Structure and Evolution of Stars

Lecture 19: White Dwarfs

- Physical Properties of White Dwarfs
    - density-mass relationship
    - mass-radius relationship
    - Chandrasekhar limiting mass
    - Cooling of White Dwarfs
    - White Dwarf Radii: Sirius A and B: gravitational redshift
- Application of Cooling Timescale Calculation
    – age of Galactic Disk
- *Star(s) of the Week* #5: The White Dwarfs in the globular cluster Omega Centauri

Mass-Radius Relationship for Degenerate Systems

- Used Hydrostatic Equilibrium in Lecture 6 to give central pressure for uniform density system:

$$P_c = \frac{GM^2}{8\pi R^4}; \quad M = \frac{4}{3}\pi R^3 \rho$$

- Eliminate radius:

$$\Rightarrow P_c \propto GM^{2/3}\rho^{4/3}$$

- Equation of state for degenerate gas of non-relativistic particles (Lecture 8):

$$P = K_{NR}\rho^{5/3}$$

- Equate expressions for the pressure:

$$\Rightarrow K_{NR}\rho^{5/3} \propto GM^{2/3}\rho^{4/3}$$

- Rearrange to give dependence of density on mass of the system:

$$\Rightarrow \rho \propto M^2$$

Mass-Radius Relationship for Degenerate Systems

- Eliminate density from the 2 equations: $\rho \propto M^2; \quad M = \frac{4}{3}\pi R^3 \rho$

$$\Rightarrow MR^3 = const$$

- To give the mass-radius relationship for a degenerate system in hydrostatic equil: $\Rightarrow R \propto M^{-1/3}$

- The shrinking of the degenerate Helium core as mass increases critical to explaining the evolution away from the main sequence of low mass (M<2M_{sun}) stars – H-burning shell – and the shrinking of the Carbon-Oxygen core explains behaviour of stars (M<10M_{sun}) away from the horizontal branch

Chandrasekhar Mass

• Relationship applies equally to the degenerate Helium or Carbon-Oxygen cores revealed following the planetary nebula phase and more massive white dwarfs thus possess smaller radii

• What is the maximum mass that a white dwarf can possess and degeneracy pressure still hold the system up against gravity?

• As the mass of a white dwarf increases, the density increases rapidly (as M^2), electrons are packed more tightly and momentum thus increases (Heisenberg Uncertainty Principle). Velocities become relativistic and form of equation of state is modified (Lecture 8)

Chandrasekhar Mass

- Repeat analysis as for non-relativistic case:

$$P_c = \frac{GM^2}{8\pi R^4}; \quad M = \frac{4}{3}\pi R^3 \rho$$

$$\Rightarrow P_c \propto GM^{2/3}\rho^{4/3}$$

- Change in form of equation of state:

$$P = K_R \rho^{4/3}$$

- Leads to relation where density term cancels on both sides! Limiting case where density approaches infinity or, put another way, there is no radius at which the system is stable

$$\Rightarrow K_R \rho^{4/3} \propto GM^{2/3}\rho^{4/3}$$

Chandrasekhar Mass

• Can include the constants, which depend on fundamental physics in the case of K_R, and on the composition of the system, which determines the number electrons (providing the pressure) per nucleon

• Evaluation of the expression produces the result $M_{Ch}=1.46M_{sun}$ for a composition where $Z/A=0.5$, which is the case for Helium and essentially any other higher mass nuclei such as Carbon and Oxygen – i.e. 0.5 of an electron per nucleon

• The Chandrasekhar mass sets the maximum achievable mass for a degenerate system supported by electron degeneracy pressure.

The Fate of White Dwarfs

• Following a planetary nebula, degenerate Carbon-Oxygen core of star revealed as a white dwarf. No internal energy generation mechanism but the white dwarf is radiating away its internal thermal energy which is stored in the form of the kinetic energy of the ions

• Pressure is due to the degenerate electrons which move large distances, conducting heat very efficiently (energy transport via conduction) and the white dwarf is an essentially isothermal, homogeneous sphere

• Leon Mestel developed a model to predict the cooling of a white dwarf, consisting of a degenerate, isothermal core with nearly all the mass, surrounded by a thin envelope consisting of non-degenerate material behaving as an ideal gas

White Dwarfs: Central Temperature

• The model is very similar in concept to the procedure we used to calculate the properties of a fully convective star with a thin outer radiative envelope

• The thin outer layer in the case of the white dwarf is going to act like a blanket, determining how much of the internal heat can escape and be radiated away

• Assume a sharp transition between the degenerate core and the thin envelope at radius r_b, with the envelop mass insignificant, i.e. $m(r=r_b)\approx M$, and demand a boundary condition such that the degeneracy pressure and ideal gas pressure are equal at the transition radius

White Dwarfs: Central Temperature

- For the envelope, the stellar structure equations give:

$$\frac{dP}{dr} = -\rho \frac{GM}{r^2}; \quad \frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L}{4\pi r^2}$$

- Adopt a Kramers opacity law, substituting for ρ using $P=nkT$:

$$\kappa = \kappa_0 \rho T^{-7/2} = \frac{\kappa_0 \mu m_H}{k} P T^{-9/2}$$

- Substitute for κ in 2nd equation above and divide result into the first equation:

$$\Rightarrow PdP = \frac{16\pi ackG}{3\kappa_0 \mu m_H}\frac{M}{L} T^{15/2} dT$$

- Integrate from surface ($P=T=0$) inward to give:

$$P(T) = \left(\frac{64\pi ackG}{51\kappa_0 \mu m_H}\right)^{1/2}\left(\frac{M}{L}\right)^{1/2} T^{17/4}$$

White Dwarfs: Central Temperature

- Obtain density as a function of T using $P=nkT$:

$$\Rightarrow \rho(T) = \left(\frac{64\pi ac\mu m_H G}{51\kappa_0 k}\right)^{1/2}\left(\frac{M}{L}\right)^{1/2} T^{13/4}$$

- At boundary radius the pressures must be equal:

$$\left[\frac{k}{\mu_e m_H}\rho T\right]_b = \left[K_1'\left(\frac{\rho}{\mu_e}\right)^{5/3}\right]_b$$

- Eliminate density between the two equations:

$$\frac{L}{M} = \frac{64\pi ac\mu m_H^4 G K_1'^3}{51k^4\kappa_0\mu_e^2} T_c^{7/2} \qquad [LMT\text{ - rel}]$$

- Include constants for a typical white dwarf with composition 50% C and O:

$$\Rightarrow T_c \approx 4\times 10^7\left(\frac{L/L_{sun}}{M/M_{sun}}\right)^{2/7}\ \mathrm{K}$$

