Numerical Galaxy Formation & Cosmology

Lecture 4: Smoothed particle hydrodynamics and baryonic sub-grid models

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Outline of the lecture course

- Lecture 1: Motivation & Initial conditions
- Lecture 2: Gravity algorithms & parallelization
- Lecture 3: Hydro schemes
- Lecture 4: SPH, Radiative cooling & heating, Subresolution physics
- Lecture 5: Halo and subhalo finders & Semi-analytic models
- Lecture 6: Getting started with Gadget
- Lecture 7: Example cosmological box
- Lecture 8: Example galaxy collision

Outline of this lecture

- Smoothed particle hydrodynamics (SPH)
  - Idea & smoothing kernel
  - Equations of motion
  - Artificial viscosity
  - Advantages and problems (fluid-mixing, SPH blobs)
  - Modern SPH techniques (Pressure-entropy, artificial diffusion, etc.)

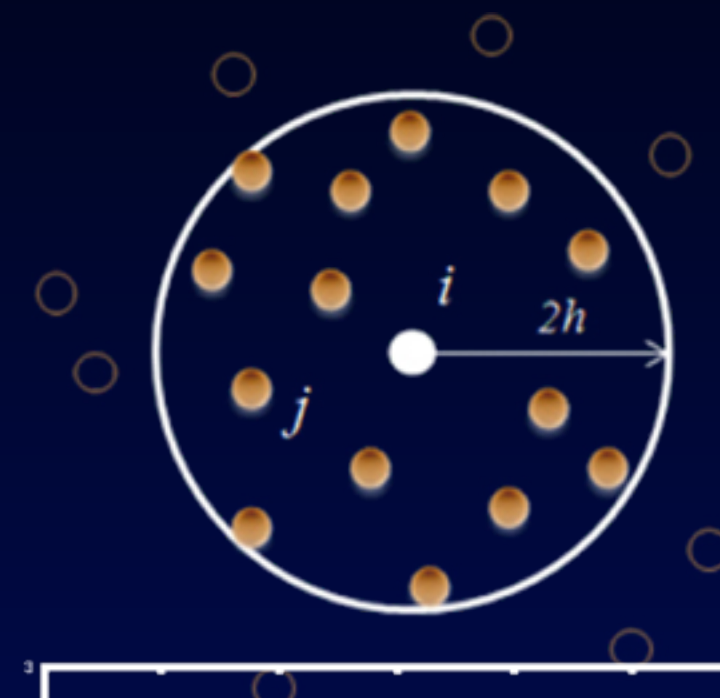
- Baryonic sub-grid physics
  - Radiative Cooling and heating
  - Star formation
  - Stellar feedback (Multi-phase model, blastwave feedback)
  - AGN feedback

Smoothed particle hydrodynamics

- Idea: treat hydrodynamics in a completely mesh-free fashion
  Use: set of sampling particles to represent the fluid

- Gas cloud of mass M_g represented by N particles of mass $m = M_g / N$

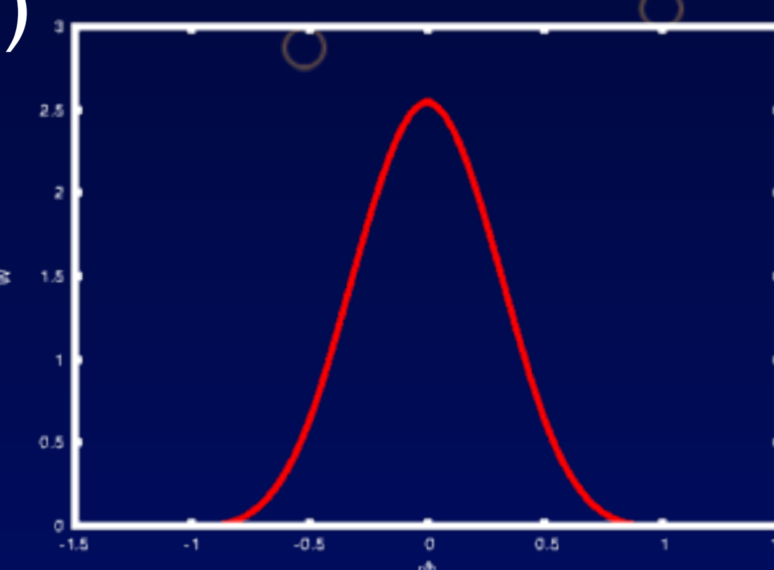
- Each particle i interacts with each other particle j through the symmetric SPH Kernel $W(r_{ij},h)$



- Depends on inter-particles distance $r = |\vec{x}_i - \vec{x}_j|$ and on ‘smoothing-length’ h (defines Kernel width)

- Most commonly the cubic spline Kernel is used:

$$W(r,h) = \frac{8}{\pi h^3} \begin{cases} 1 - 6(r/h)^2 + 6(r/h)^3 & 0 \leq (r/h) \leq 1/2 \\ 2(1 - r/h)^3 & 1/2 < (r/h) \leq 1 \\ 0 & 1 < (r/h) \end{cases}$$



Smoothed particle hydrodynamics

- The density in SPH is given by 'smoothing' the particle masses:

$$\rho(\vec{x}) = \sum_j m\, W(\vec{x} - \vec{x}_j, h)$$

- For any field F, we can define a smoothed interpolated version:

$$F_s(\vec{x}) = \int \mathrm{d}\vec{x}\; F(\vec{x}')\; W(\vec{x} - \vec{x}', h) = \sum_j \frac{m_j}{\rho_j}\; F_j\; W(\vec{x} - \vec{x}_j, h)$$

- And the derivative is given by:

$$\vec{\nabla} F_s(\vec{x}) = \sum_j \frac{m_j}{\rho_j}\; F_j\; \vec{\nabla} W(\vec{x} - \vec{x}_j, h)$$

- Usually smoothing length h is not constant: $\frac{4\pi}{3} h^3 = \frac{N_{\mathrm{ngb}} m_i}{\rho_i}$

- Pressure is related to the density via:

$$P_i = A_i \rho_i^{\gamma} = (\gamma - 1) \rho_i u_i$$

Equations of motion

- Equations of motion derived from Lagrangian: $\mathcal{L} = \sum_i \left(\frac{1}{2} m_i \vec{v}_i^2 - m_i u_i \right)$
- From $\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial \mathcal{L}}{\partial \dot{\vec{x}}_i} - \frac{\partial \mathcal{L}}{\partial \vec{x}_i} = 0$ we get $m_i \frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = -\sum_j m_j \frac{P_j}{\rho_j^2} \frac{\partial \rho_j}{\partial \vec{x}_i}$

  where $\frac{\partial \rho_j}{\partial \vec{x}_i} = \vec{\nabla} \rho_j + \frac{\partial \rho_j}{\partial h_j} \frac{\partial h_j}{\partial \vec{x}_i}$ and h is constant in the 1st term
- Differentiation of $\rho_j h_j^3 = const$: $\frac{\partial \rho_j}{\partial \vec{x}_i} = \left(1 + \frac{h_j}{3\rho_j} \frac{\partial \rho_j}{\partial h_j} \right)^{-1} \vec{\nabla}_i \rho_j$
- Using $\vec{\nabla}_i \rho_j = m_i \vec{\nabla}_i W_{ij}(h_j) + \delta_{ij} \sum_k m_k \vec{\nabla}_i W_{ki}(h_i)$

  we get: $\frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = -\sum_j m_j \left(f_i \frac{P_i}{\rho_i^2} \vec{\nabla}_i W_{ij}(h_i) + f_j \frac{P_j}{\rho_j^2} \vec{\nabla}_j W_{ij}(h_j) \right)$

  with $f_i = \left(1 + \frac{h_i}{3\rho_i} \frac{\partial \rho_i}{\partial h_i} \right)^{-1}$, mass & energy automatically constant

Artificial viscosity

- Up to now: entropy is conserved along the flow (for each particle)
- However, in shocks the entropy must increase! ➙ Artificial viscosity
- Friction force: $\left.\frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t}\right|_{\mathrm{vics}} = -\sum_j m_j \Pi_{ij} \vec{\nabla}_i \bar{W}_{ij}$
- Since antisymmetric in i and j: (angular) momentum preserved

