Structure and Evolution of Stars

Lecture 21: Compact Remnant Properties (#2)

- Supernovae lightcurves – constrain pre-explosion composition and model for explosion
- Neutron Star Radii
- Angular Momentum Conservation as Sanity Check on Relative Radii of stars, white dwarfs and neutron stars
- Minimum Period for a White Dwarf – do we need Neutron Stars?
- Pulsars
- Direct Imaging of Neutron Stars
- Maximum Mass for a Neutron Star
- Recap of Mass Function for Spectroscopic Binaries
- Current Observational Limits on the Masses of Neutron Stars and Black Holes

Supernovae – when should we be worried?

● Apparent magnitude of Sun =-26.7, absolute magnitude of supernovae -19 to -21, i.e. at 10pc distance!

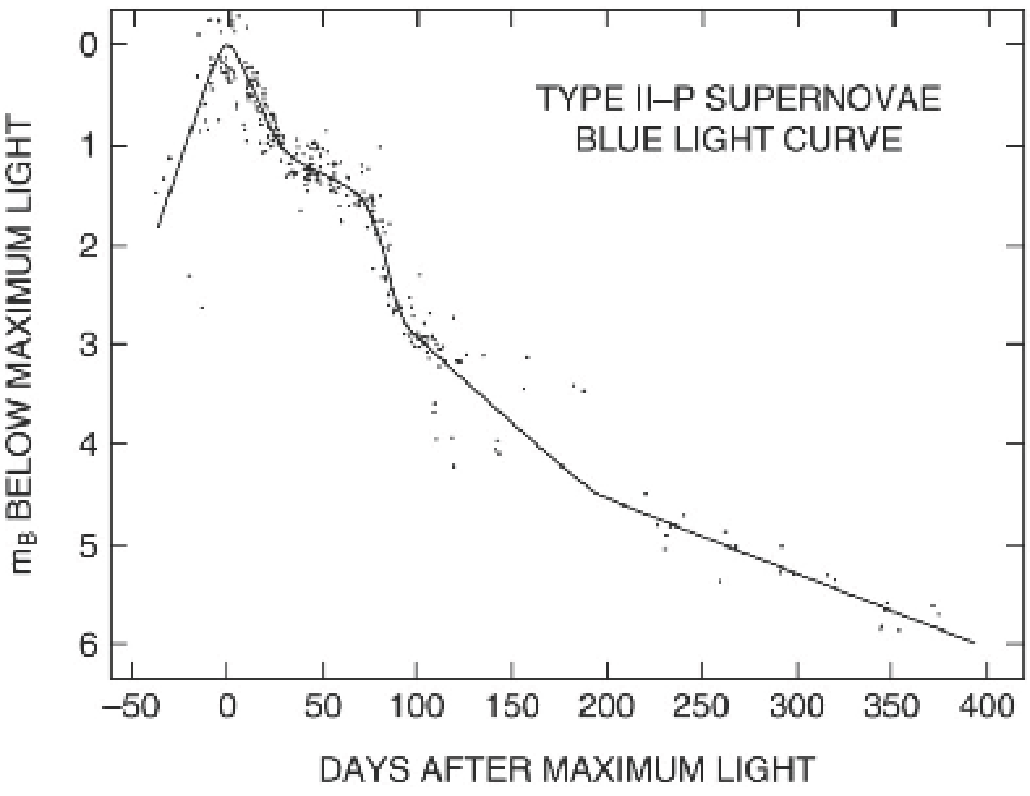
● Main concern is gamma-ray flux impact on Earth atmosphere (e.g. destroy ozone layer)

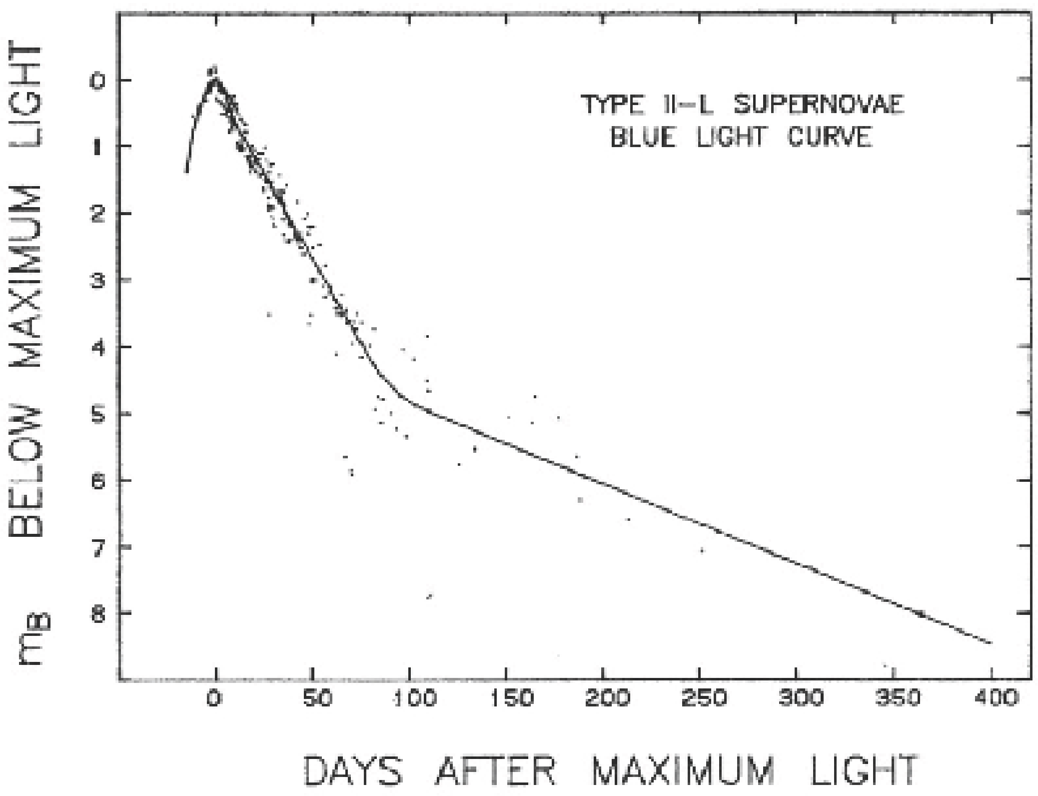
● There are published claims from late 1970s of signature of historic 1006 and 1054 SN in ice cores (excess of nitrates) but interpretation not generally believed

● Calculations suggest really not good if SN occurs within 10-15pc of the Earth but probability estimated to give one such SN every 250 million years

● Recent discovery of class of "hypernovae" that produce collimated beams of gamma-rays – massive star, where rotation important and post-core collapse explosion directed along axis of rotation. Not good if one of these goes off within 2kpc but estimated frequency one every 100000yrs per galaxy and have another factor of 100-1000 due to narrow beaming angle

Typical Light Curves of Type II SNe





Doggett & Branch 1985

Core-collapse Supernovae - Post-collapse Conditions

● Shockwave moving through star raises temperature such that explosive nucleosynthesis occurs [neutron capture important]

● Crude estimate of T from:

$$E_{Explosion} \sim Volume \; \times \; \sigma T^4$$

where have volume behind shock and the energy density

$T \approx \left(\frac{3E}{4\pi R^3 \sigma}\right)^{1/4}$; $E \approx 10^{44}$ J; for R~ 2000 km T~10^{10}K

● Source of elements from Fe → U, numerous unstable isotopes formed

Supernovae Light curves from Radioactive Decay

• Radioactive decay of isotopes, formed by shock passing through the star, powers the supernova remnant

$$\frac{dN}{dt} = -\frac{N}{\tau}$$

Halflife $T_{1/2} = \tau \ln(2)$

• Rate of decay of species *dN/dt* determines rate of energy deposition *dL/dt*

$$\frac{1}{L}\frac{dL}{dt} = -\frac{1}{\tau}$$

• Calculate slope of the resulting lightcurve

$$\frac{dln(L)}{dt} = -\frac{1}{\tau}$$

• Can then use observations to determine contribution of a species with particular decay rate to the remnant luminosity

