_0^{\rm SN} \approx 1.3$ and $\alpha_{\rm rh} \approx 2$
- Heated gas either trapped in halo, or ejected into diffuse IGM
- Fraction of ejected gas is $f_e(V_{\rm vir}) = [1 + (V_{\rm vir}/V_e)^{\alpha_e}]^{-1}$
  with $\alpha_e \approx 6$ and $V_e \approx 100 - 150\,{\rm km\,s^{-1}}$
- Ejected gas can recollapse into halo at later times and become available for cooling: $\dot{m}_{\rm re} = \chi_{\rm re}(m_e/t_{\rm dyn})$
  with $\chi_{\rm re} \approx 0.1$

AGN feedback

- Every top-level halo contains a seed black hole (usually $M_{seed} = 100 M_\odot$)
- BHs grow during galaxy mergers: $\Delta m_{\rm BH} = \frac{f_{\rm BH}\,(m_s/m_c)\,m_{cg}}{1 + (280\,{\rm km\,s^{-1}}/V_{\rm vir})^2}$
- Quasar mode: energy released during rapid BH growth drives wind.

  Outflow rate: $p_{\rm rad} = p_{\rm wind}$ or $\epsilon_w \eta_r m_{\rm acc} c = m_{\rm out} V_e$

  yields $\dot{m}_{\rm out} = \epsilon_w \eta_r (c/V_e) \dot{m}_{\rm acc}$

- Radio mode: hot gas accretes onto BH and releases energy to hot halo

  Accretion rate: $\dot{m}_{\rm radio} = \kappa_r \left(\frac{kT}{\Lambda(T,Z)}\right)\left(\frac{m_{\rm BH}}{10^8 {\rm M}_\odot}\right)$

  Energy that suppresses cooling: $L_{\rm BH} = \eta_r \dot{m}_{\rm radio} c^2$

  Modified cooling rate: $\dot{m}'_{\rm cool} = \dot{m}_{\rm cool} - 4L_{\rm BH}/3V_{\rm vir}^2$

  Cooling never allowed to fall below 0

Starbursts during mergers

- Main haloes host central galaxies, subhaloes host satellite galaxies
- Stars can be either in the disc, or in a spherical bulge component
- When satellites merge with central: merger triggered starburst

  Fraction of cold gas turned into stars: $e_{\rm sb} = e_0\,\mu^{\gamma_{\rm sb}}$ μ: mass ratio
- Slope γ_{sb} depends on bulge-to-total ratio B/T

  (more massive bulges surpress starburst efficiency)
- Normalisation e_{sb} depends on halo mass and gas fraction
- Stars created in a starburst are added to the bulge

