Structure and Evolution of Stars

Lecture 23: (i) Binary stars and mass transfer, (ii) Shell Burning Instability

- Binary stars and mass transfer – SN Ia and compact objects

- Thermal Stability for Stars in Hydrostatic Equilibrium

- Thermal Stability for Nuclear Burning in Thin Shells

- Double Shell-Burning and Thermal Pulses in AGN Stars

- Dependence of mass-loss on stellar luminosity

Binary Star System

- Binary system, masses M_1 & M_2, separation a, in circular orbit in X-Y plane
- Angular rotation velocity Ω
- Potential at (x,y,z) = Φ(x,y,z)

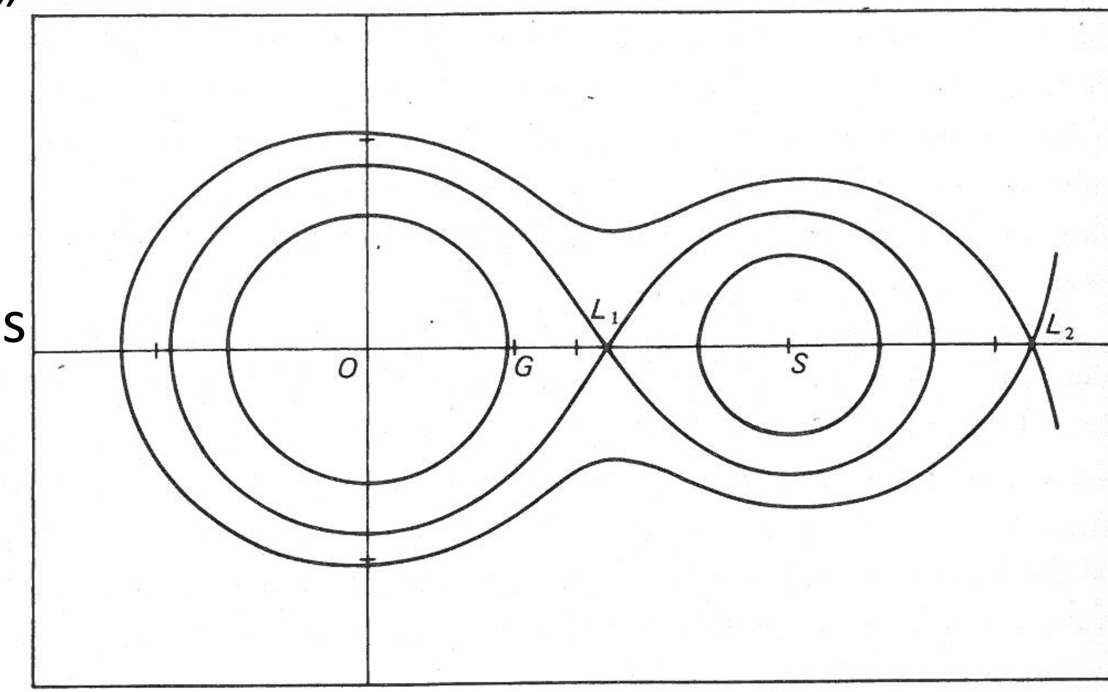
- Centrifugal potential from rotation
- Φ = Potential Star#1 + Potential Star#2
  + Centrifugal Potential

Star coordinates (0,0,0) and (a,0,0)

$$\Phi = -\frac{GM_1}{(x^2+y^2+z^2)^{1/2}} - \frac{GM_2}{((x-a)^2+y^2+z^2)^{1/2}} - \frac{\Omega^2\left((x-\mu a)^2+y^2\right)}{2}$$

• Φ= Effective gravitational potential
• Consider "equipotentials" in X-Y plane where Φ=constant
• Close to a star; star dominates & equipotentials ~circles
• Intermediate distances; two distinct equipotentials
• Large distances; rotation dominates & equipotentials ~circles

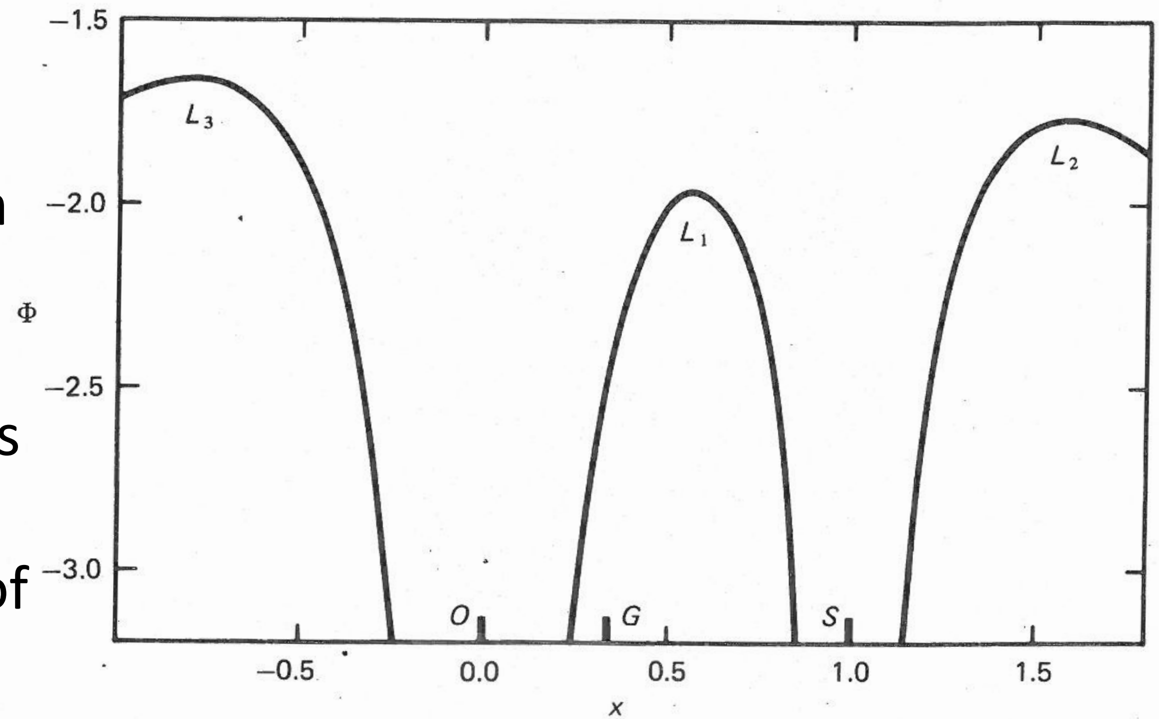
Fig. 1.2. Roche equipotentials. The centres of the stars are for star 1 at O (also the origin of the coordinates) and for star 2 at S. The centre of mass is at G. The mass ratio is $M_1/M_2 = 2$. The plane shown is in the orbital plane of the binary system. The inner Lagrangian point L_1 and the outer Lagrangian point L_2 are also marked.



• Topology of equipotentials changes with distance
• Equipotential where the two distinct surfaces touch; a distorted "Figure of 8"-shape – "Roche Lobes"

- Cross-section along line joining the binary stars
- Three Lagrangian points – equilibrium locations **BUT** unstable and small displacement moves particle into region dominated by one of the three components that make up the effective potential

Fig. 1.3. The value of the Roche potential Φ is shown as a function of distance along the line joining the two stars. Deep potential wells surround star 1 at O and star 2 at S, and between them the potential maximum occurs at L_1. At the edges additional maxima occur at L_2 and L_3 as the centrifugal potential dominates at large distances.



Figures on Slides 3,4,5 from Pringle and Wade, 1985 CUP: Interacting Binary Stars

• Three classes of binary star:

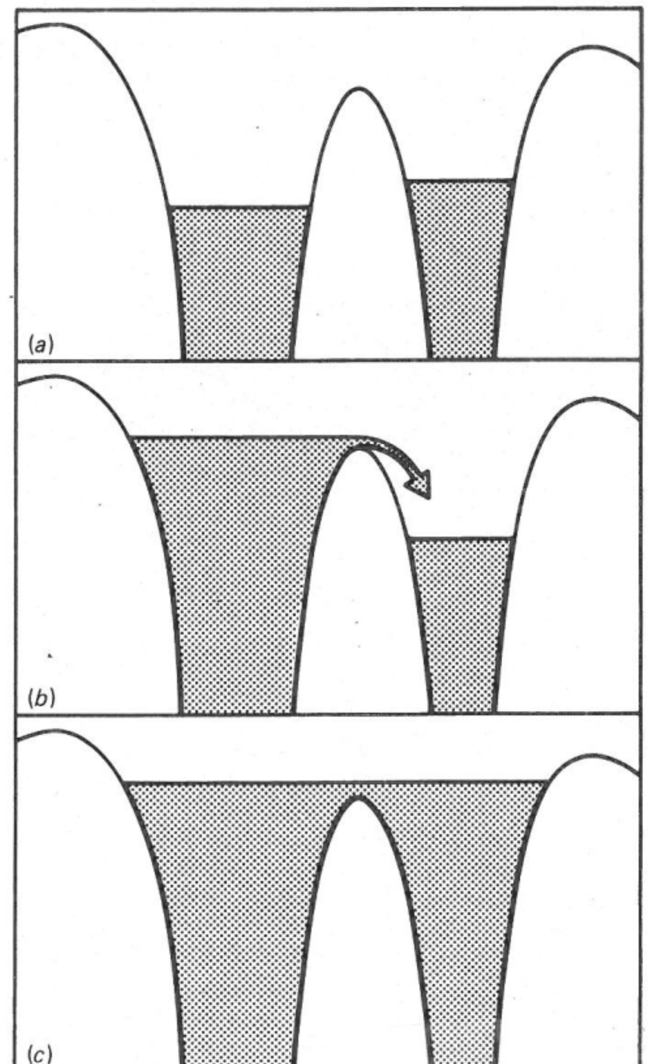
- Detached binary with no transfer of material from photospheres