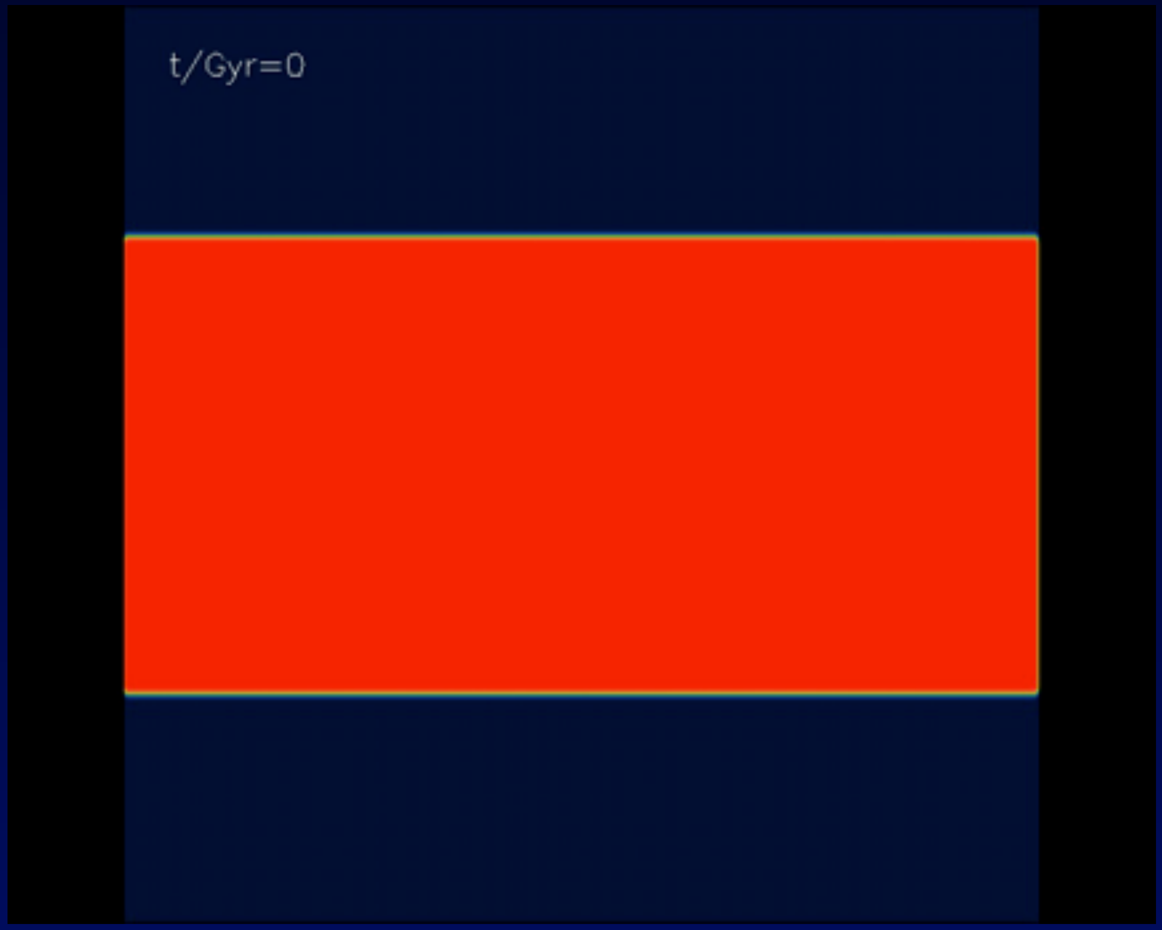
  To conserve total E: $\left.\frac{\mathrm{d}u_i}{\mathrm{d}t}\right|_{\mathrm{vics}} = \frac{1}{2}\sum_j m_j \Pi_{ij} \vec{v}_{ij} \vec{\nabla}_i \bar{W}_{ij}$
- Usual choice: $\Pi_{ij} = (-\alpha c_{ij}\mu_{ij} + \beta\mu_{ij}^2)/\rho_{ij}$ with $\mu_{ij} = \frac{h_{ij}\vec{v}_{ij}\vec{x}_{ij}}{|x_{ij}|^2 + \epsilon h_{ij}^2}$

  and $\Pi_{ij} = 0$ for $\vec{v}_{ij}\vec{x}_{ij} < 0$
- To prevent viscosity in shear flows (and conserve angular momentum):

  Balsara switch: $\Pi_{ij} \rightarrow f_{ij} \cdot \Pi_{ij}$ with $f_i^{\mathrm{AV}} = \frac{|\vec{\nabla}\cdot\vec{v}|_i}{|\vec{\nabla}\cdot\vec{v}|_i + |\vec{\nabla}\times\vec{v}|_i}$

Advantages and problems

- Advantages of SPH:
  - Numerically very robust
  - excellent conservation properties
  - Lagrangian nature → trace flow & Galilean-invariant
  - Inherently adaptive (no ad-hoc refinement needed)
  - Couples properly to N-body gravity methods



- Problems?
  - SPH doesn‘t mix fluids (well)
  - ‚Kaufmann-blobs‘ form
  - Impacts accretion rates, SFRs disc sizes, gas fractions, etc...

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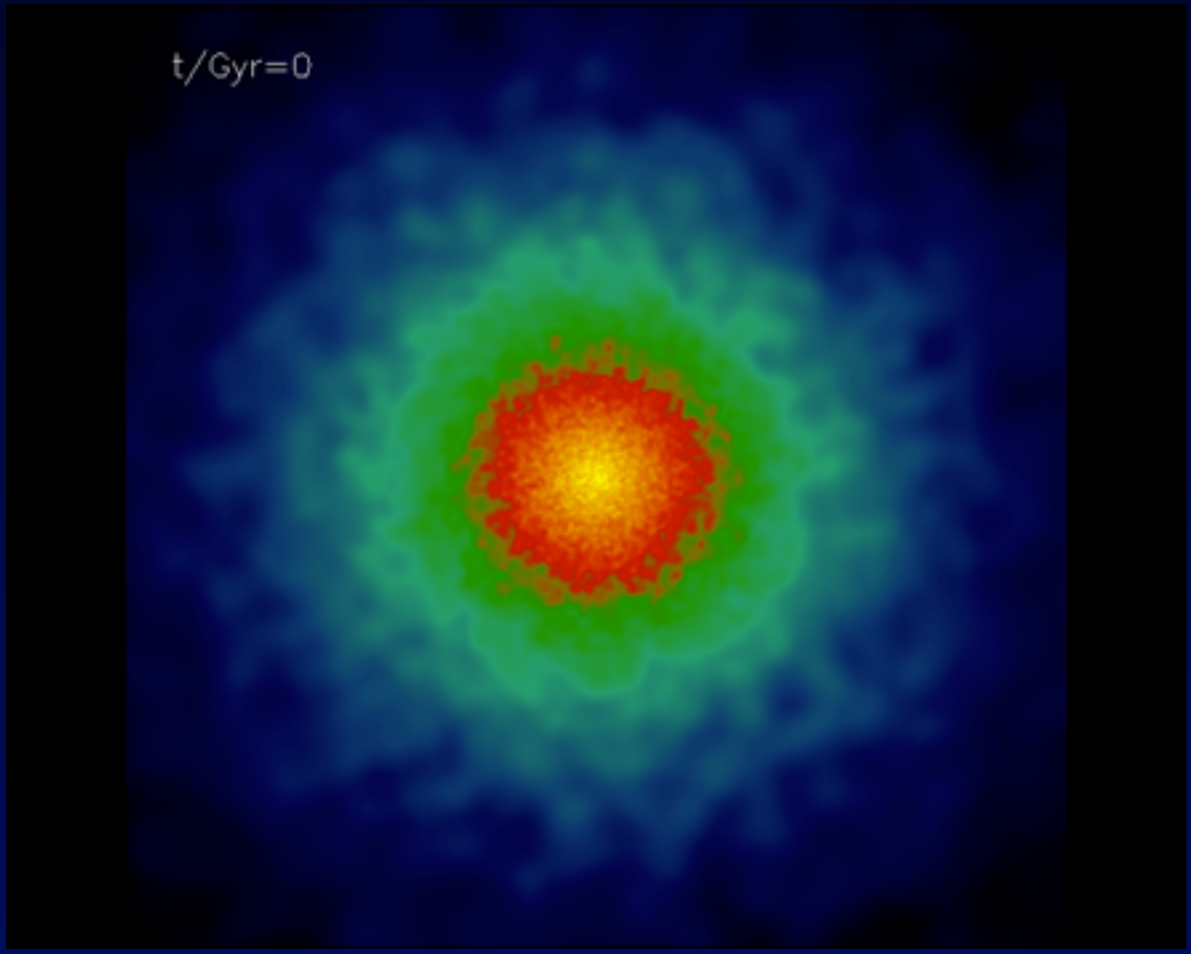
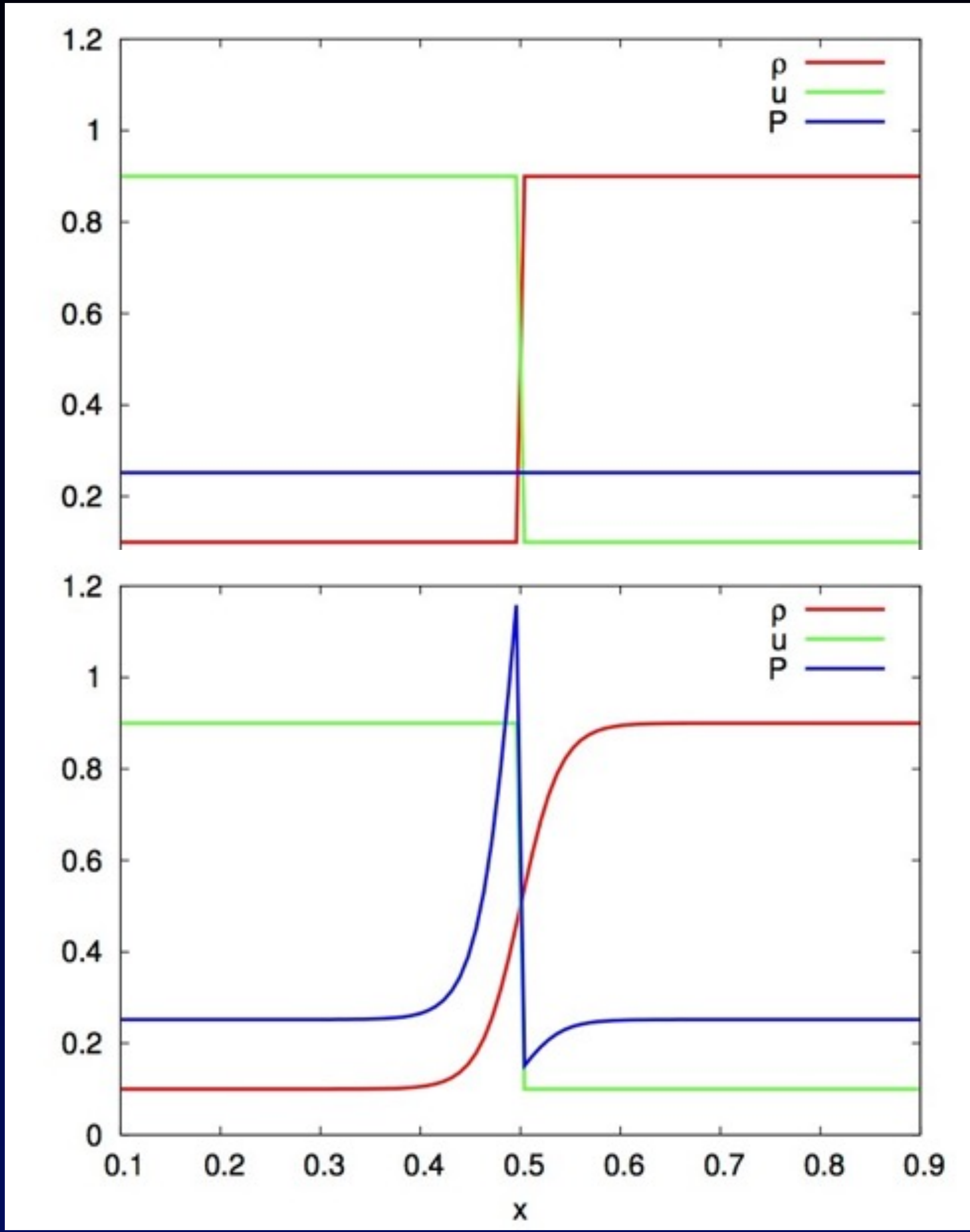


