Numerical Galaxy Formation & Cosmology

Lecture 1: Motivation and Initial Conditions

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Outline of the lecture course

- Lecture 1: Motivation & Initial conditions
- Lecture 2: Gravity algorithms & parallelization
- Lecture 3: Hydro schemes
- Lecture 4: Radiative cooling, photo heating & Subresolution physics
- Lecture 5: Halo and subhalo finders & Semi-analytic models
- Lecture 6: Getting started with Gadget
- Lecture 7: Example galaxy collision
- Lecture 8: Example cosmological box

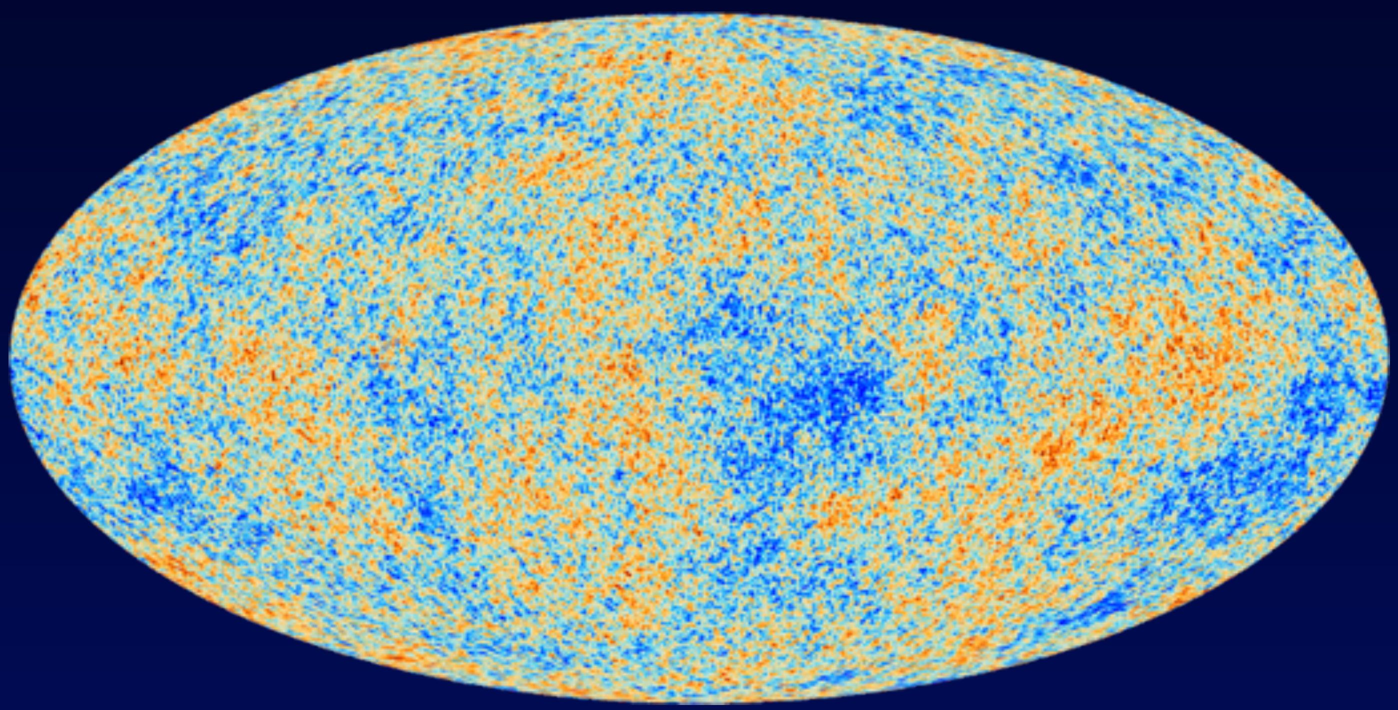
Outline of this lecture

- Motivation for simulations and semi-analytic models
  - Observations at high redshift (CMB) and low redshift (SDSS)
  - Linear density perturbations
- Initial conditions for cosmological simulations
  - Zel'dovich Approximation
  - Putting particles in a simulation box
  - Zoom simulations
- Initial conditions for disc galaxy simulations
  - Properties of disc galaxies
  - Creating a particle representation of a disc galaxy

Observing large scale structure

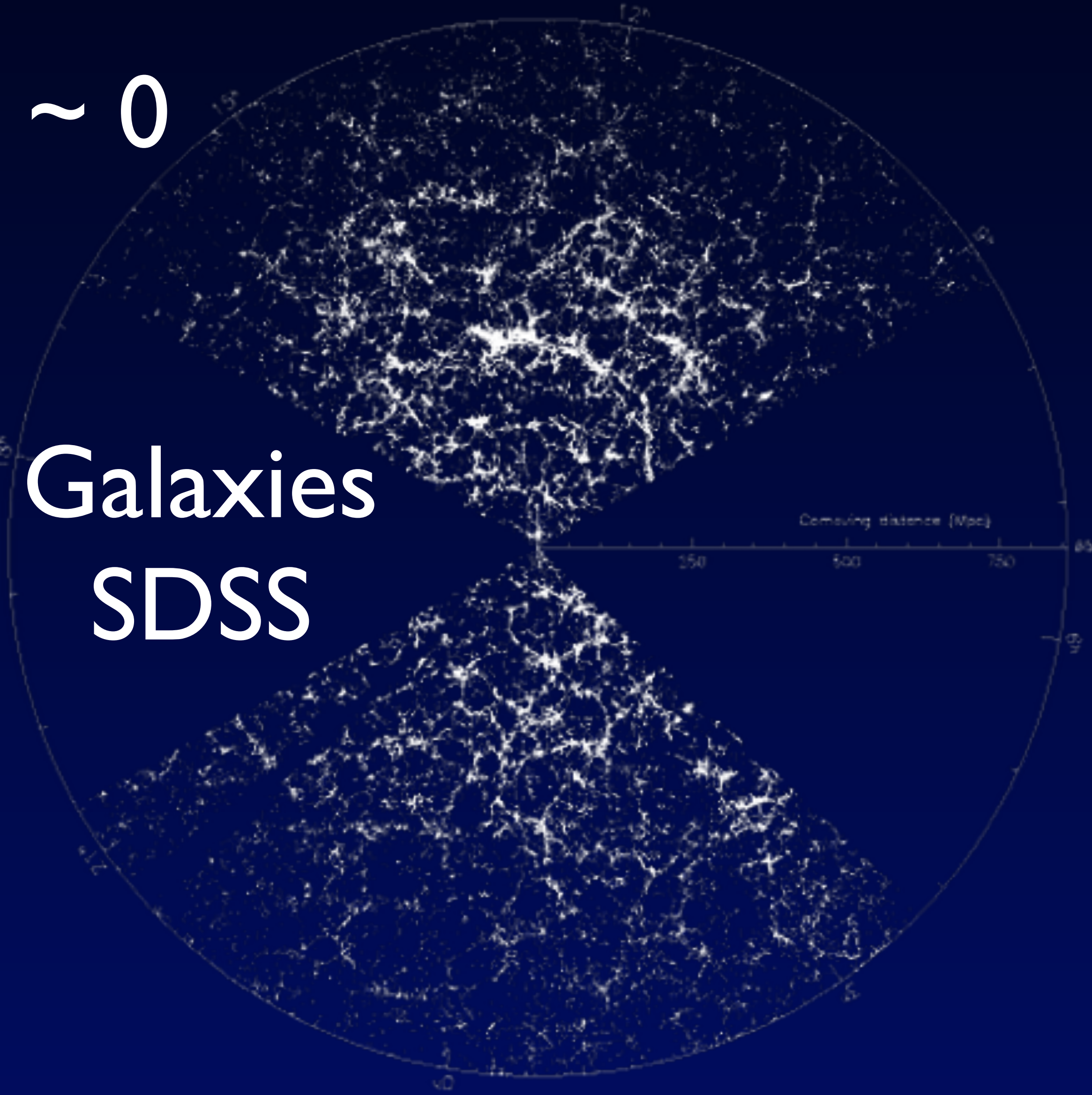
- Cosmic structure can be observed at very high redshift (z>1000): CMB
  Very smooth, only small perturbations (10^{-5})
- At low redshift: galaxies are very clustered forming a ‘cosmic web’.