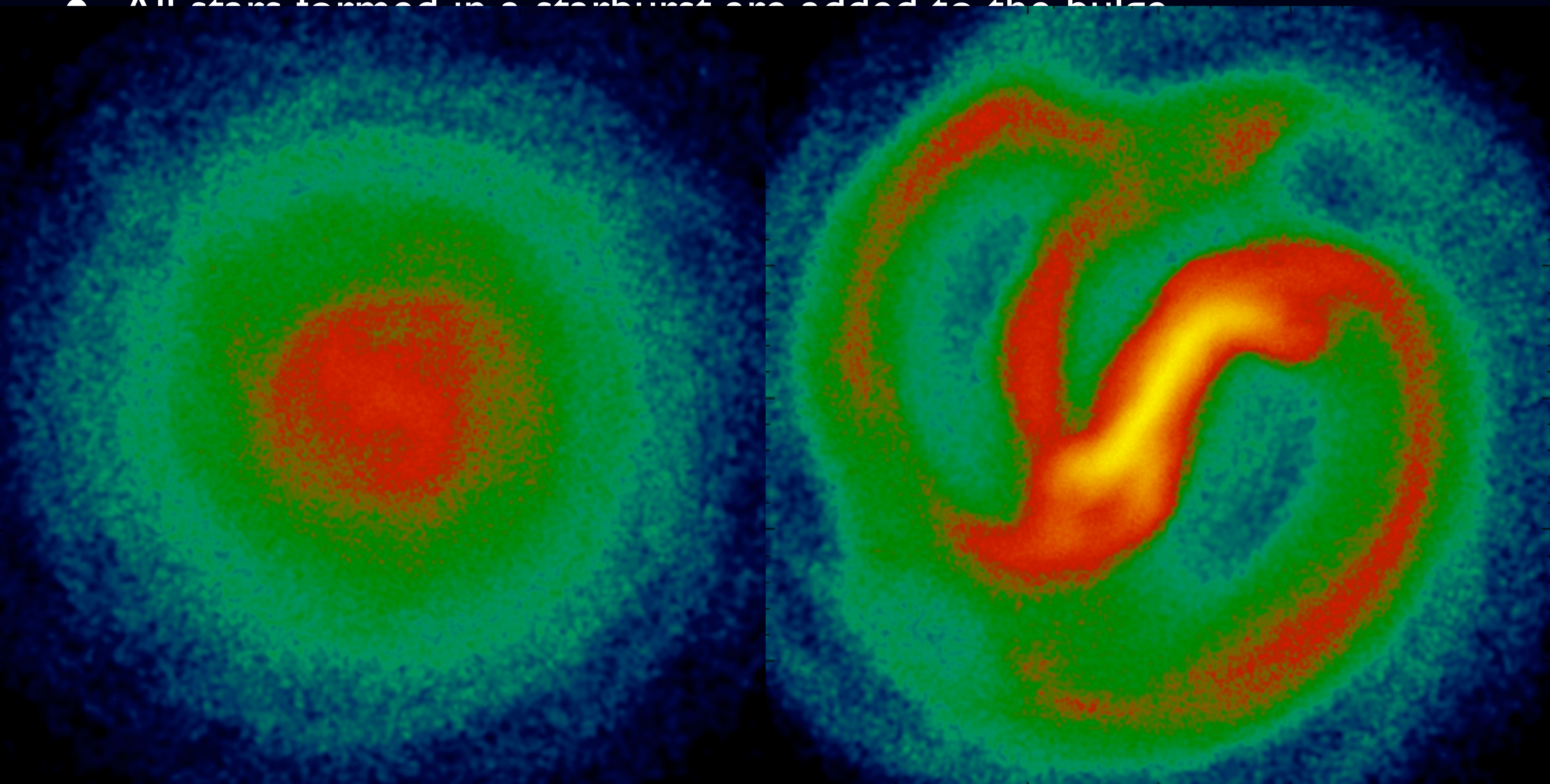
Morphological transformations

- All stars formed from the cold gas disc are added to the stellar disc
- All stars formed in a starburst are added to the bulge
- In a galaxy merger: a fraction of the disc is moved to the bulge:
  Major merger: all stars are moved to the bulge
  Minor merger: Stars from satellite are added to the bulge
  Fraction of central stars added to bulge: f(μ)
- Secular evolution: if stellar disc becomes too massive for its size, a disc instability develops (bar forms and gets destroyed ➛ bulge)
  If $\epsilon = \frac{v_{\rm max}}{\sqrt{{\rm G}M_{\rm disc}/R_{\rm disc}}}$ a fraction of disc stars are moved to the bulge

  (either total stellar disc, or just the ‘unstable mass’)