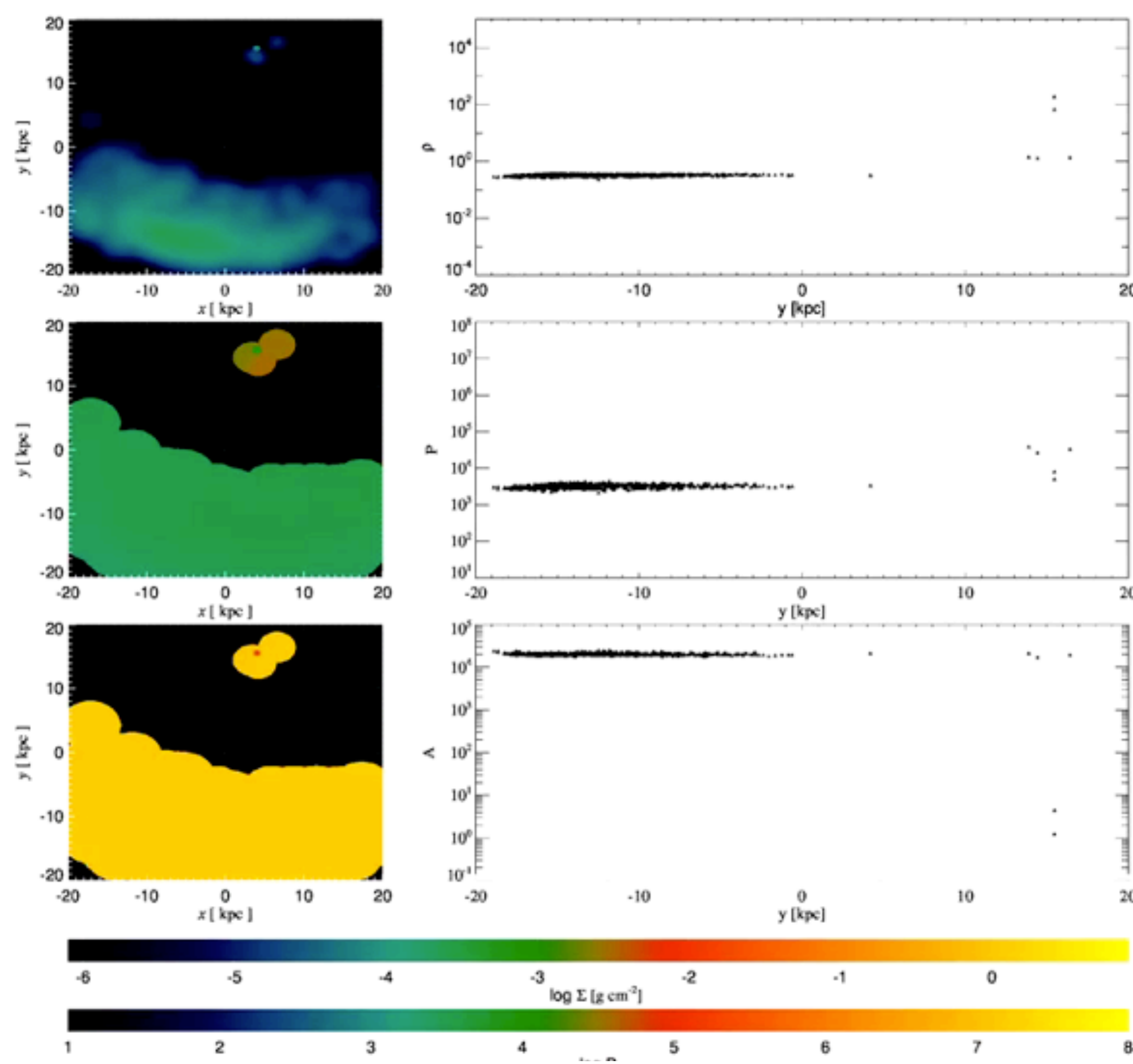
The mixing problem

- $P \propto u\,\rho$
- Contact discontinuity in hydrostatic eq.: P = cst.
- Left: $u_1\ \rho_1$, Right: $u_2\ \rho_2$
  → $u_1 = \rho_2/\rho_1\ u_2$
- SPH: smooth density gradient
  Energy not smoothed
- Pressure blib introduced
  → surface tension prevents mixing



The blob problem

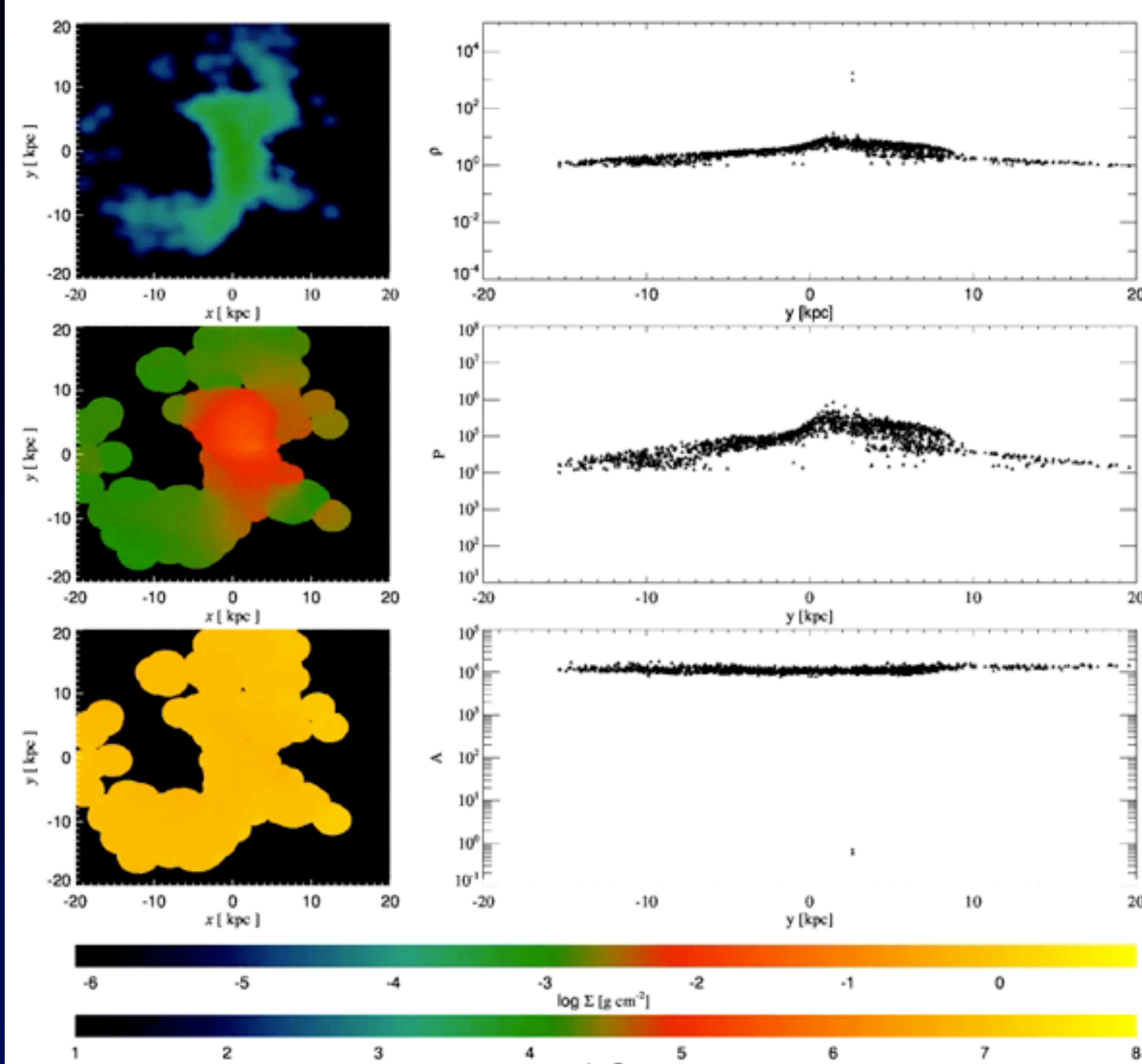
- Two gas clouds collide
- Density increases
  Energy decreases



Credit: A. Hobbs & J. Read

The blob problem

- Two gas clouds collide
- Density increases
  Energy decreases
- Density is smoothed
  Energy isn‘t smoothed
  → Pressure blip