$$\frac{dlogL}{dt} = -\frac{0.434}{\tau}$$

• Have used *M = -2.5 Log L*

$$\frac{dM}{dt} = -\frac{1.086}{\tau}$$

Supernovae Light curves from Radioactive Decay

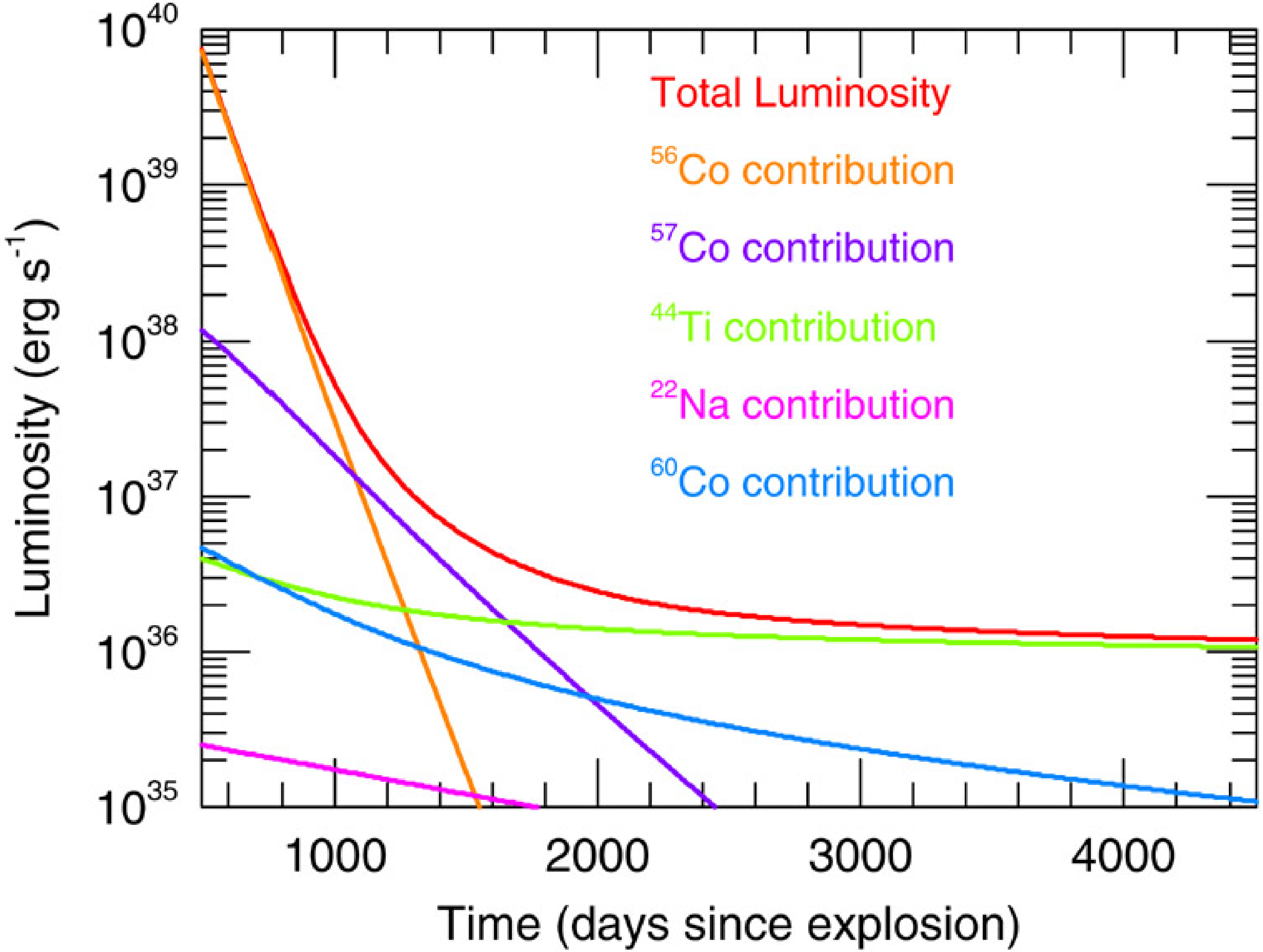
● Significant mass of species around the iron-peak, includes $^{56}_{28}Ni$ with halflife of 6.1 days and $^{56}_{27}Co$, with halflife of 77.7 days. Also $^{57}_{27}Co$, halflife 271 days

● Distinctive signatures in the time-magnitude plot – short halflife results in a steep, rapid brightness decrease

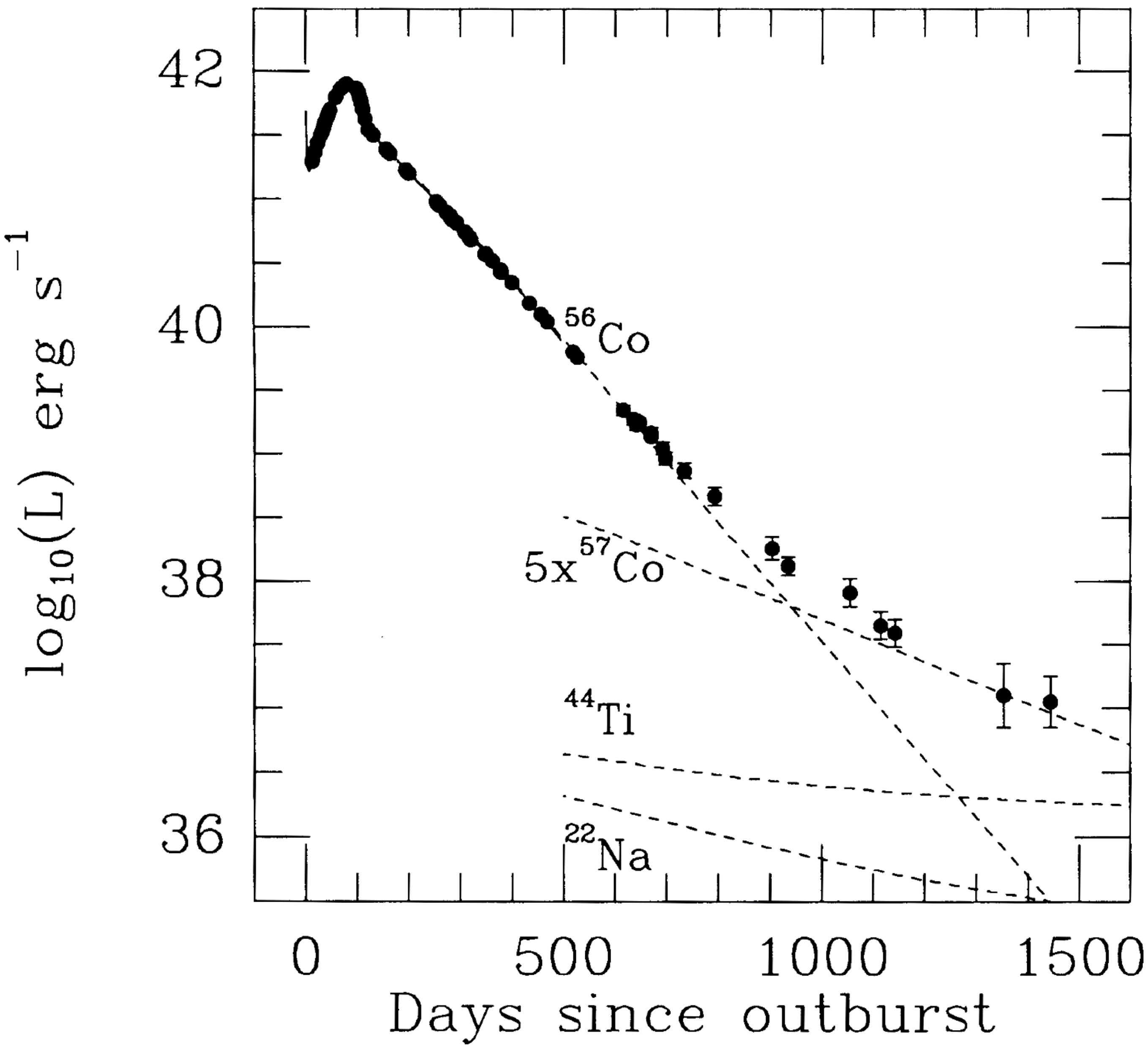
$$^{56}_{28}Ni \longrightarrow {}^{56}_{27}Co + e^{+} + \nu + \gamma$$

$$^{56}_{27}Co \longrightarrow {}^{56}_{26}Fe + e^{+} + \nu + \gamma$$

● Can construct census of main unstable isotopes produced and leads to good estimate of total mass of remnant