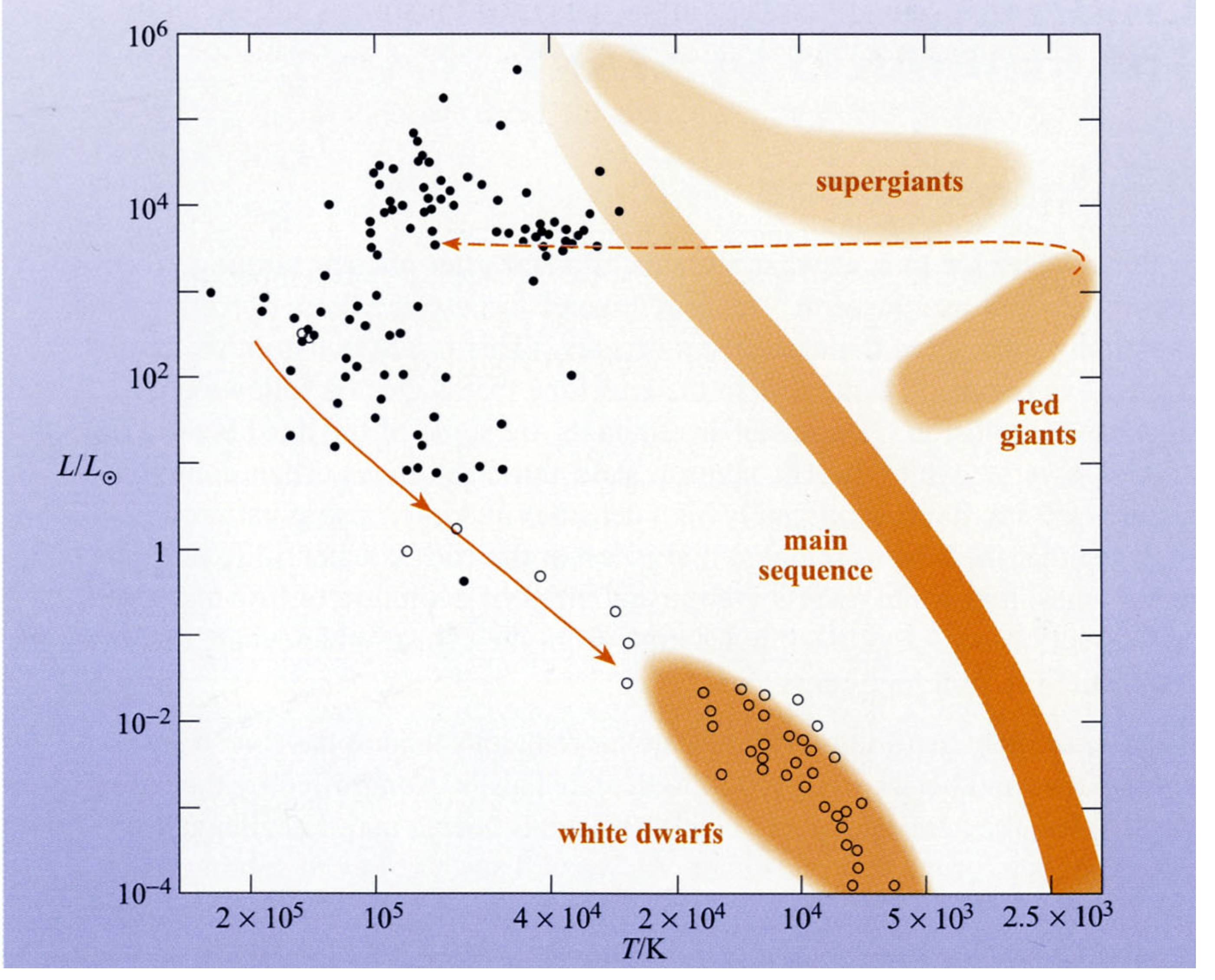


Figure 9.1 The positions of central stars associated with planetary nebulae (dots) and of white dwarfs (open circles) on the H–R diagram. Also shown (solid line) is the evolutionary track that would be followed by a star of constant radius as it cools and (dashed line) a schematic evolutionary track between the regions occupied by AGB stars (Section 8.2.1) and by the central stars of planetary nebulae.

White Dwarfs: Cooling Time

• Total energy is due to the thermal motions of the nuclei in the core:

$$KE_{nuclei} = \frac{3}{2}\frac{k}{\mu_{nuc} m_H} M T_c$$

• White dwarf luminosity is equal to the rate of change of the thermal energy:

$$L = -\frac{dKE_{nuc}}{dt} = -\frac{3}{2}\frac{k}{\mu_{nuc} m_H} M \frac{dT_c}{dt}$$

$$= -\frac{3}{7}\frac{k}{\mu_{nuc} m_H} M \frac{T_c}{L}\frac{dL}{dt}$$

• Using the *LMT*-relation see that the change in luminosity is a strong function of *T*, or, in other words, the cooling rate declines rapidly as *T* decreases

$$\Rightarrow -\frac{dL}{dt} \propto M T_c^6$$

White Dwarfs: Cooling Time

• Integrate penultimate equation on last Slide to calculate cooling time:

$$\tau_{cool} = 0.6 \frac{k}{\mu_{nuc} m_H} M \left(\frac{T_c}{L} - \frac{T_c^{'}}{L^{'}} \right)$$

• For significant cooling the equation simplifies (use *LMT*-relation), giving a simple relation in terms of the mass (which is constant) and the current luminosity:

$$\text{For } T_c^{'} >> T_c,\ T_c^{'} / L^{'} << T_c / L$$

$$\Rightarrow \tau_{cool} \approx 2.5 \times 10^6 \left(\frac{M / M_{sun}}{L / L_{sun}} \right)^{5/7} \text{yr}$$

$$\approx 2 \times 10^9 \text{yr for WD of } 1\text{M}_{\text{sun}} \text{ to reach } 10^{-4}\,\text{L}_{\text{sun}}$$

• Very long timescales to reach luminosities that are observed for many white dwarfs