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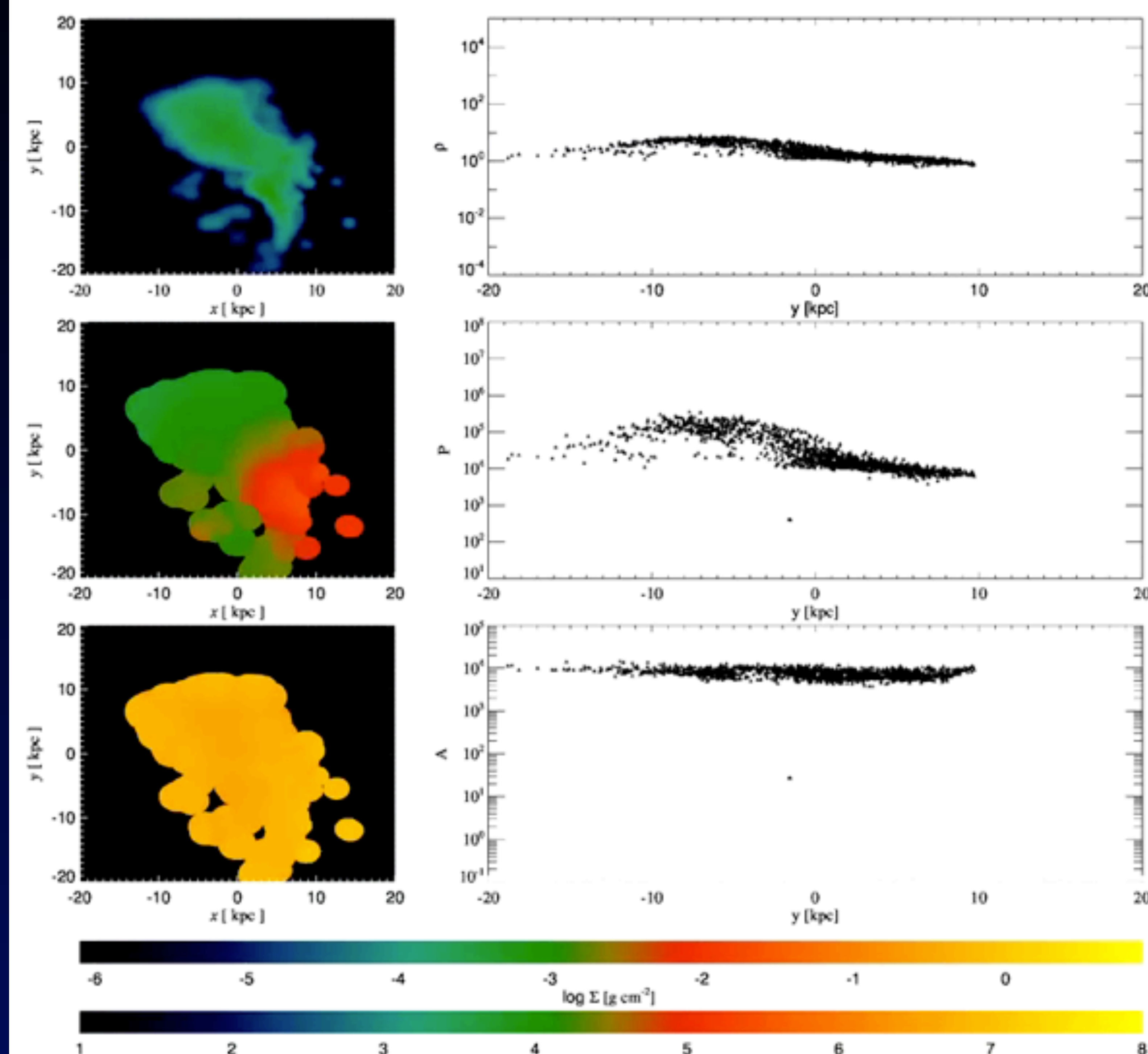
- Pressure minimum causes nearby particles to collapse
  → blob



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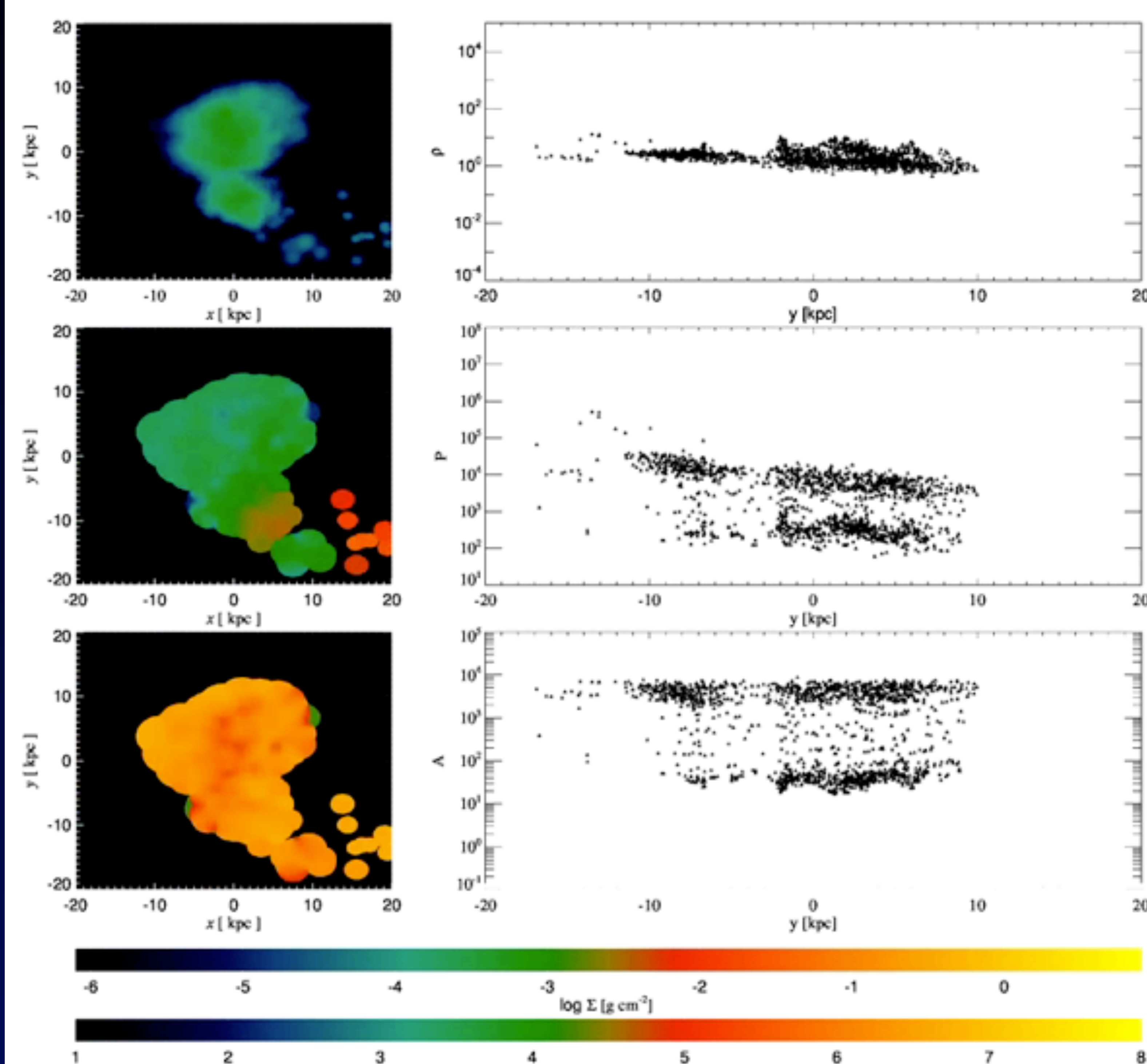