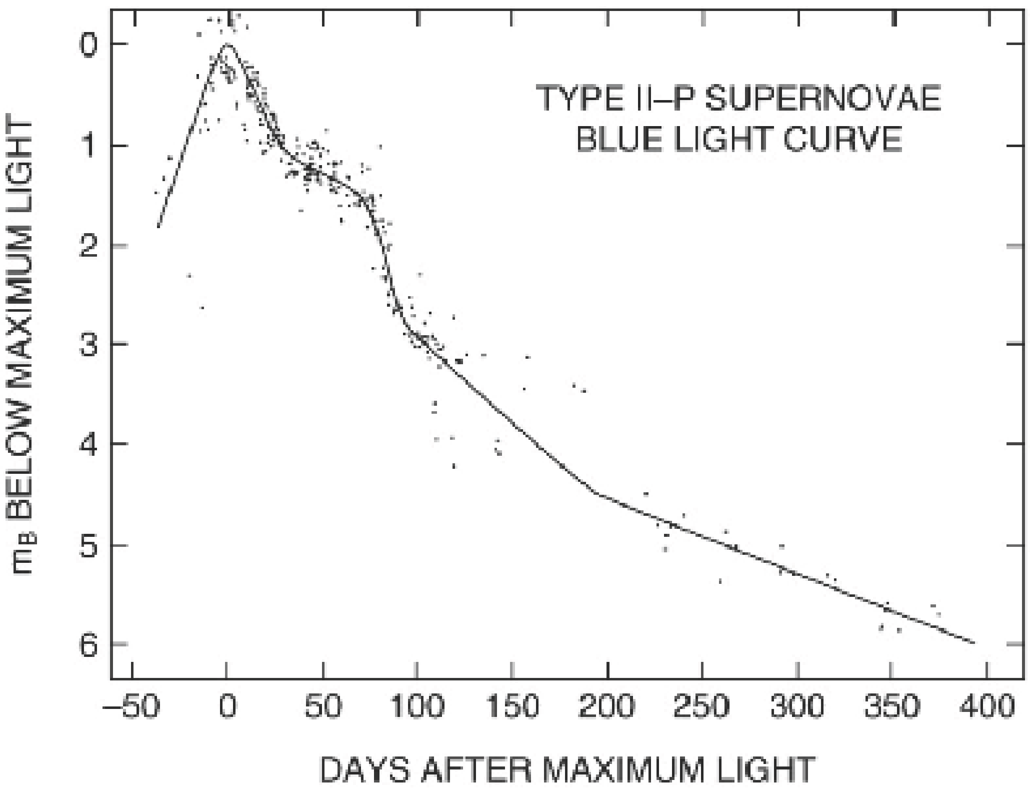


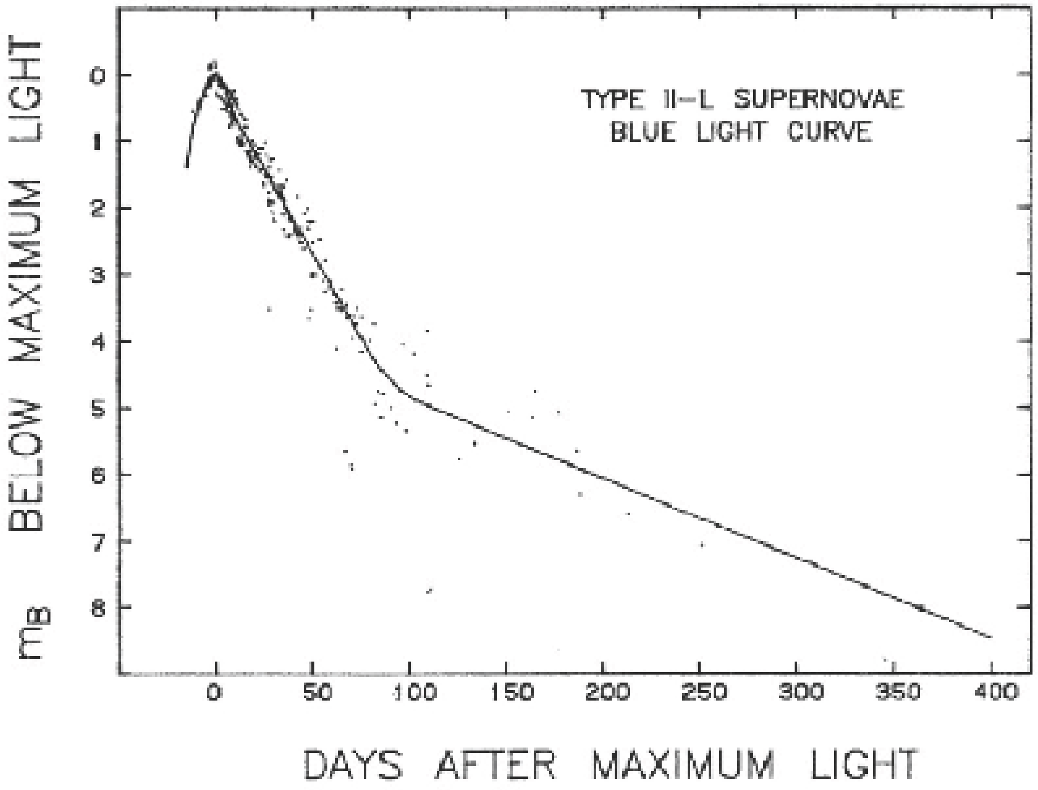
Total Luminosity
56Co contribution
57Co contribution
44Ti contribution
22Na contribution
60Co contribution
Luminosity (erg s-1)
Time (days since explosion)
10^40
10^39
10^38
10^37
10^36
10^35
1000
2000
3000
4000



SN1987A light curve

Typical Light Curves of Type II SNe

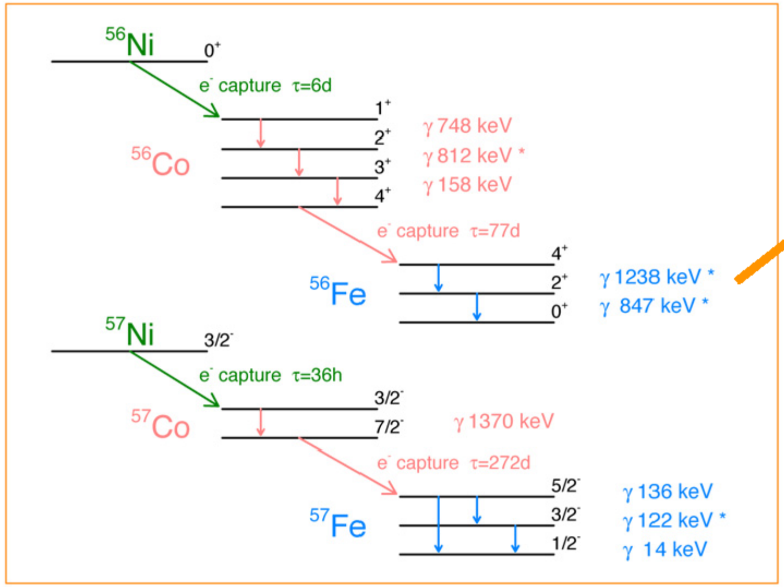




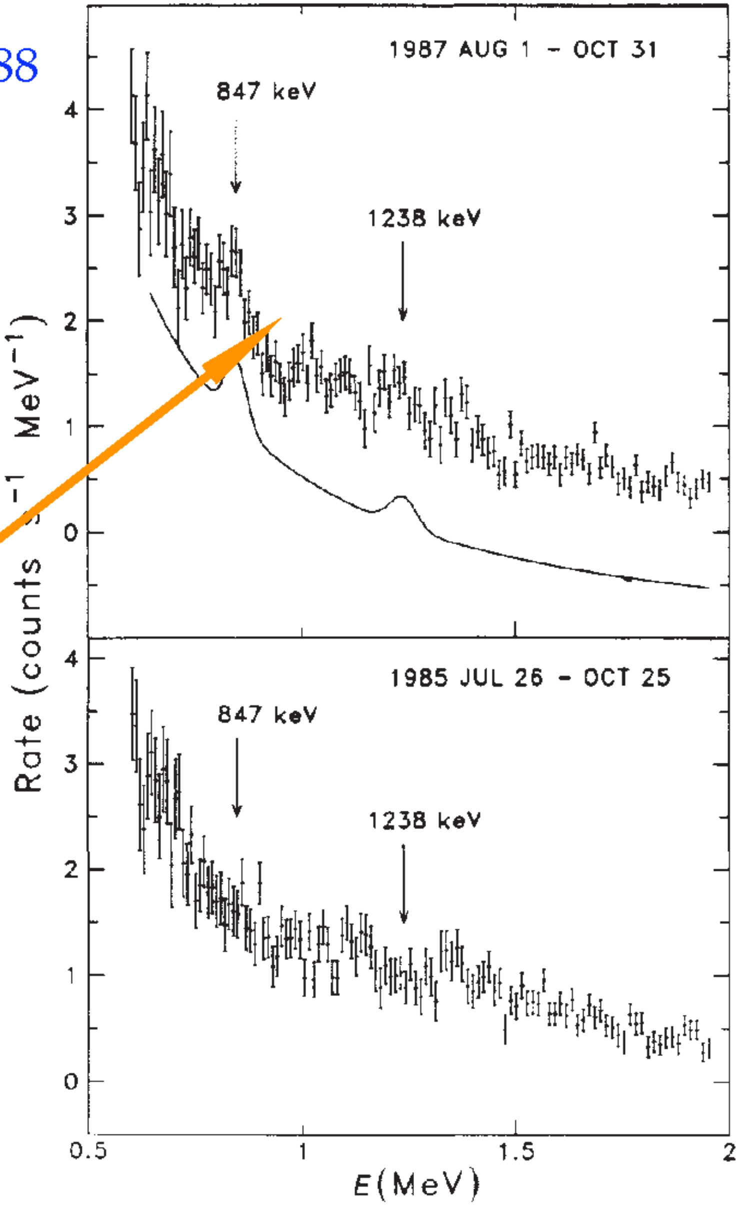
The II-P "plateau" due to large mass of ${}^{56}_{28}Ni$ produced

Doggett & Branch 1985

● Direct confirmation of radioactive decay from X-ray observations of the region in the LMC containing SN1987A before and after the supernova explosion