z > 1000



CMB - Planck

z ~ 0

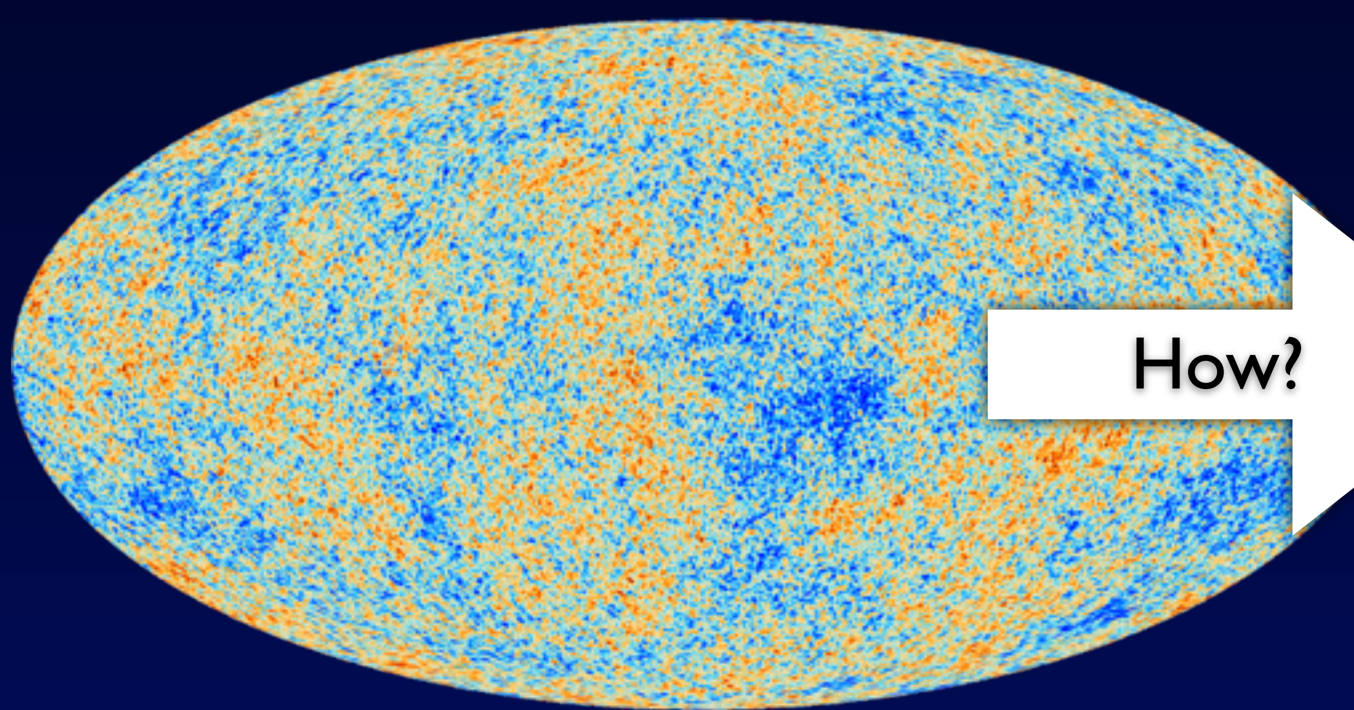


Galaxies SDSS

Observing large scale structure

- Things to keep in mind:
  - Structure formation process is dominated by gravity
  - Galaxies are only tracers of cosmic structure (<3% of all mass)
  - Galaxy formation depends on 'baryonic physics'

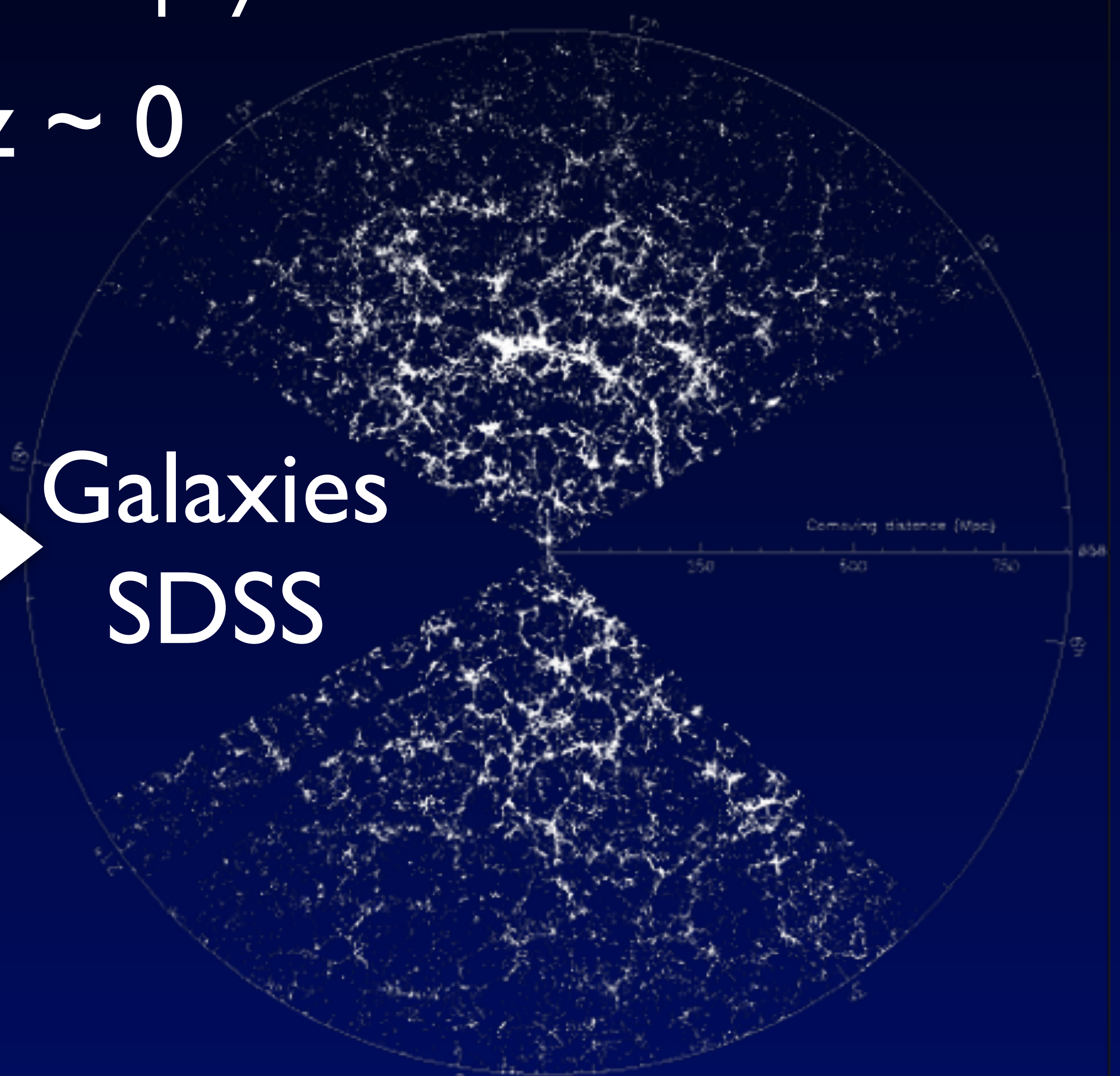
z > 1000



How?

z ~ 0

Galaxies
SDSS



CMB - Planck

What about linear perturbation theory?

- How far can we push it?
- A quick recap of cosmological perturbation theory:

$\vec{r} = a\vec{x}$ — $\vec{x}$ comoving position

$\vec{v} = \dot{\vec{r}} = \vec{u} + \frac{\dot{a}}{a}\vec{r}$ with $\vec{u} = a\dot{\vec{x}}$ — $\vec{u}$ peculiar velocity

momentum conservation:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\,\vec{\nabla}_{phys})\vec{v} = -\vec{\nabla}_{phys}\phi$$

comoving (1st order):

$$\frac{\partial \vec{u}}{\partial t} + \frac{\dot{a}}{a}\vec{u} = -\frac{1}{a}\vec{\nabla}\phi$$

continuity equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}_{phys}(\rho\vec{v}) = 0$$

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}\vec{u} = 0$$

with

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

Poisson equation:

$$\vec{\nabla}^2_{phys}\phi = 4\pi G\rho$$

$$\vec{\nabla}^2\phi = 4\pi G a^2 \bar{\rho}\delta$$

Linear growth of structures

- Combining this we get the evolution of the density contrast δ

$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G \frac{\bar{\rho}_c}{a^3}\delta$ solved by growth function $D(a)$: $\delta(a) = \delta_0 D(a)$

- For a matter dominated universe we have $D(a) \sim a$

- Can be decomposed into waves: $\delta(\vec{x}) = \int \hat{\delta}(\vec{k}) e^{-i\vec{k}\vec{x}} d^3k$

  Power spectrum is $P(k) = \langle |\hat{\delta}(k)|^2 \rangle$ (where $\langle\rangle$: ensemble average)

  and it grows like $P(k) = P_0 D^2(a)$

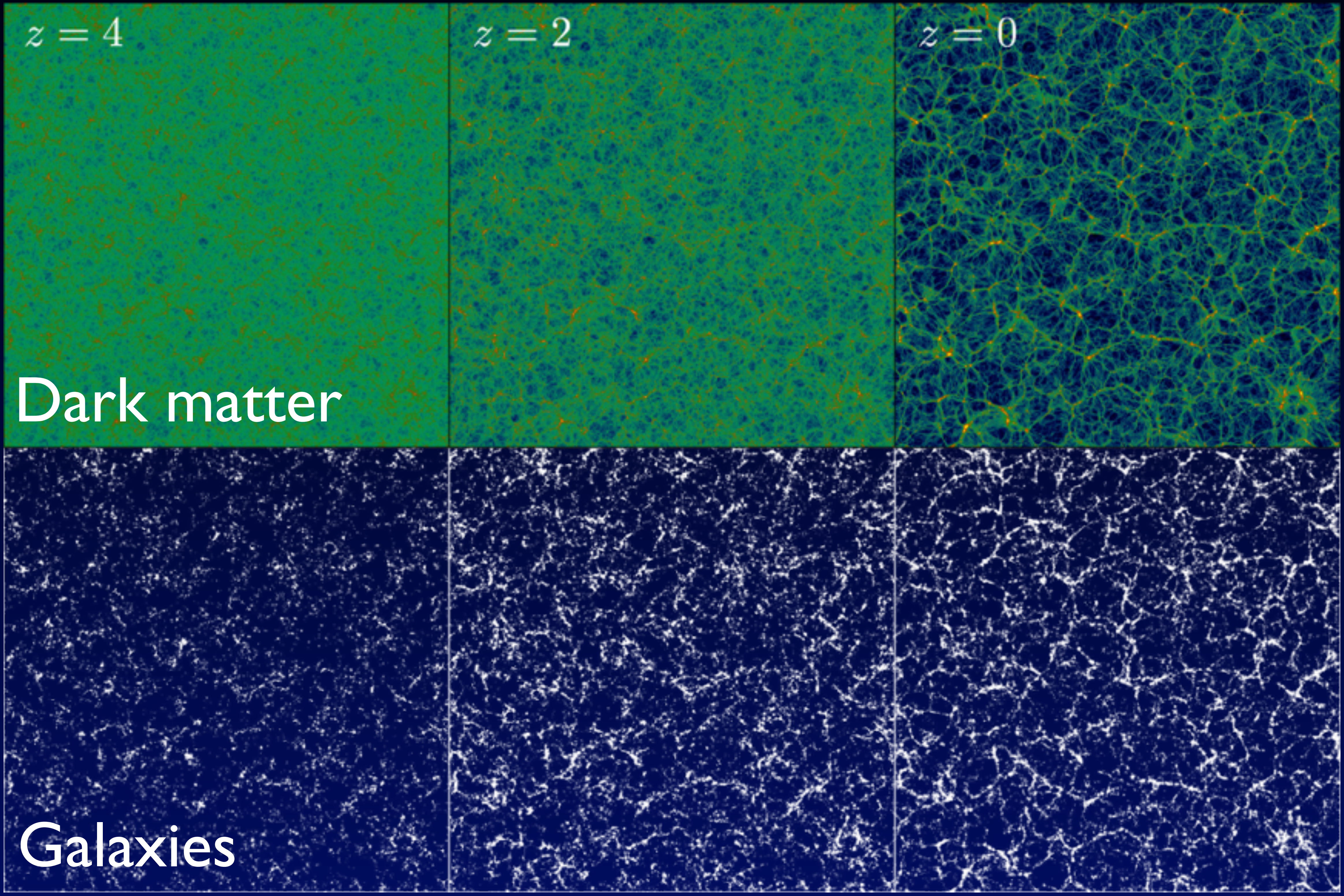
- Formalism works as long as $\delta \ll 1$