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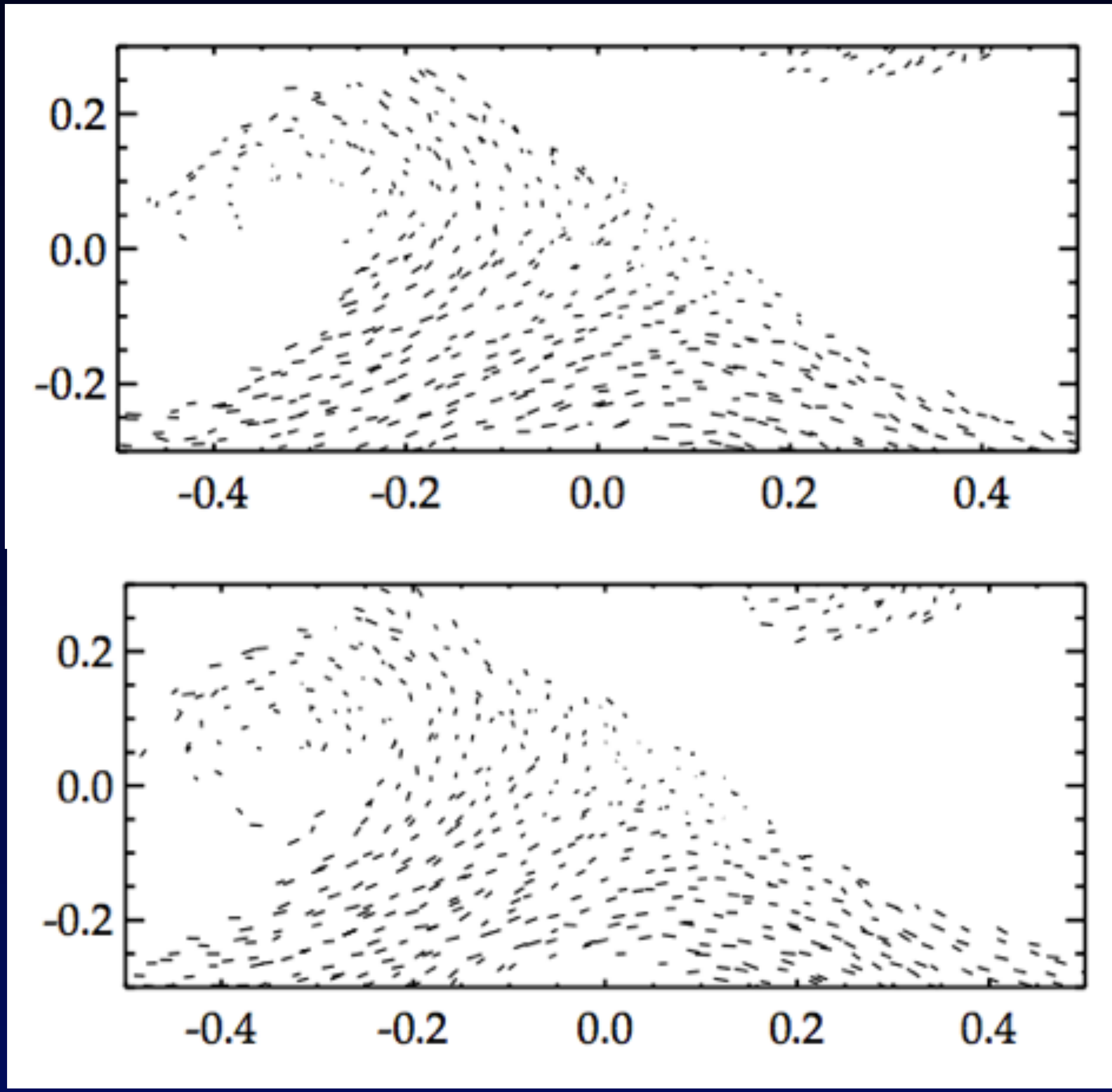
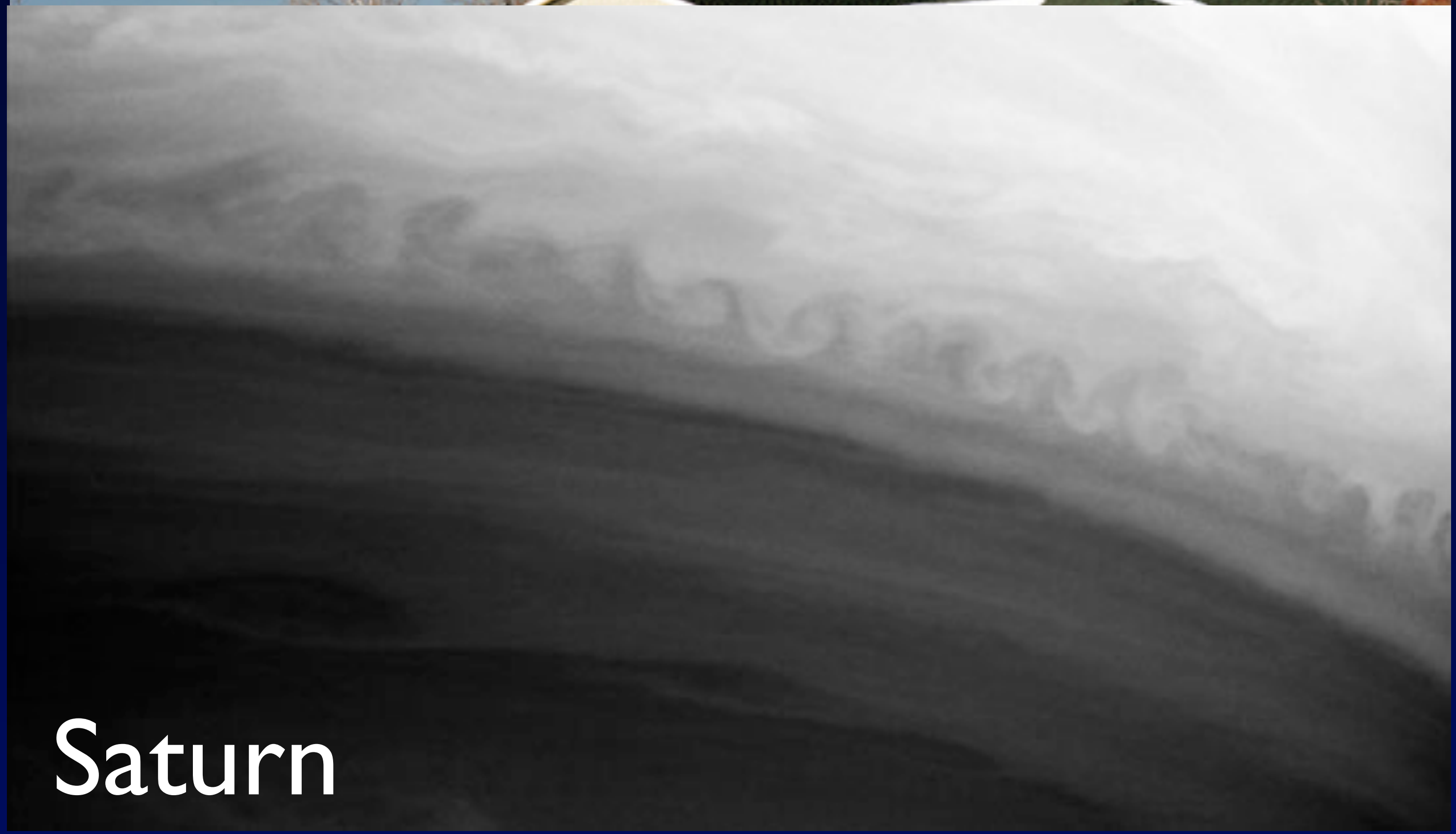
Artificial Diffusion

- Physical Diffusion should be included in simulations (shouldn‘t include metal diffusion without energy diffusion
- du/dt = −P/ρ ∇v
- Diffusion removes the energy minimum ➙ pressure smooth
- Surface tension vanishes
  Fluids can mix / no blobs form
- However: Unclear how much diffusion to be added



Kelvin Helmholtz Instability



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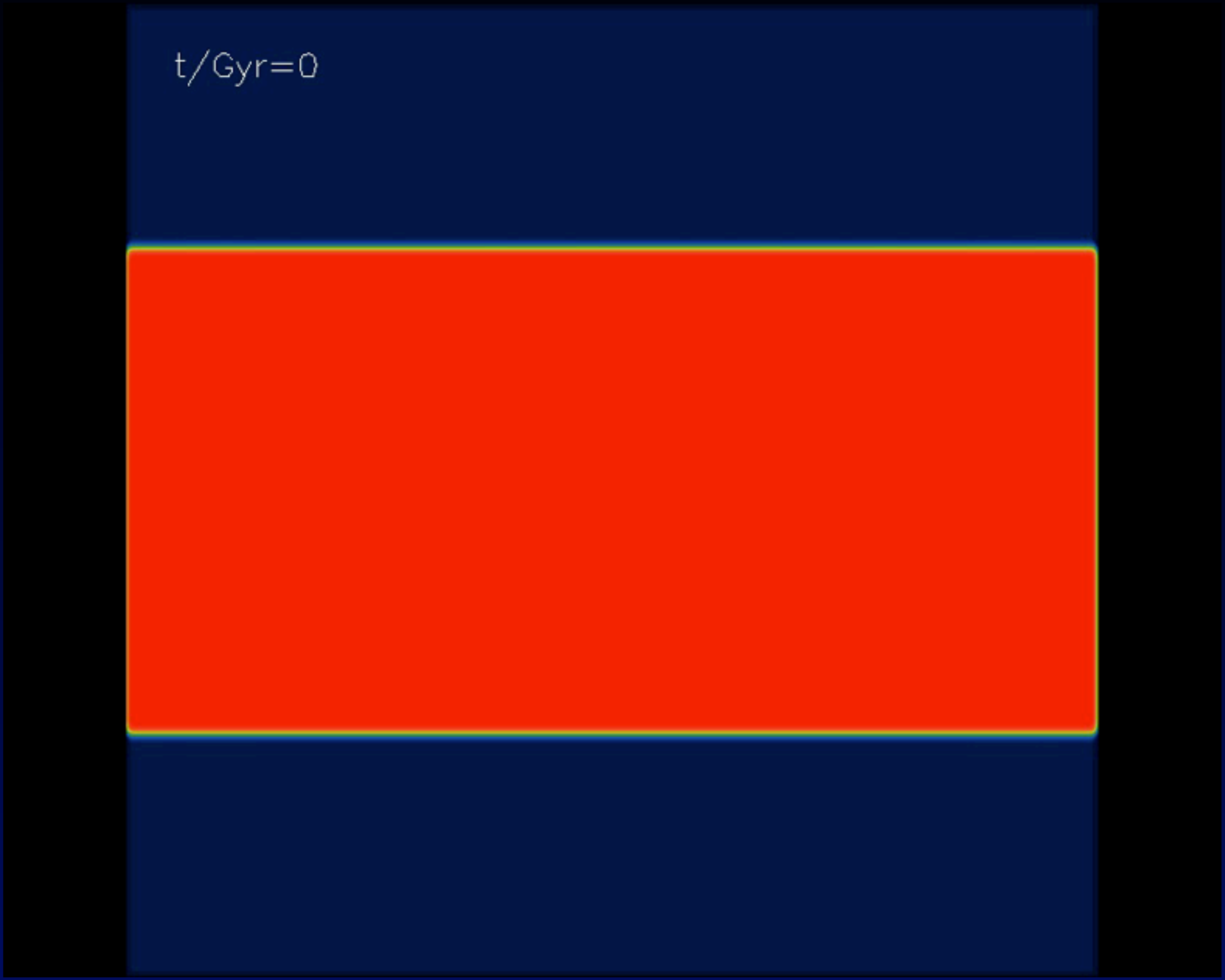
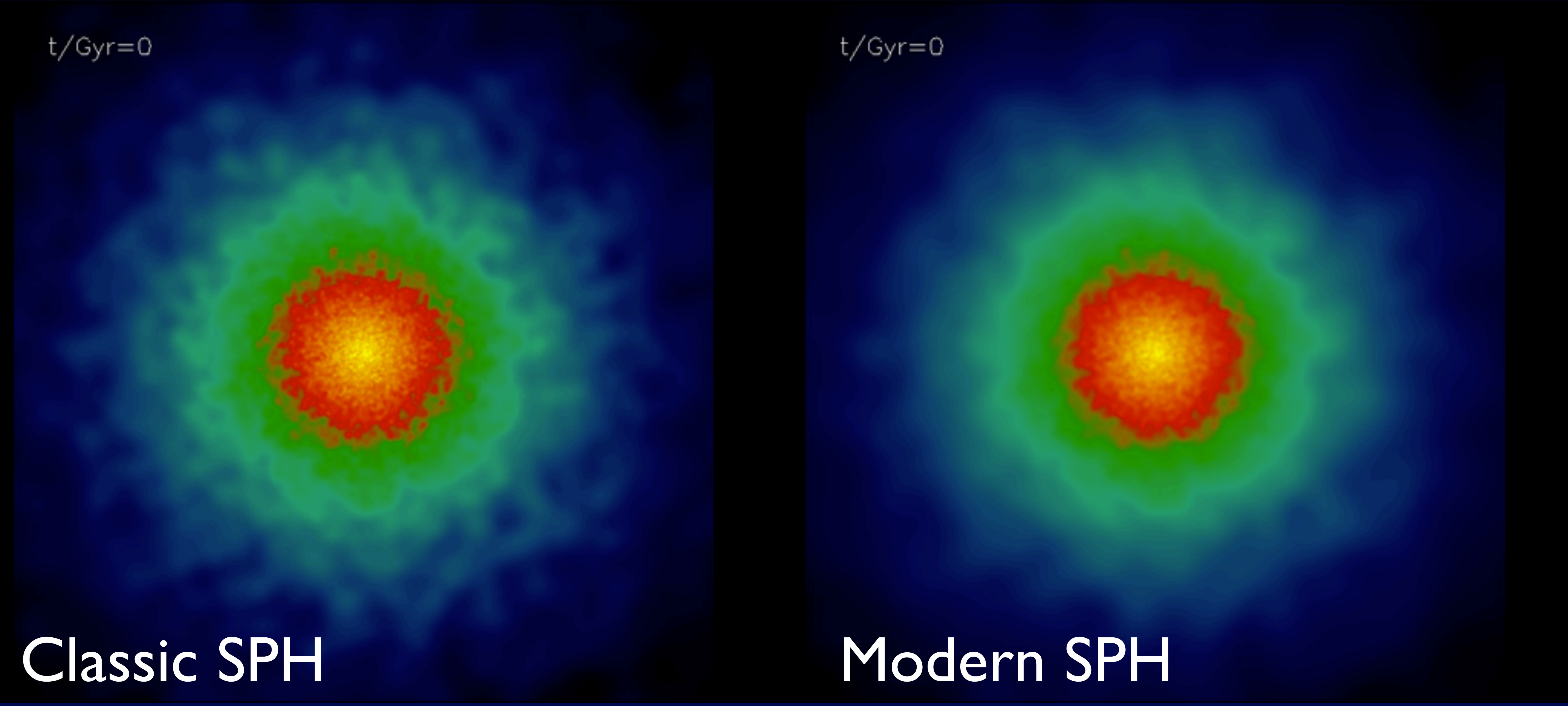