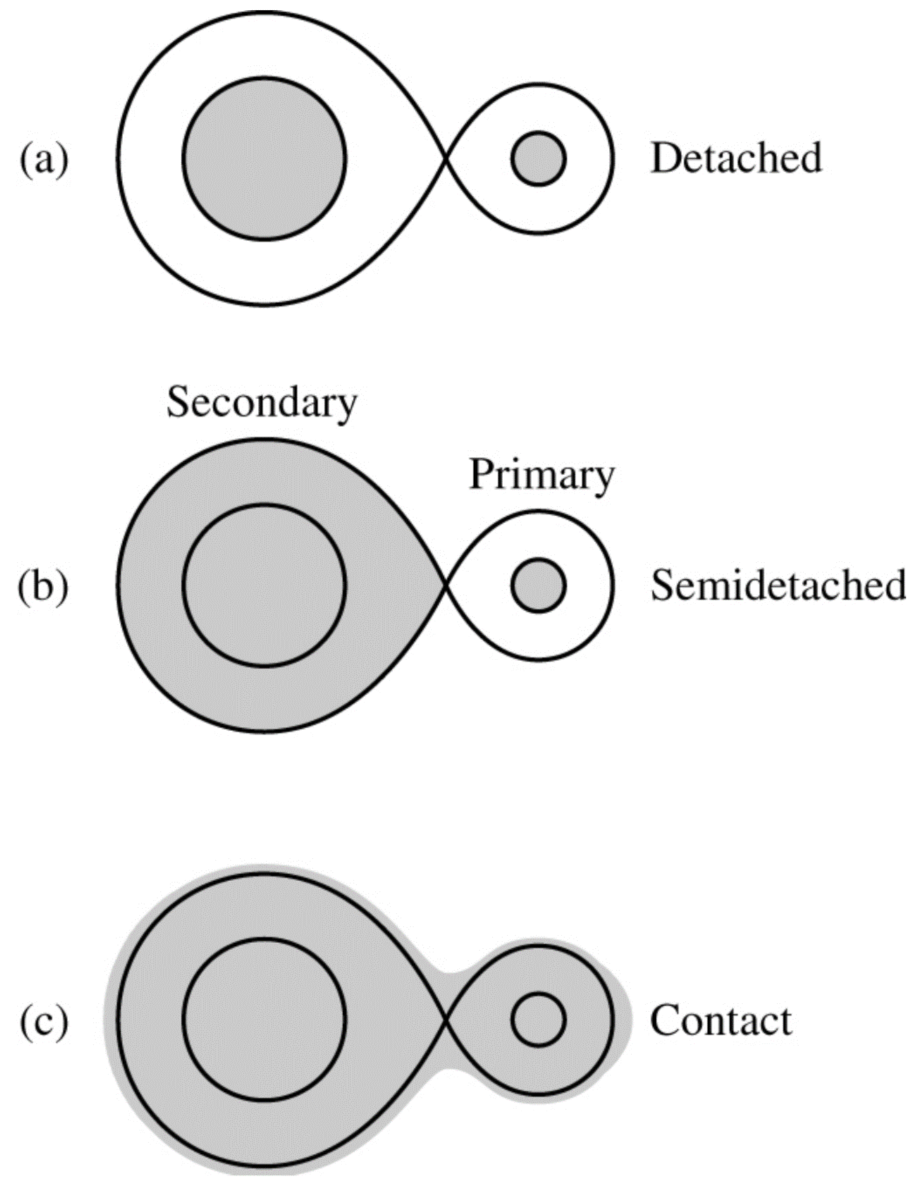
- Semi-detached binary where one star fills its Roche Lobe

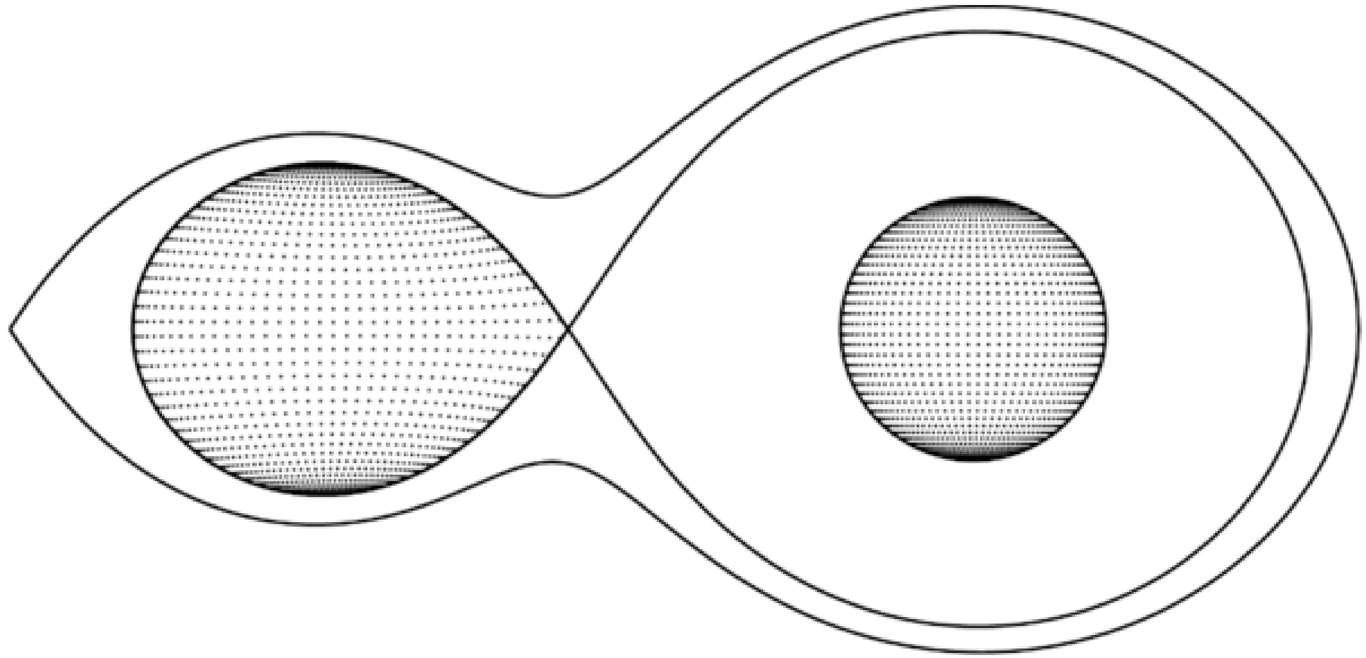
- Contact-binary where stars overfill Roche Lobes and exist within a common envelope

• Most binaries are "detached" but small separations or phases of stellar evolution with large R_{star} can result in mass transfer

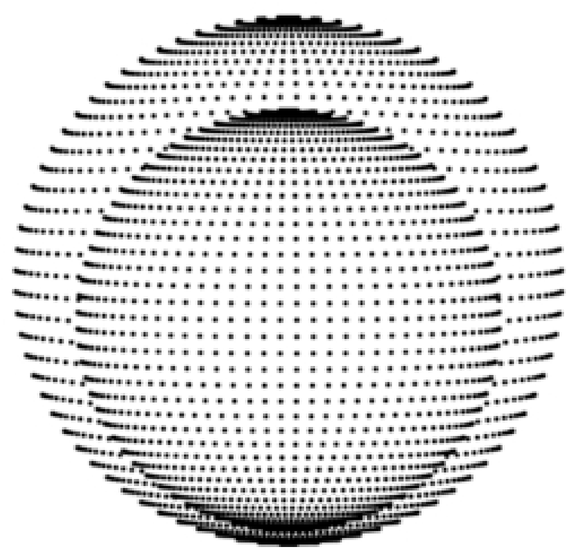
Fig. 1.4. The potential is shown as in Figure 1.3. (*a*) In a detached binary neither star is large enough that its surface potential approaches the value at L_1. (*b*) In a semi-detached binary one of the stars is large enough that its surface potential reaches the value at L_1. Then matter on the surface is able to flow through the L_1-point down the potential well onto the other star. (*c*) In a contact binary both stars have expanded beyond the L_1-point and have a common envelope. A contact binary cannot expand beyond the L_2-point and still rotate in a uniform manner.