- Breaks down when $\delta \sim 1$ (negative densities)

- Either go to higher order (still no 'baryonic physics') or use simulations

From high to low redshift

- Cosmological model + initial conditions + simulation code = galaxies



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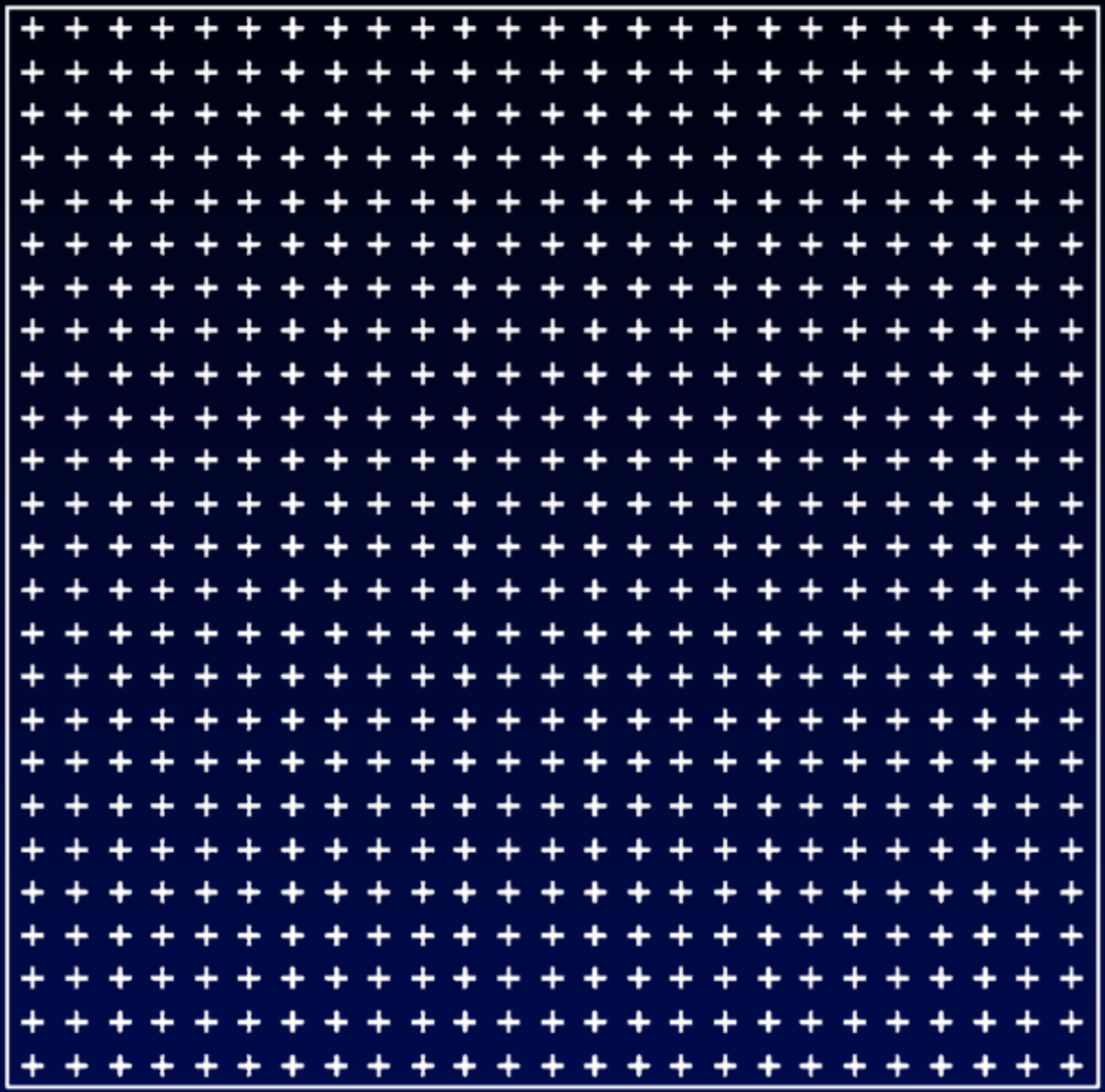
Cosmological Principle

- On large scales, the properties of the Universe are the same to all observers
- Homogeneity: Universe looks the same at every location
- Isotropy: Universe looks the same in every direction
- Discretize density field into particles