Blob test

Classic SPH

Modern SPH

Isolated discs in hot halo



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How to form a galaxy…

- Up to now we have included Gravity (N-body), and Hydrodynamics
- All other processed happen below the scale we can resolve
- Include effective models to describe these physics:
  - Radiative cooling and heating
  - Star formation
  - Feedback from supernovae
  - Feedback from AGN
  - Feedback from other sources?
- All processes are in general rather poorly understood…

Radiative Cooling & Heating

- Stars can only form when gas is dense enough (i.e. cold enough)
- Gas can cool via a number of mechanisms:
    - Inverse Compton scattering of CMB photons by electrons in the hot gas. Long cooling time (only important in the early Universe).
    - Excitation of rot. & vib. levels in molecular hydrogen through collisions, and subsequent decay (important in low mass haloes).
    - Photon emission after collisions between atoms and electrons, excited levels decay radiatively (important in int. mass haloes).
    - Bremsstrahlung: electrons are accelerated in an ionized plasma (important in massive clusters)
- Gas is heated by a background of high energy UV photons from quasars and massive stars

Radiative Cooling & Heating

- Each particle (or cell) carries information on density and temperature
- Need to compute abundances of the different ionic species of the gas

  Assume: gas optically thin,

  & Ionisation equilibrium

$$\Gamma_{e\mathrm{H}0} n_e n_{\mathrm{H}0} + \Gamma_{\gamma\mathrm{H}0} n_{\mathrm{H}0} = \alpha_{\mathrm{H}+} n_{\mathrm{H}+} n_e \,,$$

$$\Gamma_{e\mathrm{He}0} n_{\mathrm{He}0} n_e + \Gamma_{\gamma\mathrm{He}0} n_{\mathrm{He}0} = (\alpha_{\mathrm{He}+} + \alpha_d) n_{\mathrm{He}+} n_e \,,$$

$$\Gamma_{e\mathrm{He}+} n_{\mathrm{He}+} n_e + \Gamma_{\gamma\mathrm{He}+} n_{\mathrm{He}+} + (\alpha_{\mathrm{He}+} + \alpha_d) n_{\mathrm{He}+} n_e$$
$$= \alpha_{\mathrm{He}++} n_{\mathrm{He}++} n_e + \Gamma_{e\mathrm{He}0} n_{\mathrm{He}0} n_e + \Gamma_{\gamma\mathrm{He}0} n_{\mathrm{He}0} \,,$$

$$\alpha_{\mathrm{He}++} n_{\mathrm{He}++} n_e = \Gamma_{e\mathrm{He}+} n_{\mathrm{He}+} n_e + \Gamma_{\gamma\mathrm{He}+} n_{\mathrm{He}+} \,.$$

- Compute cooling rate:

$$\Lambda = \sum_i \Lambda_i(n_i, T)$$

COOLING RATES

Process	Species	Λ Rate[a] (ergs s^{-1} cm^{-3})
Collisional excitation	H^0	$7.50 \times 10^{-19} e^{-118348.0/T} (1 + T_5^{1/2})^{-1} n_e n_{\mathrm{H}_0}$
	He^+	$5.54 \times 10^{-17} T^{-0.397} e^{-473638.0/T} (1 + T_5^{1/2})^{-1} n_e n_{\mathrm{He}_+}$
Collisional ionization	H^0	$1.27 \times 10^{-21} T^{1/2} e^{-157809.1/T} (1 + T_5^{1/2})^{-1} n_e n_{\mathrm{H}_0}$
	He^0	$9.38 \times 10^{-22} T^{1/2} e^{-285335.4/T} (1 + T_5^{1/2})^{-1} n_e n_{\mathrm{He}_0}$
	He^+	$4.95 \times 10^{-22} T^{1/2} e^{-631515.0/T} (1 + T_5^{1/2})^{-1} n_e n_{\mathrm{He}_+}$
Recombination	H^+	$8.70 \times 10^{-27} T^{1/2} T_3^{-0.2} (1 + T_6^{0.7})^{-1} n_e n_{\mathrm{H}_+}$
	He^+	$1.55 \times 10^{-26} T^{0.3647} n_e n_{\mathrm{He}_+}$
	He^{++}	$3.48 \times 10^{-26} T^{1/2} T_3^{-0.2} (1 + T_6^{0.7})^{-1} n_e n_{\mathrm{He}_{++}}$
Dielectric recombination	He^+	$1.24 \times 10^{-13} T^{-1.5} e^{470000.0/T} (1 + 0.3 e^{-94000.0/T}) n_e n_{\mathrm{He}_+}$
Free-free	All ions	$1.42 \times 10^{-27} g_{ff} T^{1/2} (n_{\mathrm{H}_+} + n_{\mathrm{He}_+} + 4 n_{\mathrm{He}_{++}}) n_e$

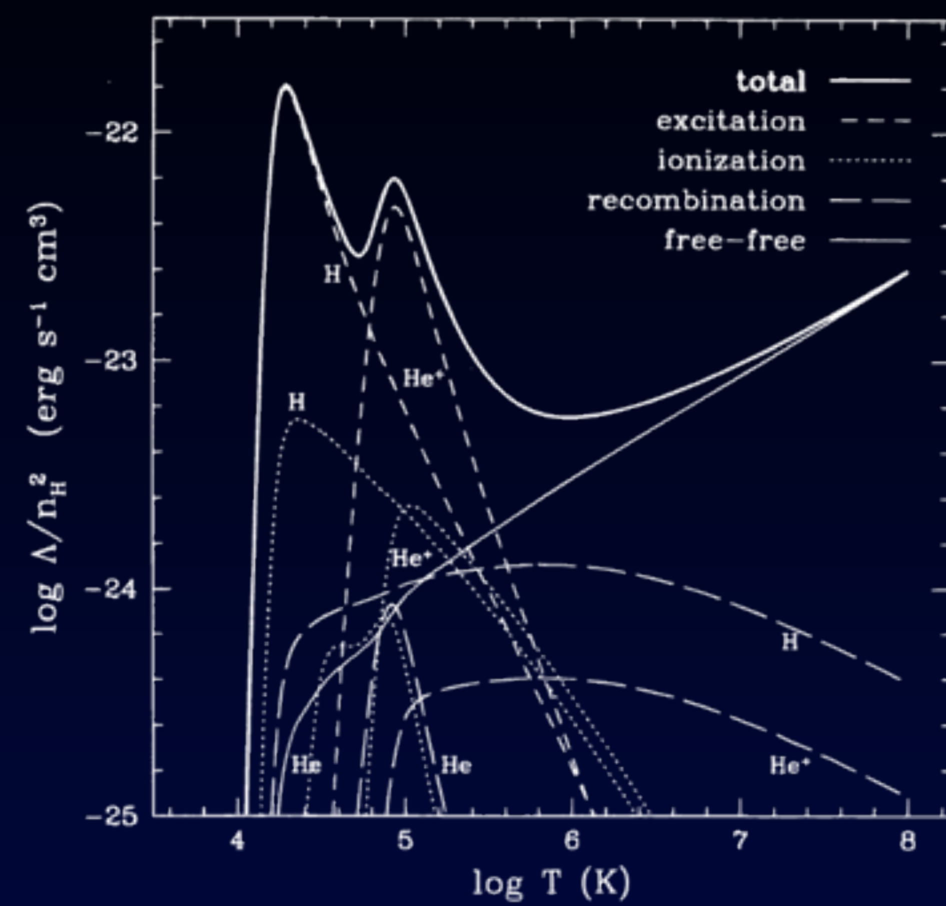
[a] $T_n = T/10^n$ K.