White Dwarfs: Cooling Time

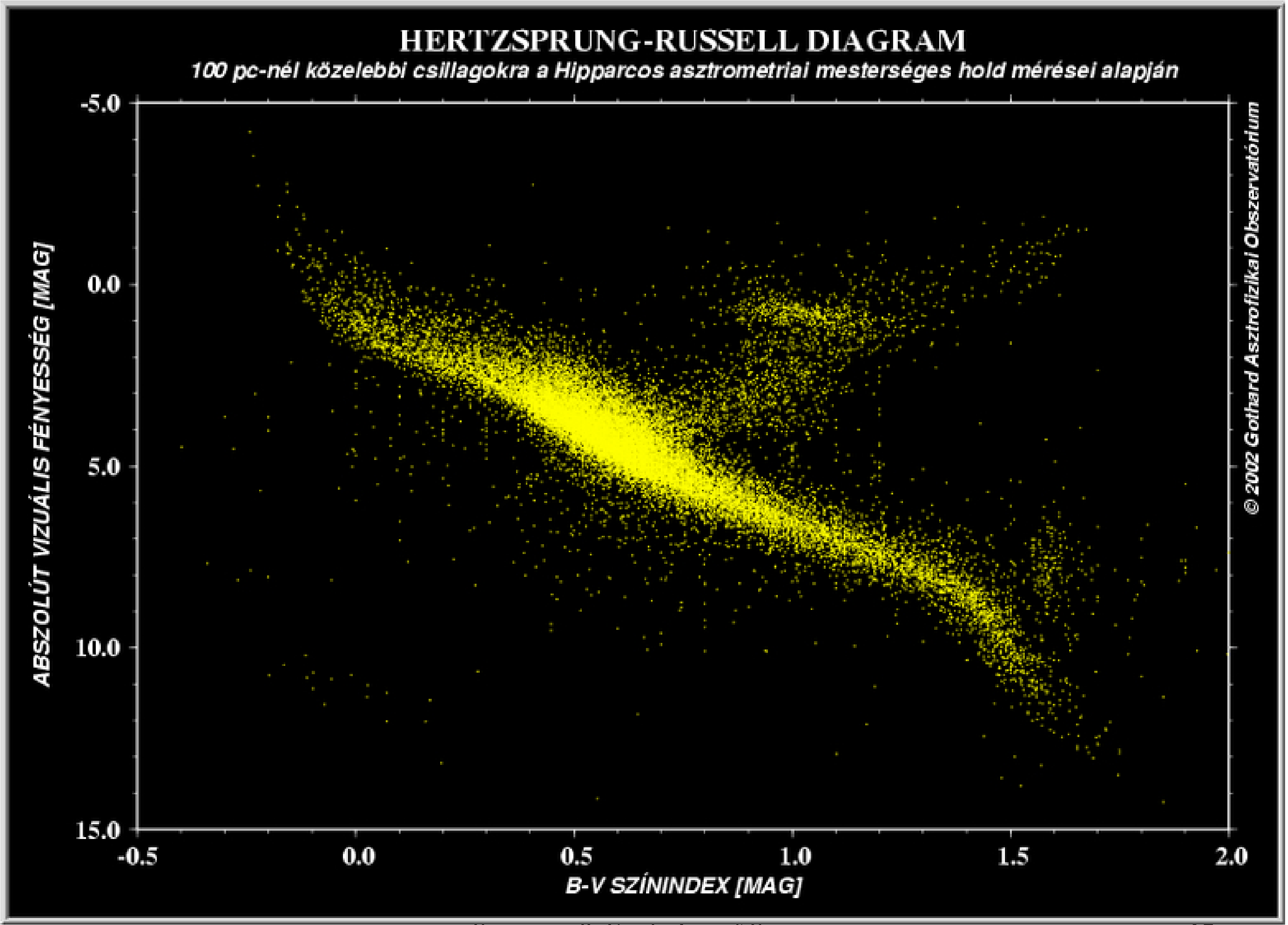
• Note that for once a simple estimate of the cooling time gives a poor result. Using the *LMT*-relation to give L in terms of M & T, with all the constants included in "*const*", along with the calculation of the initial T_c

$$\tau_{cool} \approx \frac{KE_{nuc}}{L} = \frac{3}{2}\frac{k}{\mu_{nuc} m_H}\frac{1}{const T_c^{5/2}}$$

$$\approx 10^8 \,\mathrm{yr} \text{ for } T_c = 4 \times 10^7 \,\mathrm{K} \text{ from earlier}$$

• Cooling behaviour is well-understood. Need to know make-up of degenerate core (He or mix of C and O) and composition of thin envelope (H or He) for detailed predictions

• For systems where can observe very faint (i.e. old) white dwarfs, have "clock" to estimate age – e.g. Galactic Disk


HERTZSPRUNG-RUSSELL DIAGRAM
100 pc-nél közelebbi csillagokra a Hipparcos asztrometriai mesterséges hold mérései alapján
-5.0
0.0
5.0
10.0
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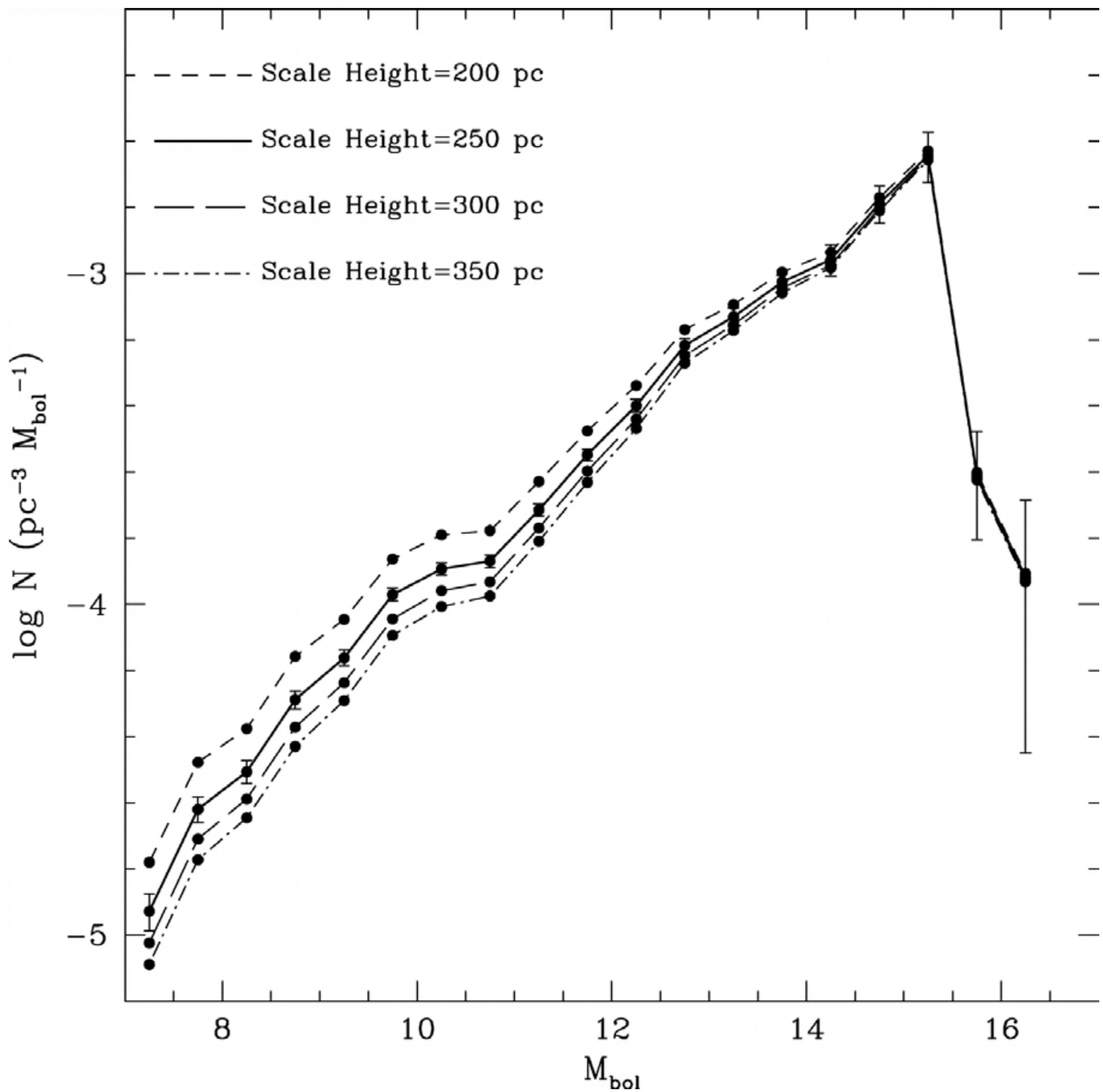
ABSZOLÚT VIZUÁLIS FÉNYESSÉG [MAG]
-0.5
0.0
0.5
1.0
1.5
2.0
B-V SZÍNINDEX [MAG]
© 2002 Gothard Asztrofizikai Obszervatórium