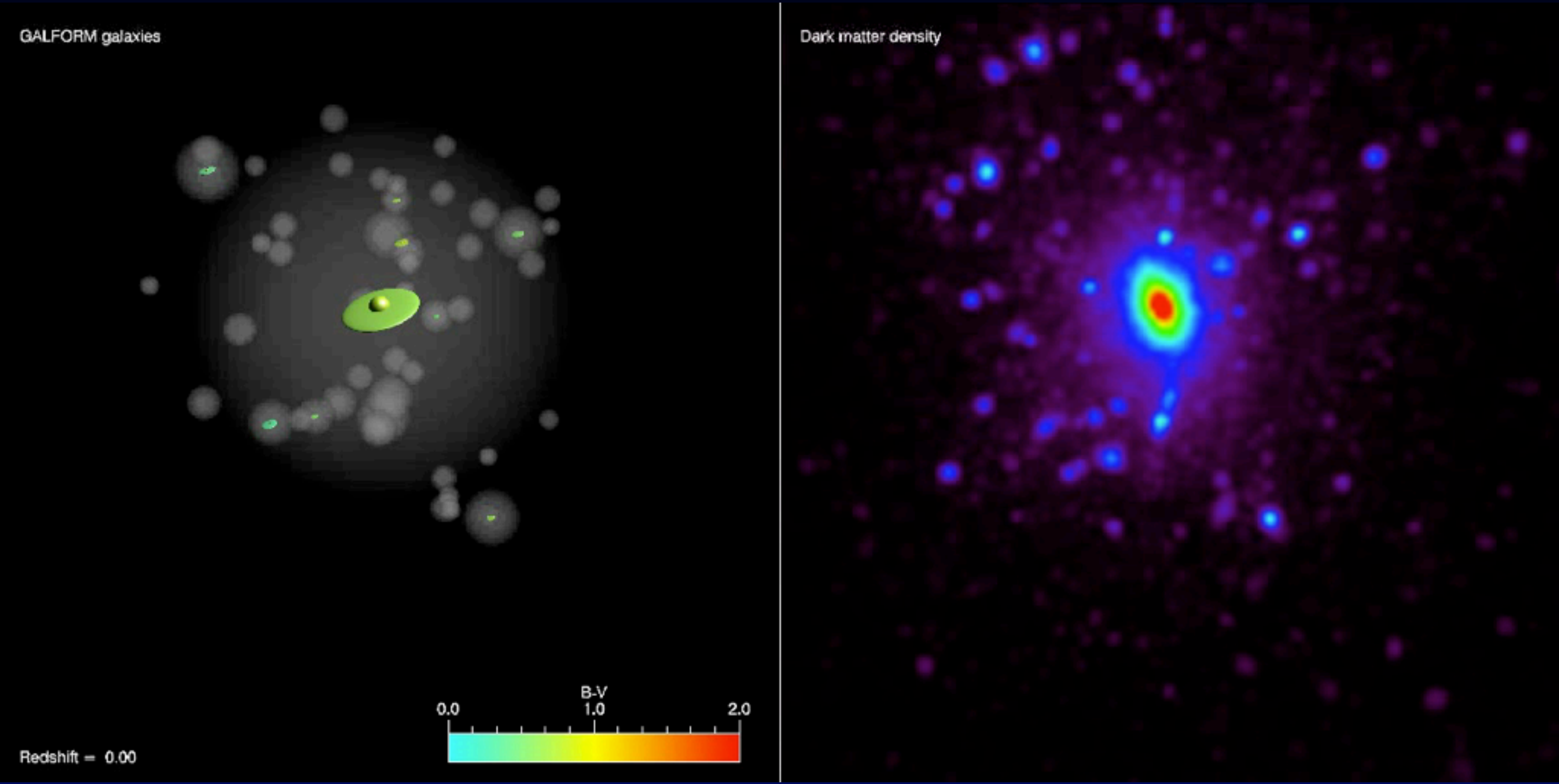
Morphological transformations

- All stars formed from the cold gas disc are added to the stellar disc
- All stars formed in a starburst are added to the bulge