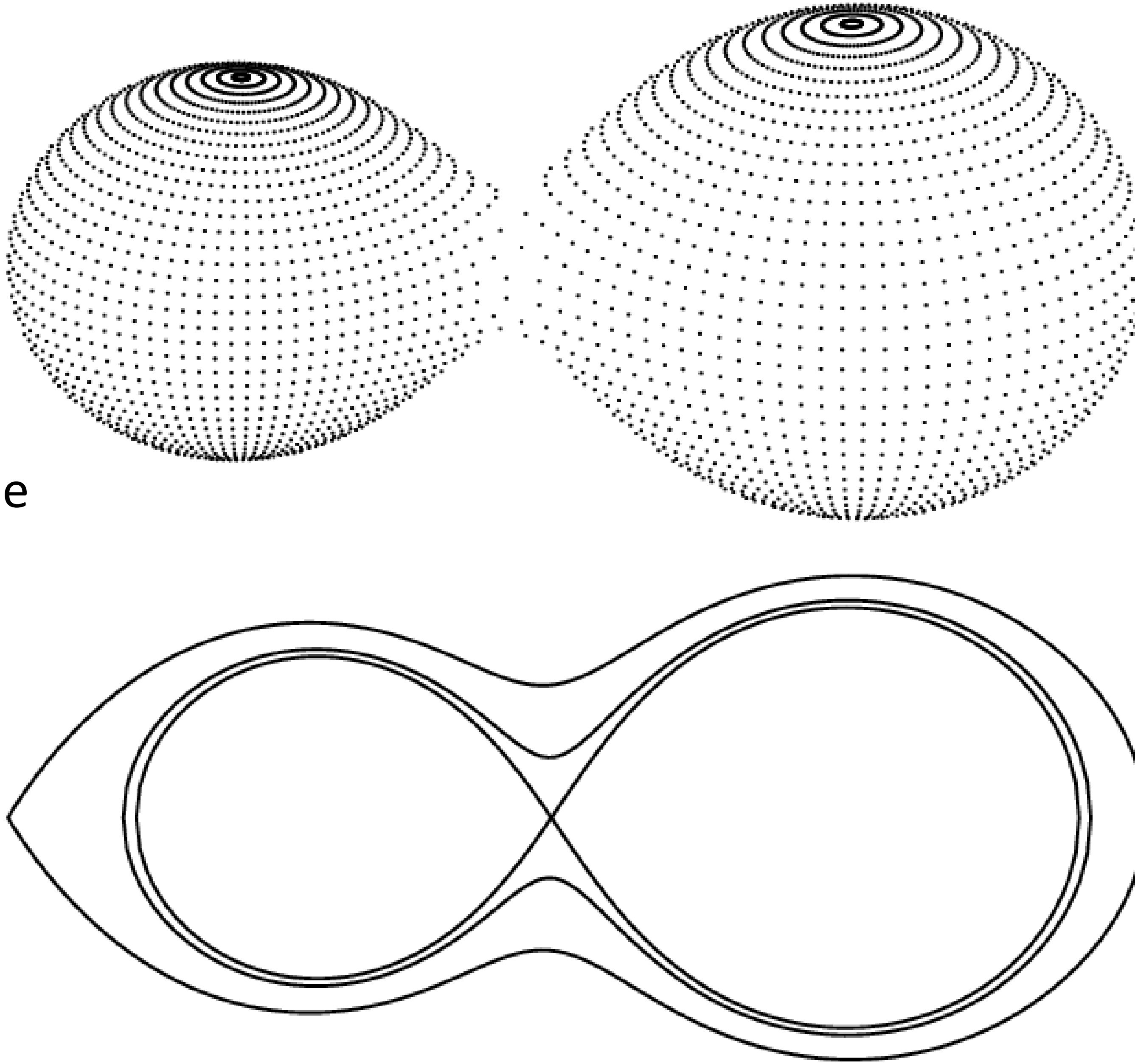

(a)
Detached
Secondary
Primary
(b)
Semidetached
(c)
Contact



3D view of semi-detached binary with one star filling Roche Lobe



3D view of common-envelope binary



Binary Star Mass Transfer: Angular Momentum

- Binary system
- Neglect angular

momentum of individual stars

Know $M_1 a_1 = M_2 a_2$; $a_1 + a_1 = a$; $M_{Tot} = M_1 + M_2$

Angular momentum $J = (M_1 a_1^2 + M_2 a_2^2)\Omega$

$$\Rightarrow J = \frac{M_1 M_2}{(M_1 + M_2)} a^2 \Omega$$

Suppose, angular momentum conserved $\dot{J} = 0$

And no mass loss $\dot{M}_{Tot} = 0$

Binary Star Mass Transfer: Angular Momentum

differentiate $\Rightarrow \dot{J} = \frac{\dot{M}_1 M_2 a^2 \Omega + M_1 \dot{M}_2 a^2 \Omega + 2 M_1 M_2 a \dot{a} \Omega + M_1 M_2 a^2 \dot{\Omega}}{(M_1 + M_2)} = 0$

Kepler's 3rd law: $P^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)} \quad \Rightarrow \quad \frac{\dot{P}}{P} = -\frac{\dot{\Omega}}{\Omega} = \frac{3\dot{a}}{2a}$

Substitute in
Expression for $\dot{J}$ $\quad \Rightarrow \frac{3\dot{M}_1 (M_1 - M_2)}{M_1 M_2} = -\frac{\dot{\Omega}}{\Omega} = \frac{\dot{P}}{P}$

Can now relate $M_1, M_2, \dot{M}_1$ to the period and change in period

Clearly $\dot{P} \propto |\dot{M}|$ but what happens?

Binary Star Mass Transfer: Angular Momentum

If star M_1 losing mass ($\dot{M}_1 < 0$):

$M_1 < M_2$ then $\dot{P} > 0$ and orbit radius increases

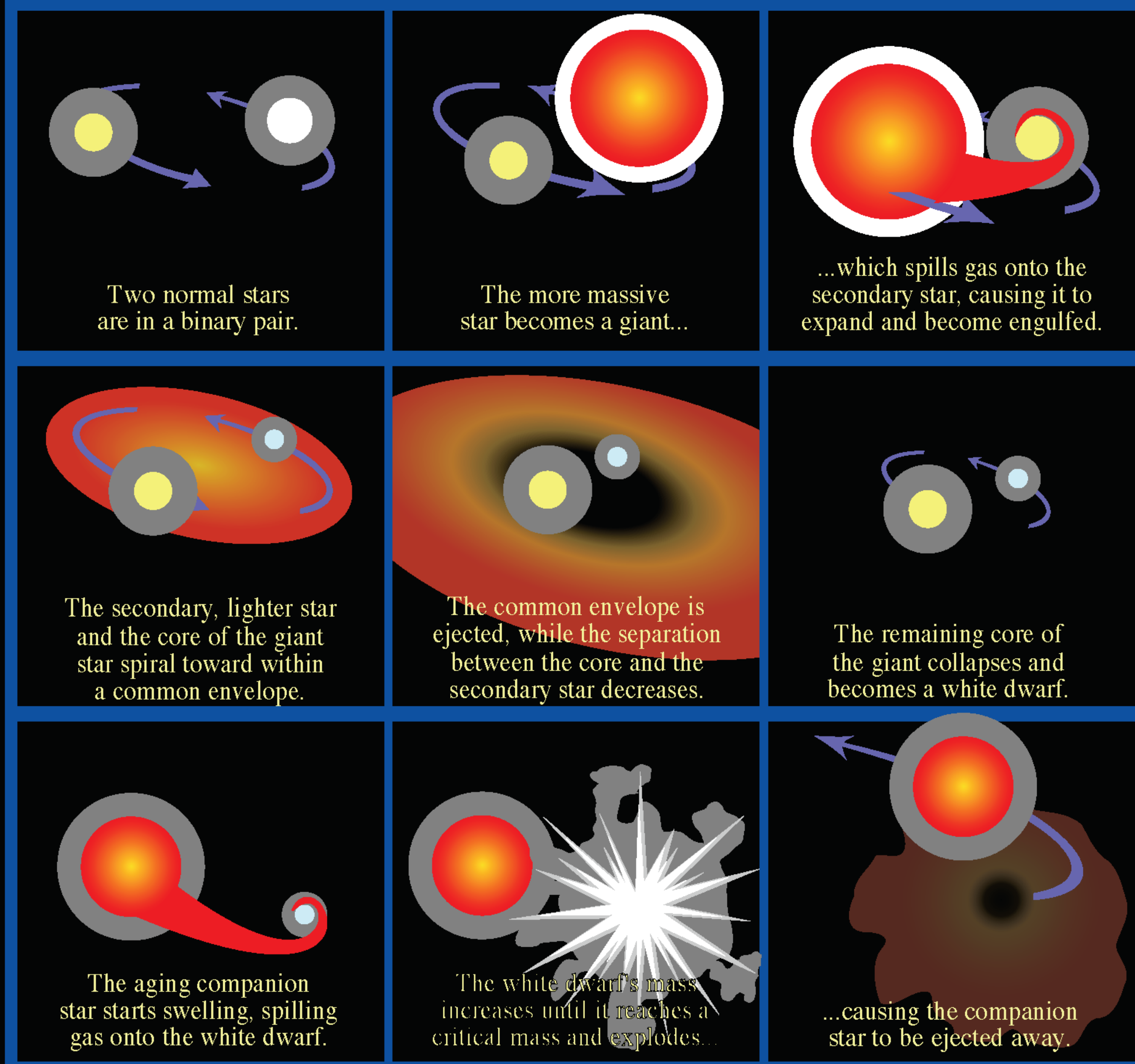
$M_1 > M_2 \;\Rightarrow \dot{P} < 0$ and orbit radius decreases

For case $\dot{P} > 0$ mass-transfer is stable **BUT**
where $\dot{P} < 0$ mass-transfer is unstable
Orbit shrinks, mass-transfer increases…