• Age of Galactic Disk from discontinuity in white dwarf luminosity function at $M_{bol}\approx15$ gives age=9±1.5Gyr

WD LF from Harris et al. 2006 AJ 131 571
Note sudden decline at $M_{bol}\approx15$
Uncertainty in scale height of the Galactic disk affects the determination at the bright end



White Dwarfs: Radii

- Discovery of companion Sirius B to Sirius A in 1862. Visual binary with known distance, parallax measurable = 0.377 arcsec, provides masses for the individual stars via orbit (Period 49.9 years)

- Properties of Sirius A:

$$B = -1.5; \quad M = 2.3\mathrm{M}_{\mathrm{sun}} \quad T_{eff} = 9900\mathrm{K}$$

- Properties of Sirius B:

$$B = +5.7; \quad M = 1.05\mathrm{M}_{\mathrm{sun}} \quad T_{eff} = 27000\mathrm{K}$$

- Take familiar *LRT* relation:

$$L = 4\pi R^2 \sigma T_{eff}^4; \quad L_A / L_B = 780$$

- Results in very small radius, equivalent to 5500km, less than radius of the Earth

$$\Rightarrow R_B = 0.0048 R_A \quad (R_B = 0.008\mathrm{R}_{\mathrm{sun}})$$

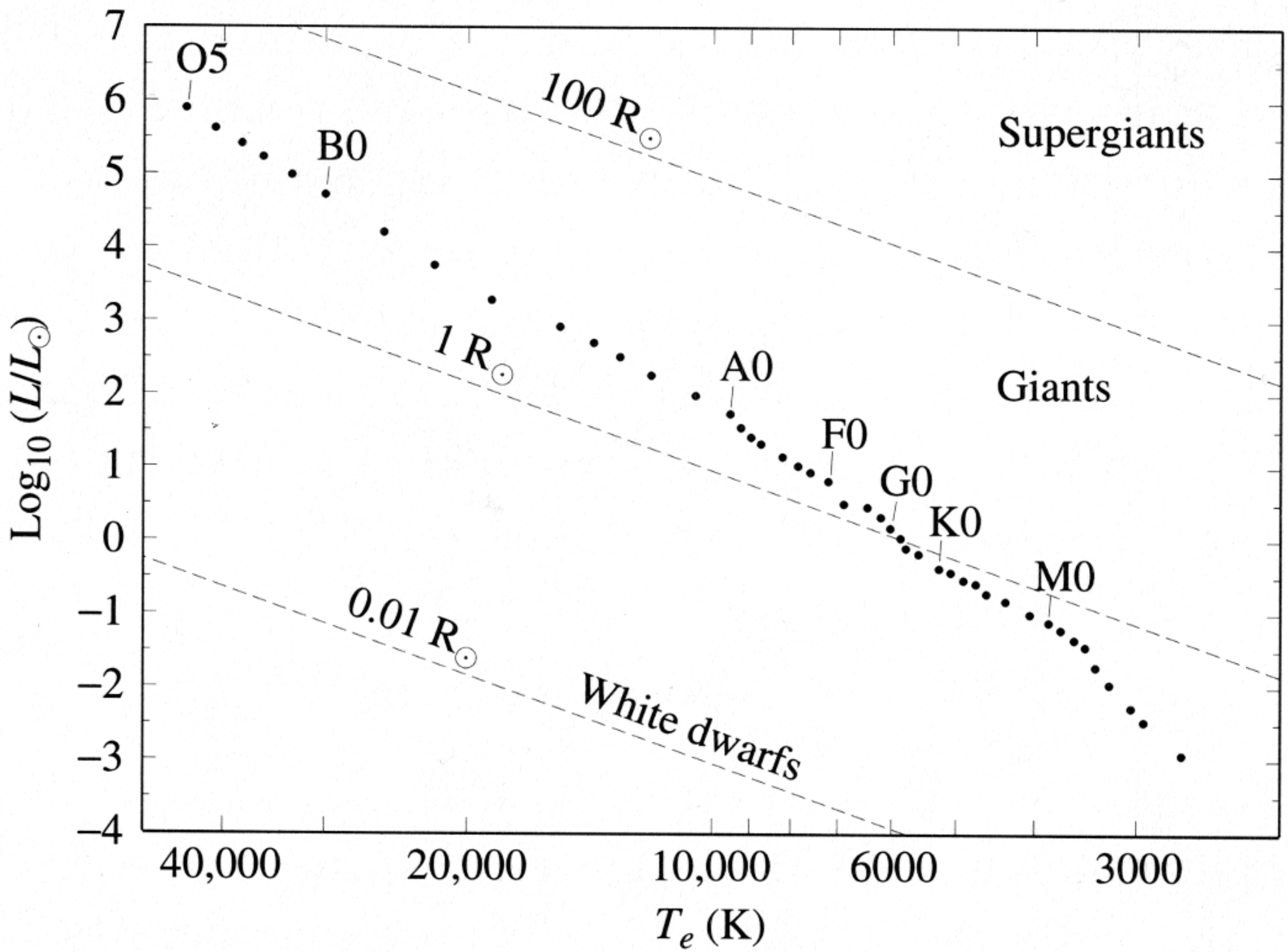


Figure 8.13 Lines of constant radius on the H–R diagram.

