Structure and Evolution of Stars

Lecture 13: Homology Solutions (#1)

- Equations of Stellar Structure with mass as the independent variable
- Review of Observed relations between *L*, *T* and *M*
- Homology – self-similar solutions to stellar structure equations
  - Provides dependencies of physical properties on mass
  - Mass-Luminosity relationship
- Star of the Week #3: The current search for “failed” supernovae – Candidate #1 in NGC 6946

Equations of Stellar Structure

• Reached stage where we have set of 4 differential equations with 4 unknowns (incorporating dependence of the pressure, opacity and energy generation rate in terms of density, temperature and composition from our understanding of physics) and well-defined boundary conditions – solution now possible in principle

• Note that there is no time dependence involved and thus the solutions to the equations can tell us about stellar structure but not, on their own, about stellar evolution

• The change in the composition of a star is critical for understanding how a star evolves. Change in mass with time due to nuclear reactions is very small (<1%) but mass-loss resulting from exceeding Eddington Luminosity or cataclysmic events (supernovae are an extreme example) is also key

Equations of Stellar Structure

- Our 4 familiar equations of stellar structure for case of radiative energy transport and, for simplicity, opacity κ=constant

- More tractable to solve using m as the independent variable rather than the radial distance r

- Have boundary conditions:

$r=0$ and $L=0$ at $m=0$

$P=0$ and $T=0$ at $m=M_*$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{dT}{dm} = -\frac{3}{4ac}\frac{\kappa}{T^3}\frac{L}{(4\pi r^2)^2}$$

$$\frac{dL}{dm} = \varepsilon = \varepsilon_0 \rho T^n$$

Explaining the Main Sequence

• Observationally we have seen that there is a well-defined relation between luminosity and effective temperature for stars in the HR-diagram

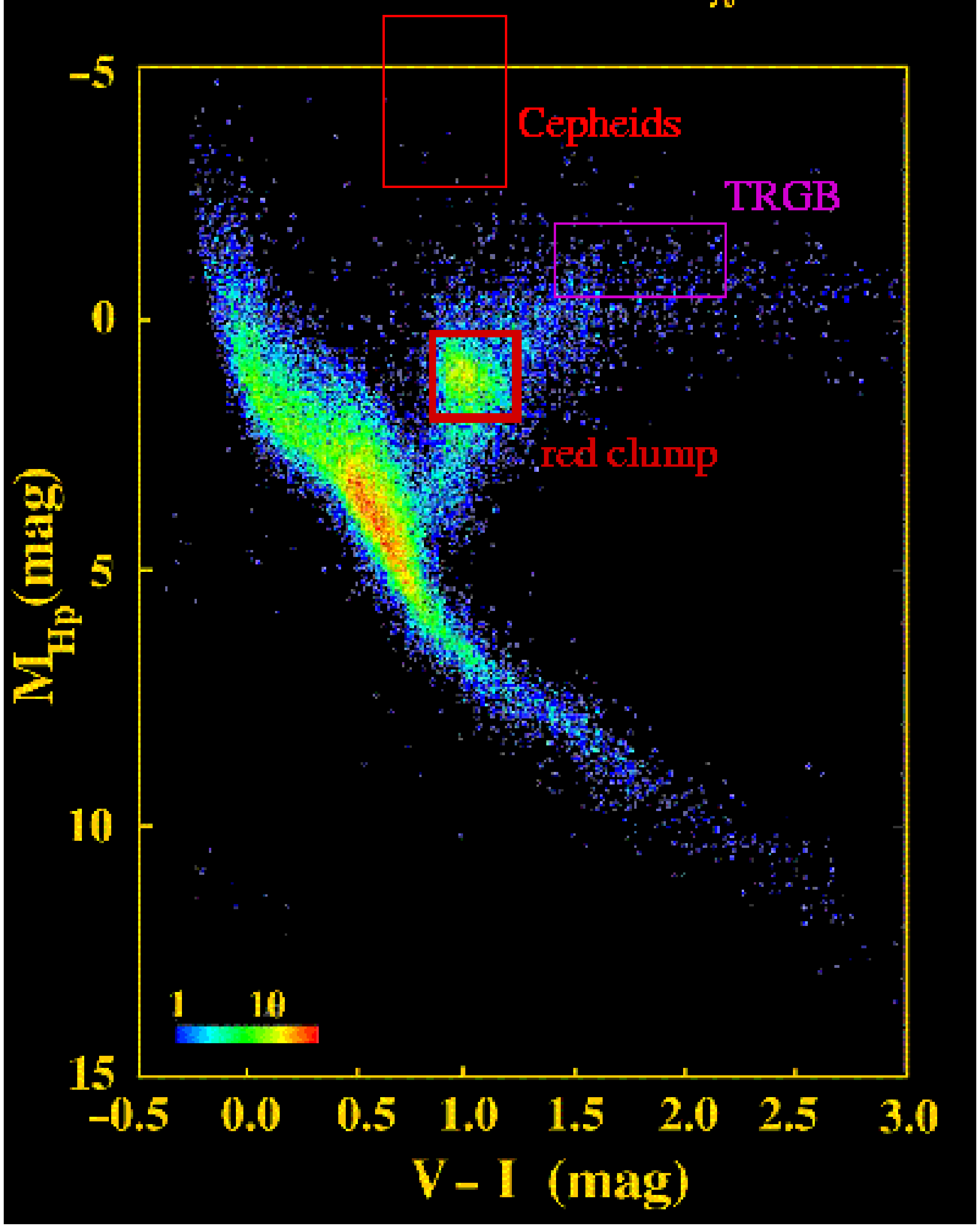
$$\log L = \alpha \log T_{eff} + \text{constant}$$