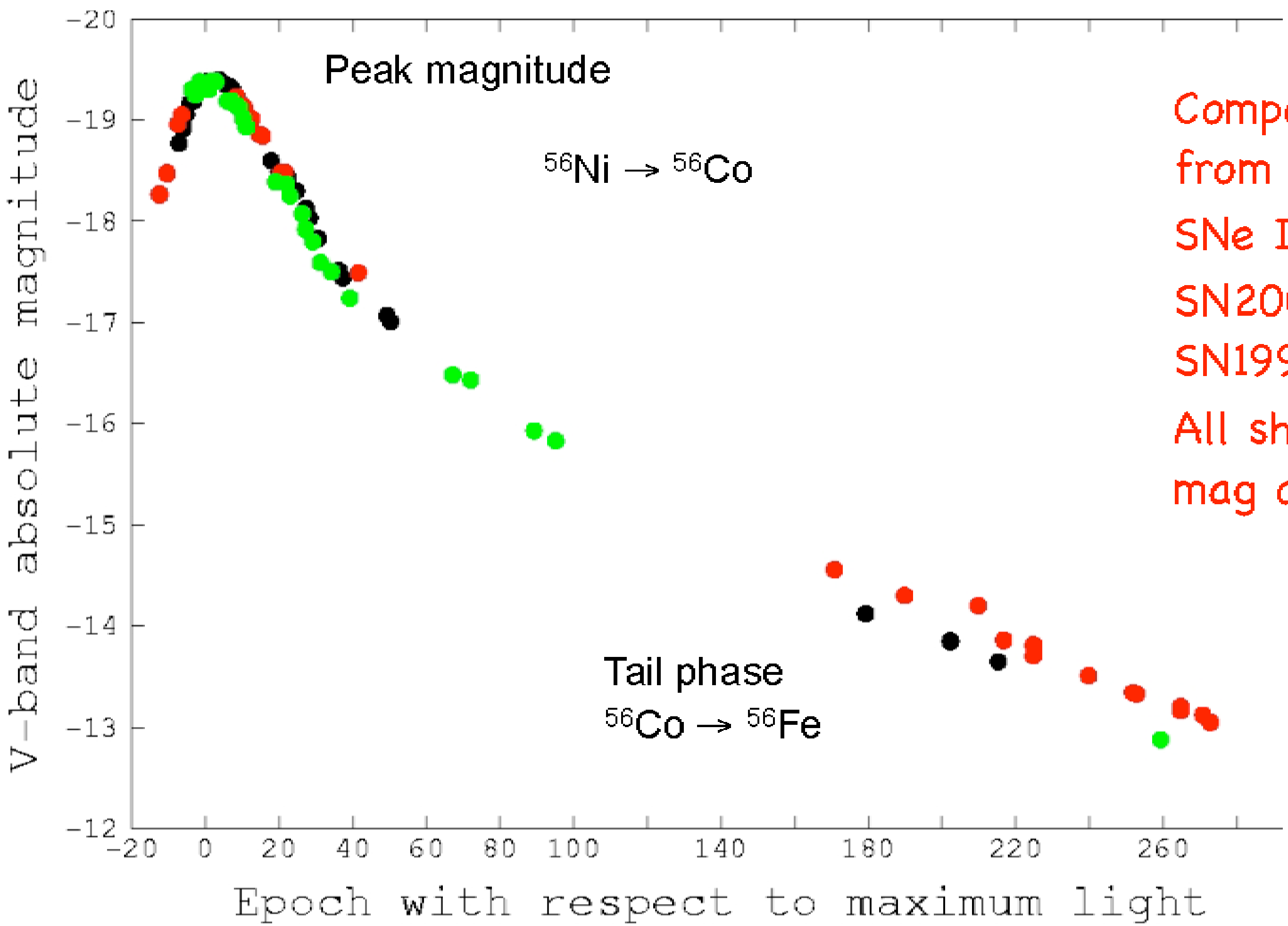
● Significant changes to period of orbit and masses of the two stars. For case of normal stars with $M_1 > M_2$ rapid transfer of mass occurs until $M_1 \approx M_2$ when a more stable configuration achieved.

● For pairs of normal stars with $M_1 > M_2$ rapid transfer of mass until $M_1 \sim M_2$ - more massive star evolves fastest

The progenitor of a Type Ia supernova



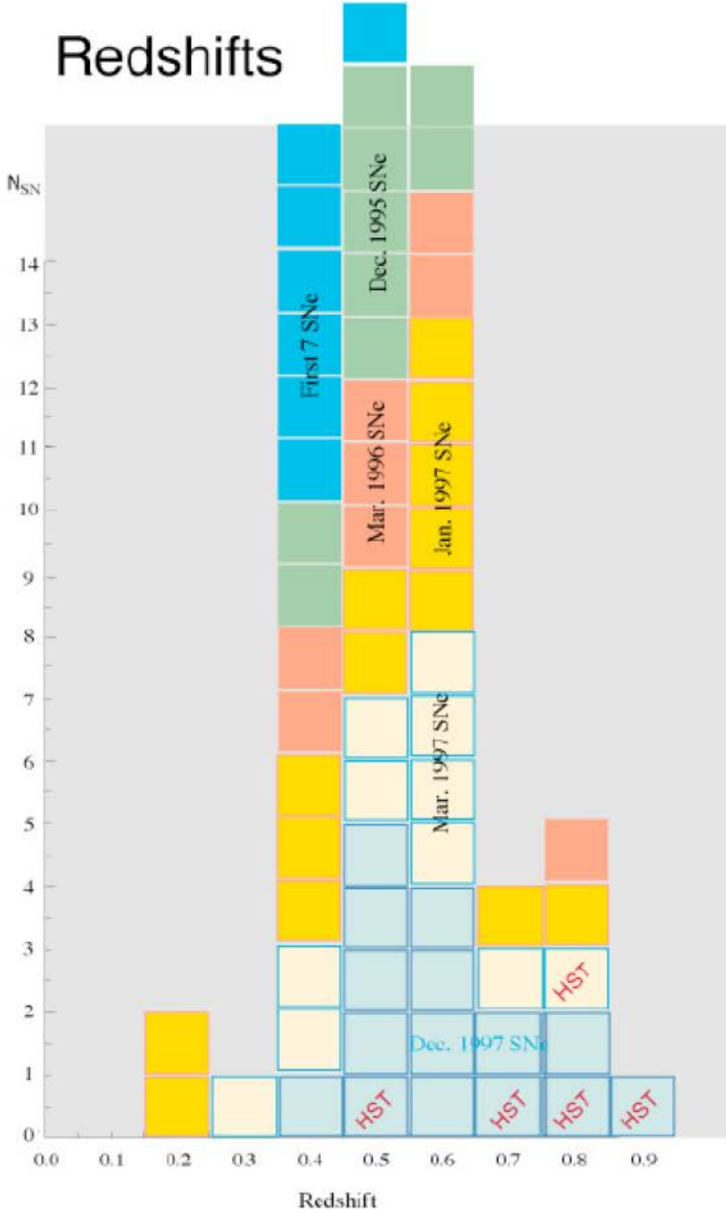
Typical lightcurve for Type Ia SNe

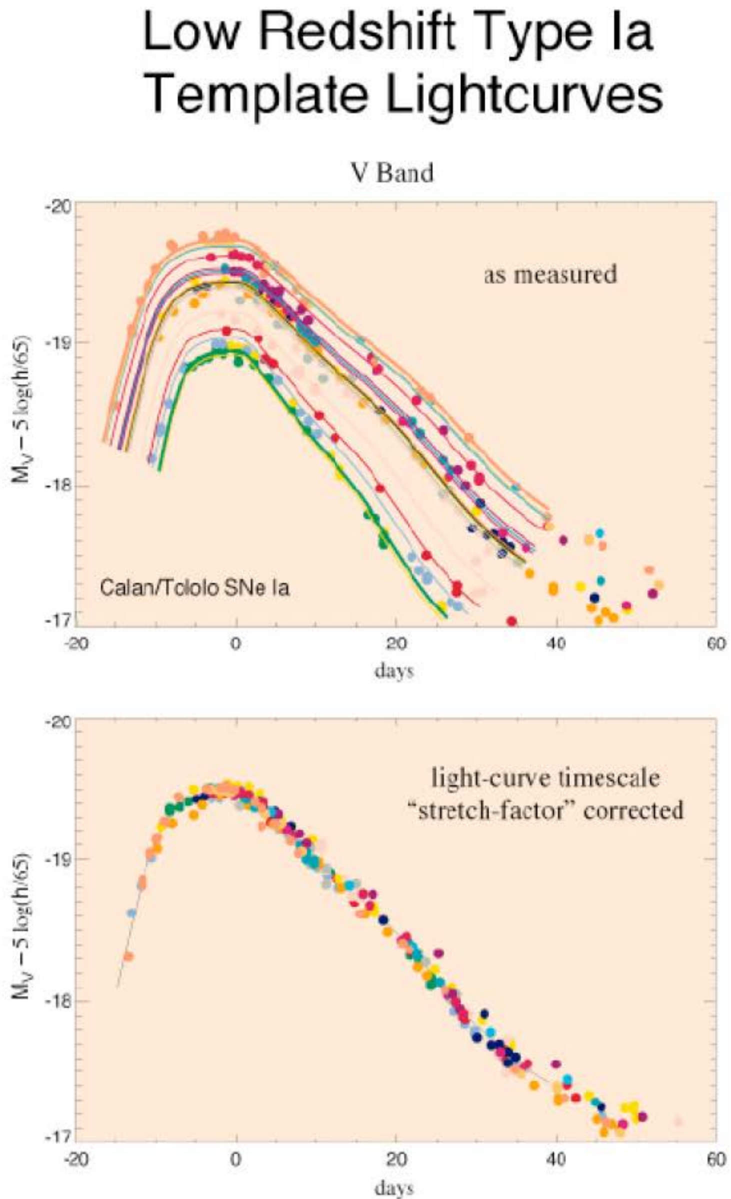


Composite lightcurve from three typical SNe Ia.

SN2002er, SN1990N, SN1996X.

All shifted to peak mag of SN2002er





● All is not simple! Empirical transformation of SNIa –lightcurves to allow determination of individual peak absolute magnitudes