White Dwarfs: Radii

• White dwarfs of a given mass lie on tracks of constant radius in the HR-diagram:

$$\log L - 4\log T = const$$

• Recall the mass-radius relationship for degenerate systems and as mass remains unchanged, radius remains unchanged. Position along a locus of constant radius depends on the age of the white dwarf, via the cooling track

• Behaviour in HR-diagram in good agreement with theoretical predictions but more direct measurement of white dwarf radius would be reassuring

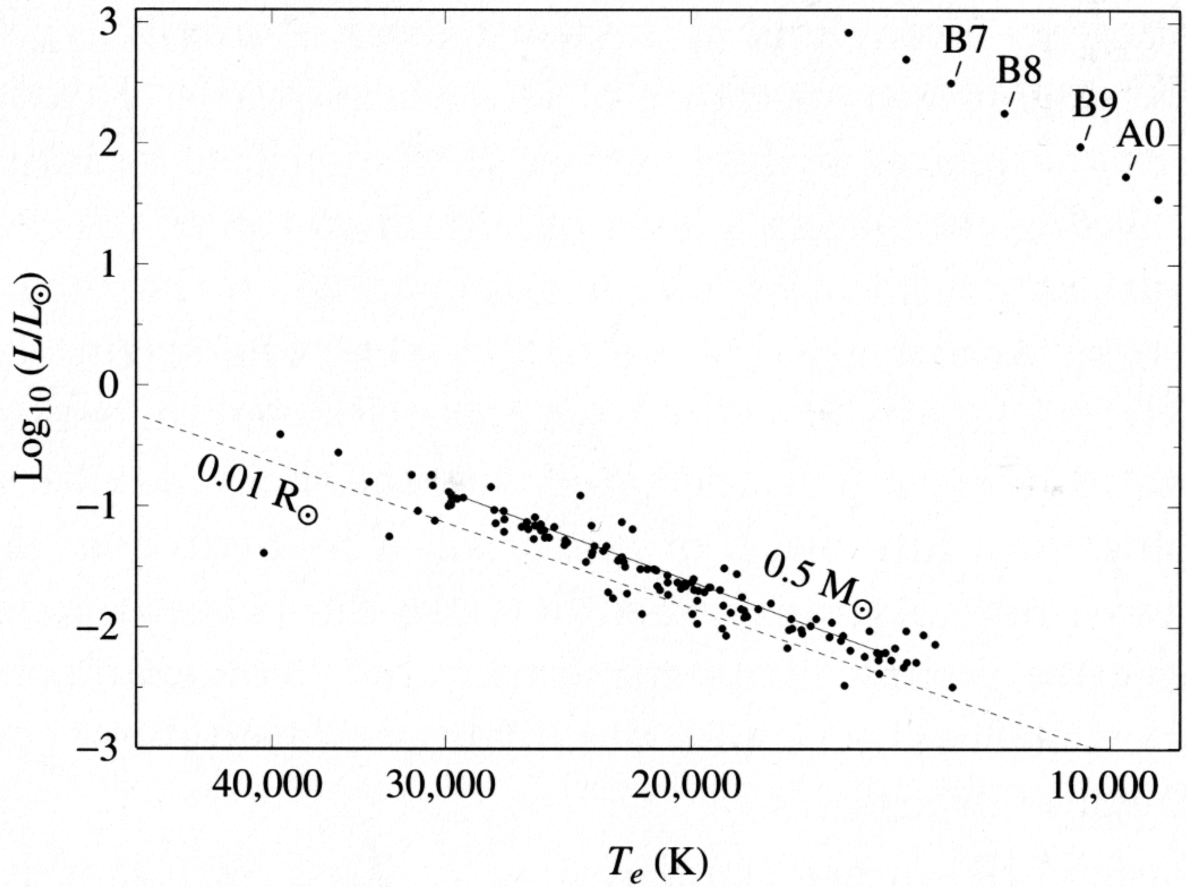


Figure 15.3 DA white dwarfs on an H–R diagram. A line marks the location of the 0.50 $M_\odot$ white dwarfs, and a portion of the main sequence is at the upper right. (Data from Bergeron, Saffer, and Liebert, *Ap. J.*, *394*, 228, 1992.)

White Dwarfs: Gravitational Redshift

- Gravitational redshift for photon escaping from gravitational potential from General Relativity:

$$z = \frac{\Delta\lambda}{\lambda} = \left[1 - \frac{2GM}{Rc^2}\right]^{-1/2} - 1 \approx \frac{GM}{Rc^2}$$

- More intuitively, photon "mass": $m = h\nu / c^2$

- Total energy of photon on surface:

$$h\nu - \frac{GmM}{R}$$

- Conserve energy as photon climbs out of the potential:

$$\Delta\nu = -\frac{GmM}{Rh} = -\frac{GM\nu}{Rc^2}$$

- Observations of Sirius B (need binary to provide wavelength zero-point). Direct confirmation of white dwarf radius:

$\mathrm{H}\alpha$(6564Ang) appears at 6566Ang

$$\Rightarrow \mathrm{z} = 3.0 \times 10^{-4} = \frac{GM}{Rc^2}$$

Stars of the Week #5: WDs in Omega Centauri

(Monelli et al. 2005, *Astrophysical Journal*, 621, L117, Bellini et al. 2013, *Astrophysical Journal*, 769, L32)

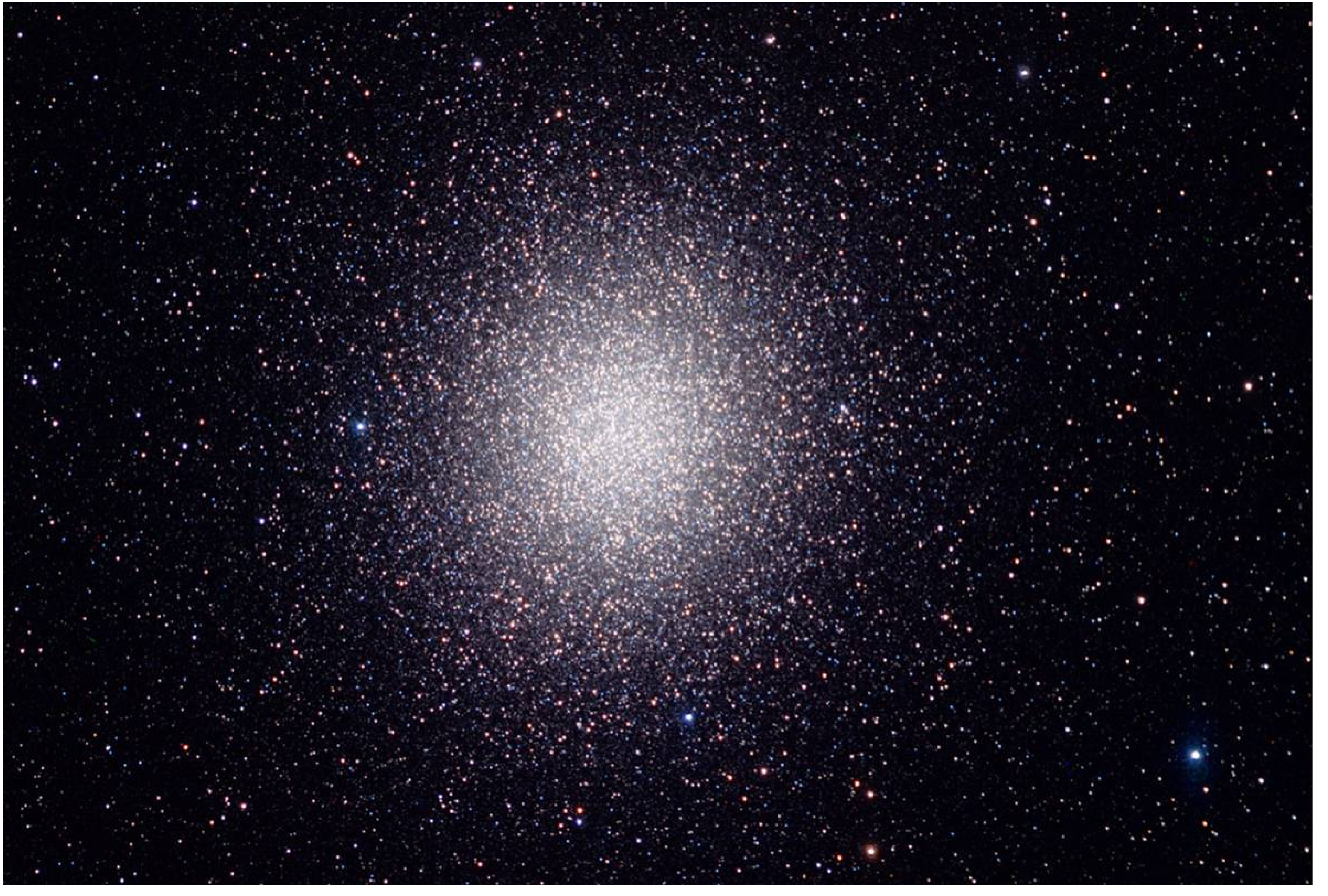
• Omega Centauri is the most massive globular cluster in the Galaxy: $M=5\times10^6 M_{sun}$