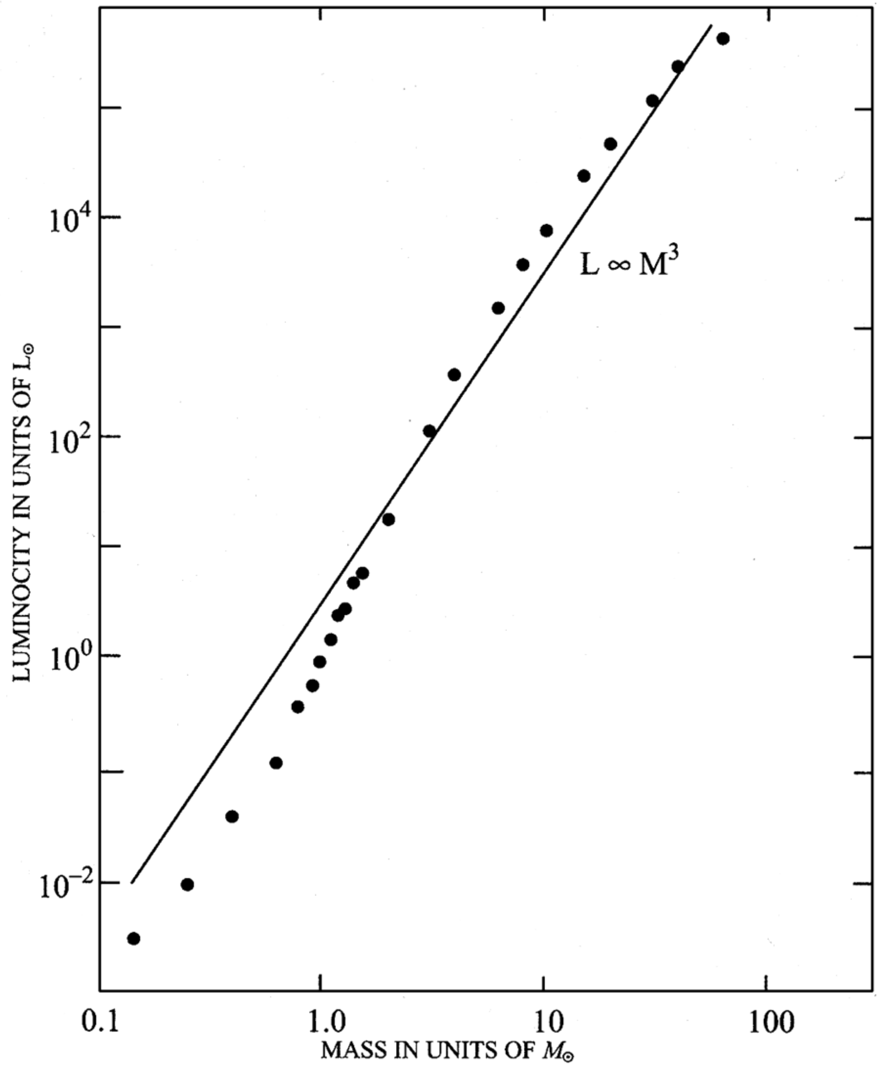
where the slope, α, becomes steeper at higher L

• From measurement of stellar masses in binary systems have also established the existence of a mass-luminosity relation that is well approximated by

$$L \propto M^{\beta}$$

• Hypothesis is that the main-sequence is the locus of stars of different masses burning hydrogen in their cores. Can we now use the stellar structure equations to reproduce the observational data?


-5
0
5
10
15
Cepheids
TRGB
red clump
M_{Hp} (mag)
1
10
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
V - I (mag)



The mass–luminosity relation for hydrogen burning stars with a chemical composition similar to the sun. The data on representative main sequence stars is taken from Table 3.13 in the Astronomy and Astrophysics section of the *Physics Vade Mecum* compiled by Fredrick (1989)

Homology

• Even assuming that stars on the main sequence have constant composition, are in hydrostatic and thermal equilibrium, possess constant opacity, with pressure due only to that of a perfect gas – still have a set of complex nonlinear differential equations (Slide 3) that are difficult to solve for the variables $r(m)$, $P(m)$, $\rho(m)$, $T(m)$ and $L(m)$ over range $0<m<M$

• Instead, consider the relationship between variables as a function of mass by using dimensional analysis of the equations

• Homology, or self-similar, solutions to the equations describe how a particular solution to the equations scales as a function of a dimensionless variable – fractional mass in our case