Energetics once material has escaped M_1

● Suppose: material from star 1 falls to surface of star 2

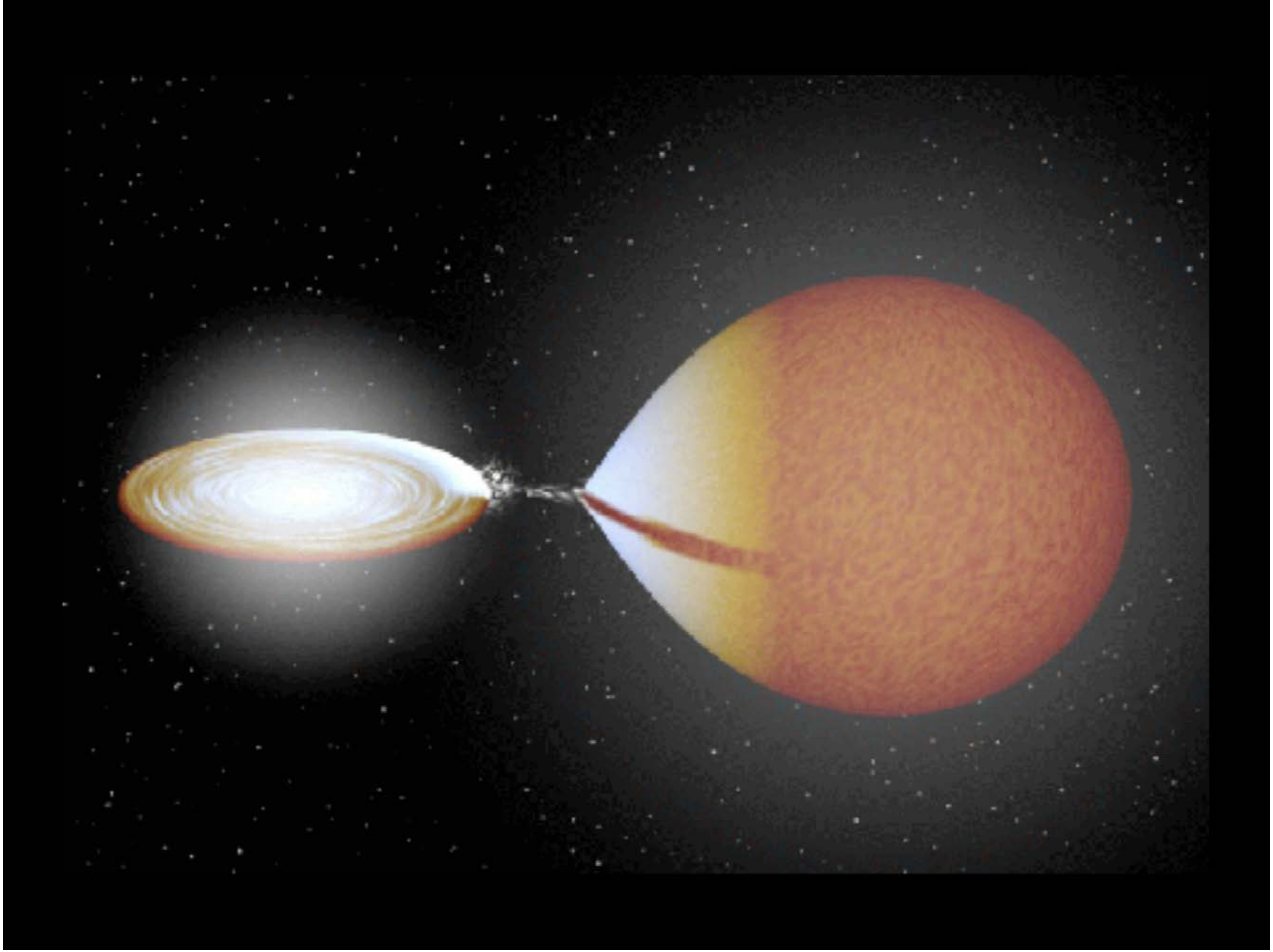
Consider: spherical accretion with energy thermalized on impact

Energy ~ PE $\approx \frac{GM_2}{R_2}$ per unit mass

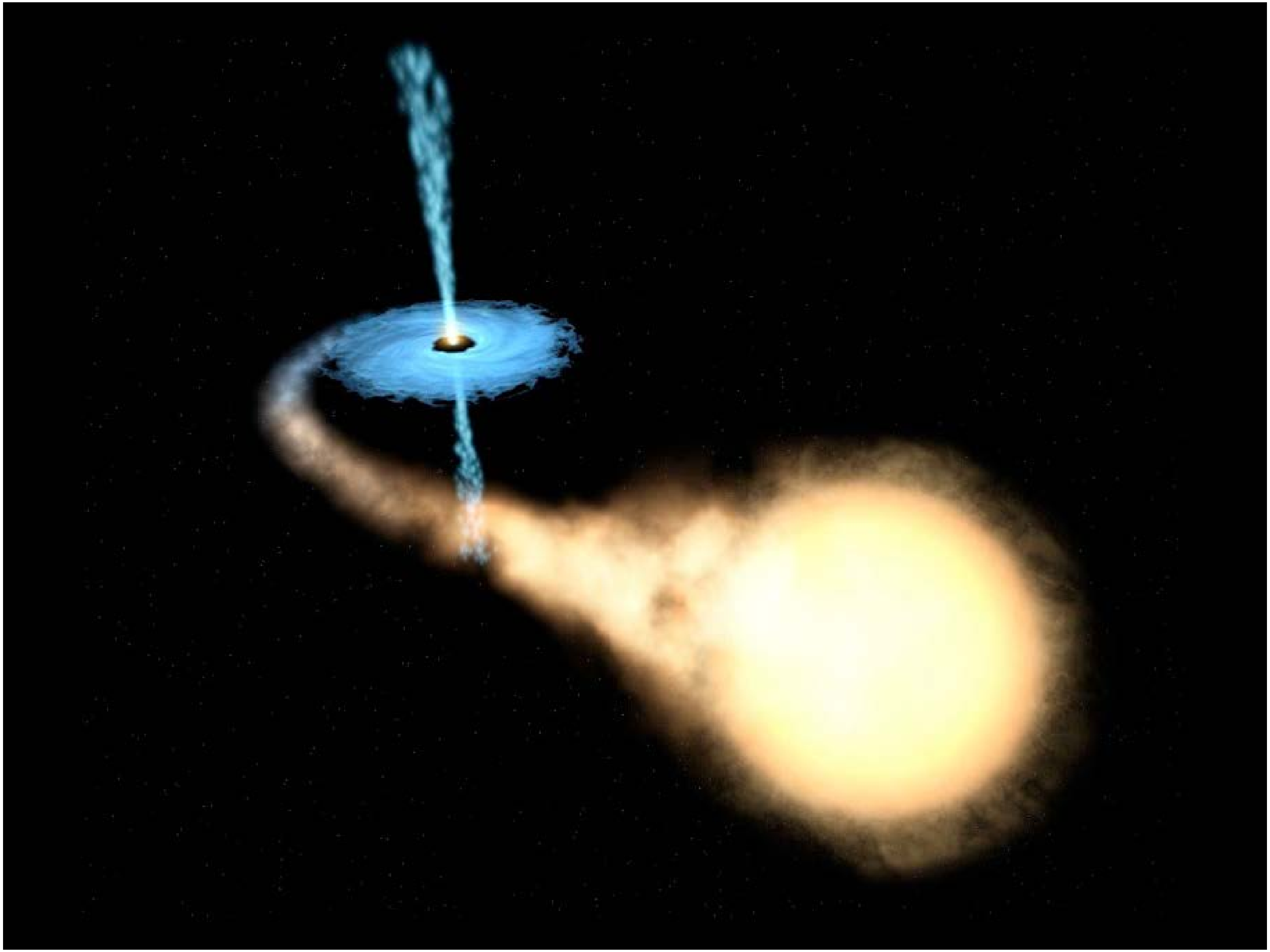
Luminosity $\approx \frac{GM_2\dot{M}}{R_2} \sim 4\pi R_2^2\sigma T^4$ blackbody radiation from star 2 surface

Accretion onto a normal star or even a white dwarf generates relatively little energy

For small R, neutron star with $R \approx 10^4$m and $M \approx 1.5\mathrm{M_{Sun}}$ then 20% of rest-mass (mc^2) liberated as material reaches neutron star surface, cf energy from nuclear fusion, ≈1% of rest-mass



- Accretion is actually via an accretion-disk [angular momentum]



- Energetics very similar to spherical accretion. Large luminosity due to enormous gravitational PE, hence very high Temperature → X-rays

Eddington Limit for Accretion

• Looked at Eddington luminosity in Lecture 12. Consider competition between gravity (due to the compact object) and the radiation pressure (due to the luminosity generated from infalling material)

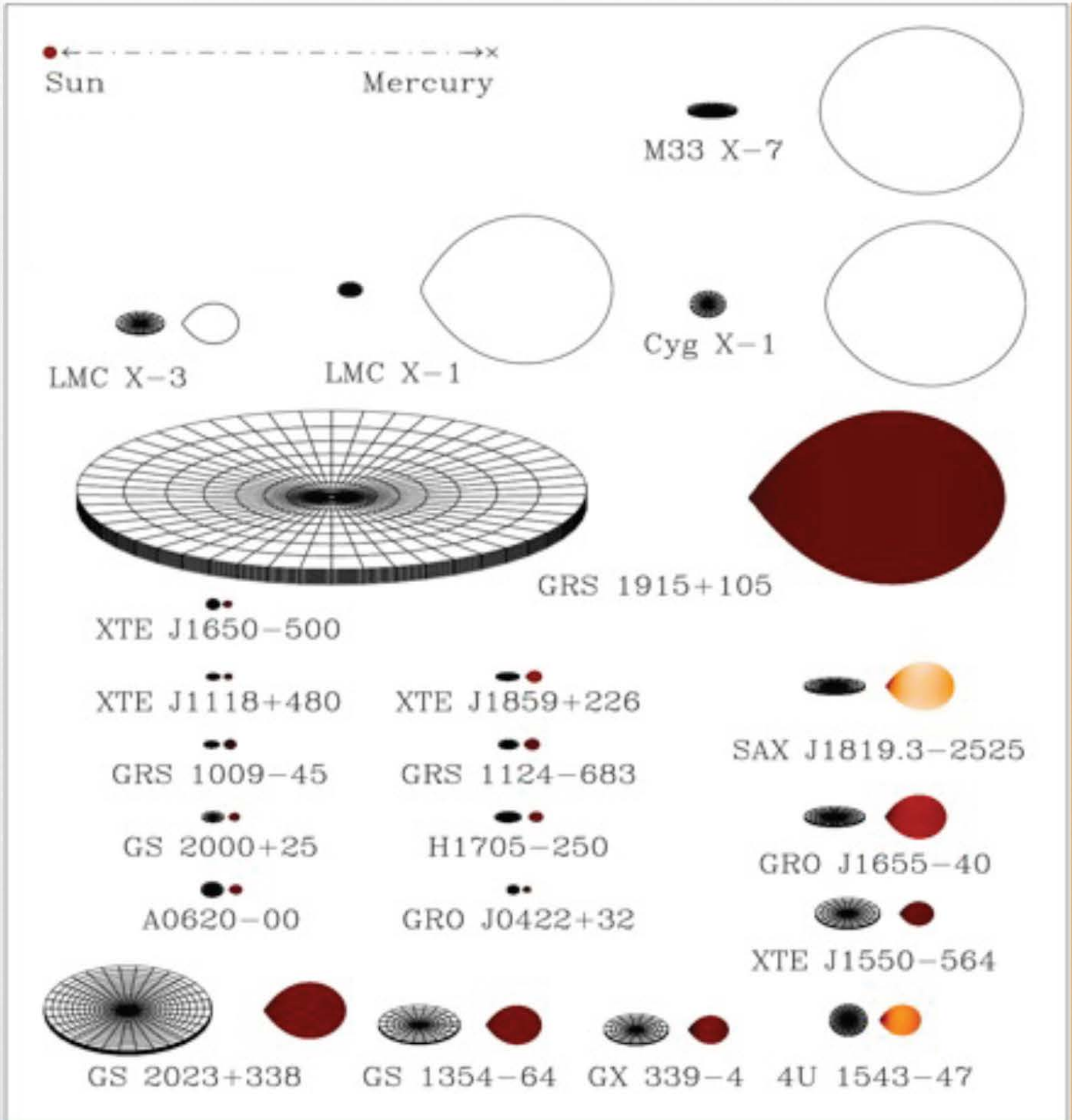
• Infalling material is ionized, so the Thomson cross-section of the electrons dominates the radiation-matter interaction cross-section. So, for balance between gravity and radiation momentum transfer, have:

$$\left(GMm_p - \frac{L\sigma_T}{4\pi c}\right)\frac{1}{r^2} = 0$$

$$L_{Edd} = \frac{4\pi GMm_p c}{\sigma_T} \approx 1.3 \times 10^{31}\left(\frac{M}{M_{Sun}}\right)\text{Watts}$$

• For neutron star can obtain luminosity $L \sim 10^5 L_{Sun}$

The Black Hole Binary Census



Courtesy: J. Orosz

Thermal Stability in Stars

- For a star in hydrostatic equilibrium, the Virial Theorem applies:

$$3\int_0^M \frac{P}{\rho} dm = -E_{Grav}$$

and in the case of a perfect gas with negligible radiation pressure:

$$P_{rad} << P_{gas} \Rightarrow U = -\frac{1}{2} E_{Grav}$$

giving the familiar expression for the total energy of the system:
(where U is the internal energy of the star)

$$\Rightarrow E_{tot} = \frac{1}{2} E_{Grav} = -U$$

Thermal Stability in Stars

- In the case of significant radiation pressure there are two contributions to the total pressure:

$$\frac{P}{\rho} = \frac{P_{gas}}{\rho} + \frac{P_{rad}}{\rho}$$

$$= \frac{k}{\mu m_H} T + \frac{aT^4}{3\rho}$$

- Incorporate into the Virial Theorem to give:

$$U_{gas} = -\frac{1}{2}(E_{Grav} + U_{rad})$$

where the radiation pressure leads to a reduction in the gravitational attraction

$$\Rightarrow E_{tot} = \frac{1}{2}(E_{Grav} + U_{rad}) = -U_{gas}$$

Thermal Stability in Stars

• In either case, a contraction of the star leads to an increase in U_{gas}, and hence T, while expansion leads to cooling

• Change in energy of star is determined by the difference in the nuclear energy production rate and the rate of emission of radiation:

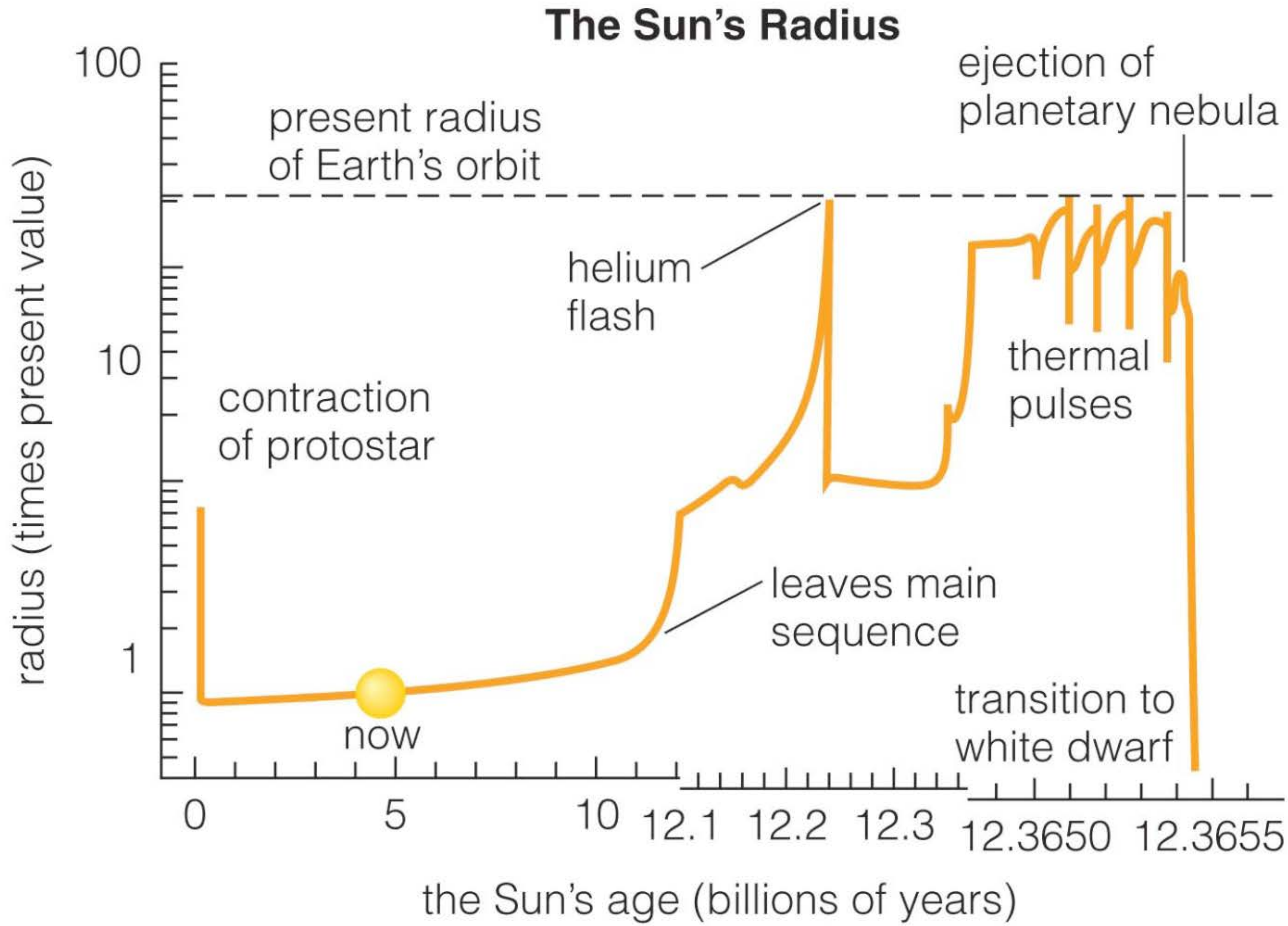
$$\dot{E}_{tot} = L_{nuc} - L$$

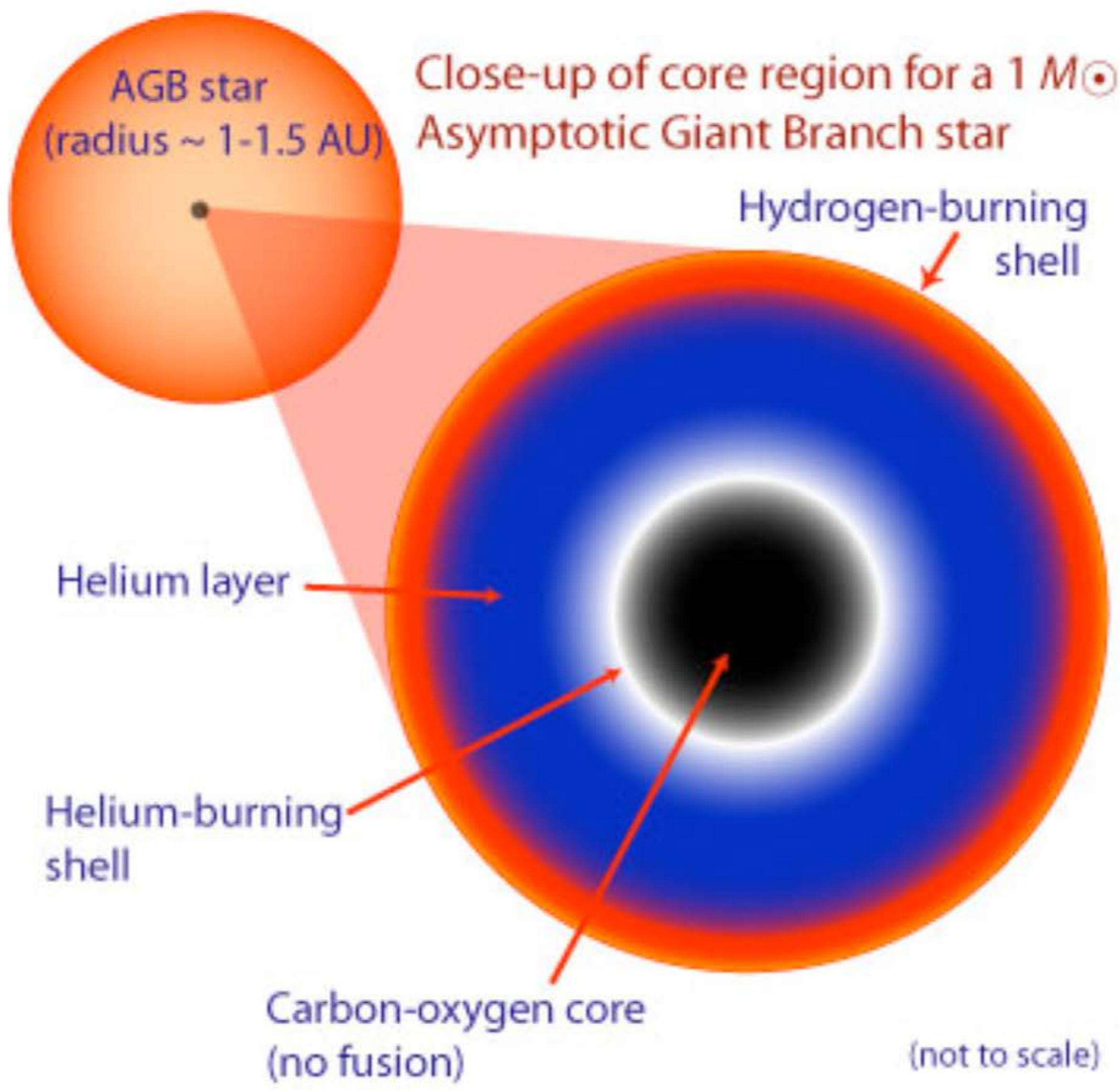
• Thermal equilibrium achieved when energy change is zero