• Metallicity distribution shows 3 peaks [Fe/H]=-1.7:-1.5:-1.2 with a small fraction of relatively metal-rich stars near [Fe/H]=-0.5. Includes multiple He-normal MSs with [Fe/H] differences and a He-rich MS

• Omega Cen is not a simple globular cluster but is much studied in order to understand properties of stellar populations of different ages, metalicities and Helium-abundances

• Hubble Space Telescope ACS images covering 9×9 arcmin in *B* and *R* with ~1000s total integrations. Point-spread-function photometry obtained for 1.2×10^6 stars. *B* versus *B*-*R* colour-magnitude diagram

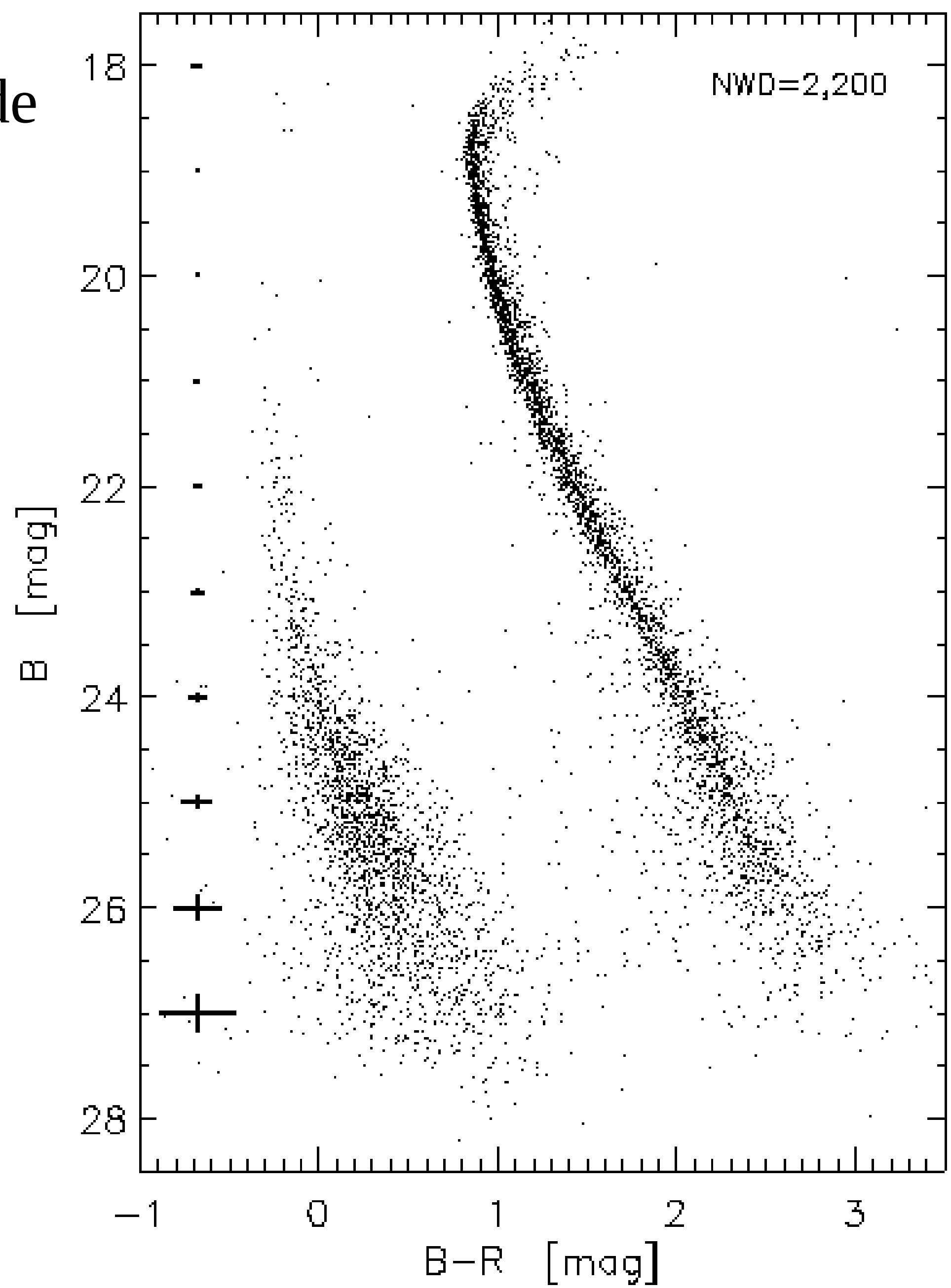
Globular Cluster Omega Centauri



• Omega Cen colour-magnitude diagram showing main-sequence and 2200 candidate white dwarfs

• Contamination?
    - Field galaxies (~60)
    - Field stars (~20)
    - Field white dwarfs (~15)

• Largest sample of cluster WDs observed to date (>2000 objects)



White Dwarfs in Omega Centauri

- Omega Cen has distance modulus =14.2, giving a distance of ≈6950pc
- White dwarf cooling sequence seen from M_B=+7 to +13 with L_{WD}~$10^{-3}L_{sun}$ at faint limit
- For luminosities that correspond to cooling times that are comparable to the lifetime of stars on the horizontal branch – the Helium-core burning main sequence that gives rise to the recently created white dwarfs – can perform direct test of stellar evolution theory
- Evolution of stars from horizontal branch to white dwarf is almost instantaneous relative to lifetime on horizontal branch (HB)
- Only a very small change in the mass of stars now on horizontal branch compared to the progenitors of the white dwarfs now cooling
- Ratio of #HB to #white dwarfs within a given cooling time should be equal to the ratio of the HB-lifetime to the cooling time