Homology

Define the dimensionless variable x, the fractional mass:

$$x = \frac{m}{M}$$

Replace the full solutions to the equations of stellar structure:

$$r(m), P(m), \rho(m), T(m), L(m)$$

with dimensionless functions of x: $f_1(x)$, $f_2(x)$… employing definitions:

$$r = f_1(x)R_*$$

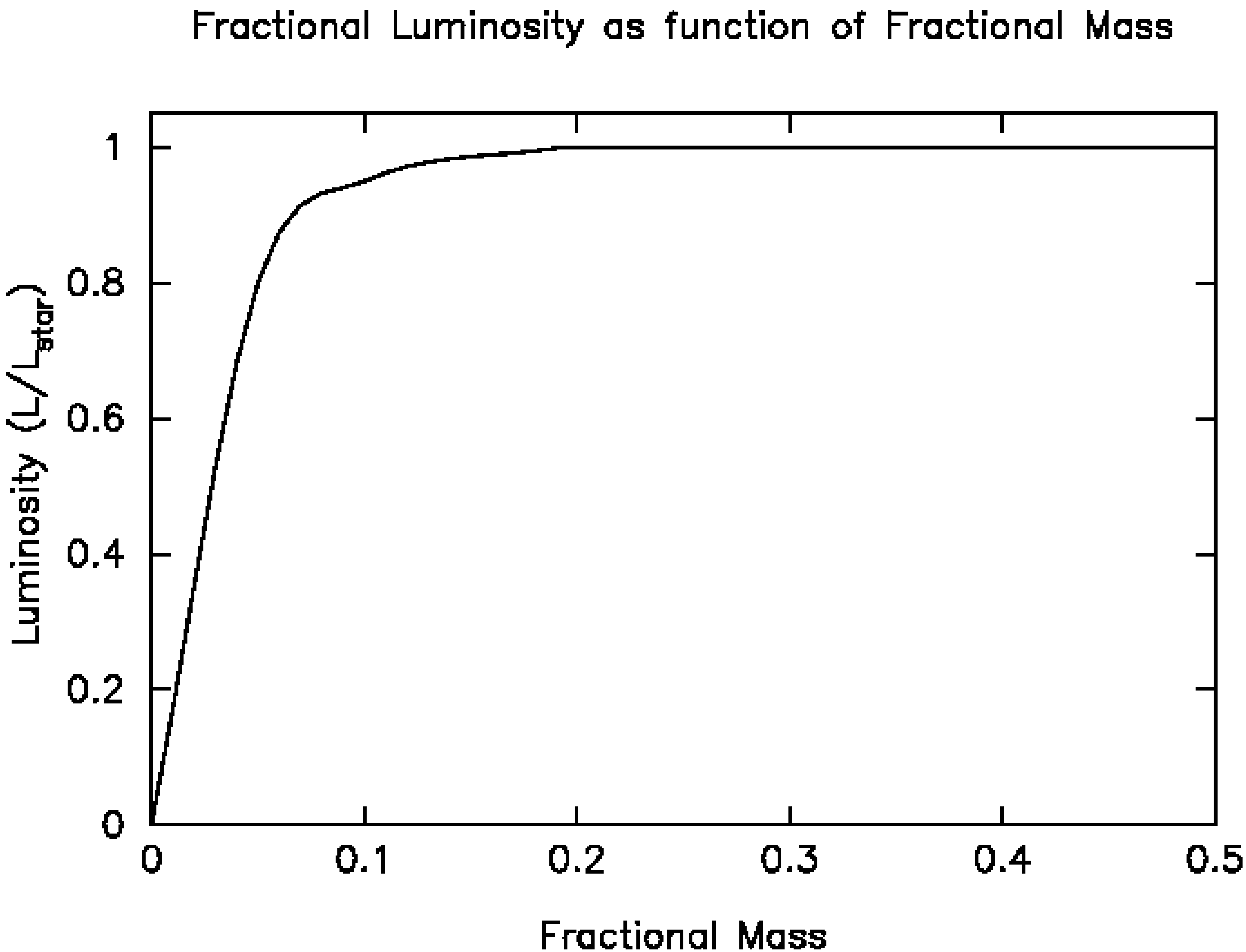
$$P = f_2(x)P_*$$

$$\rho = f_3(x)\rho_*$$

$$T = f_4(x)T_*$$

$$L = f_5(x)L_*$$

where the coefficients N_* have the dimensions of the original functions


Fractional Luminosity as function of Fractional Mass
Luminosity (L/L$_{star}$)
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
Fractional Mass

Homology

• Now substitute for x, r and P in the hydrostatic equilibrium equation to give:

$$\frac{P_*}{M}\frac{df_2}{dx} = -\frac{GMx}{4\pi f_1^{\,4} R_*^{\,4}}$$

x, f_1 and f_2 are dimensionless

the dimensions on both sides must be equal

$$\Rightarrow P_* \propto \frac{GM^2}{R_*^{\,4}}$$

can separate the equation into 2 parts, adopting a proportionality constant of unity

$$\frac{df_2}{dx} = -\frac{x}{4\pi f_1^{\,4}}\,;\;\; P_* = \frac{GM^2}{R_*^{\,4}}$$