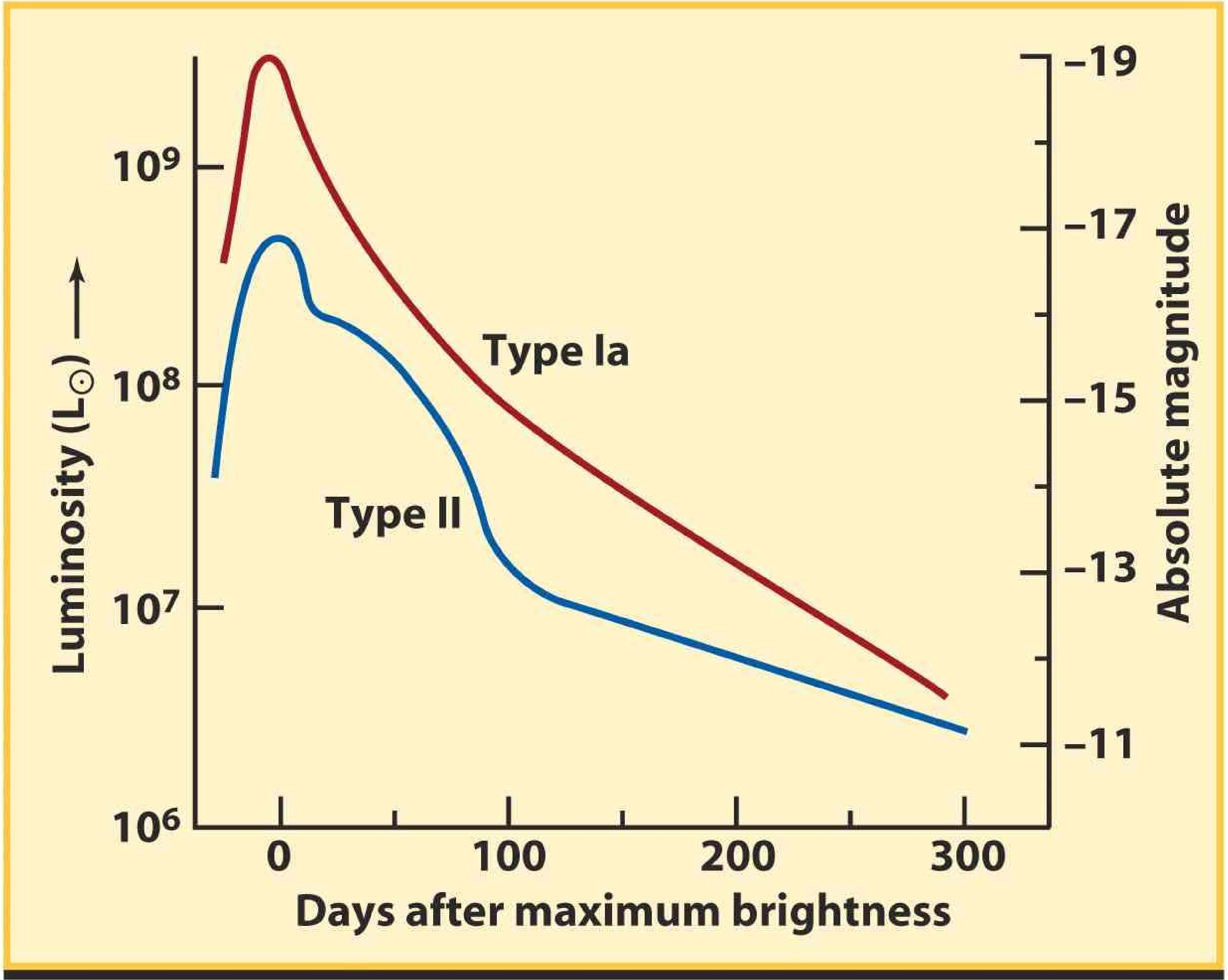


Matz et al. 1988 (SMM)



• Thermonuclear supernovae, white dwarf detonation as Chandrasekhar Mass ($1.4M_{Sun}$) exceeded – light curves far more similar as a class and has led to the most well known “standard candle”, visible out to very high-redshifts – history of the expansion rate, Dark Energy and Nobel Prize.

Will look at the possibilities for mass transfer in binary systems to explain origin of $1.4M_{Sun}$ white dwarfs in a later lecture



Core-collapse SN Compact Remnants
Neutron Star Radius

• In Lecture 8 we derived an expression for the pressure in a degenerate system with non-relativistic particle velocities. We were considering the effects of degeneracy pressure due to electrons and found that the pressure depended on the inverse of the degenerate particle mass and also the mean molecular weight of the degenerate particles:

$$P_{e,\deg} = \frac{h^2}{20m_e}\left(\frac{3}{\pi}\right)^{2/3} \frac{1}{m_H^{5/3}}\left(\frac{\rho}{\mu_e}\right)^{5/3}$$

where $\mu_e = \dfrac{\rho}{n_e m_H}$; $\mu_e = 2$ for $\dfrac{Z}{A} = 0.5$

Neutron Star Radius

• In Lecture 19 we combined the value of the central pressure for a system in hydrostatic equilibrium with the density-mass relationship for a degenerate system to show $RM^{1/3}$= const – the mass-radius relationship

• Propagate the dependence on the electron mass and #electrons per nucleon through into the right-hand-side:

$$R_{wd} M^{1/3} \propto \frac{1}{m_e} \left(\frac{1}{\mu_e m_H} \right)^{5/3}$$

• Same exercise for the neutron star, where #neutrons = #nucleons:

$$R_{ns} M^{1/3} \propto \frac{1}{m_n} \left(\frac{1}{m_H} \right)^{5/3}$$

• Can calculate ratio of radii for white dwarf and neutron star of the same mass:

$$\frac{R_{wd}}{R_{ns}} \approx \frac{m_n}{m_e} \left(\frac{1}{\mu_e} \right)^{5/3} \approx 500$$

Stars and Compact Objects: Sanity Check

• Angular momentum must be conserved for (isolated) star as evolves from main sequence through to white dwarf or neutron star phase

• Observations of the Sun and other main sequence stars show surface rotations of typically tens of days. Not solid body but observations of stars on horizontal branch (when significant fraction of envelope has been ejected during time on giant branch) also produce rotation periods of tens of days

• What are predicted rotation velocities of objects with radii of white dwarfs and neutron stars?

• What is the maximum rotation rate for a white dwarf? – do we need neutron stars?

Stars and Compact Objects: Sanity Check

- Moment of inertia for a sphere:

$$I = cMR^2 \quad c = 2/5 \text{ for uniform } \rho$$

- Conserve angular momentum (to give maximal value):

$$I_{star}\omega_{star} \approx I_{wd}\omega_{wd} \approx I_{ns}\omega_{ns}$$

$$M_{star}R_{star}^{\ 2}\omega_{star} \approx M_{wd}R_{wd}^{\ 2}\omega_{wd} \approx M_{ns}R_{ns}^{\ 2}\omega_{ns}$$

- Typical periods of main sequence and horizontal branch stars:

$$\Rightarrow P_{wd} \approx P_{star}\left(\frac{R_{wd}}{R_{star}}\right)^2; \quad P_{star} \approx 2\times 10^6\text{s}$$

- Fastest rotating white dwarfs have $P \approx 1000$s – good consistency

$$R_{wd} = 0.01R_{star} \quad \Rightarrow P_{wd} \approx 200\text{s}$$

Stars and Compact Objects: Sanity Check

• Exactly the same argument for white dwarf to neutron star radius reduction leads to impressively rapid rotational periods having started with observations of main sequence stars

$$P_{ns} \approx P_{wd}\left(\frac{R_{ns}}{R_{wd}}\right)^2; \quad P_{wd} \approx 1000\mathrm{s}$$

$$R_{ns} = 0.002R_{wd} \quad \Rightarrow P_{ns} \approx 4\times10^{-3}\mathrm{s}$$

• Predicted period from angular momentum considerations in excellent agreement with periods of young pulsars but is it necessary to invoke such small objects to explain pulsars? Could one spin-up a white dwarf to provide an alternative explanation for pulsars?

Stars and Compact Objects: Sanity Check

• Maximum rotation rate from equating centripetal force at surface to gravity at surface:

$$\omega^2 R = \frac{GM}{R^2}; \quad P = 2\pi / \omega$$

rearrange relation to obtain minimum period in terms of mass and radius of the object:

$$\Rightarrow P_{\min} = 2\pi \sqrt{\frac{R^3}{GM}}$$

substituting now familiar numbers for mass and radius of white dwarf gives:

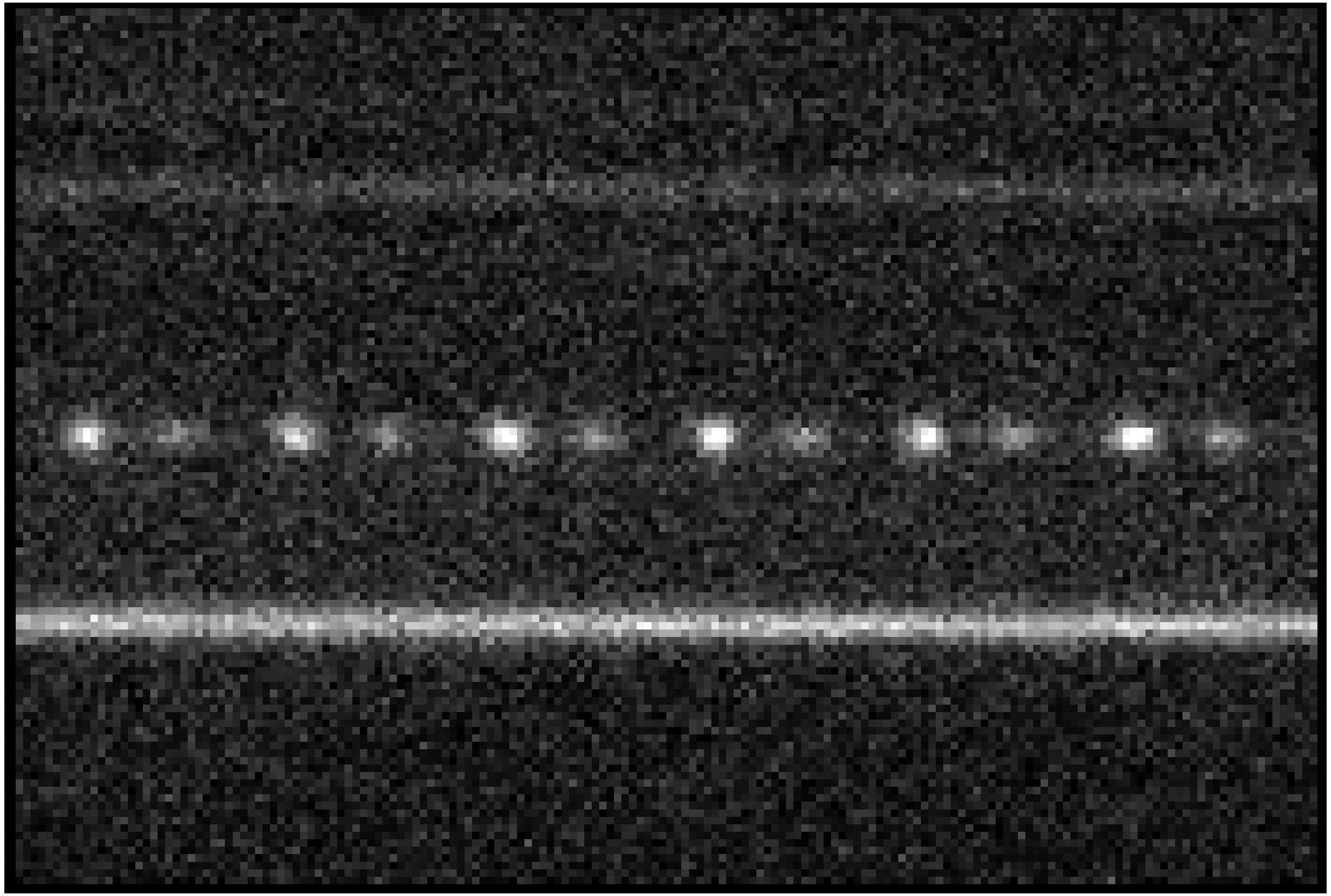
$$\approx 7\text{s}$$

• Minimum period is 3- to 4-orders of magnitude too large to explain observed pulsar periods

SN1987A in the LMC – no pulsar detected to date

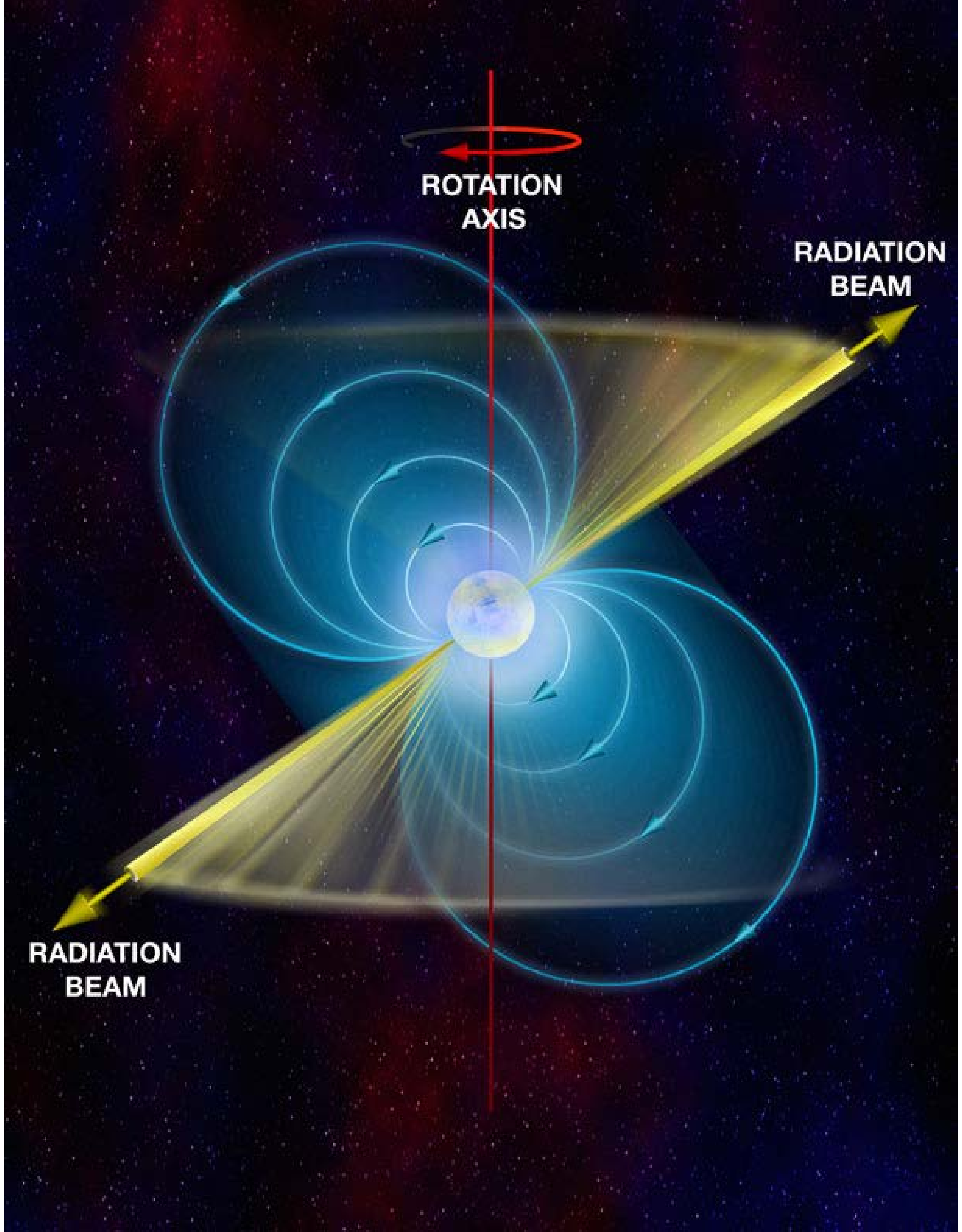
- Crab Supernova Remnant, powered by the rotational kinetic energy of the rapidly spinning neutron star (pulsar) created in the core collapse that produced the supernova

Optical time sequence (x-axis) of the Crab Pulsar and nearby (constant) objects showing double-pulse (per orbit) with a period of 0.033s

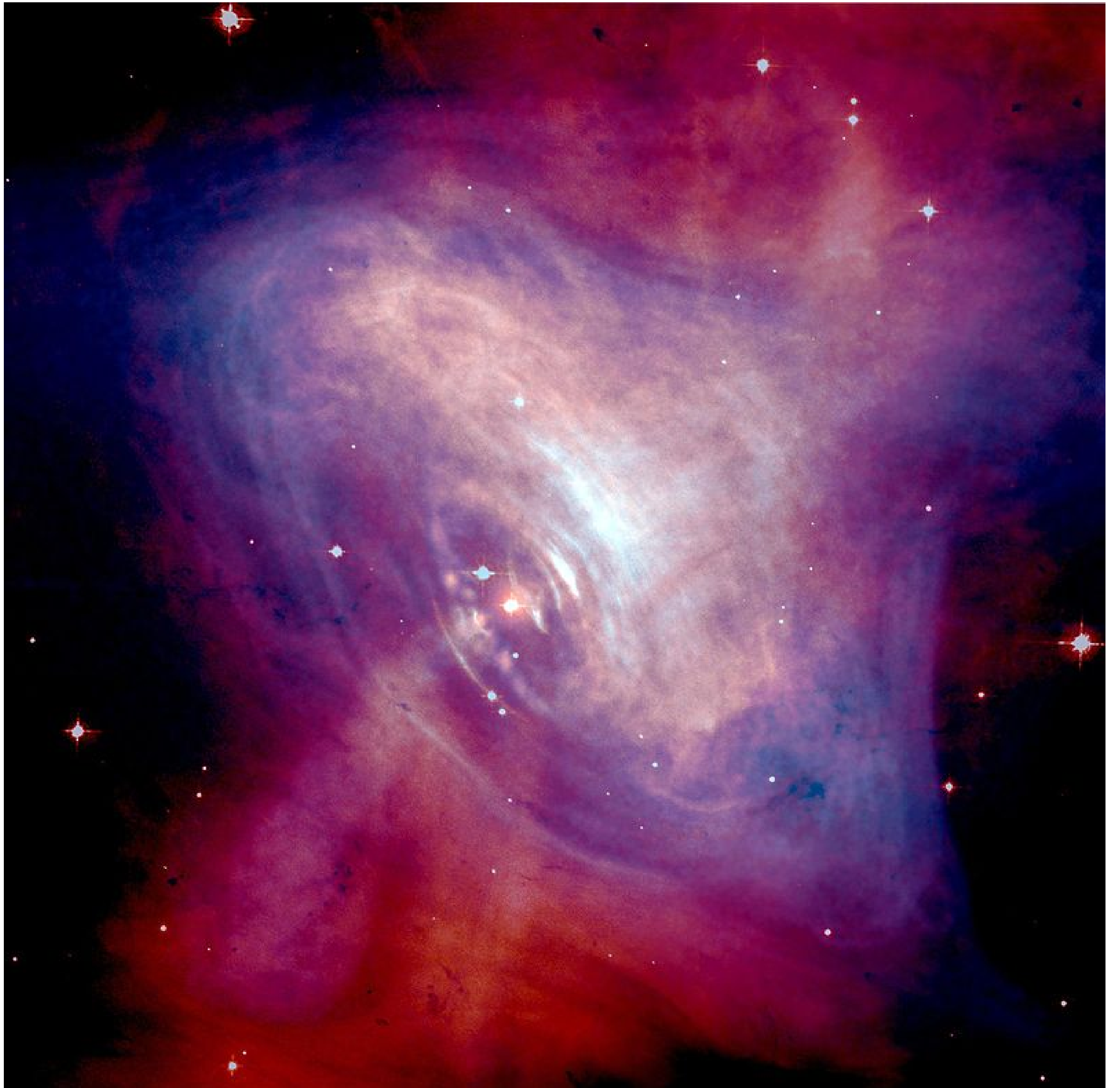


Time Sequence of Crab Pulsar (VLT KUEYEN + FORS2 + FIERA)

ESO PR Photo 40h/99 (17 November 1999)


ROTATION
AXIS
RADIATION
BEAM
RADIATION
BEAM

X-ray optical (Chandra and HST) image of the Crab Pulsar.
2.5 arcmin across with distance ~2kpc – radius of approx. 0.5pc



Direct Detection of Neutron Stars (Pictures!)