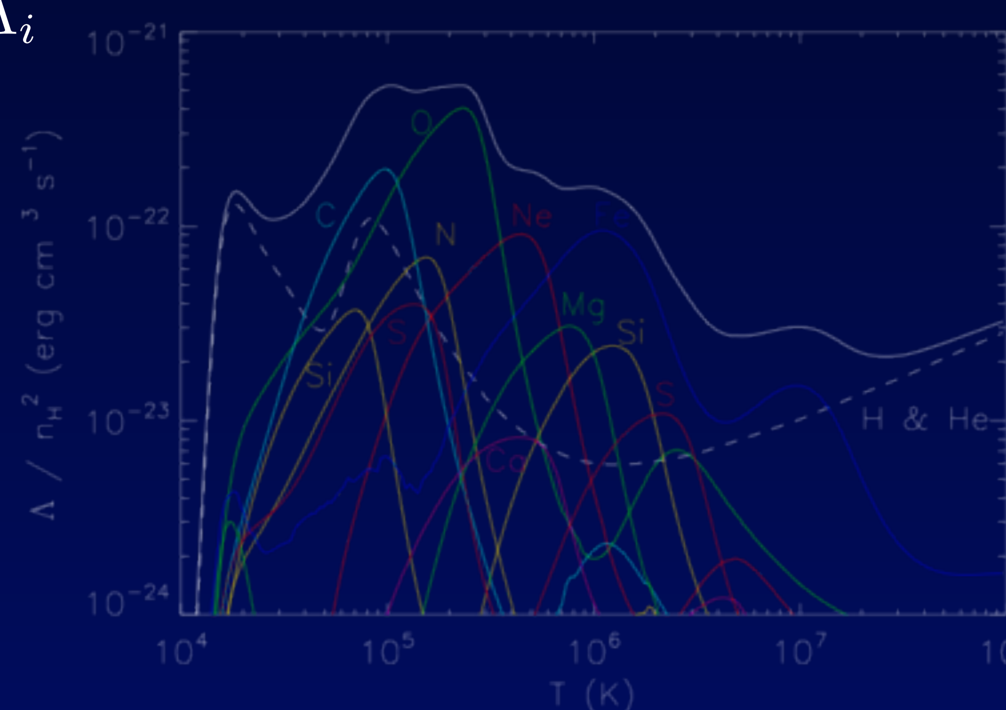
Katz+96

- Heating rate from photo-ionisation: $\mathcal{H} = \sum_i n_i \, \epsilon_i$

Change in thermal energy

- With the cooling rate Λ and the heating rate $\mathscr{H}$, we can compute the change in thermal energy: $$\frac{\mathrm{d}u_i}{\mathrm{d}t} = \frac{\mathcal{H}_i - \Lambda_i}{\rho_i}$$
- Up to now, only primordial cooling
- In practice, cooling is computed on an element-by-element basis: $\Lambda = \Lambda_{\mathrm{H,He}} + \sum_{i>He} \Lambda_i$ where Λ_i is tabulated for n_H, T, and z
- Need to follow the creation of elements in SNe (usually 11 elements are tracked)





Star formation

- Stars should form in a gas cloud if:

  - the gas density is high enough (threshold density n_{min})

  - the region is Jeans unstable

- The star formation rate is given by $\frac{\mathrm{d}\rho_*}{\mathrm{d}t} = \epsilon \frac{\rho_g}{t_{\mathrm{sf}}}$

  where ε is an efficiency and t_{sf} is a characteristic timescale

- Often the dynamical timescale is used for t_{sf}

- Implemented in the code in a stochastic way: $p = \frac{m_g}{m_*}\left(1 - e^{-\epsilon \Delta t / t_{\mathrm{sf}}}\right)$

  Draw random number r: if r < p create new star particle

- No feedback ➙ gas can form stars on dynamical time ➙ overcooling

Stellar feedback: Multiphase model

- Springel & Hernquist (2003) developed a multi-phase model
- Each gas particle is treated as a two-phase medium: cold and hot
- Gas density is $\rho = \rho_c + \rho_h$ with specific energy $\varepsilon = \varepsilon_c \rho_c + \varepsilon_h \rho_h$
- Star formation: cold clouds are converted to stars: $\frac{\mathrm{d}\rho_*}{\mathrm{d}t} = (1-\beta)\frac{\rho_c}{t_*}$
  where β is the fraction of short-lived stars and t_* is a timescale
- SNe release energy and move gas from cold to hot phase.
  Cloud mass evaporated proportional to SN mass: $\left.\frac{\mathrm{d}\rho_c}{\mathrm{d}t}\right|_{\mathrm{EV}} = A\beta\frac{\rho_c}{t_*}$
- Cold clouds grow via radiative cooling $\left.\frac{\mathrm{d}\rho_c}{\mathrm{d}t}\right|_{\mathrm{RC}} = -\left.\frac{\mathrm{d}\rho_h}{\mathrm{d}t}\right|_{\mathrm{RC}} = \frac{\Lambda_{\mathrm{net}}(\rho_h, \epsilon_h)}{\epsilon_h - \epsilon_c}$

Stellar feedback: Multiphase model

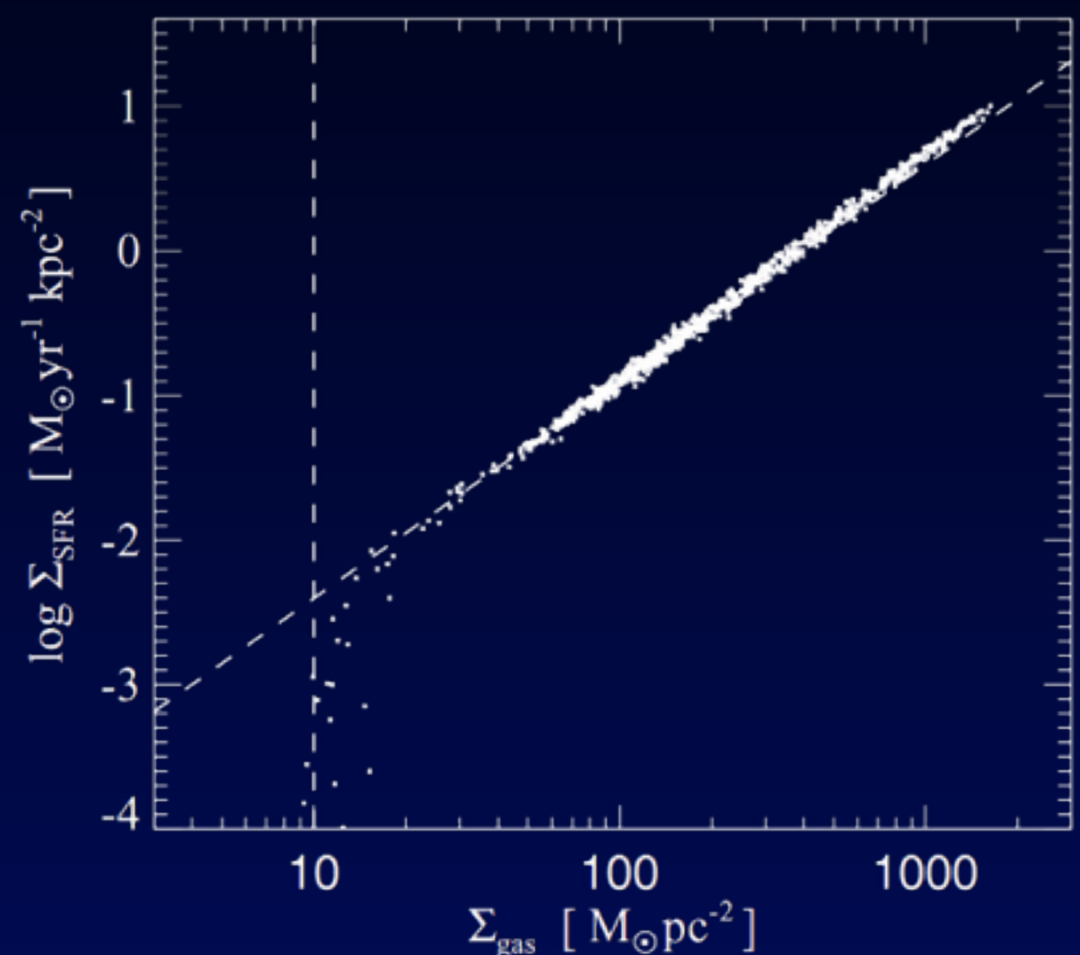
- The rates at which the hot and the cold phases evolve are:

$$\frac{\mathrm{d}\rho_c}{\mathrm{d}t} = -\frac{\rho_c}{t_*} - A\beta\frac{\rho_c}{t_*} + \frac{\Lambda_{\mathrm{net}}(\rho_h, \epsilon_h)}{\epsilon_h - \epsilon_c}$$

$$\frac{\mathrm{d}\rho_h}{\mathrm{d}t} = \beta\frac{\rho_c}{t_*} + A\beta\frac{\rho_c}{t_*} - \frac{\Lambda_{\mathrm{net}}(\rho_h, \epsilon_h)}{\epsilon_h - \epsilon_c}$$

- The star formation timescale t* is chosen to reproduce the observed Schmidt-Kennicutt relation:

$$t_*(\rho) = t_0^*\left(\frac{\rho}{\rho_{\mathrm{th}}}\right)^{-1/2}$$



- Free parameters are chosen such that

  - KS-relation reproduced

  - threshold density ρ_{th} corresponds to observed one

Galactic winds

- Feedback is still very inefficient (overcooling)
- Introduce 'mechanical' feedback given by galactic winds
- The mass-loss rate is proportional to the SFR: $\dot{M}_w = \eta \dot{M}_*$

  η is called 'mass loading factor', observations suggest η = 2
- Wind carries fixed fraction of SN energy: $\chi \epsilon_{\rm SN} \dot{M}_*$

  Set equal to the kinetic wind energy: $$v_w = \sqrt{\frac{2\beta\chi\epsilon_{\rm SN}}{\eta(1-\beta)}}$$
- Particle added to the wind if random number is below

  $$p_w = 1 - \exp\left(-\frac{\eta(1-\beta)\rho_c \Delta t}{\rho t_*}\right)$$ and $$\vec{v}' = \vec{v} + v_w \vec{n}$$
- Numerical trick: decouple winds from hydro for short amount of time

Blastwave feedback

- Thermal feedback is usually quickly radiated away (in dense gas)
- Stinson et al. (2006) implemented feedback based on a SN blast wave
- In each SN explosion a fraction of the SN energy is added to the ISM
- Cooling is temporarily switched of in neighboring particles within a radius of: $R_E = 10^{1.74}\ E_{SN}^{0.32}\ n^{-0.16}\ P^{-0.2}\ \mathrm{pc}$

  for a time: $t_E = 10^{5.92}\ E_{SN}^{0.31}\ n^{0.27}\ P^{-0.64}\ \mathrm{yr}$

  based on work by Chevalier (1974) and McKee & Ostriker (1977)
- Neighboring particles will become hot, reducing the SFR
- High temperature leads to high pressure and drives a galactic wind

AGN feedback

- In massive haloes, SN feedback cannot reduce the SF efficiency
  Potential well too deep to drive massive winds

- Energy released from accretion of gas onto BHs can be considerable

- Relevant scales for the BH cannot be resolved, hence effective model:

- Accretion rate given by Bondi-Hoyle rate: $\dot{M}_B = \frac{4\pi\alpha G^2 M_{\rm BH}^2 \rho}{(c_s^2 + v^2)^{3/2}}$
  limited by Eddington rate.