Homology

- Repeat the exact same procedure for all 4 stellar structure equations along with *P=nkT* for a perfect gas to give a set of 5-pairs equations (*P=nkT* gives the 3rd equation as listed here)

- lh set consists of nonlinear differential equations independent of *M* for the functions $f_1, f_5\ldots$

- rh set consists of *algebraic* equations that relate the dimensional coefficients $P_*, R_*\ldots$

$$\frac{df_2}{dx} = -\frac{x}{4\pi f_1^{\,4}}; \quad P_* = \frac{GM^2}{R_*^{\,4}}$$

$$\frac{df_1}{dx} = \frac{1}{4\pi f_1^{\,2} f_3}; \quad \rho_* = \frac{M}{R_*^{\,3}}$$

$$f_2 = f_3 f_4; \qquad T_* = \frac{\mu m_H}{k}\frac{P_*}{\rho_*}$$

$$\frac{df_4}{dx} = -\frac{3 f_5}{4 f_4^{\,3}(4\pi f_1^{\,2})^2}; \quad L_* = \frac{ac}{\kappa}\frac{T_*^{\,4} R_*^{\,4}}{M}$$

$$\frac{df_5}{dx} = f_3 f_4^{\,n}; \qquad L_* = \varepsilon_0 \rho_* T_*^{\,n} M$$

Homology

• Can combine the solutions of the algebraic and the differential equations to provide behaviour of $r, P\ldots$ as a function of M

• Key point is that the shape of the profiles as a function of the fractional mass, x, is the same in all stars, with the profiles differing only by a constant factor determined by the mass

• The similarity property is termed *homology*

• Solving only the algebraic equations can derive dependence of the physical properties on mass

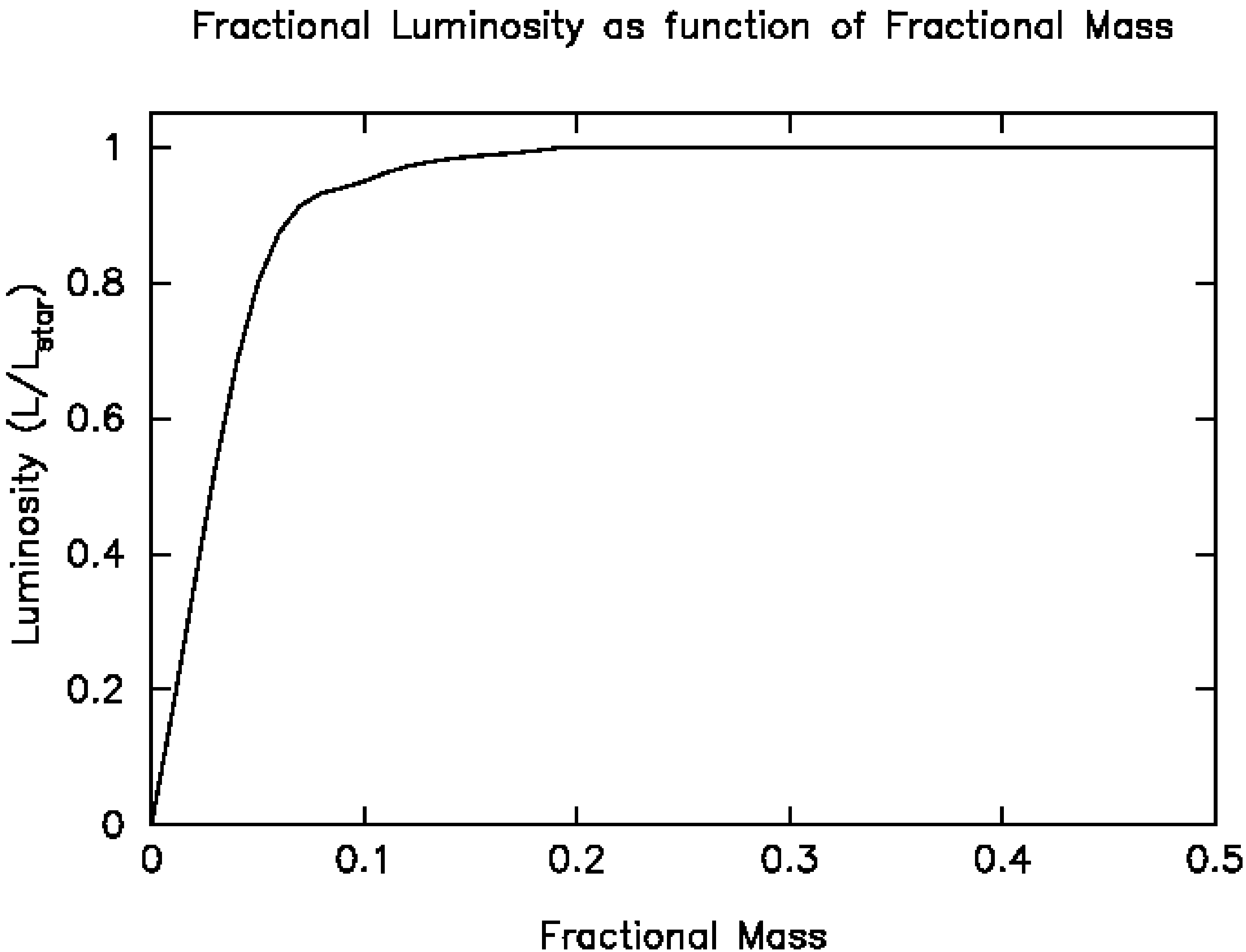
$$r = f_1(x) R_*$$

$$P = f_2(x) P_*$$

$$\rho = f_3(x) \rho_*$$

$$T = f_4(x) T_*$$

$$L = f_5(x) L_*$$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0
0.1
0.2
0.3
0.4
0.5
Fractional Mass