• If there is an imbalance, say L_{nuc} increases, then total energy becomes less negative and thus T decreases (using either relation at bottom of previous two Slides). Energy generation then decreases and thermal equilibrium is restored

• Virial Theorem provides stability via thermostatic control

The Life of the Sun




AGB star
(radius ~ 1-1.5 AU)
Close-up of core region for a 1 $M_\odot$
Asymptotic Giant Branch star
Hydrogen-burning shell
Helium layer
Helium-burning shell
Carbon-oxygen core (no fusion)
(not to scale)

Thermal Stability in Thin Shells

• Nuclear burning in post-main sequence stars occurs in shells. Are such shells always in stable thermal equilibrium?

• Consider a thin shell, mass Δm, temperature T, density ρ, in a star, radius R with shell boundaries r_0 and r, such that $l=r-r_0<<R$

• If shell is in thermal equilibrium, then the energy produced within the shell equals the energy flow out of the shell

$$\frac{dL}{dr} = 4\pi r^2 \rho\varepsilon$$

• Now suppose that the energy generation rate exceeds the rate at which energy can escape from the shell. The shell will expand, pushing the overlying material in the star out to larger radii

Thermal Stability in Thin Shells

- Equation of hydrostatic equilibrium determines change in pressure as the radius changes:

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4} \Rightarrow P \propto r^{-4}$$

$$\Rightarrow \frac{dP}{P} = -4\frac{dr}{r}$$

- Mass of shell (=constant) determines relation between thickness and density:

$$\Delta m = 4\pi r_0^2 l \rho$$

$$\Rightarrow \frac{d\rho}{\rho} = -\frac{dl}{l} = -\frac{dr}{l} = \frac{dr}{r}\frac{r}{l}$$

- Substitute for *dr/r* in 2nd equation from top to give:

$$\frac{dP}{P} = 4\frac{l}{r}\frac{d\rho}{\rho}$$

Thermal Stability in Thin Shells

- Take equation of state including gas and radiation pressure:

$$P = \frac{k}{\mu m_H} \rho T + \frac{a}{3} T^4$$

take log and differentiate:

$$\Rightarrow \frac{dP}{P} = c_1 \frac{d\rho}{\rho} + c_2 \frac{dT}{T}$$

combine the two expressions for *dP/P* to eliminate pressure:

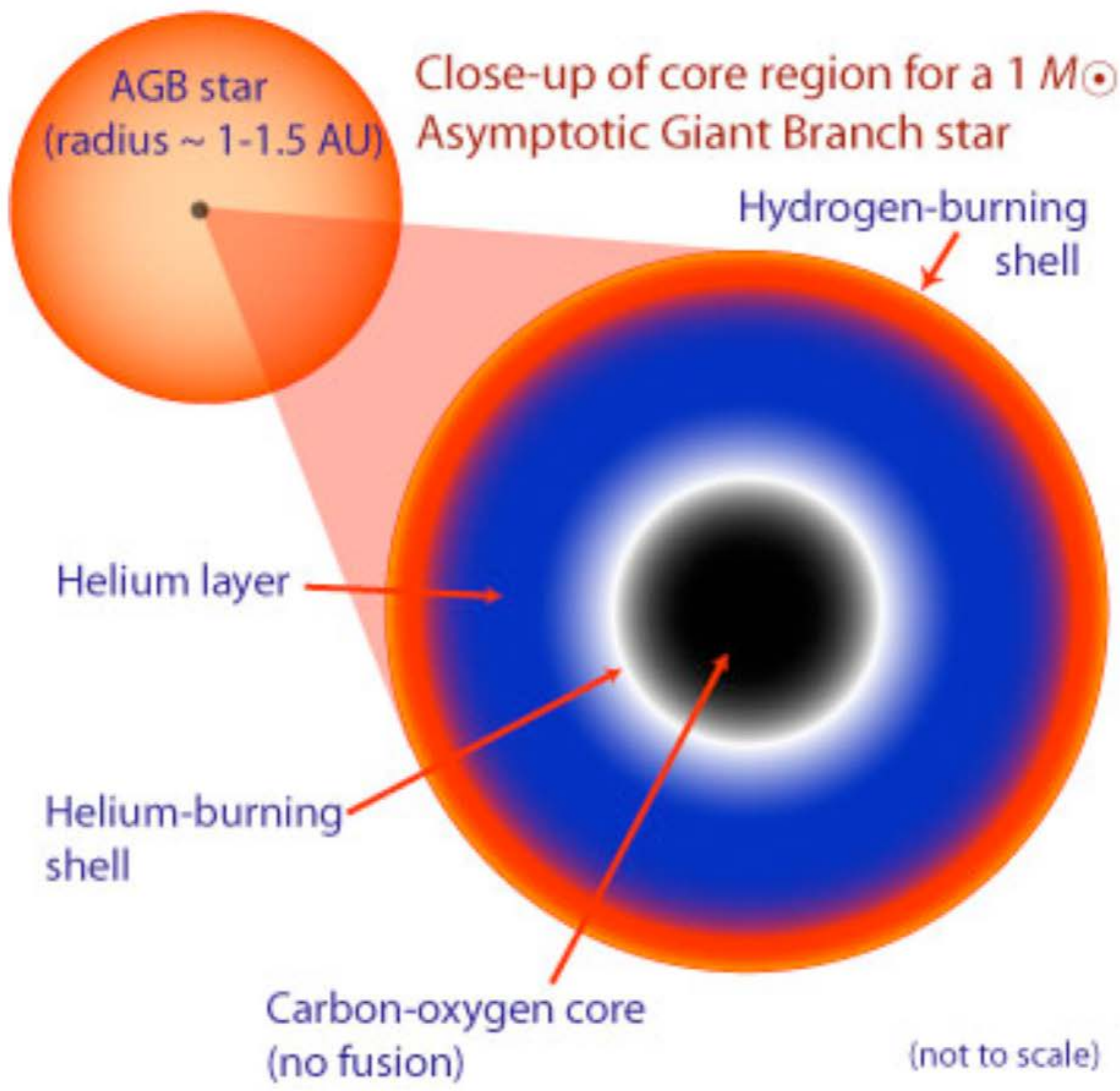
$$\Rightarrow 4 \frac{l}{r} \frac{d\rho}{\rho} = c_1 \frac{d\rho}{\rho} + c_2 \frac{dT}{T}$$

rearrange to give:

$$\Rightarrow \left(4 \frac{l}{r} - c_1 \right) \frac{d\rho}{\rho} = c_2 \frac{dT}{T}$$

constant on rhs is positive and thus there is a constraint on *l/r* in terms of the constant on lhs. Violated as $l \to 0$

$$c_2 > 0 \quad \therefore \quad 4 \frac{l}{r} > c_1$$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(no fusion)
(not to scale)

Shell Burning: Thermal Pulses

- Analysis shows shell burning must be unstable at some thickness

- Consequence of nuclear energy generation in two thin shells, with very different dependencies on T and ρ – stars on the asymptotic giant branch

- Outer shell burns H to He via p-p chain or CNO cycle

- Inner shell burns He to Carbon and Oxygen via triple-α reaction

- In principle, the configuration could be stable if the nuclear burning fronts advanced outwards at the same rate

- Not true in practice and the result is cyclic behaviour that involves large changes in the energy generation rate – **thermal pulses**

Shell Burning: Thermal Pulses

- Cycle involves H-burning in the outer shell for much of the time

- No burning in the inner shell to begin (*T* not high enough)

- He-shell grows in mass as H-burning produces more He

- He-shell contracts, *T* at base of shell increases and He-burning ignites. Similar to the ignition of Helium burning in stellar core at the tip of the giant branch and sometimes termed **shell flash**

- Energy generation rate huge, heat flow cannot keep pace, He-shell and overlying layers (including H-burning shell) expand and cool and H-burning essentially stops

- He-burning front moves out rapidly through the He-shell until reaches H-shell, which reignites