- Some fraction ε_f of the radiated luminosity can couple thermally to surrounding gas: $\dot{E}_{\rm FB} = \epsilon_f L_r = \epsilon_f \epsilon_r \dot{M}_{\rm BH} c^2$ (ε_f: M-E conversion)

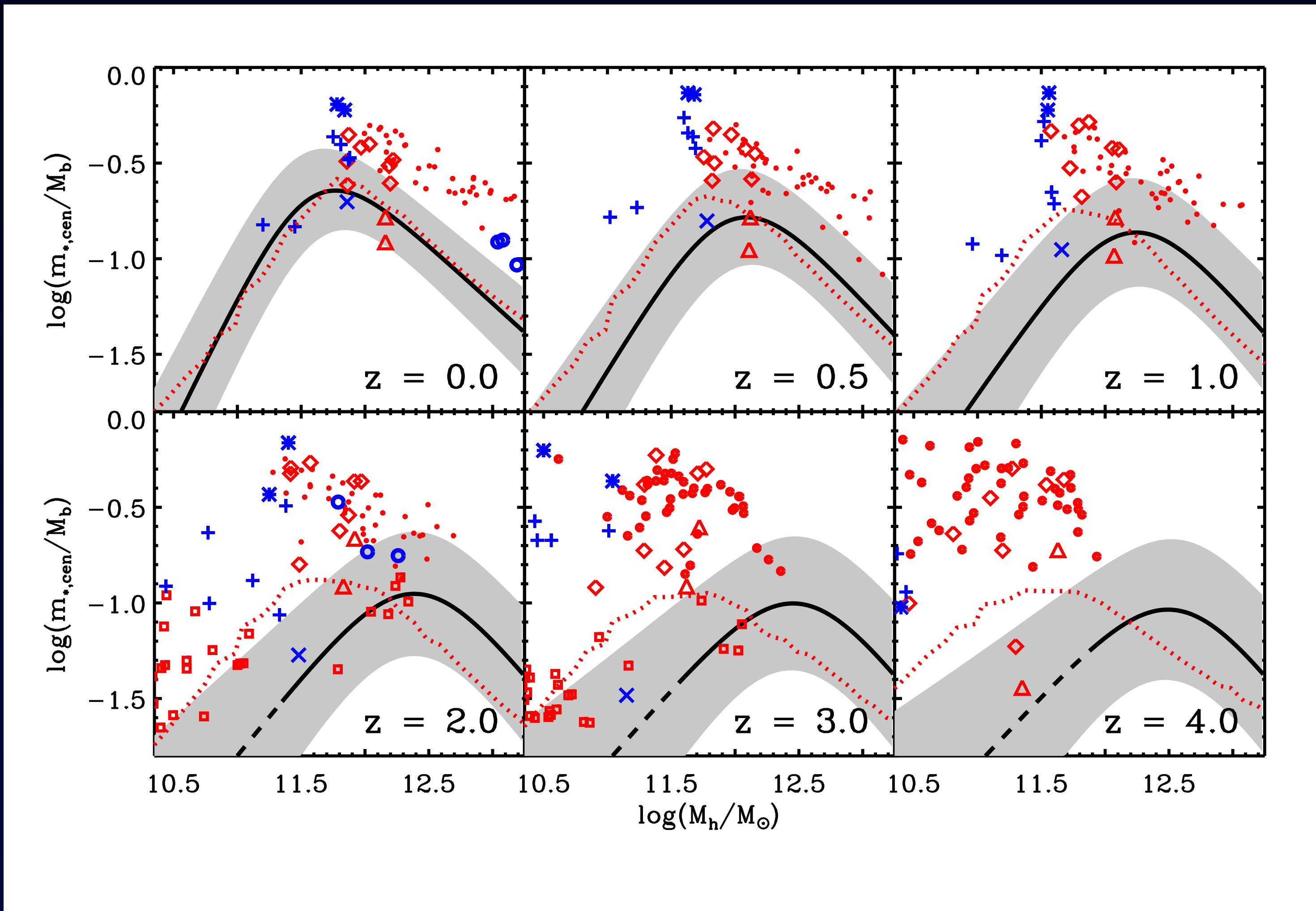
- Usually $\varepsilon_f = 0.05$ ➛ 0.5% of the accreted rest mass energy is available

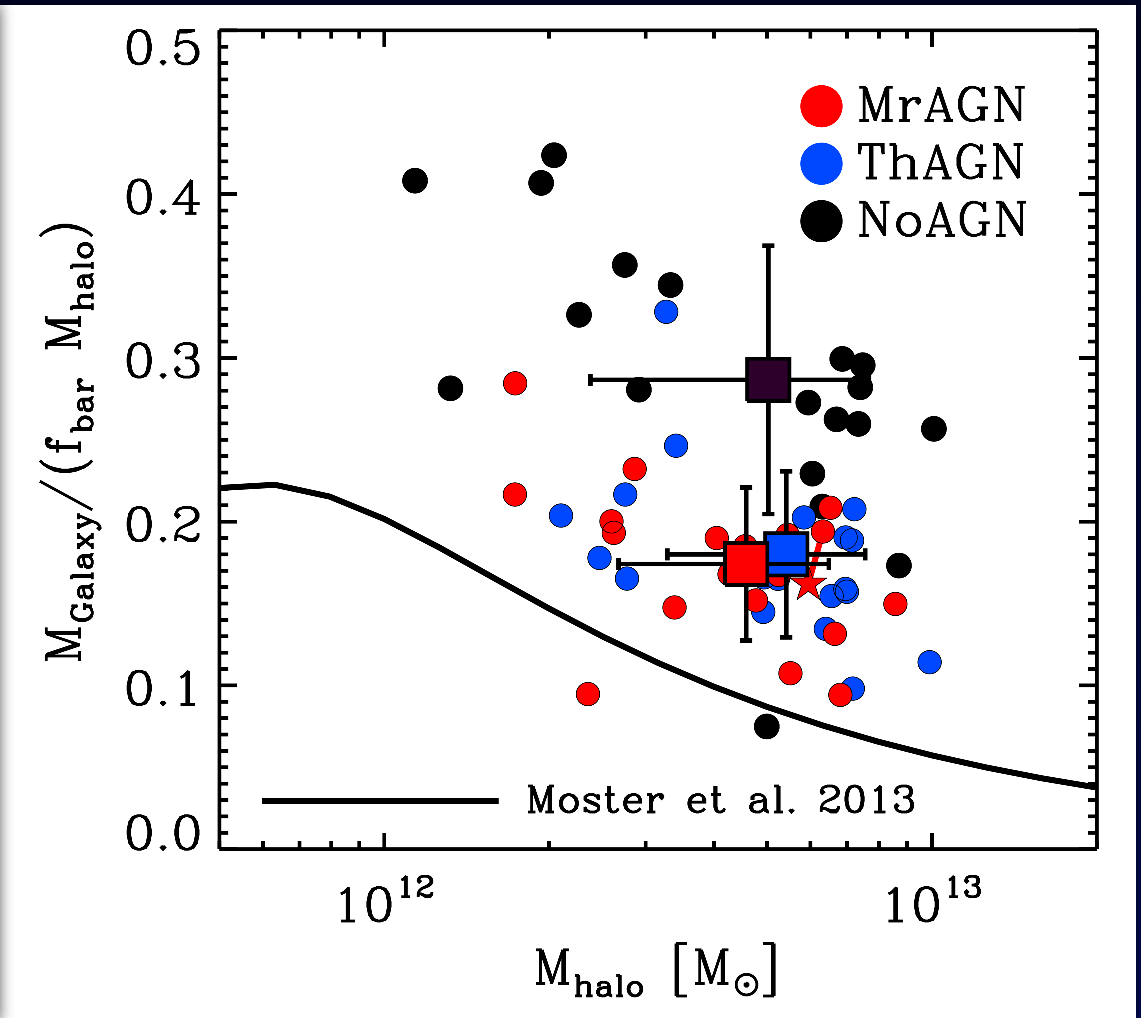
AGN feedback

- Implementation: BH particles are colissionless sink particles though compute gas density & temperature for each BH
- For each gas particle around BH compute: $p = \frac{w \dot{M}_{\mathrm{BH}} \Delta t}{\rho}$ with w = kernel weight. If random number < p absorb gas particle in BH (including momentum)
- Add energy kernel-weighted to the thermal energy of the surrounding gas particles.
- Merge BHs if they are within one smoothing length of each other and if the relative velocity is smaller than the sound speed

Results of feedback

- In low mass haloes SN feedback is very efficient
- In massive haloes AGN feedback can reduce the ‘conversion efficiency’
- To simulate realistic massive galaxies, a combination of both is needed





Final notes

- Text Books:
  - Cosmology: Galaxy Formation and Evolution (Mo, vdBosch, White)
- Papers:
  - Hopkins (2013), MNRAS, 428, 2840
  - Hu et al. (2014), MNRAS, 443, 1173
  - Springel & Hernquist (2003), MNRAS, 339, 289
  - Stinson et al. (2006), MNRAS, 373, 1074
  - Springel et al. (2005), MNRAS, 62, 79
- Gadget and N-GenIC website:

  http://www.mpa-garching.mpg.de/gadget/

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