Homology

- Substitute for P_* and ρ_* (using first two equations on previous but two slide) into the 3rd equation gives:

$$T_* = \frac{\mu m_H G}{k} \frac{M}{R_*}$$

- Now substituting for T_* into the 4th equation gives:

$$L_* = \frac{ac}{\kappa} \left(\frac{\mu m_H G}{k} \right)^4 M^3$$

- Relation is true at any value of the fractional mass but putting fractional mass equal to unity and we have the mass-luminosity relationship:

$$L \propto M^3$$

The Search for Failed Supernovae – Candidate #1 in NGC 6946

- Gerke et al. 2015 MNRAS 450 3289
- Adams et al. 2017 MNRAS 469 1445

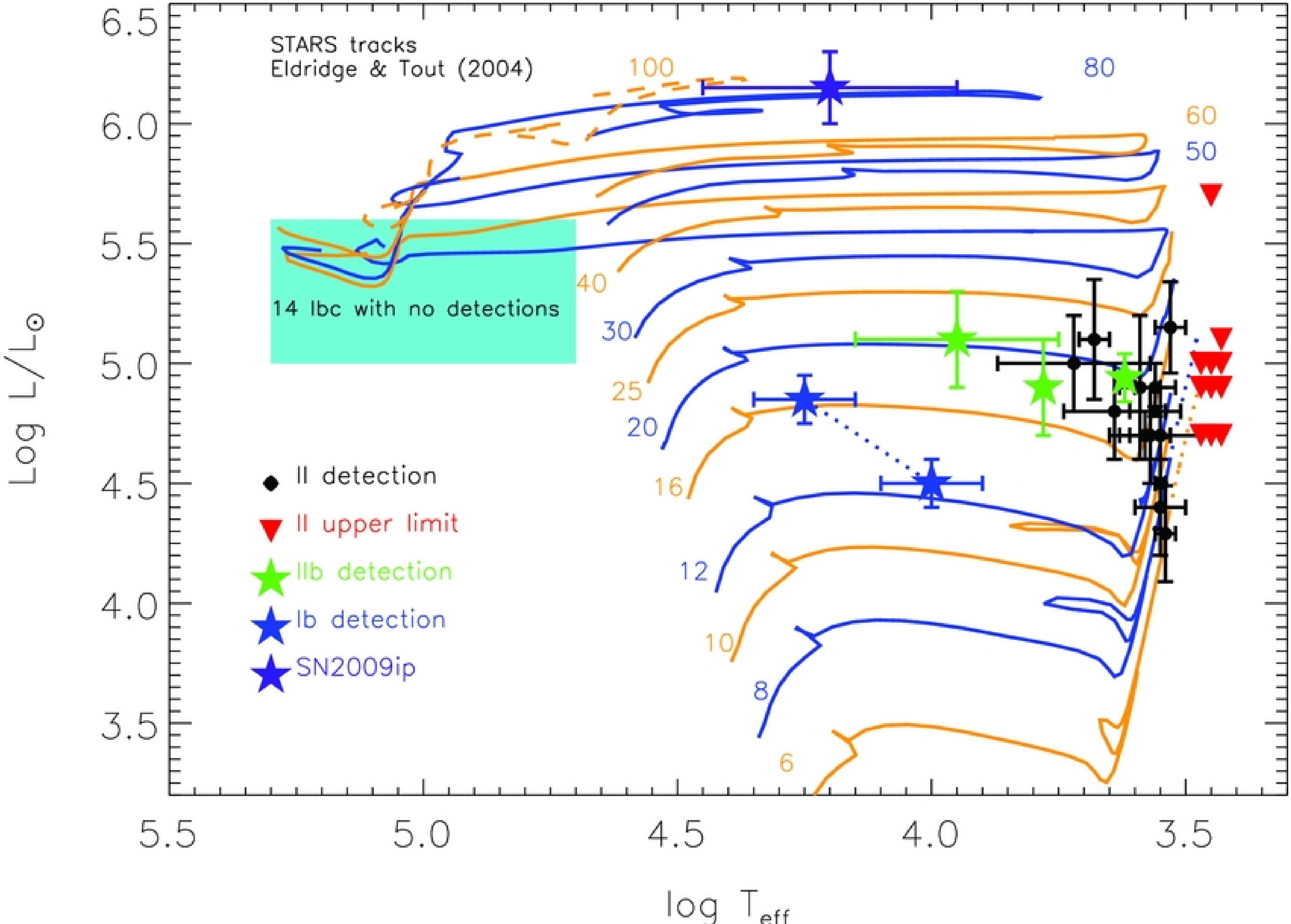
- Monitoring 27 galaxies with distance <10Mpc. Four years of data

- After “failed” supernovae and quantitatively search for objects where luminosity has declined by more than $10^4 L_{sun}$

- Check viable via distance-modulus (Lecture 2)

$d = 10^{(m-M+5)/5}$ pc - need a large telescope!