homogeneous & isotropic

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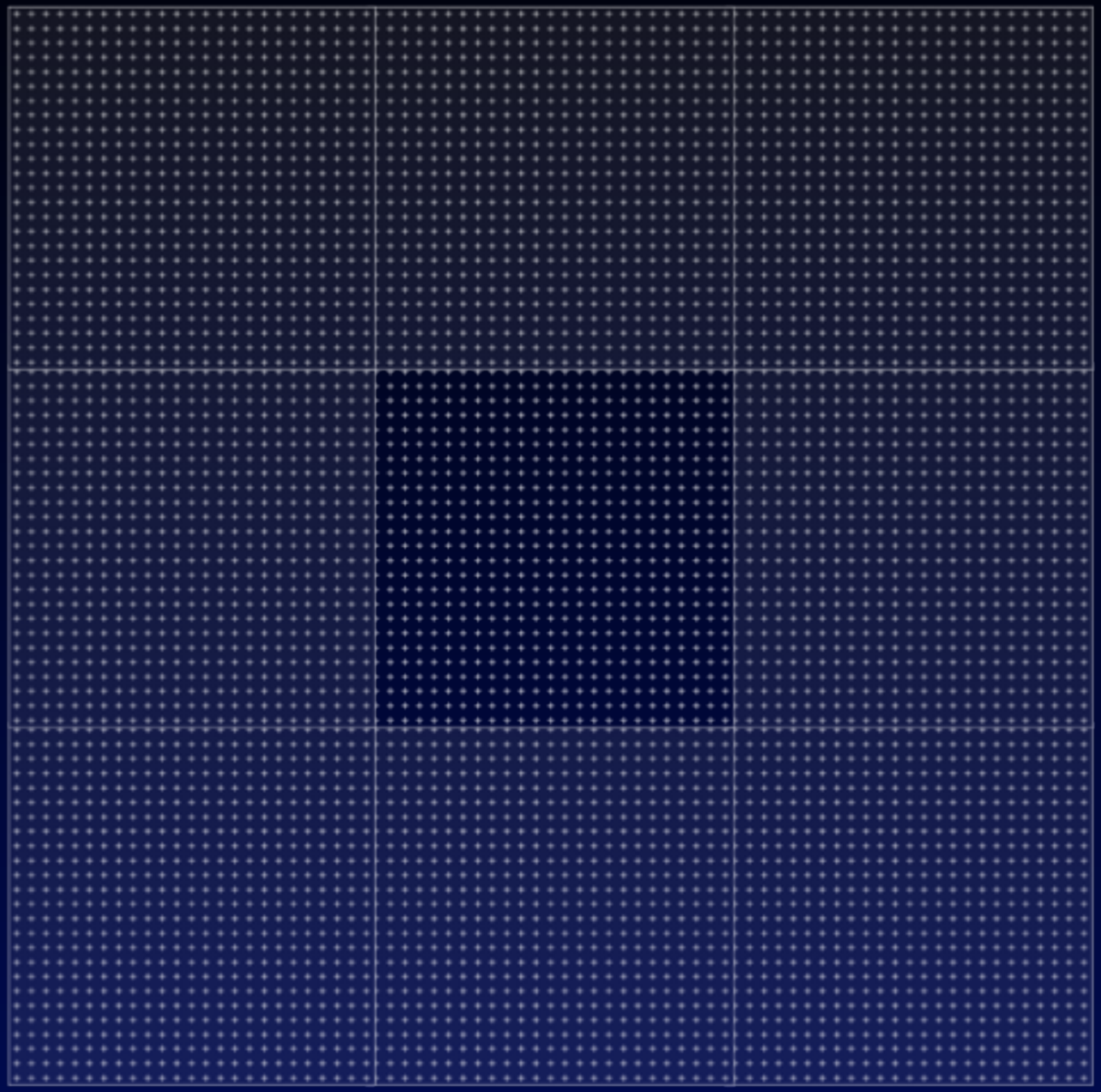
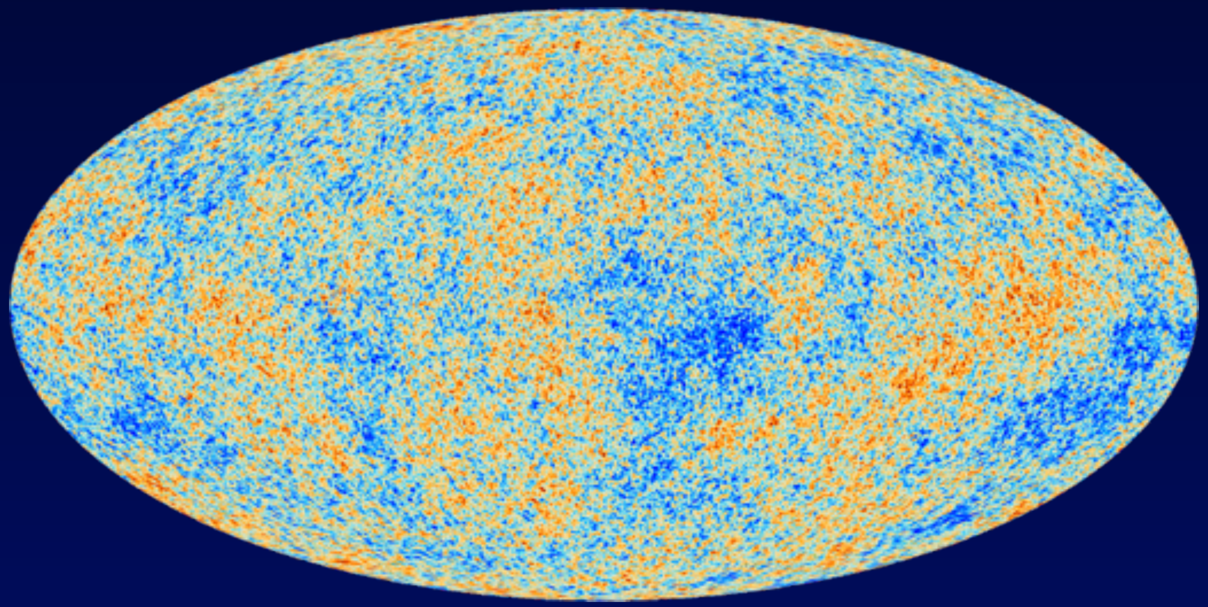
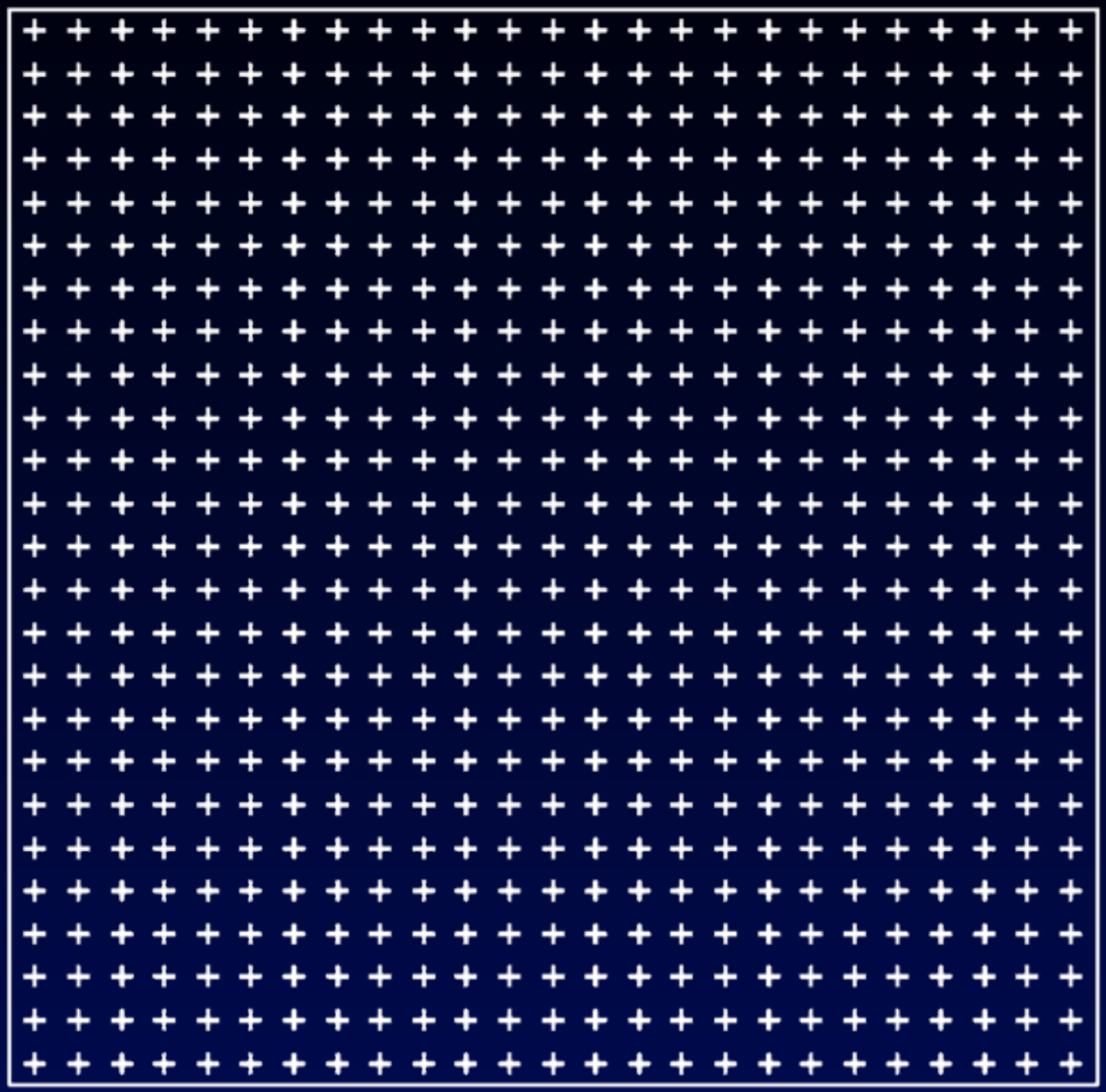


infinite
(periodic boundary conditions)

Perturbations

- Universe shows small density fluctuations already at high z (see CMB)
- Convert CMB fluctuations to density perturbations