White Dwarfs in Omega Centauri

• Calculated cooling time for $0.5M_{sun}$ white dwarf with CO core at M_B=+11.3 (B=25.5) is 3.5×10^8yr

• Predicted horizontal branch lifetime 1-2×10^8yr

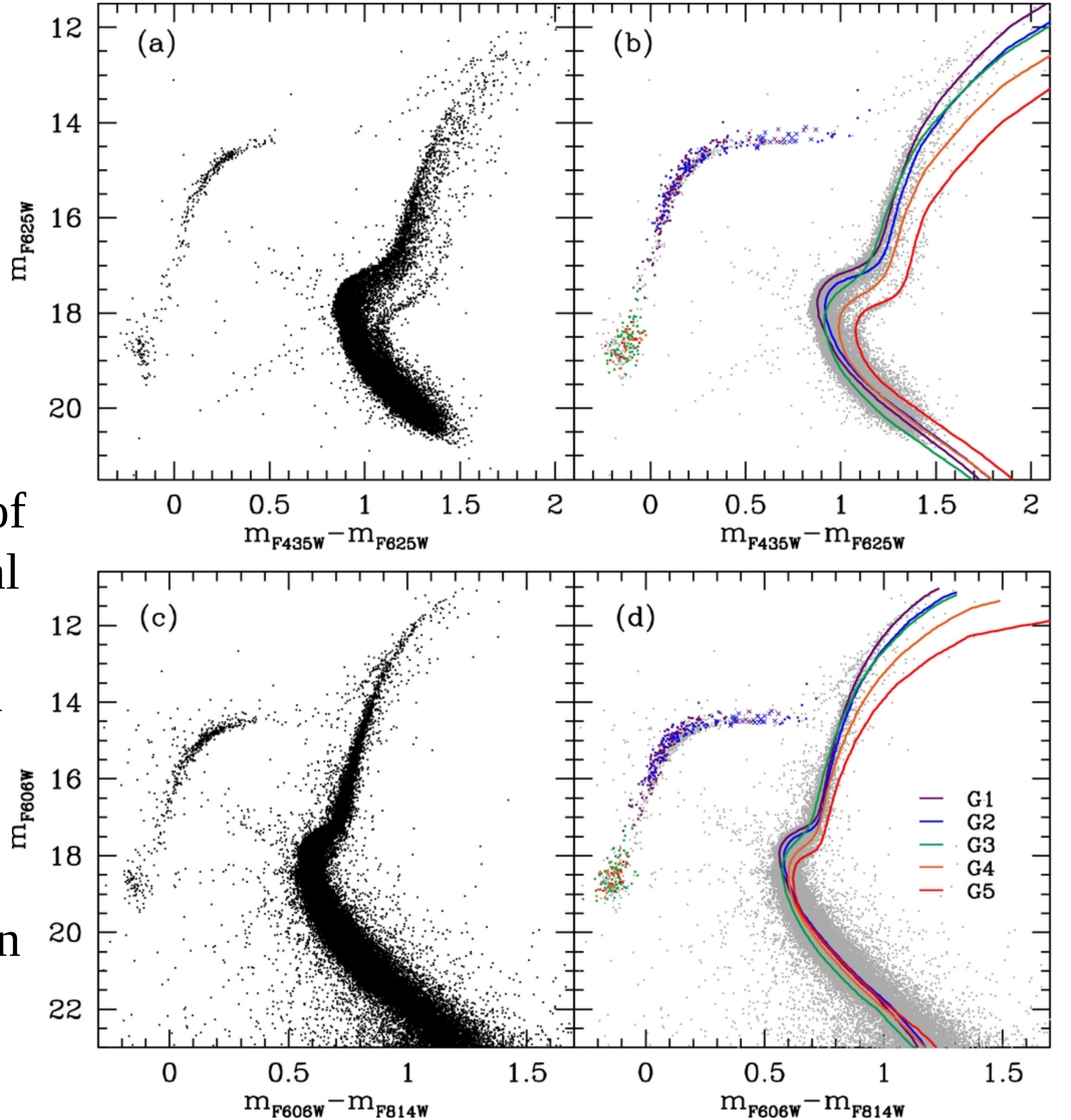
• Find ~630 horizontal branch stars and 1200 white dwarfs brighter than B=25.5 – consistent with predictions.

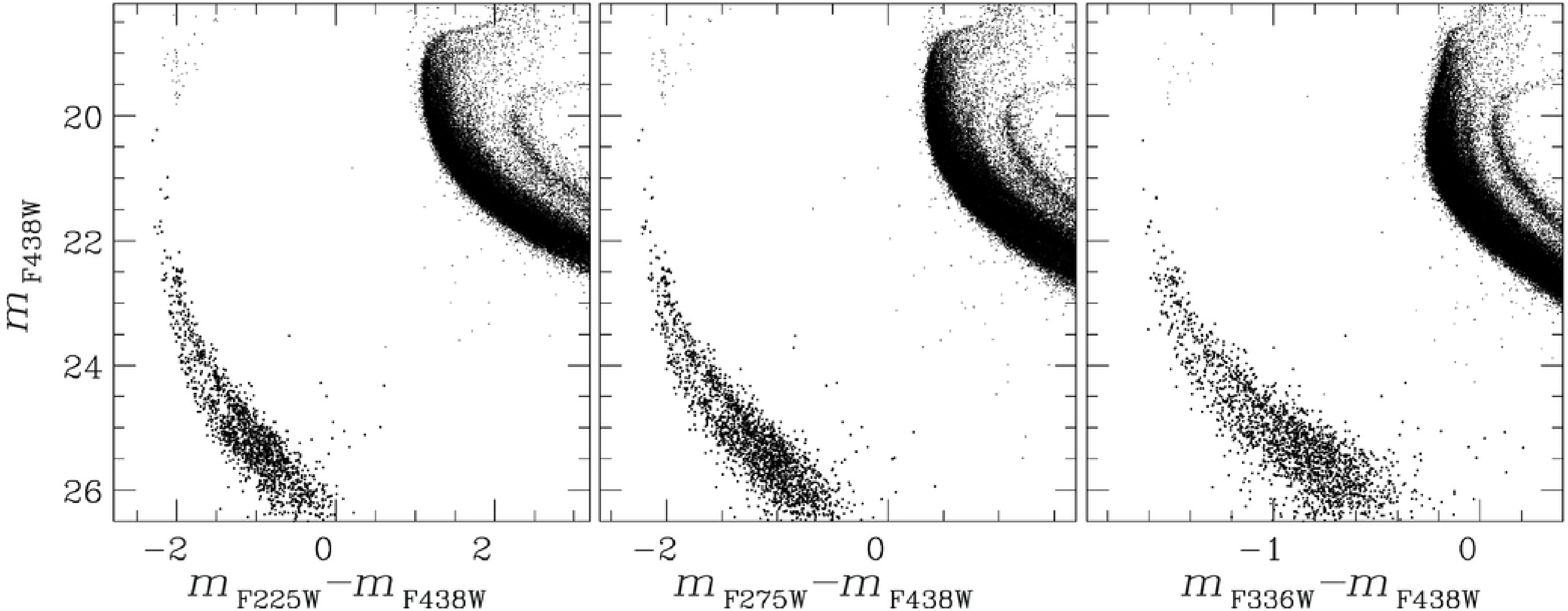
• What is the origin of the apparent spread in the white dwarf colours at faint magnitudes?