- Practical problems include “false positives” due to limitations/features of the imaging/data and astrophysical supernovae imposters – what might be examples?



S Smartt (2015, PASA, 32, 16 >30 progenitors)

Subaru optical image of NGC 6946

The Search for Failed Supernovae – Candidate #1 in NGC 6946

• LBT in Arizona (two 8.4m mirrors). CCD detector provides 23 by 23 arcmin images

• Well matched to covering most of galaxy, with depths R~26 mag with S/N~5 and seeing of 1 arcsec. Pixels 0.23 arcsec on a side

• Variable stars, due to many different phenomena, add confusion. Use a deep image in good seeing as the reference.

• Main search-tool is to create a “difference”-image by subtracting the reference image from each exposure. Number of epochs ~6 per year and galaxies have between 6-30 epochs

The Search for Failed Supernovae – Candidate #1 in NGC 6946

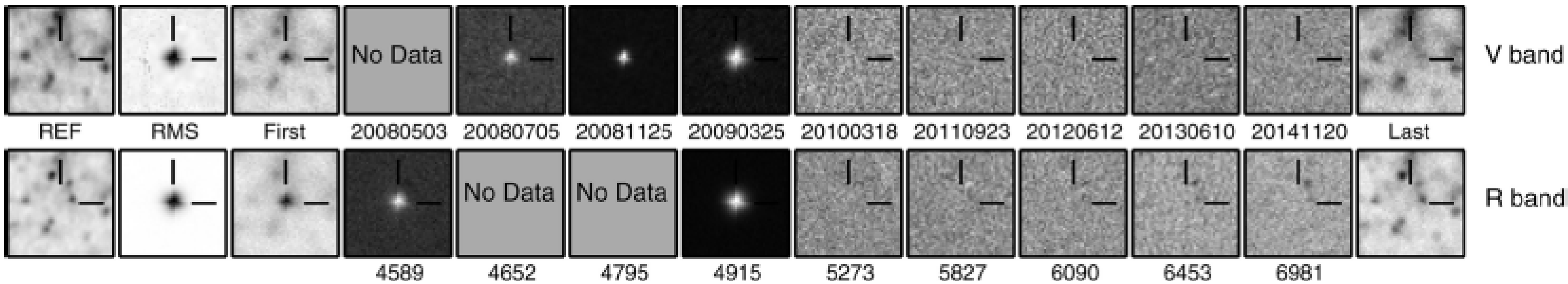


Figure 14. Select *V* and R_c band observations for Candidate 1 in NGC 6946. We have 19 epochs for this galaxy and do not show them all. The selected observations give a clear picture of the source's variability. The 'First' observation in the *V* band (R_c band) is on 2008 July 5 (2008 May 3) and the 'Last' observation is on 2014 November 20. The format is the same as in Fig. 2.

The only viable candidate from the Gerke etal. search

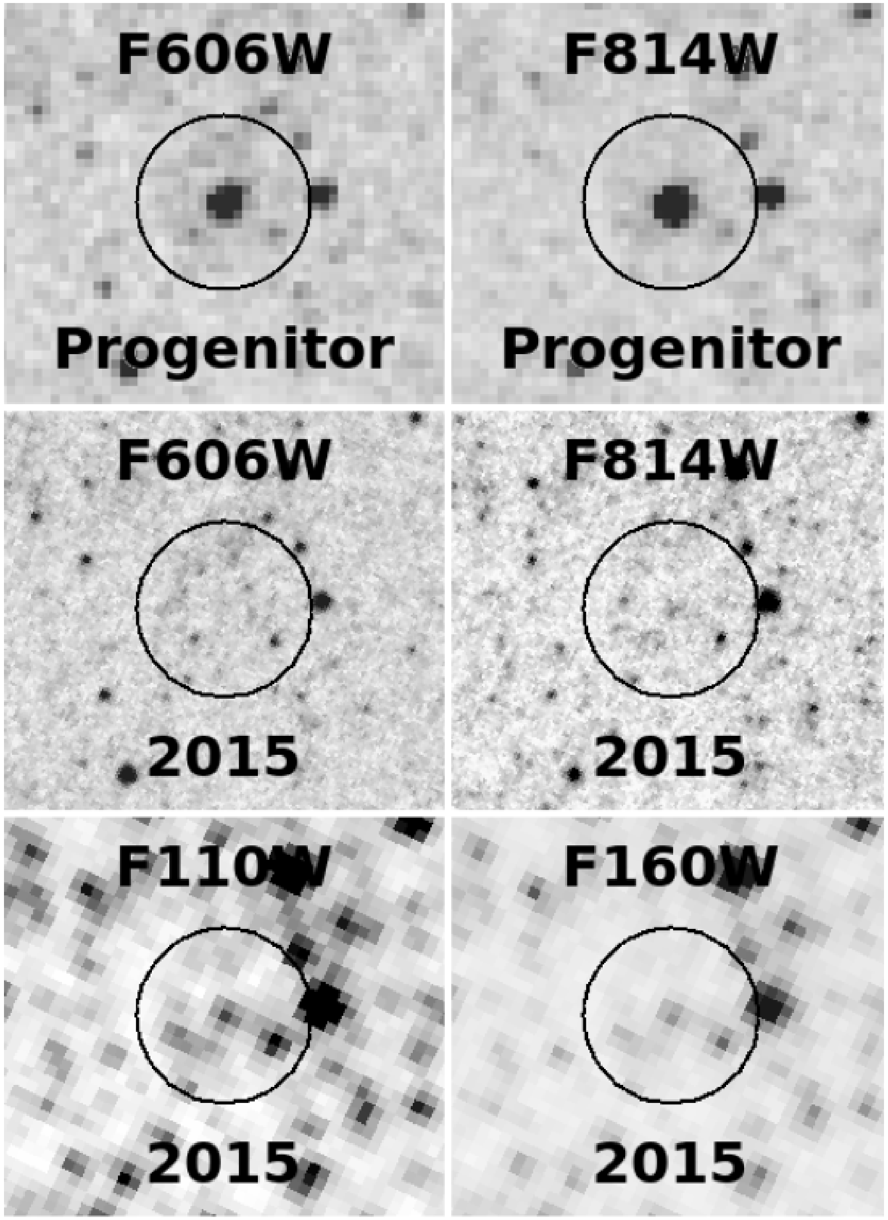
Have temperature from *V-R* colour and luminosity from magnitude and galaxy distance (5.96Mpc)