• Prognosis appears unpromising: $L = 4\pi R^2 \sigma T^4; \quad R_{ns} \approx R_{sun} / 50000$

but early in life before cooled: $T \geq 10^6\,\mathrm{K} \quad \Rightarrow L_{ns} \geq L_{sun}$

however Wien's Law $\Rightarrow \lambda_{\mathrm{max}} \leq 3\mathrm{nm} \qquad \mathrm{X\text{-}rays}$

• Rich variety of phenomena now known. Powerful tests of General Relativity using close binary pulsars now possible – multiple Nobel Prizes!

• Pictures - can obtain direct optical picture of neutron star – it moves as well!

Distance: 117pc±12

Proper-Motion: 0.3 arcsec per year

Velocity 185km/s

Age: 500000yr

Neutron Stars: Maximum Mass

• In Lecture 19 we combined the value of the central pressure for a system in hydrostatic equilibrium with the density-mass relationship for a degenerate system with relativistic particle velocities to show there is a maximum mass for which such a system is stable – the Chandrasekhar Mass $\approx 1.46 M_{sun}$ for a system supported by electron degeneracy pressure

• What is the equivalent mass for a neutron star?

• Answer is of interest because if the collapse of the stellar core is not halted by degeneracy pressure then a **black hole** will be created with radius:

$$R_{Sch} = \frac{2GM}{c^2} \approx 3 \frac{M}{M_{sun}} \text{km}$$

• Note radius very similar to that of neutron star, cf, white dwarf to neutron star

Neutron Stars: Maximum Mass

• Degeneracy pressure for relativistic particle velocities from Lecture 8:

$$P_{e,\deg} = \frac{hc}{8}\left(\frac{3}{\pi}\right)^{1/3} \frac{1}{m_H^{4/3}}\left(\frac{\rho}{\mu_e}\right)^{4/3}$$

• Result of the attempt to calculate the mass-radius relation for such a system from Lecture 19. Now included the dependence on the #electrons per nucleon ($1/\mu_e$):

$$\left(\frac{\rho}{\mu_e}\right)^{4/3} \propto GM^{2/3}\rho^{4/3}$$

• Evaluate Chandrasekhar mass, retaining dependence on $1/\mu_e$. For neutron star $\mu_e = 1$ and maximum mass is 4 times mass of white dwarf:

$$M_{Chand} \approx 1.46\left[\frac{2}{\mu_e}\right]^2 \mathrm{M_{sun}}$$

$$M_{Chand,ns} \approx 5.8\mathrm{M_{sun}}$$

Neutron Stars: Maximum Mass

• The simple-minded argument produces a value for the maximum neutron star mass that is too large

• The argument is sound as far as it goes but two effects have not been incorporated in the simple treatment

- particle kinetic energies are by definition comparable to the rest-mass energies and require General Relativistic treatment, which in fact lowers the estimate by a factor of ~8
- at nuclear densities the particles no longer behave like a perfect gas (particle interactions are important) and the effect raises the estimate by a factor ~4

• Upshot is a predicted value of 2-3M_{sun} but the equation of state for material at such densities is not fully understood and the prediction remains uncertain

Spectroscopic Binaries (Lecture 4)

• For binary in a circular orbit the velocity around the orbit is constant and $v=2\pi r/P$

• Measured radial velocity $v_r = v \sin i$, where again, the inclination is unknown

• Stars take same time to complete orbit and

$$\frac{v_1 \sin i}{v_2 \sin i} = \frac{v_{1r}}{v_{2r}} = \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

and, as for visual binaries, for double-lined objects without known distance, can determine the mass ratio for the stars

Spectroscopic Binaries

- $vP = 2\pi r$ for both components in orbit and $vP \sin i = 2\pi r \sin i$
- Substitute for r in Keplers 3rd Law and multiply by $\sin^3 i$:

$$(m_1 + m_2)\sin^3 i = \frac{4\pi^2 (r_1 + r_2)^3 \sin^3 i}{GP^2} = \frac{P}{2\pi G}(v_{1r} + v_{2r})^3$$

$$\Rightarrow m_1 + m_2 \geq \frac{P}{2\pi G}(v_{1r} + v_{2r})^3$$

where quantities on rhs are observable but only a limit on the sum of the masses for double-lined systems

Spectroscopic Binaries

Single-lined system, can determine only one velocity, v_{1r} , however, write v_{2r} in terms of mass and v_{1r}

$$v_{2r} = \frac{m_1}{m_2} v_{1r}$$

Keplers 3rd Law then can be written

$$(m_1 + m_2)\sin^3 i = \frac{P}{2\pi G} v_{1r} \left(1 + \frac{m_1}{m_2}\right)^3$$

rearrange

Gives the "mass function" on the lhs in terms of observables on the rhs

$$\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{P}{2\pi G} v_{1r}^3$$

Spectroscopic Binaries

- Can obtain a lower limit to mass of the unseen companion object, m_2, from noting that $m_1 > 0$ and $\sin i \leq 1$ and then

$$\frac{m_2^3}{(m_1 + m_2)^2} \sin^3 i = \frac{P}{2\pi G} v_{1r}^3 < m_2$$

- Observations of single-lined spectroscopic binaries provide constraints on the masses of compact remnants – neutron stars and black holes
- In case of black holes, spectroscopic binaries provide the only constraints on the masses of such objects
- Double pulsar binaries provide "radio"-version of double-lined spectroscopic binaries
- PSR J1638-4715 shows eclipses – so inclination known to be =90° and mass tightly constrained

Compact Remnant Mass Determinations

• The number and space density of neutron stars and black holes in binary systems is small:

- Initial Mass Function is steep and only a small fraction of stars possess ZAMS M>10M_{sun}
- Binary systems have to survive supernovae explosion

• Distance to systems is large, typically several kiloparsecs, and objects are generally close to the mid-plane of the Galaxy [Why?] and interstellar dust produces significant extinction

• Systems detected via high-energy radiation emitted due to accretion by compact object as companion star overflows Roche Lobe – X-Ray Binaries – or via radio pulses from pulsars

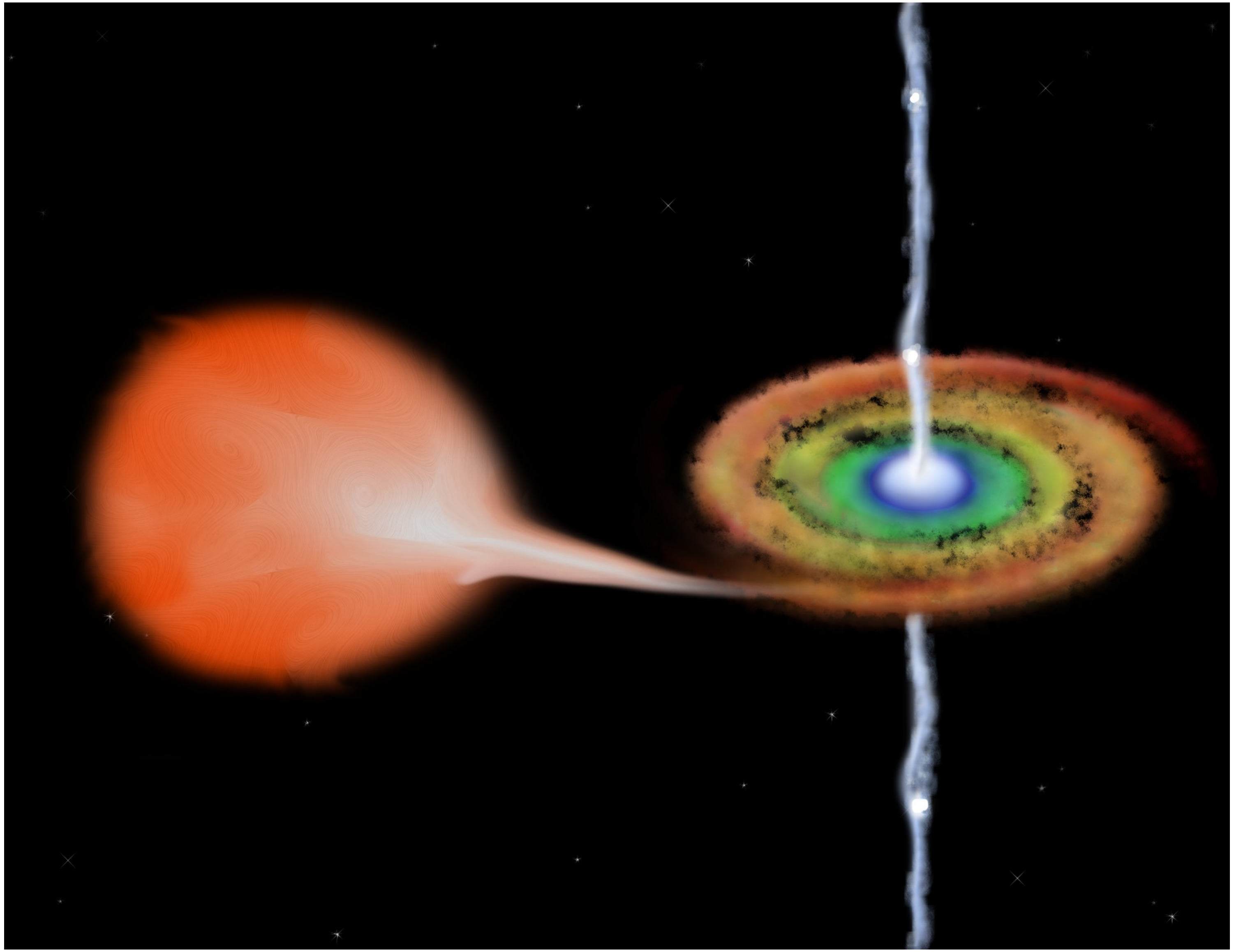
Compact Remnant Mass Determinations

• Generally, can only observe systems with high-luminosity companions (necessary in order to measure radial velocity around orbit), which are either in an advanced state of post main sequence evolution, or have such large masses that their main sequence masses are not well-determined