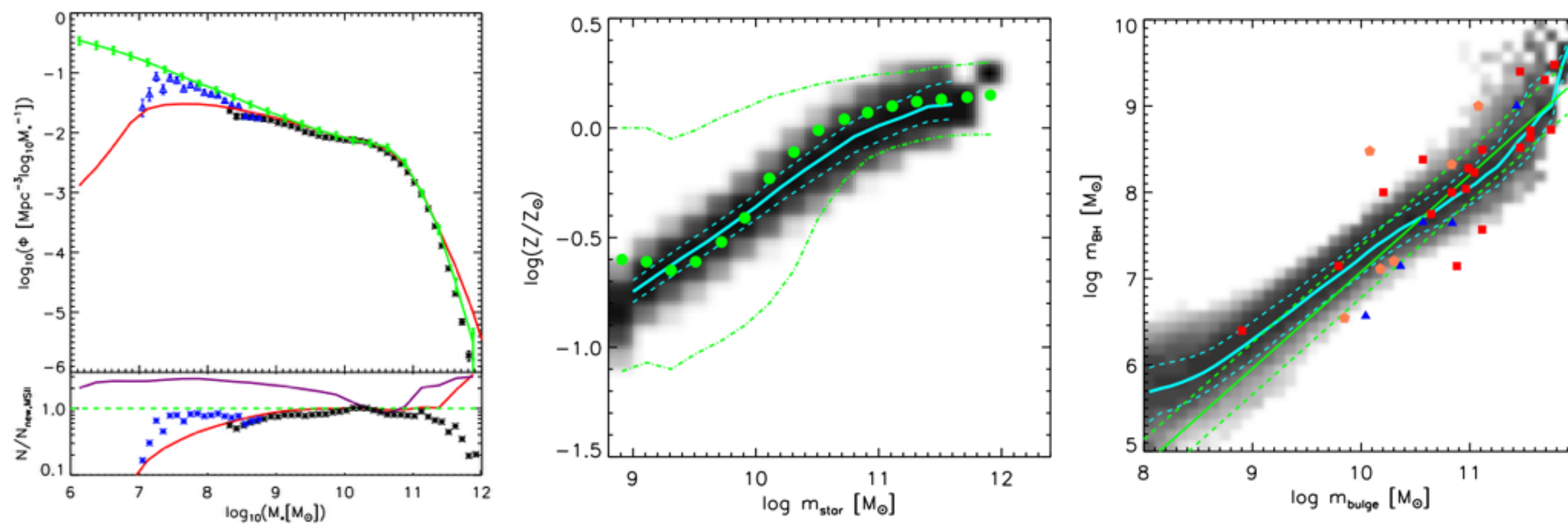
Semi-analytic models in action

- The gaseous & stellar structure are not modelled in detail
  But: profiles & positions of galaxies are given and can be visualised:



Global properties

- Semi-analytic models are able to reproduce a large number of observed statistical galaxy properties
- Advantage: flexibility, fast, large number of systems, statistics
- Disadvantage: little spatial information, simplified approximations, many free parameters



Outline of this lecture

- Halo and subhalo finders
  - Friends-of-Friends (FOF)
  - Density peak finders
  - Unbinding paricles
  - Examples: BDM, AHF, Subfind, Rockstar
- Semi-analytic models
  - Radiative Cooling
  - Star formation & feedback
  - Mergers & morphology
- Empirical galaxy formation models (abundance matching)

Subhalo abundance matching

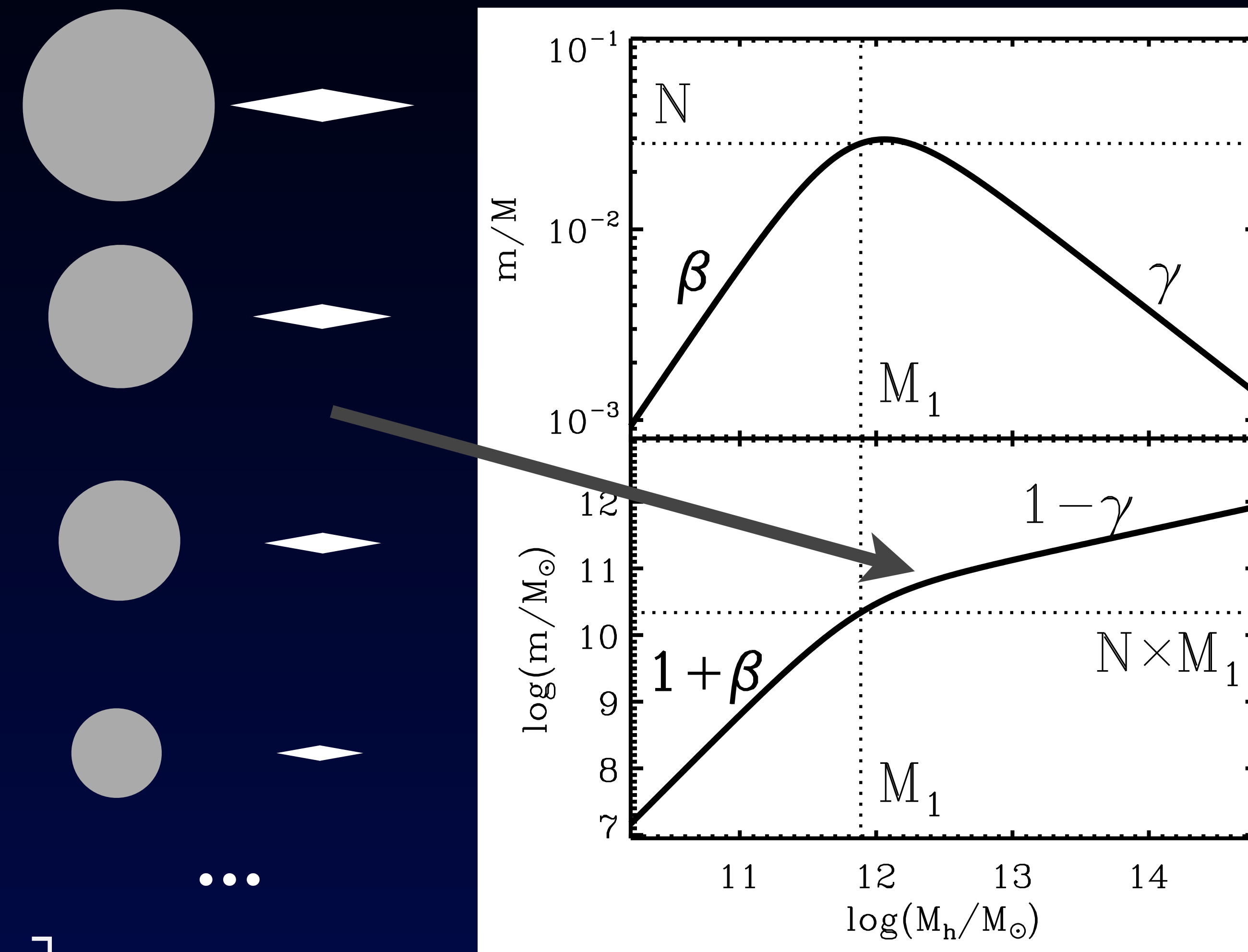
- In CDM paradigm: galaxies form in dark matter haloes
- Galaxy properties (luminosity, stellar mass) are coupled to depth of potential well and depend on halo mass
- Don't model baryonic physics ➙ link m_* and M_{halo} statistically
  Subhalo abundance matching: extract positions and masses of haloes and subhaloes from N-body simulations
- Link galaxies and haloes using simple assumption:
  Most massive galaxies live in most massive haloes
- Connection can be done in 2 ways:

  a) assuming a non-parametric monotonic relation

  b) assuming parameterized functions

Abundance matching & parameterized linking

- Produce galaxy catalogue from observed SMF in same volume as halo catalogue
- Match galaxies-haloes by mass
- Optional: Use fitting-function to get $m_*(M_h)$



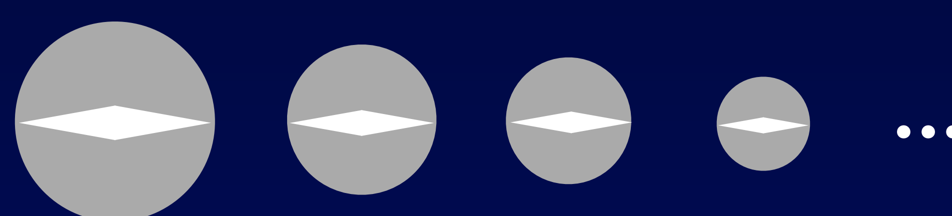
$$m_*(M_h) = 2\ R\ M_h \left[\left(\frac{M_h}{M_1} \right)^{-\beta} + \left(\frac{M_h}{M_1} \right)^{\gamma} \right]$$

Abundance matching & parameterized linking

- Produce galaxy catalogue from observed SMF in same volume as halo catalogue
- Match galaxies-haloes by mass
- Optional: Use fitting-function to get $m_*(M_h)$

- Assume function for $m_*(M_h)$
- Populate haloes with galaxies
- Compute model SMF
- Fit parameters to observed SMF

$$m_*(M_h) = 2\,R\,M_h \left[\left(\frac{M_h}{M_1} \right)^{-\beta} + \left(\frac{M_h}{M_1} \right)^{\gamma} \right]$$