New HST observations at ultraviolet wavelengths – sensitive to [Fe/H] variations.

Note extended form of [not very] "Horizontal Branch". Blue-clump due to stars with high mass-loss on RGB.

Consequences for mass of He-core when mass loss extreme?





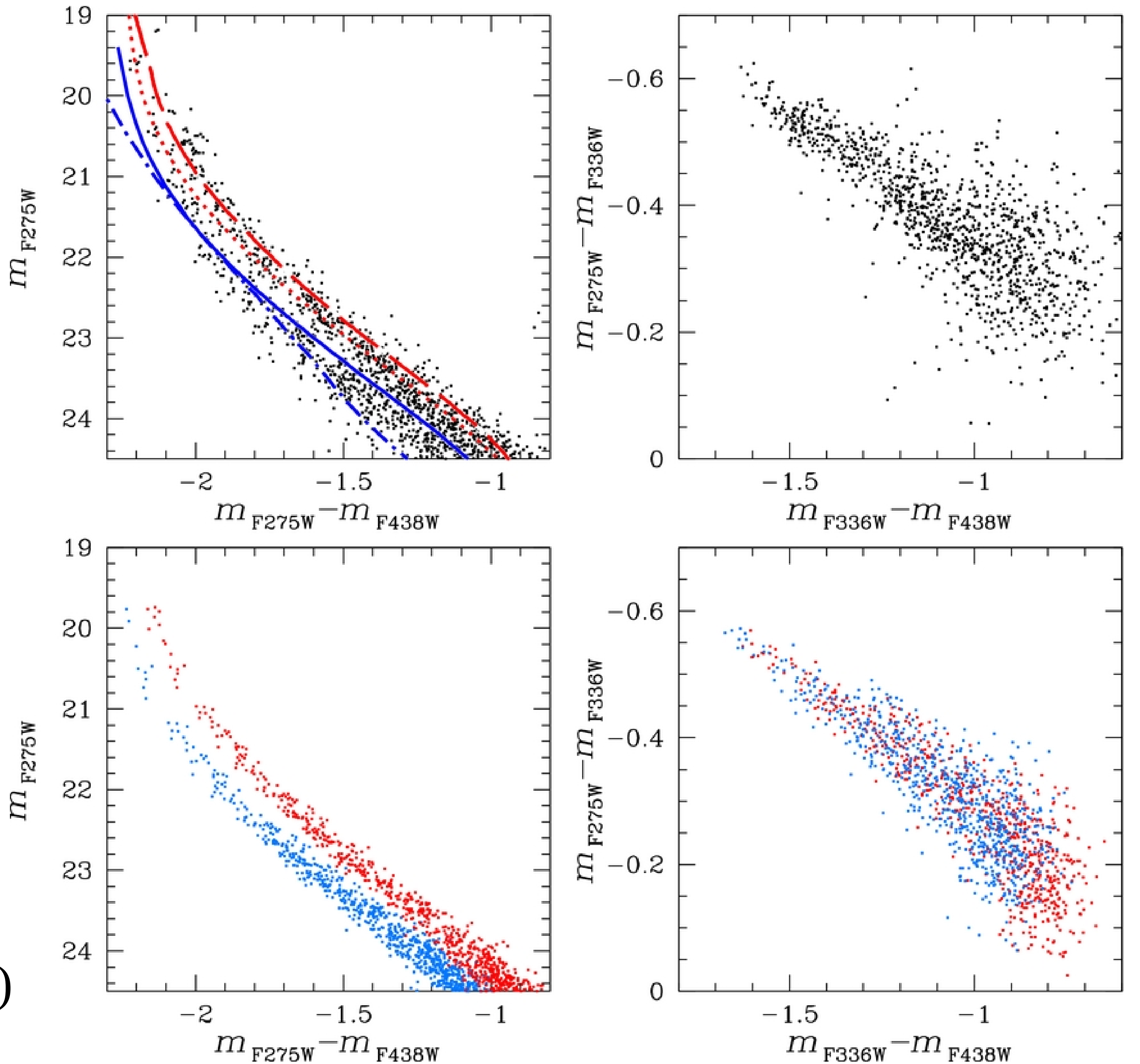
HST ultraviolet observations more accurate and clear bifurcation of white-dwarf cooling sequence visible – origin?

- Binary white dwarfs [what would happen at bright end?]
- Different outer atmospheres – H or He?
- Different masses/compositions; implications for evolution of stars from RGB to the white-dwarf cooling sequence?

Data (top panels) and models (with photometric errors) in bottom panels.

Solid and dashed $0.54\ M_{sun}$ CO models with H and He atmospheres (blue)

$0.46 M_{sun}$ He models with H and He atmospheres (red)



White Dwarfs in Omega Centauri

- Can analyse the main-sequence populations for the two dominant stellar populations He-rich bMS (Y=0.4, [Fe/H]=-1.3) & He-normal [Fe/H]-poor rMS (Y=0.25, [Fe/H]=-1.6)

- Compare to the ratio of the populations in the "Blue Clump" compared to the rest of the population on the Horizontal Branch – only 20% of the He-rich bMS stars reach the Horizontal Branch

- Some 80% of the population don't experience the Helium-flash at top of RGB, hence skip Horizontal Branch and AGB phase, moving direct from RGB to reveal He-cores as hot white dwarfs

Lecture 19: Summary

• Degenerate systems, both white dwarfs and stellar cores, supported by electron pressure, are a major feature of stellar evolution
• Degenerate equation of state leads to mass-radius relationship where object shrinks as mass increases
• Once electrons become relativistic the systems become dynamically unstable, gamma=4/3, and the Chandrasekhar Mass quantifies the limiting mass for this to occur and thus is the maximum mass that may be obtained by a white dwarf
• Cooling behaviour can be calculated using a simple model with a thin overlying radiative envelope. White dwarfs cool rapidly to start but then increasingly slowly
• If very faint, i.e. old, white dwarfs can be detected then white dwarfs can be used to date the system
• Current research enables properties of white dwarf populations to constrain later phases of stellar evolution for low-mass stars

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