homogeneous & isotropic

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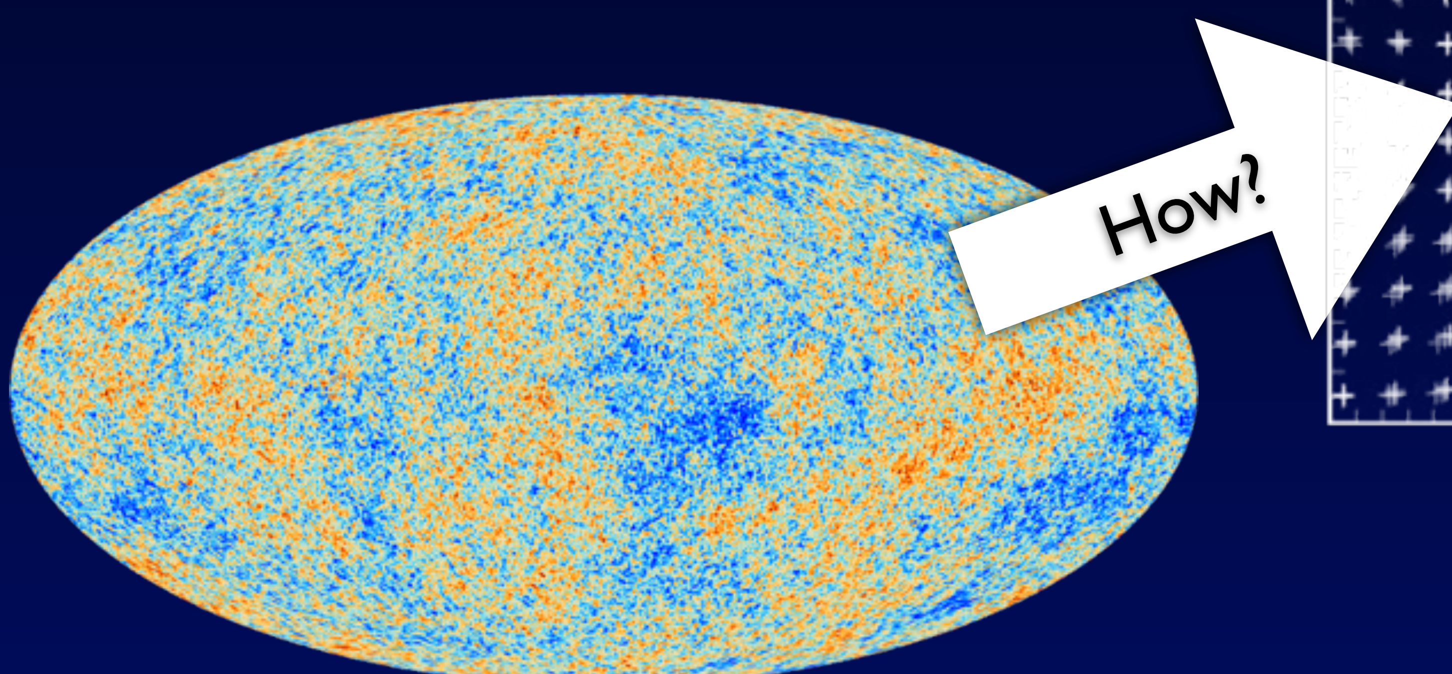
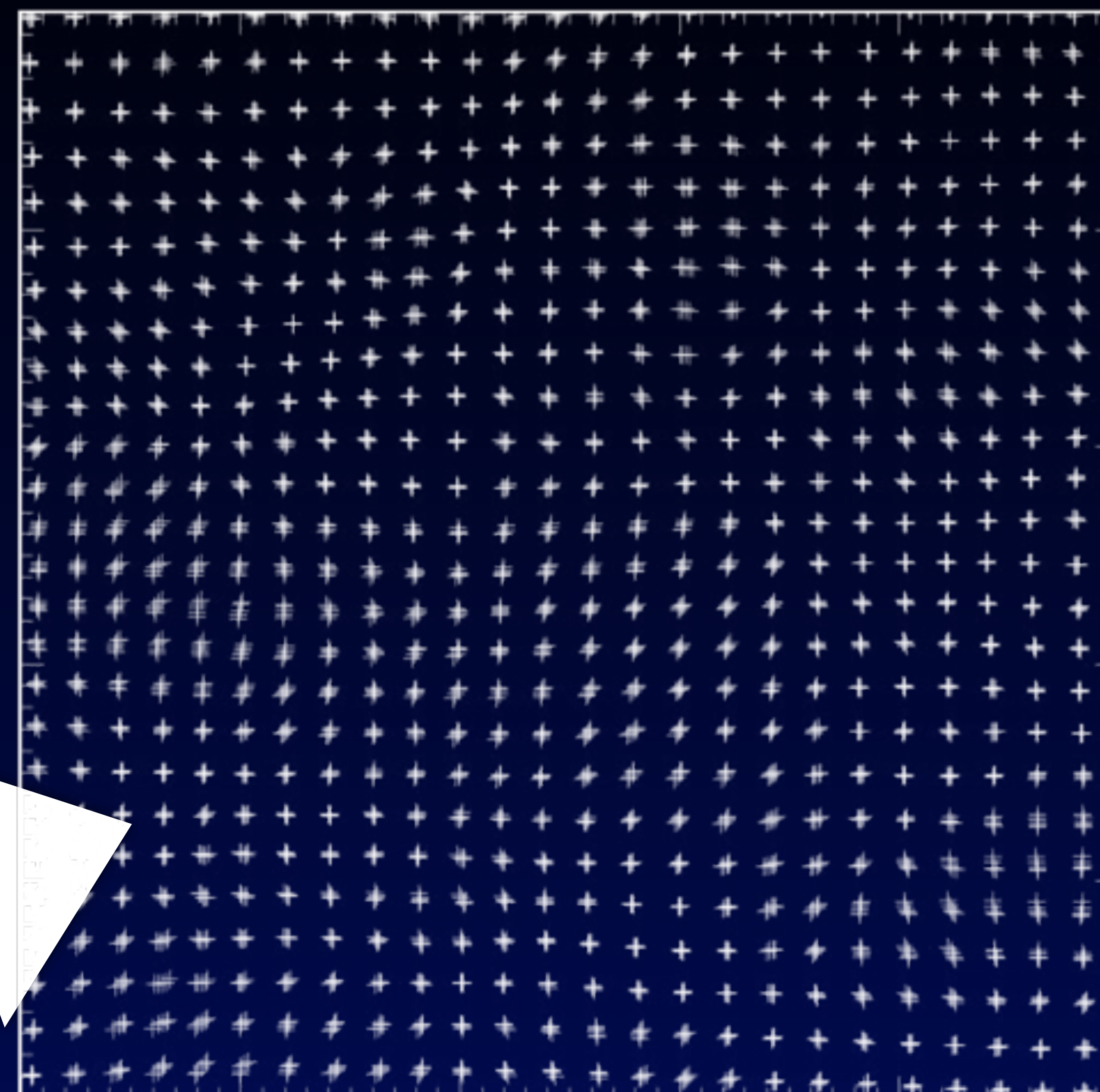
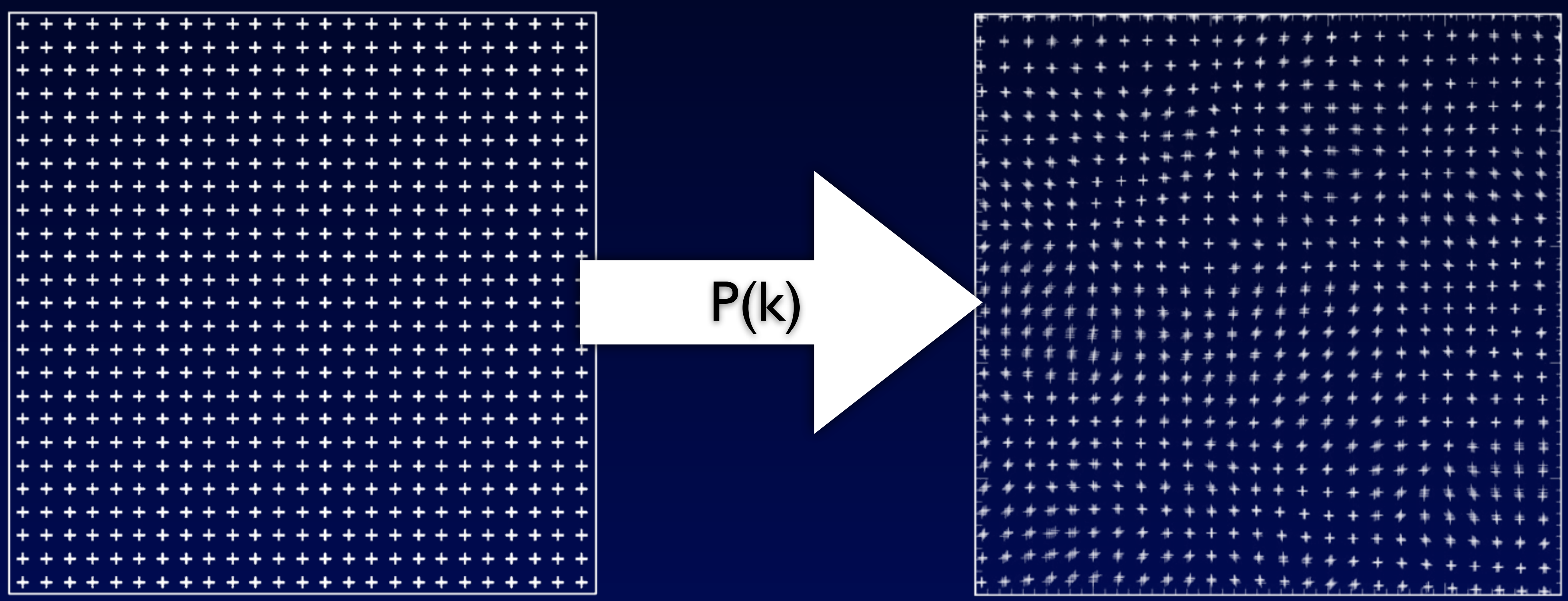




perturbed density field

The power spectrum

- Use the power spectrum to describe the density fluctuations
- From inflation: $P_i(k) = Ak^n$
- Temporal evolution (in the linear regime): $P(k,t) = P_0(k) D^2(t)$



- How to impose a spectrum of fluctuations on a particle distribution?

The Zel'dovich Approximation

- Substitute $\delta(a) = \delta_0 D(a)$ into Poisson equation (with $\bar{\rho} = \bar{\rho}_0 / a^3$):

$$\vec{\nabla}^2 \phi = 4\pi G a^2 \bar{\rho} \delta = 4\pi G a^2 \frac{\bar{\rho_0}}{a^3} D(a) \delta_0 = \frac{D(a)}{a} \vec{\nabla}^2 \phi_0$$

- So the potential also grows as $\phi(a) = \frac{D(a)}{a} \phi_0$
- In a matter dominated universe: $D(a) \sim a \rightarrow \phi(a) = \phi$
- Integrate continuity equation $\frac{\partial \vec{u}}{\partial t} + \frac{\dot{a}}{a} \vec{u} = -\frac{1}{a} \vec{\nabla} \phi$ or $\frac{\partial (a\vec{u})}{\partial t} = -\vec{\nabla} \phi$

$$\vec{u} = -\frac{\vec{\nabla} \phi_0}{a} \int \frac{D}{a} \mathrm{d}t$$

- Growth function satisfies the growth equation:

$$\ddot{D} + 2\frac{\dot{a}}{a} \dot{D} = 4\pi G \frac{\bar{\rho_c}}{a^3} D$$ or rewritten: $$\frac{1}{a^2} \frac{\partial (a^2 \dot{D})}{\partial t} = 4\pi G \frac{\bar{\rho_c}}{a^3} D$$

The Zel'dovich Approximation

- Integrate this yields $$\frac{a^2\dot{D}}{4\pi G\bar{\rho}_c} = \int \frac{D}{a}dt$$

- Use this with the integrated continuity equation to get
$$\dot{\vec{x}} = \frac{\vec{u}}{a} = -\frac{\vec{\nabla}\phi_0}{a^2}\frac{a^2\dot{D}}{4\pi G\bar{\rho}_c} = -\frac{\dot{D}\vec{\nabla}\phi_0}{4\pi G\bar{\rho}_c}$$