- Derive $m_*(M_h)$ individually for a set of redshifts

Abundance matching & parameterized linking

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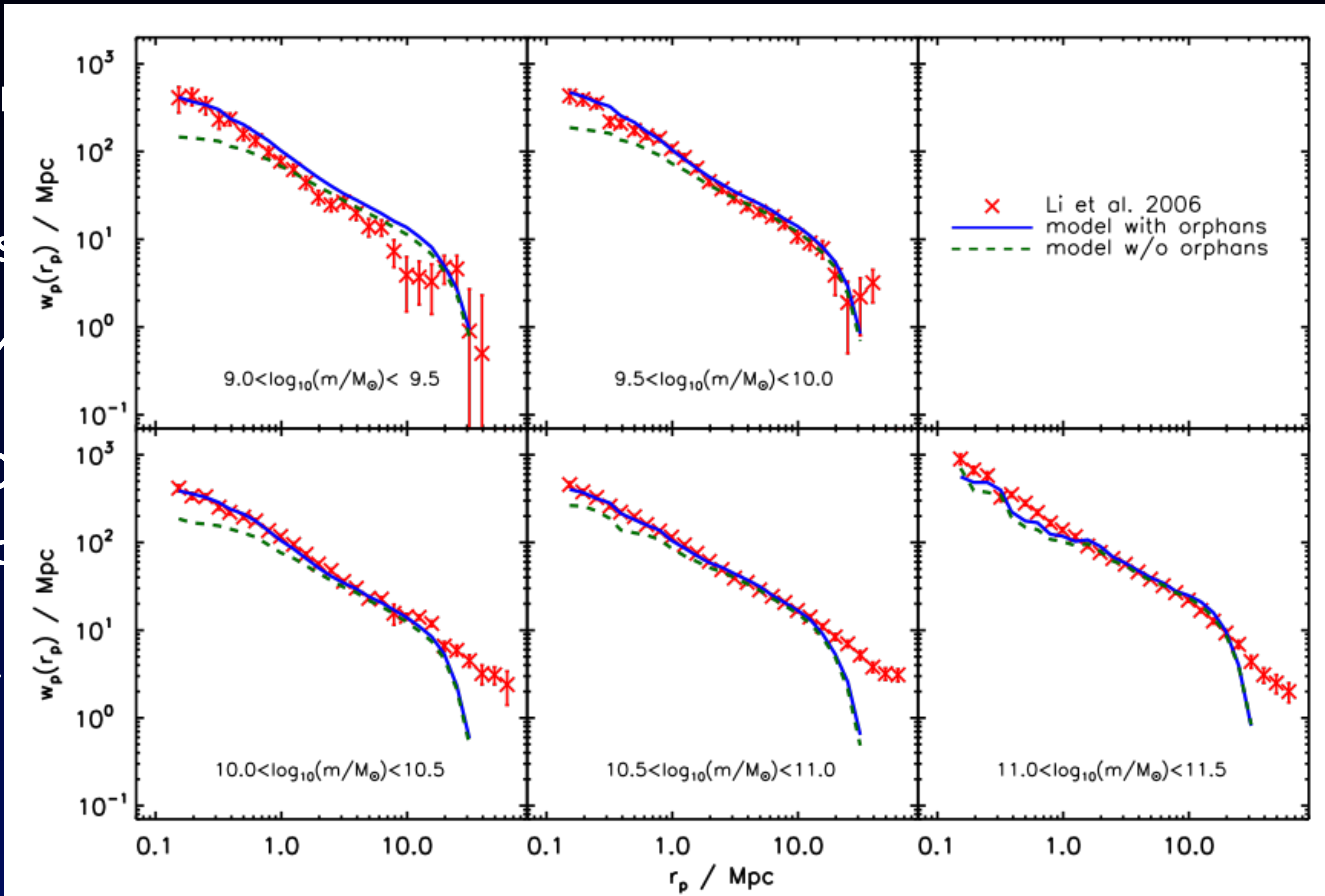
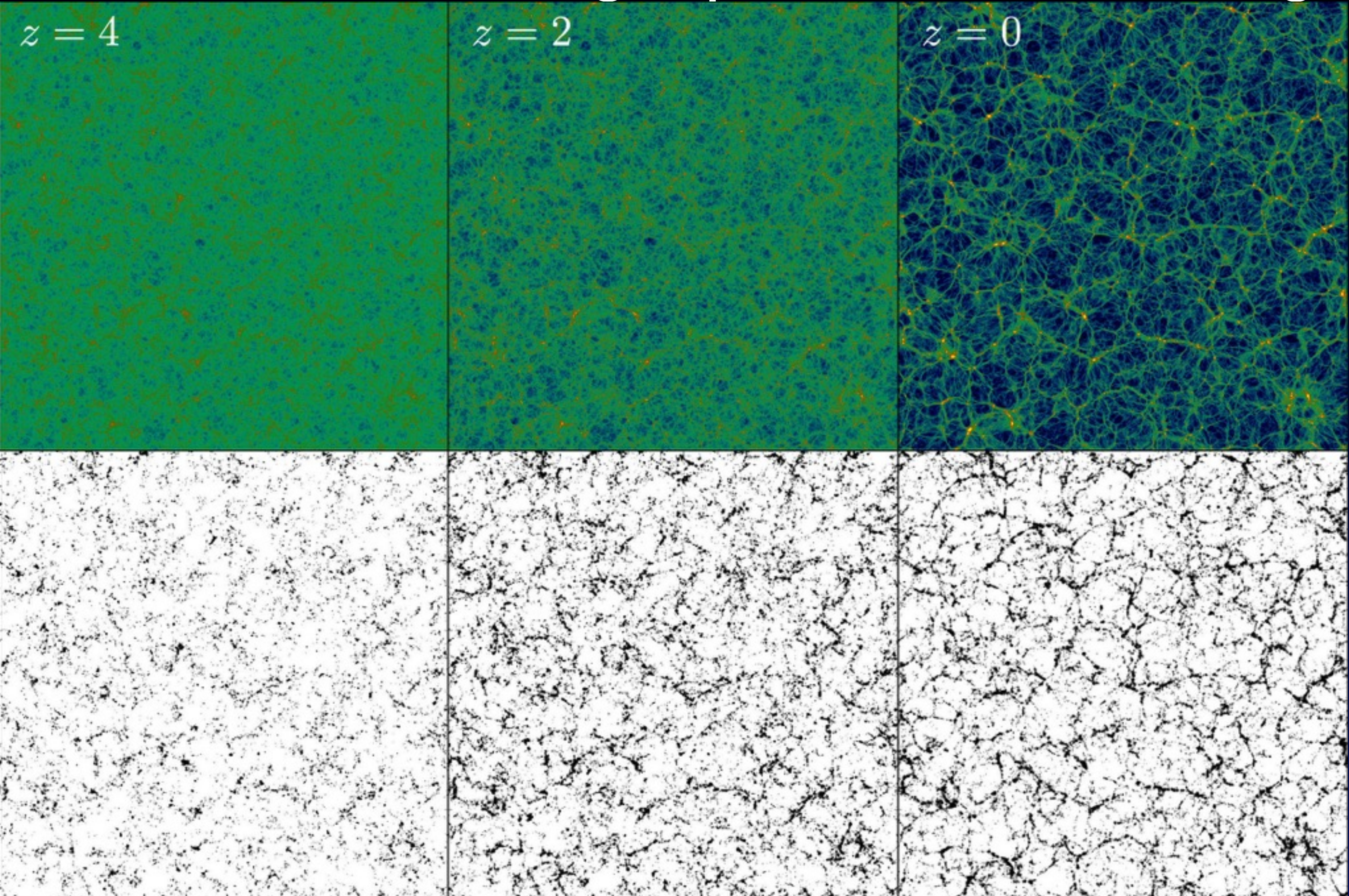


Abundance matching & parameterized linking



Final notes

- Text Books:
  - Cosmology: Galaxy Formation and Evolution (Mo, vdBosch, White)
- Reviews:
  - White (1993): astro-ph/9410043
  - Baugh (2006): astro-ph/0610031
- Papers:
  - Knebe et al. (2011), MNRAS, 415, 2293
  - Guo et al. (2011), MNRAS, 435, 897
- Gadget and N-GenIC website:

  http://www.mpa-garching.mpg.de/gadget/

Up next

- Lecture 1: Motivation & Initial conditions
- Lecture 2: Gravity algorithms & parallelization
- Lecture 3: Hydro schemes
- Lecture 4: Radiative cooling, photo heating & Subresolution physics
- Lecture 5: Halo and subhalo finders & Semi-analytic models
- Lecture 6: Getting started with Gadget
- Lecture 7: Example galaxy collision
- Lecture 8: Example cosmological box