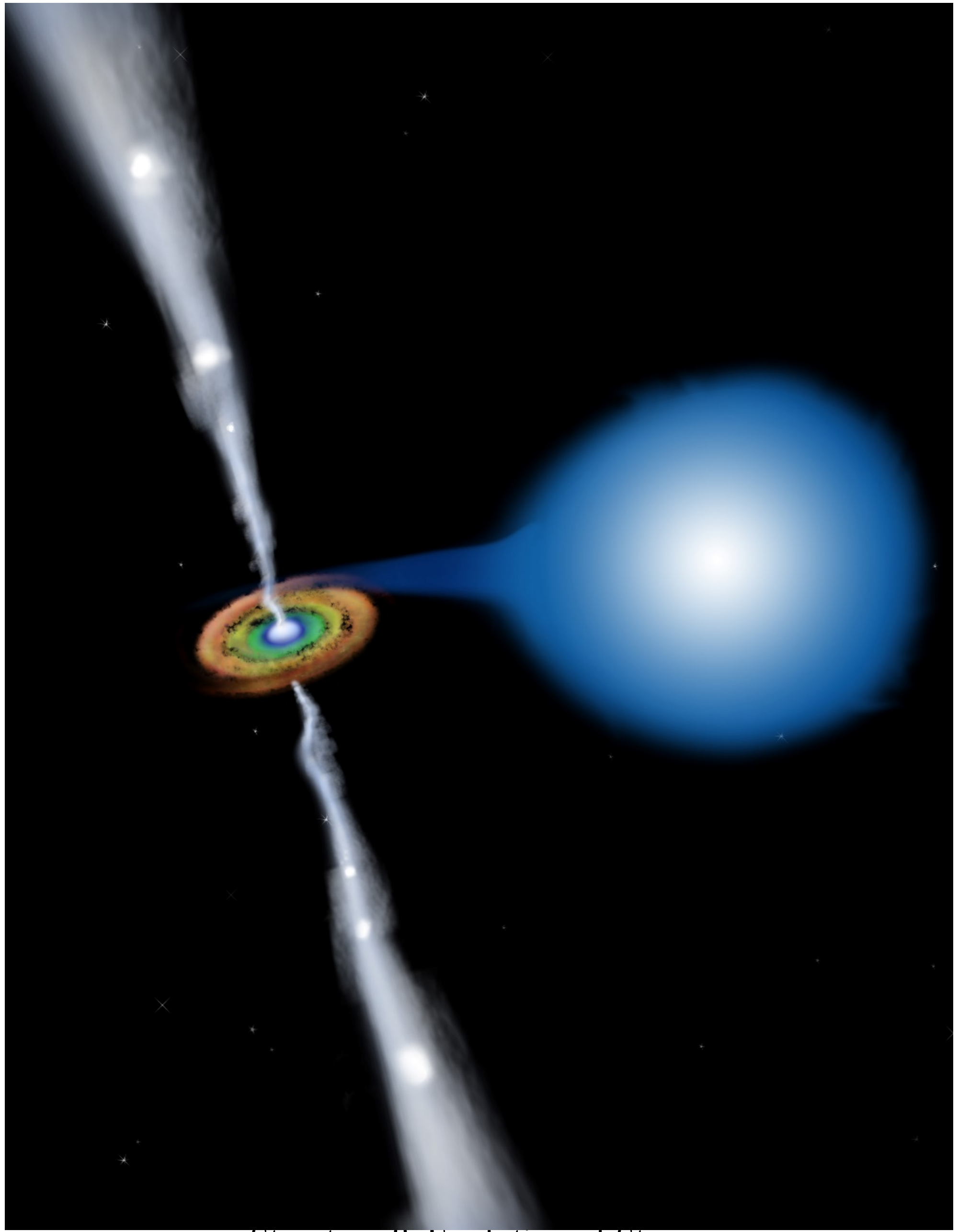
• Even when visible, mass estimates for luminous companion object not straightforward and thus there is significant uncertainty in constraining m_2 from the “mass function”

• Inclination always a difficulty unless eclipses present, however, because accretion is taking place, the luminous companion fills Roche Lobe and modelling of photometric variations (due to projected shape change of distorted photosphere around the orbit) can give quite tight constraints on the inclination

LMXB



HMXB



Neutron Star Mass Estimates from Binary Systems

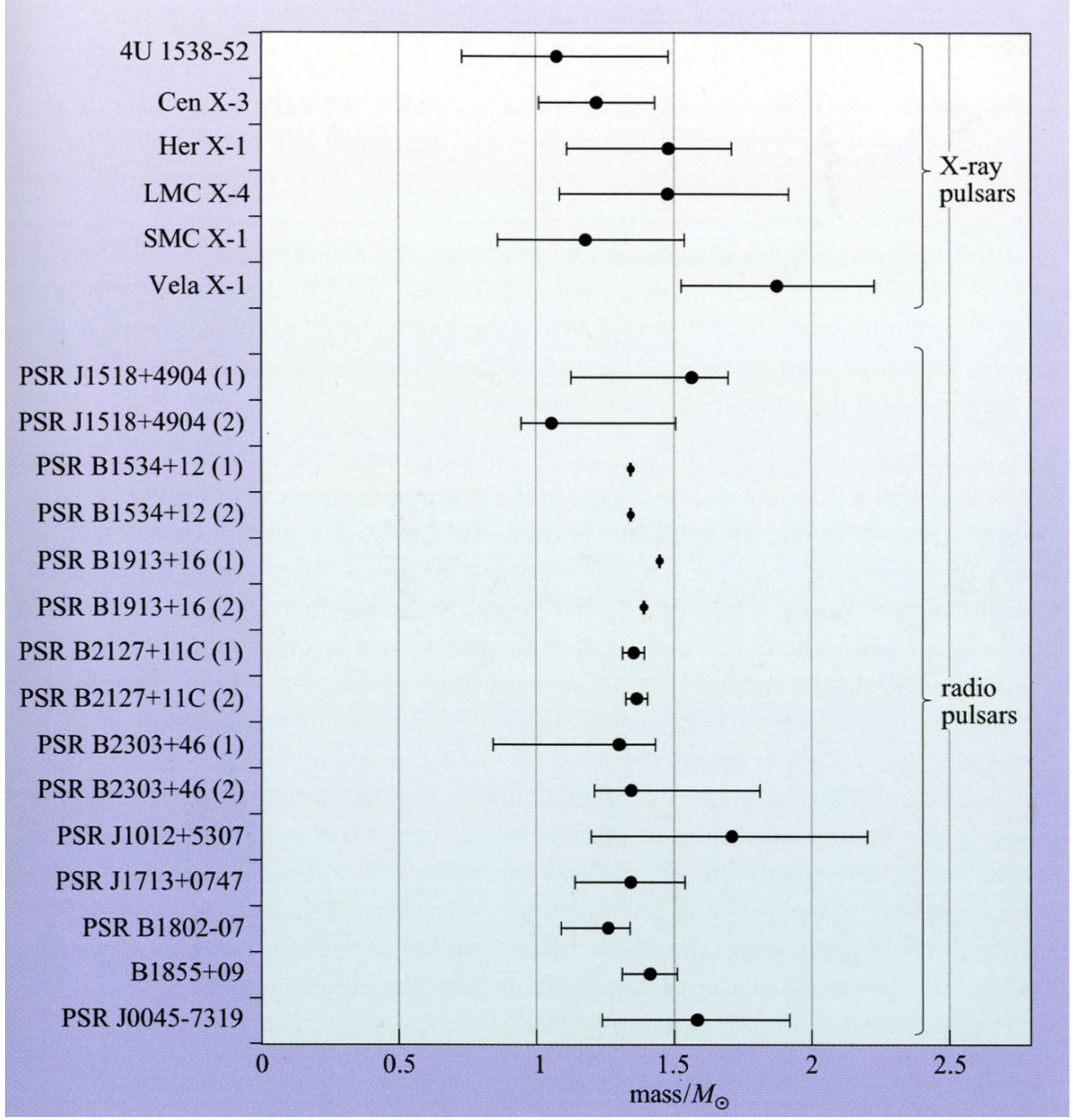


Figure 9.21 The measured masses of neutron stars. The upper six cases are X-ray pulsars that are in interacting binaries, the lower ten are radio pulsars that are in non-interacting binaries. In the case of radio pulsars, five of the systems are double neutron star systems, and the masses of each of these neutron stars (distinguished by (1) or (2) in the name) are shown. (Data compiled by S. Clark (University College London))