- Integrate this to get the Zel'dovich approximation:
$$\vec{x} - \vec{x}_0 = -\frac{D\vec{\nabla}\phi_0}{4\pi G\bar{\rho}_c} = -\frac{a\vec{\nabla}\phi(a)}{4\pi G\bar{\rho}_c}$$ with the unperturbed position $\vec{x}_0$

- This formulation can be used to extrapolate the evolution of structures into the regime where displacements are no longer small

Displacing particles on a grid

- How apply the Zel'dovich approximation to a grid?
- Define $\vec{\psi} = \vec{x} - \vec{x}_0$ and its Fourier transform $\vec{\psi}_{\vec{k}} = -\frac{a(i\vec{k})\phi_{\vec{k}}}{4\pi G\bar{\rho}_c}$
- With Poisson's equation $-k^2\phi_{\vec{k}} = \frac{4\pi G\bar{\rho}_c\delta_{\vec{k}}}{a}$ we get $\vec{\psi}_{\vec{k}} = i\vec{k}\frac{\delta_{\vec{k}}}{k^2}$
- Use the power spectrum $P(k) = \langle|\hat{\delta}(k)|^2\rangle$: $\delta_{\vec{k}} = \sqrt{P(k)}R_{\vec{k}}e^{i\phi_{\vec{k}}}$

  with $R_{\vec{k}}e^{i\phi_{\vec{k}}} = R_1 + iR_2$ where R_1 and R_2 are drawn from Gaussian with standard deviation of 1 (P is only ensemble average)
- Initial velocities are given by $\dot{\vec{x}} = \frac{\dot{D}}{D}\vec{\psi} \rightarrow \vec{v}_{\vec{k}} = a\frac{\dot{D}}{D}\vec{\psi}_{\vec{k}}$

Limitations of this method

- There are a number of parameters that have to be chosen:
  - Box size B
  - Number of particles N
  - Starting redshift z_i

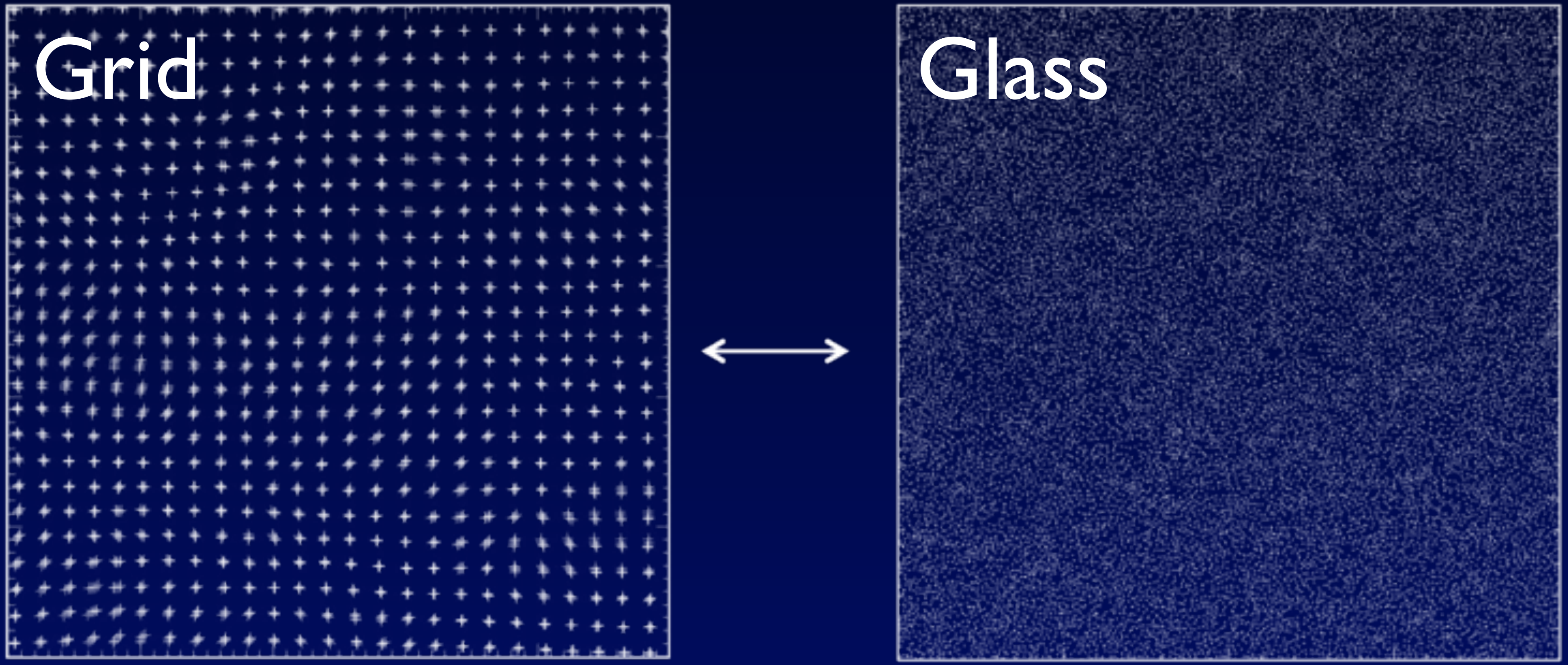
- In practice there are several constraints on these:
  - Minimal modes that are included: $2B/\sqrt[3]{N}$
  - Largest mode has to stay linear: $B \geq 2\pi/k_{nl} \sim 20 Mpc$
  - Starting redshift (typically $40 < z_i < 80$):

    Too late: shell crossing not taken into account

    Too early: numerical noise is integrated

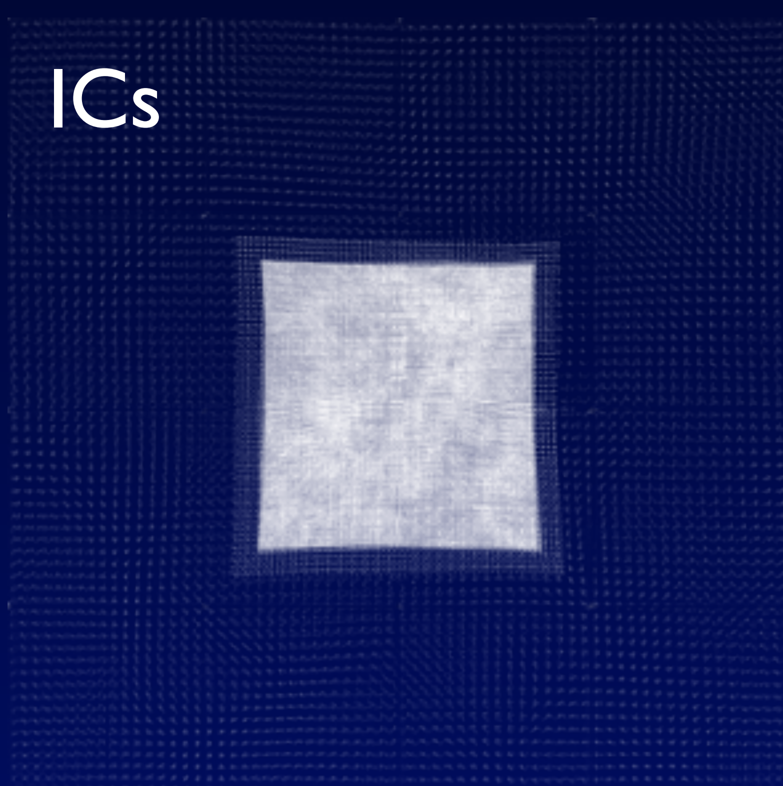
Grid vs. Glass ICs

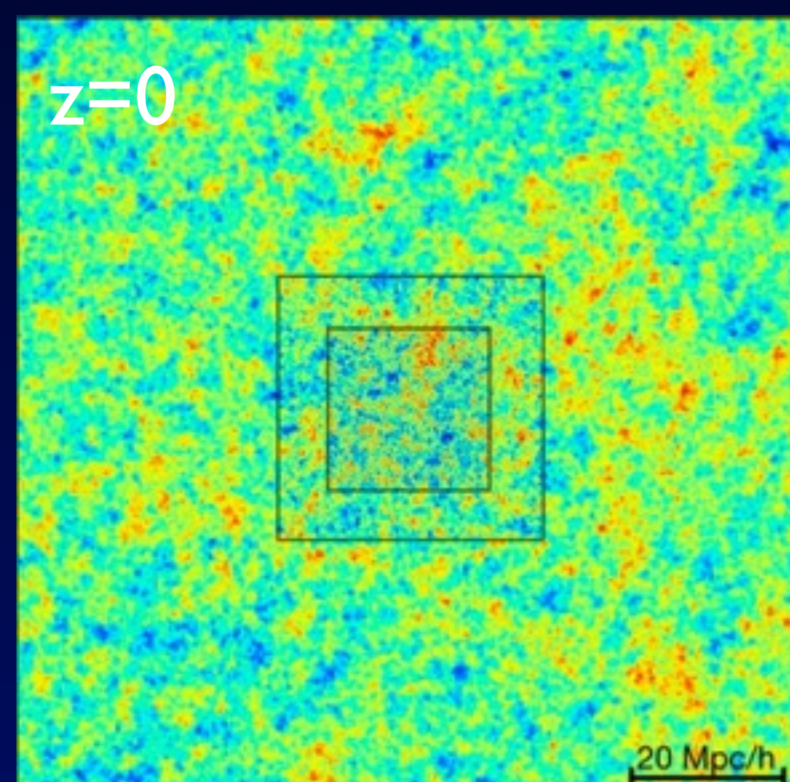
- Alternative to Grid ICs are Glass ICs
  - Start with random positions for particles in the box
  - Evolve box forward in time under gravity BUT with reversed sign
  - Use resulting particle positions for Zel'dovich approximation
- Just cosmetics?