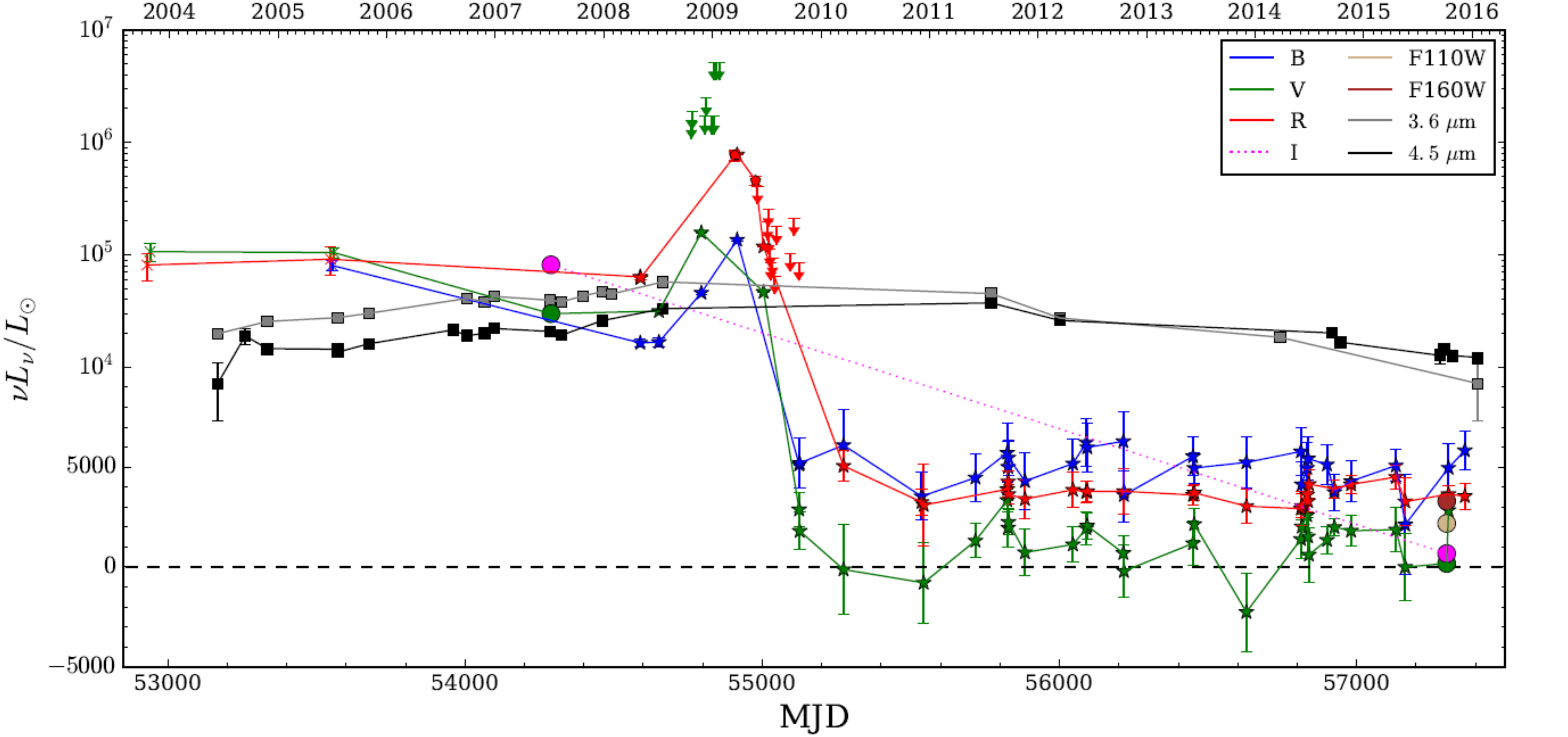
Mass estimate 18-25 solar masses (from models)

Follow-up Hubble Space Telescope observations in Adams et al.