Black Hole Mass Estimates from Binary Systems

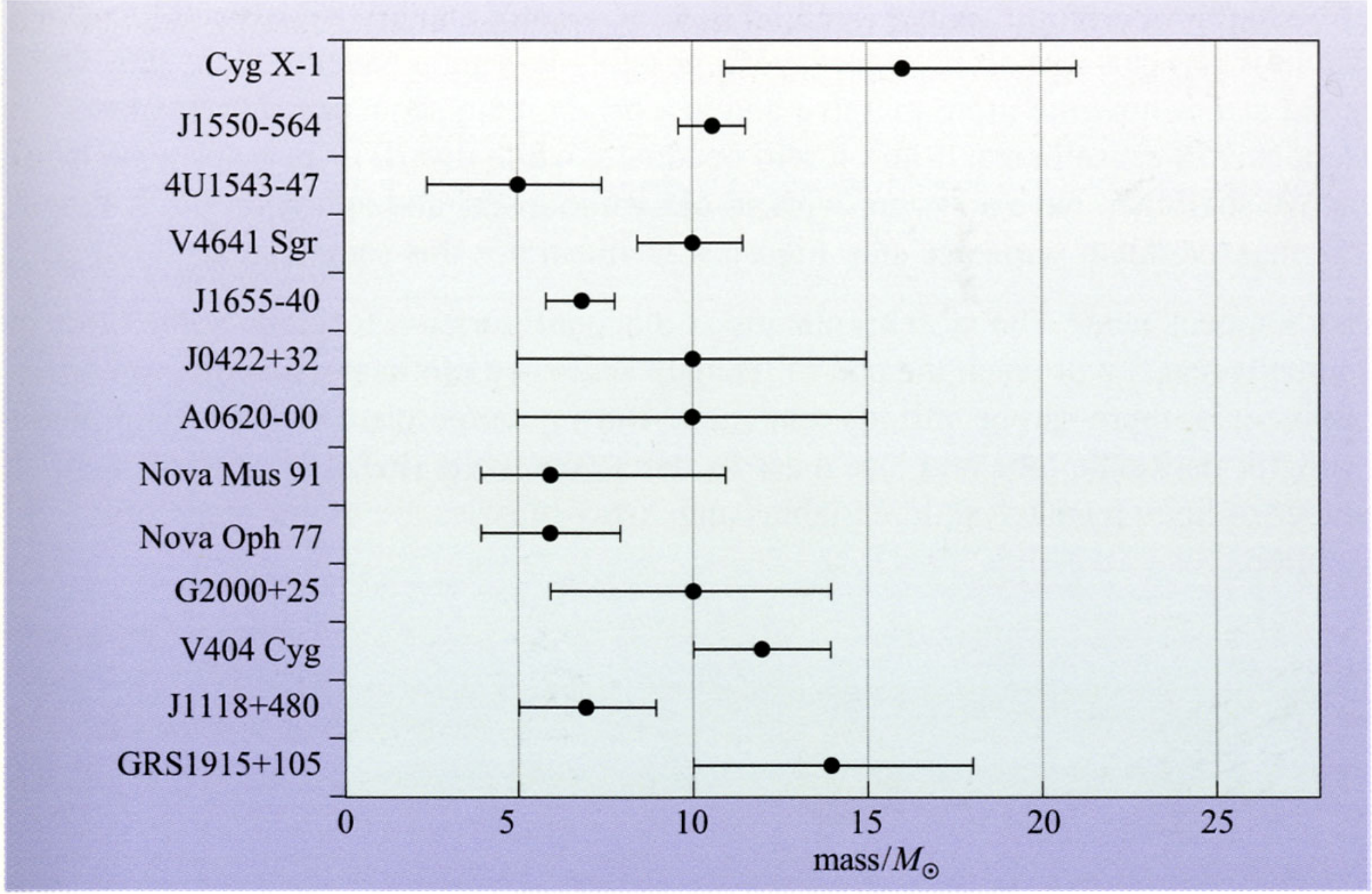


Figure 9.22 A summary of mass determinations, with uncertainties, of a selection of black hole candidates. (Data compiled by S. Clark (University College London))

Observations of Gravitational Waves from a Binary Black Hole Merger
2016 Abbott et al.
Phys. Rev. D 116 160211

Source GW150914

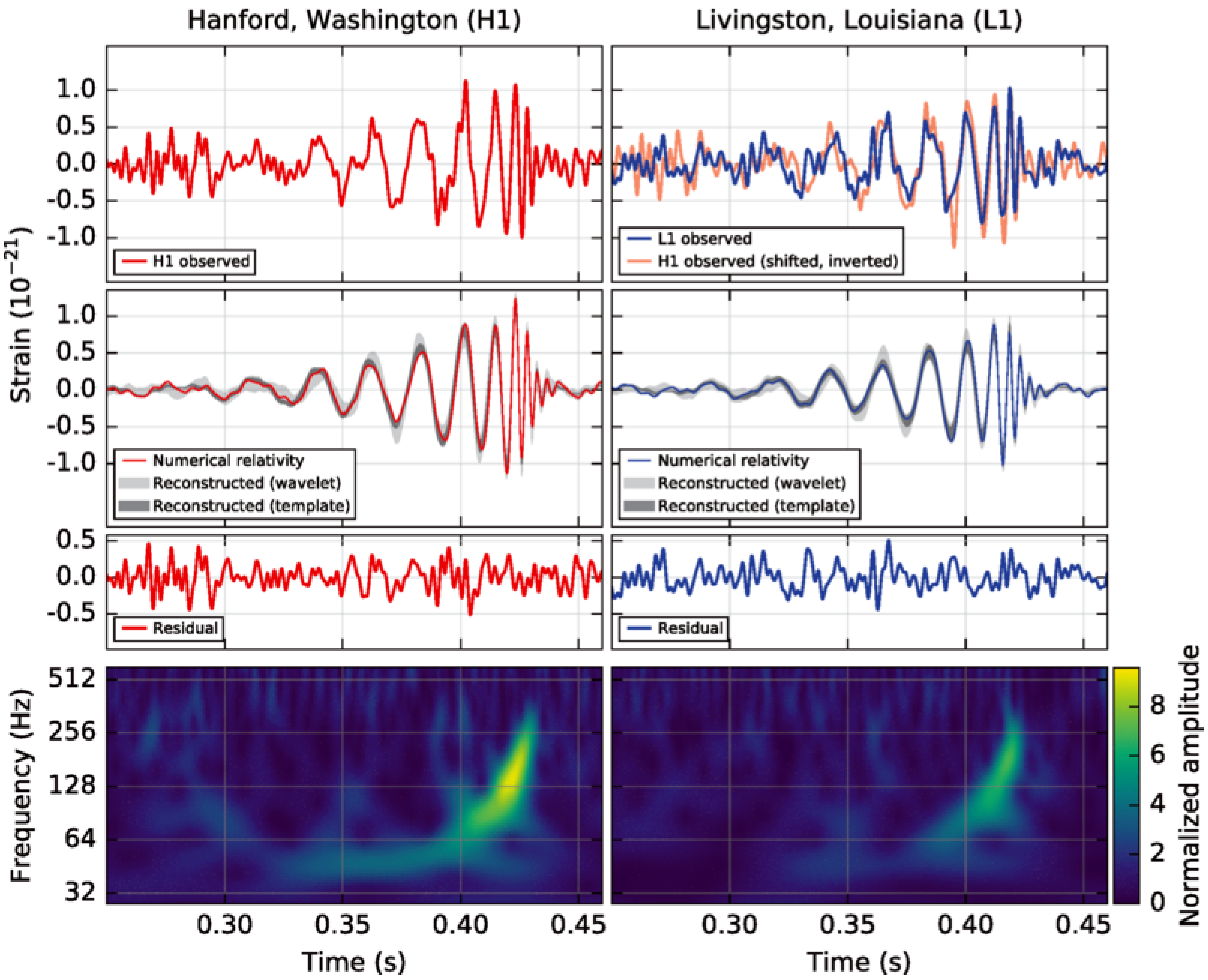


TABLE I.

Source parameters for GW150914. We report median values with 90% credible intervals that include statistical errors, and systematic errors from averaging the results of different waveform models. Masses are given in the source frame; to convert to the detector frame multiply by (1+z). The source redshift assumes standard cosmology.

Primary black hole mass	$36+5-4 M_\odot$
Secondary black hole mass	$29+4-4 M_\odot$
Final black hole mass	$62+4-4 M_\odot$
Final black hole spin	0.67+0.05−0.07
Luminosity distance	410+160−180 Mpc
Source redshift z	0.09+0.03−0.04

More coming: GW151226 - 14 and $7M_\odot$ black holes
GW170608 – 12 and $7M_\odot$ black holes

Connection with “failed” supernovae? Star of the Week #3

Lecture 21: Summary

• Simple scaling of dependence of degeneracy pressure on mass and number of particles gives radius of neutron stars only ~10km
• Check on the observed rotational periods of stars, white dwarfs and neutron stars (pulsars) produces reassuring consistency based on conservation of angular momentum
• Spinning white dwarfs incapable of explaining observed pulsar periods and now excellent observational evidence confirming properties of neutron stars
• Maximum mass of a neutron star is not well determined, although simple scaling argument based on Newtonian gravity and a perfect gas gives value of the right order
• Current limits on the masses of both neutron stars and stellar mass black holes rely on determination of the mass-function in binaries plus a few pulsar-pulsar binary systems. Gravitational waves coming though.

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