Zoom simulations

- Increasing the resolution of the box becomes very expensive
- Alternative: just increase the resolution in the area of interest:
  1) Run low resolution simulation and identify interesting object(s)
  2) trace back particles of that object to initial positions in ICs
  3) resample this area with more particles and rerun the simulation
- Disadvantage: only one system simulated - no statistics





N-GenIC

```
Nmesh          128      % This is the size of the FFT grid

Nsample        128      % sets the maximum k that the code uses,
                        % Ntot =  Nsample^3, where Ntot is the

Box            150000.0  % Periodic box size of simulation

FileBase       ics          % Base-filename of output files
OutputDir      ./ICs/       % Directory for output

GlassFile      glass.dat    % File with unperturbed glass or grid

TileFac        8        % Number of times the glass file is tiled

Omega              0.3     % Total matter density  (at z=0)
OmegaLambda        0.7     % Cosmological constant (at z=0)
OmegaBaryon        0.0     % Baryon density        (at z=0)
HubbleParam        0.7     % Hubble paramater
Redshift           63      % Starting redshift
Sigma8             0.9     % power spectrum normalization
```

N-GenIC

```
SphereMode          1     % if "1" only modes with |k| < k_Nyquist are used
WhichSpectrum       0     % "1" selects Eisenstein & Hu spectrum,
                          % "2" selects a tabulated power spectrum
                          % otherwise, Efstathiou parametrization is used

FileWithInputSpectrum   spectrum.txt      % tabulated input spectrum
InputSpectrum_UnitLength_in_cm  3.085678e24        % defines length unit
ReNormalizeInputSpectrum   1 % if zero, spectrum assumed to be normalized

ShapeGamma          0.21          % only needed for Efstathiou power spectrum
PrimordialIndex     1.0           % may be used to tilt the primordial index

Seed                123456        %  seed for IC-generator

NumFilesWrittenInParallel 2       % limits the number of files that are written

UnitLength_in_cm          3.085678e21    % defines length unit (in cm/h)
UnitMass_in_g           1.989e43     % defines mass unit (in g/cm)
UnitVelocity_in_cm_per_s  1e5         % defines velocity unit (in cm/sec)
```

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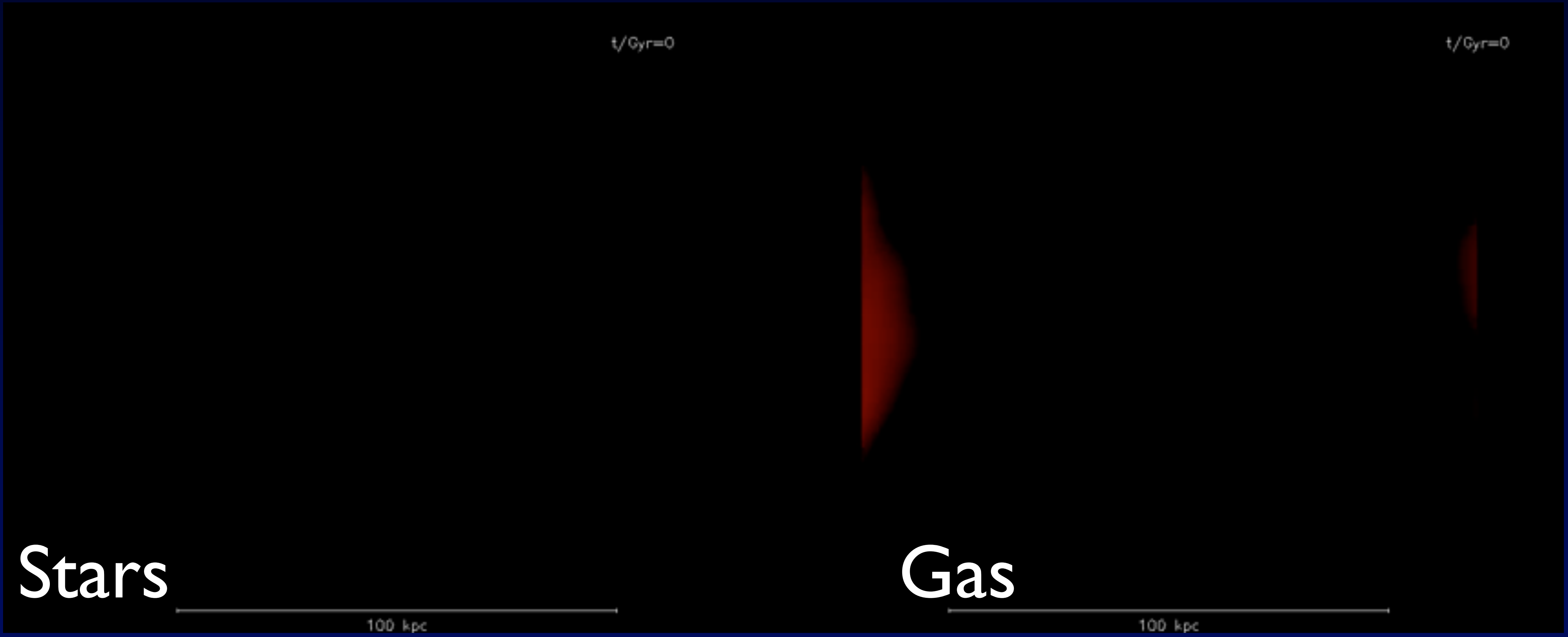
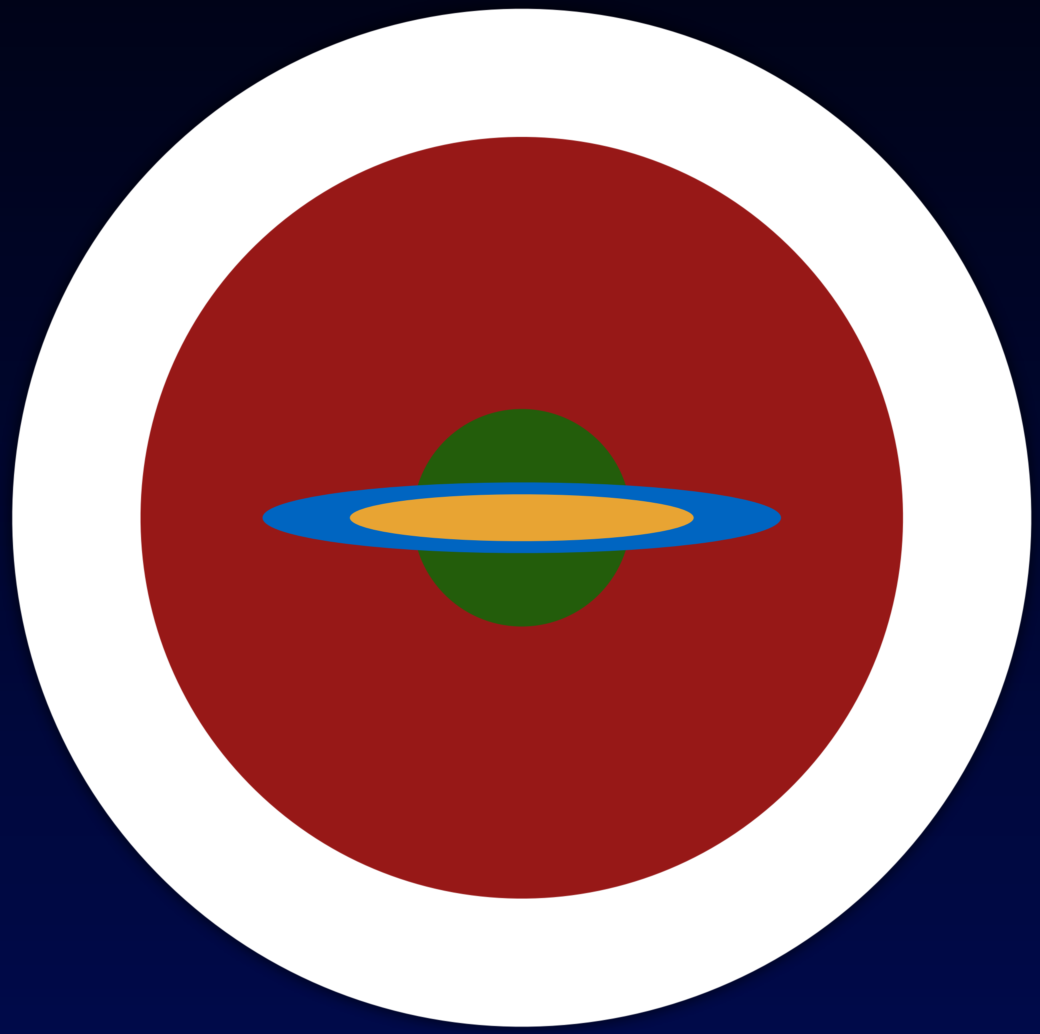
Why simulate isolated disc systems?