Existing HST precursor images (top), new 2015 images (middle) and near-infrared images (bottom)

Good or bad for the failed supernovae hypothesis?



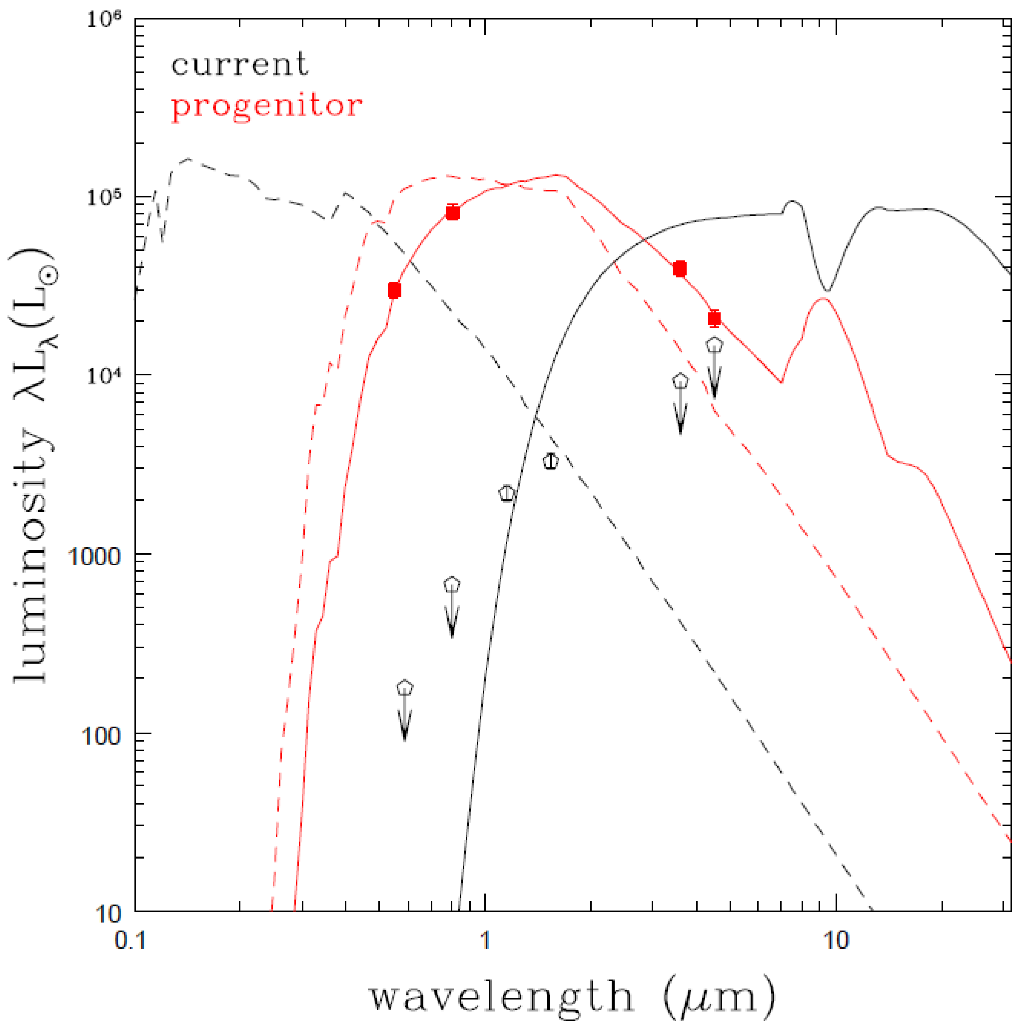


Photometric history, lightcurve, of the candidate. Y-axis flips from linear to logarithmic at 10^4

Spitzer near-infrared magnitudes challenging given resolution!

Observations by an amateur constrain peak brightness.

Photometry and model for progenitor (red) with dashed curve unreddened (without extinction)
Now called N6946-BH1 !
Need star (T=4480K and dust T=1800K) L=$10^{5.3}$ L_{sun}
Photometry now (black) with model for spectrum of star T=14500K obscured by a dusty wind T=1500K
Qualitative explanation for shape/luminosity of obscured model?



Lecture 13: Summary

- Equations of stellar structure more tractable in form with mass as the independent variable
- Homology, self-similar, solutions allows the calculation of the dependence of the key physical parameters on the mass of the stars
- First well-defined dependencies to explain are the mass-luminosity and luminosity-temperature relationships for the main sequence
- Homology approach provides dependence of luminosity-mass relationship straightforwardly
- Observations of nearby galaxies offers best prospect of identifying failed supernovae

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