Shell Burning: Thermal Pulses

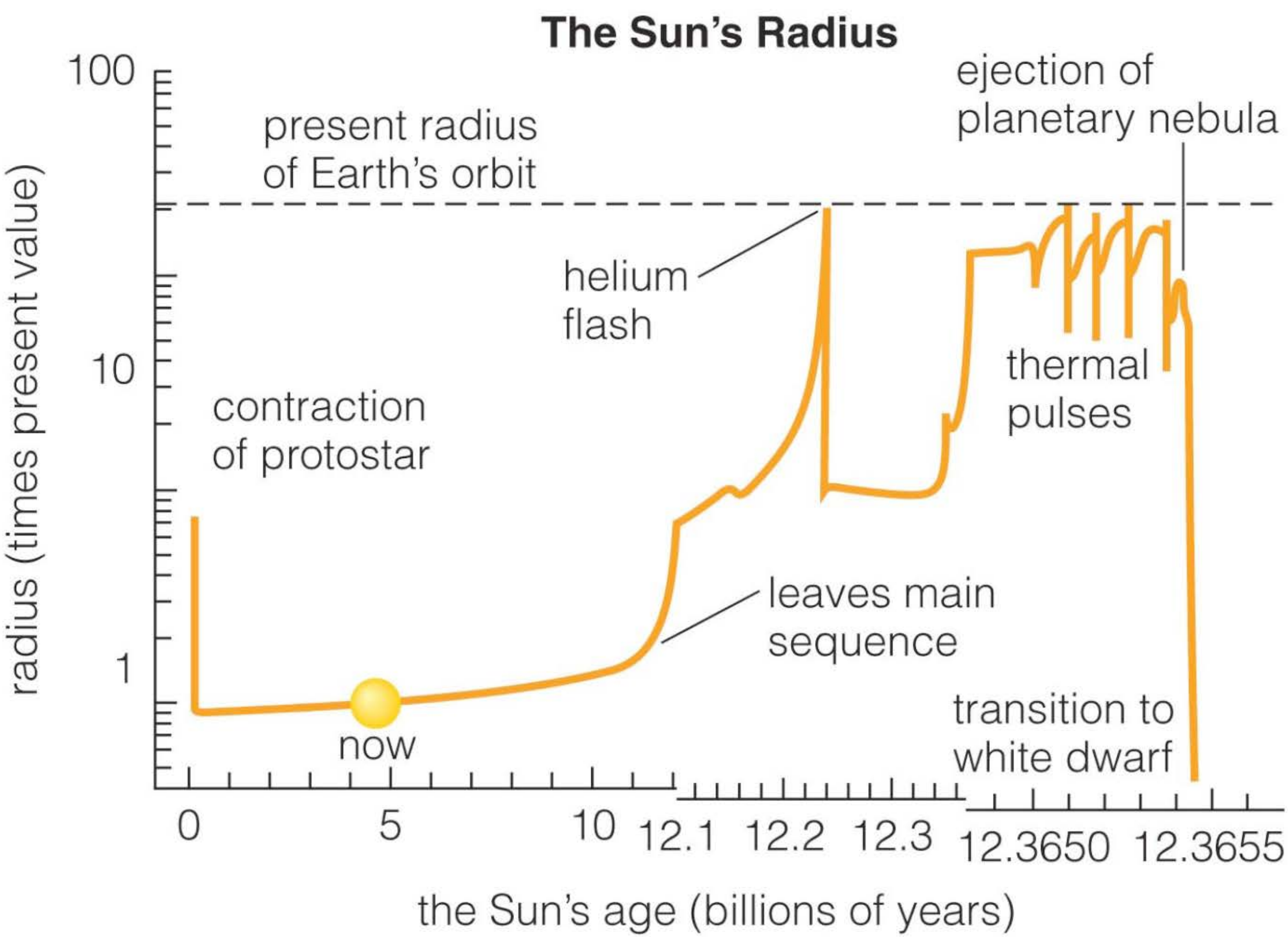
• The high T achieved in the cycles of burning in the He-shell lead to the production of many neutrons and the bombardment of metals in the surrounding layers in the star is the primary location where the build-up of elements via the s-process (Lecture 22) takes place

• Temperature in the He-burning shell drops below the point necessary to maintain burning and the He-burning ceases

• The C+O core has grown in mass, following the He-burning, and the cycle now repeats with H-burning in the outer shell continuing until the build-up of He is sufficient for the ignition of burning in the innershell

• Period of the cycle ~100s to 1000s of years

The Life of the Sun



AGB: Dredge Up and Mass Loss

• The extensive changes to the location of the interface between the nuclear burning regions and the convective envelope of the star lead to **dredge up** during which processed material, notably Carbon, is transported from the outer regions of the core to the stellar surface. Leads to overabundance of Carbon in the photospheres – **carbon stars**

• The presence of heavy elements in the photosphere exacerbates the rate of mass-loss from the surfaces of the enormously distended AGB stars. Creation of molecules and dust grains at the low photospheric temperatures (T_{eff}<3000K)

• Straightforward to estimate the mass-loss rate as a function of the main properties of a star

Mass Loss

- Mass loss rate in time interval δt: $\dot{M}\delta t$

- Radiation pressure due to transfer of momentum by photons: $(L/c)\delta t$

- Suppose that a fraction, *f*, of the total *L* is absorbed by material that is to escape. Material must reach the escape velocity to leave gravitational field of the star:

$$\Rightarrow \dot{M}\delta t v_{esc} = f(L/c)\delta t$$

$$v_{esc}^2 = \frac{2GM}{R}$$

- Substitute for the escape velocity to produce dependence of the mass loss rate on *L*, *M* and *R*

$$\Rightarrow \dot{M} = \frac{f}{2}\frac{v_{esc}}{c}\frac{LR}{GM}$$

Mass Loss

- In the case of the giant and AGB stars the key factor is the value of *f*, which is very difficult to calculate

- On the main sequence, composition is uniform and writing the dependence on *L*, *M* and *R*, combined with the Homology scalings of *L*, *M* and *R* for the uppermain sequence where $n \approx 18$ (CNO cycle), shows that the mass loss rate is almost linearly dependent on *L*

$$\dot{M} \propto LRM^{-1}$$

$$L \propto M^{3}; \quad R \propto M^{(n-1)/(n+3)}$$

$$\Rightarrow \dot{M} \propto L^{122/126} \approx L$$

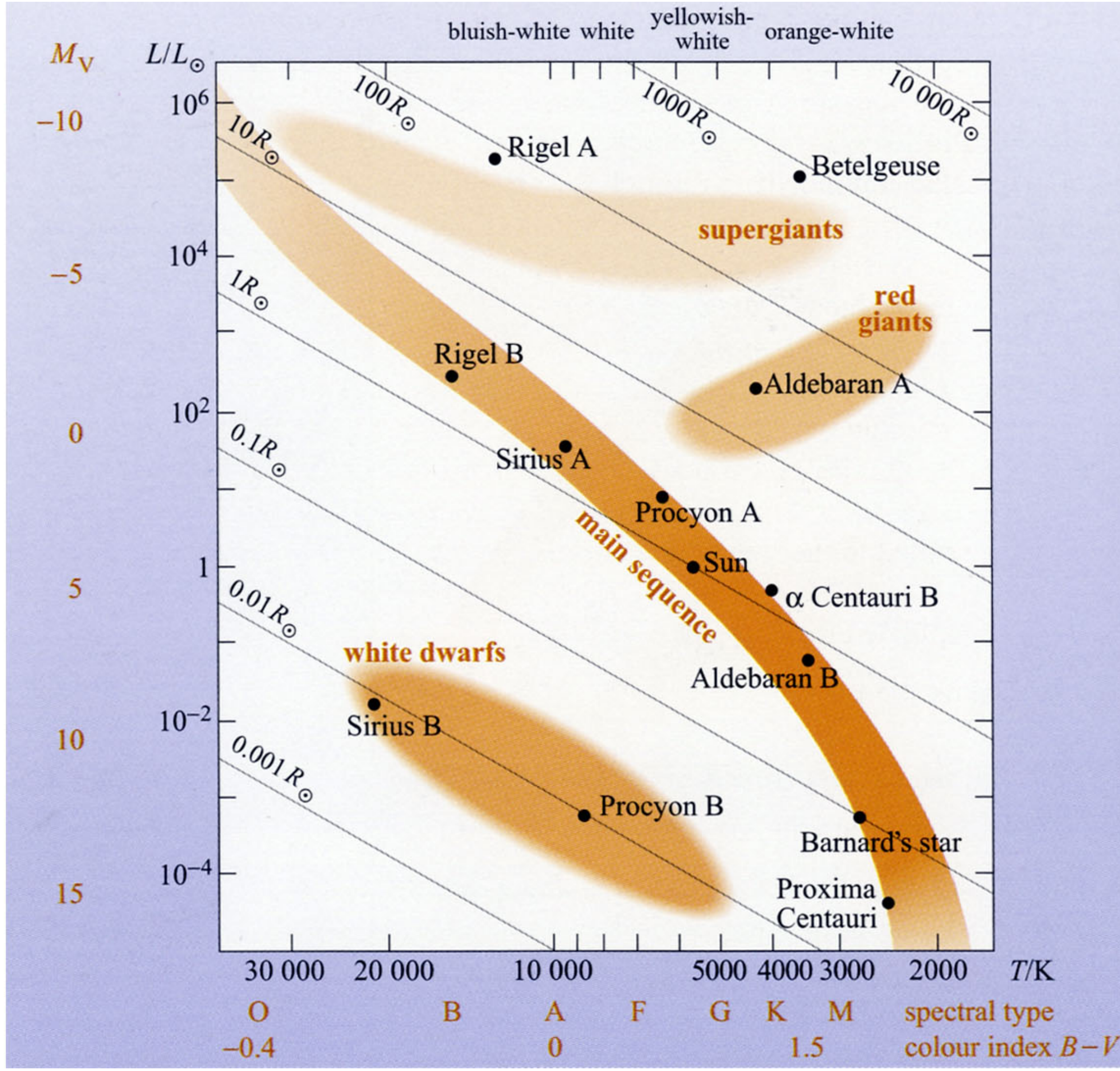


Figure 4.5 The H–R diagram in Figure 4.3, with the addition of stellar radii, and other information. (Adapted from Seeds, 1984)

Lecture 23: Summary

• Mass-transfer in binary systems has significant impact on evolution of component stars and can result in very high luminosities for binaries containing a compact object (e.g. neutron star). Type Ia supernovae result when the mass of a white dwarf approaches the Chandrasekhar Limit

•For a star in hydrostatic equilibrium the Virial Theorem means that there is a thermostatic control that maintains the system in thermal equilibrium

• Once burning is confined to shells it is possible to show that the system is thermally unstable once the shell becomes thinner than a critical thickness. Explains cyclic thermal pulses that are characteristic of stars on the AGB.

• Can obtain approximate dependence of mass-loss rate on stellar parameters using homology and simple arguments

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