- Set up individual system(s) and simulate/study those
- Advantage: much higher resolution (more particles per system)
  ‘Nice’ discs can be studied (often hard in cosmological runs)
- Disadvantage: Cosmological background is neglected (no infall, etc)



Disc Galaxies

- What does a disc galaxy system consist of?
- Stellar disc
- Gaseous disc
- Stellar bulge
- Hot gaseous halo
- Dark matter halo



The dark matter halo

- Size of the dark matter halo is given by the virial radius (with mean overdensity $200\rho_{\rm crit}$) containing the virial mass: $M_{200} = 200\rho_{crit}\frac{4\pi}{3}R_{200}^3$
- The virial velocity is $v_{200}^2 = \frac{GM_{200}}{R_{200}}$ and we have:

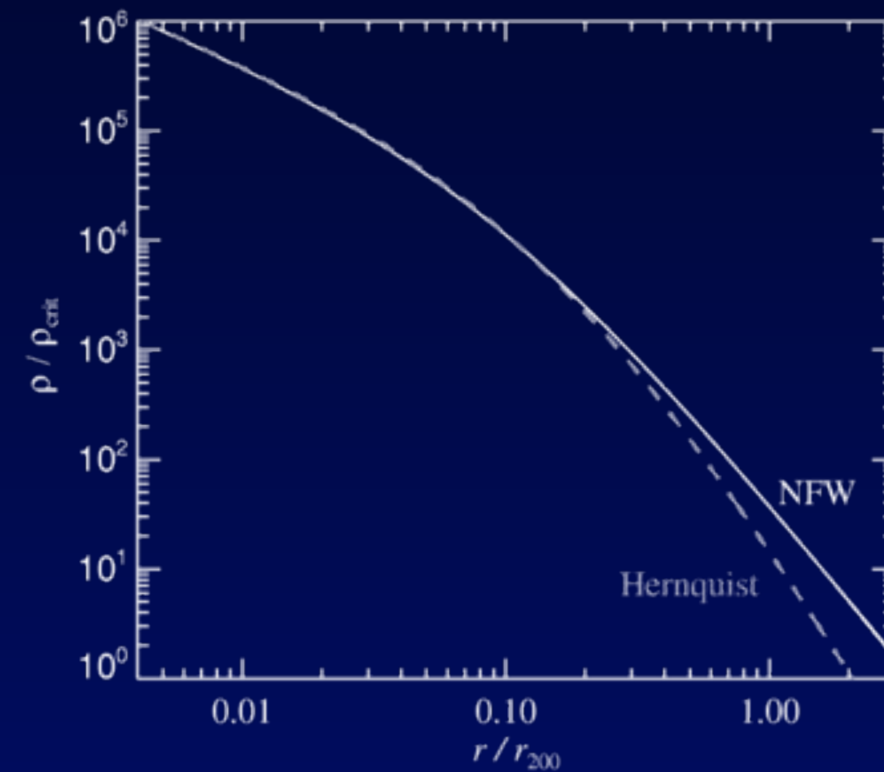
$$M_{200} = \frac{v_{200}^3}{10GH(z)} \quad \text{and} \quad R_{200} = \frac{v_{200}}{10H(z)}$$

- The density profile is the NFW profile:

$$\rho(r) = \frac{\delta_c}{(r/r_s)(1+r/r_s)^2}$$

- Often the Hernquist profile is used:

$$\rho(r) = \frac{M_{dm}}{2\pi}\frac{a}{r(r+a)^3}$$



Springel et al. (2005)

The stellar components

- Disc has exponential radial profile: $$\Sigma(R) = \frac{M_d}{2\pi R_d^2} \exp\left(-\frac{R}{R_d}\right)$$

  Disc scale length R_d is fixed by assuming the specific angular momentum of disc and dark matter halo are equal

- Vertical profile of the disc given by: $$\rho(z) = \frac{M_d}{2z_0} \operatorname{sech}^2\left(\frac{z}{2z_0}\right)$$

- For the bulge the spherical Hernquist profile is assumed:

$$\rho_b(r) = \frac{M_b}{2\pi} \frac{r_b}{r(r_b + r)^3}$$

- The bulge is typically assumed to be non-rotating

The gaseous components

- Like the stellar disc, the gaseous disc has exponential radial profile
- The vertical profile of the gas disc cannot be chosen freely because it depends on the temperature of the gas
- For a given surface density, the vertical structure of the gas disc arises as a result of self-gravity and pressure: $$-\frac{1}{\rho_g}\frac{\partial P}{\partial z} = \frac{\partial \Phi}{\partial z}$$
- Hot gaseous halo follows a beta-profile: $$\rho(r) = \rho_0 \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-\frac{3}{2}\beta}$$
- The gas temperature is determined using hydrostatic equilibrium:

$$T(r) = \frac{\mu m_p}{k_B}\frac{1}{\rho_{\rm hot}(r)}\int_r^\infty \rho_{\rm hot}(r)\frac{GM(r)}{r^2}\mathrm{d}r$$

Creating a particle representation of a galaxy

- We know the density profiles of each component.
  How can we put the particles such that these are reproduced?
- First step compute mass profile: $M(r) = \int_0^r \rho(r')d^3r'$
- Invert M(r), possibly analytically, otherwise numerically
- Draw random number *q* from uniform distribution between 0 and 1
- Radius is given by: $r = r(q\ M_{tot})$
- For spherical distribution:
  (with random θ and ϕ)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = r \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

Velocity structure

- To get the velocities: solve the collisionless Boltzmann equation
- Alternative: assume that velocity distribution can be approximated by a multivariate Gaussian ➙ only 1st and 2nd moments needed
- For static + axisymmetric system: E and L_z are conserved along orbits

  If distribution function depends only on E and L_z the moments are:

$$\bar{v_R} = \bar{v_z} = \bar{v_R v_z} = \bar{v_z v_\phi} = \bar{v_R v_\phi} = 0$$

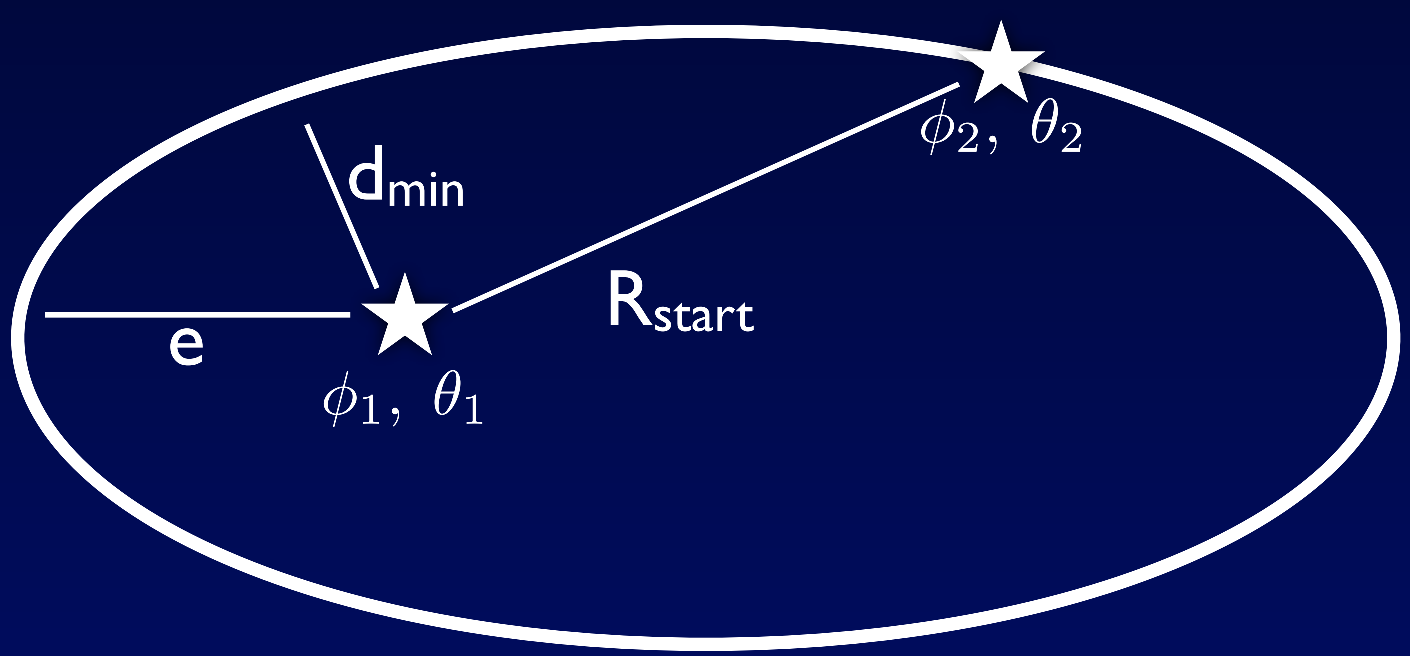
$$\bar{v_R^2} = \bar{v_z^2} = \frac{1}{\rho} \int_z^\infty \mathrm{d}z' \rho(R, z') \frac{\partial \Phi}{\partial z'}(R, z')$$

$$\bar{v_\phi^2} = \bar{v_R^2} + \frac{R}{\rho} \frac{\partial}{\partial R}(\rho \bar{v_R^2}) + R \frac{\partial \Phi}{\partial R}$$

- Must be solved numerically

The orbit of galaxy mergers

- For simulating galaxy mergers: put 2 systems on orbit.
- Parameters that specify the merger orbit:
  - Initial separation R_{start} (typically around the virial radius R_{200})
  - Pericentric distance d_{min} (if galaxies were point masses)
  - Orbital eccentricity e (usually parabolic orbits, i.e. e=1)
  - Orientation of discs with respect to orbital plane: $\phi_1,\ \theta_1,\ \phi_2,\ \theta_2$



Final notes

- Text Books:
  - Cosmology: Galaxy Formation and Evolution (Mo, vdBosch, White)
  - Galactic Structure: Galactic Dynamics (Binney, Tremaine)

- Papers:
  - Bertschinger (2001), ApJS, 137, 1
  - Springel & White (1999), MNRAS, 307, 162
  - Springel et al. (2005), MNRAS, 62, 79

- Gadget and N-GenIC website:

  http://www.mpa-garching.mpg.de/gadget/

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