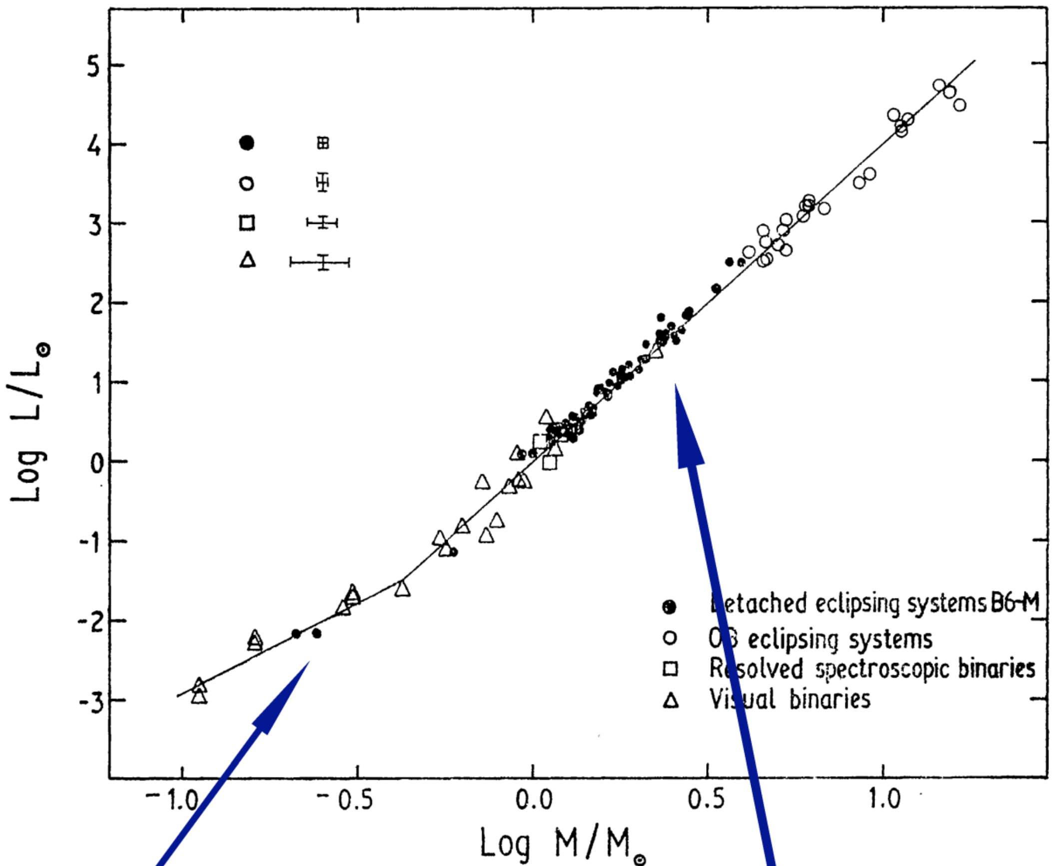
Structure and Evolution of Stars

Lecture 5: Mass-Luminosity Relation, Additional Features of the HR-diagram and Key Timescales

- Mass-Luminosity relation and the Main sequence
- Variable stars, standard candles and photometric distances
- Sparsely populated regions of the Hertzsprung-Russell diagram
- Operational definition of a star
- Key timescales for stellar evolution
  - Dynamical
  - Thermal
  - Nuclear

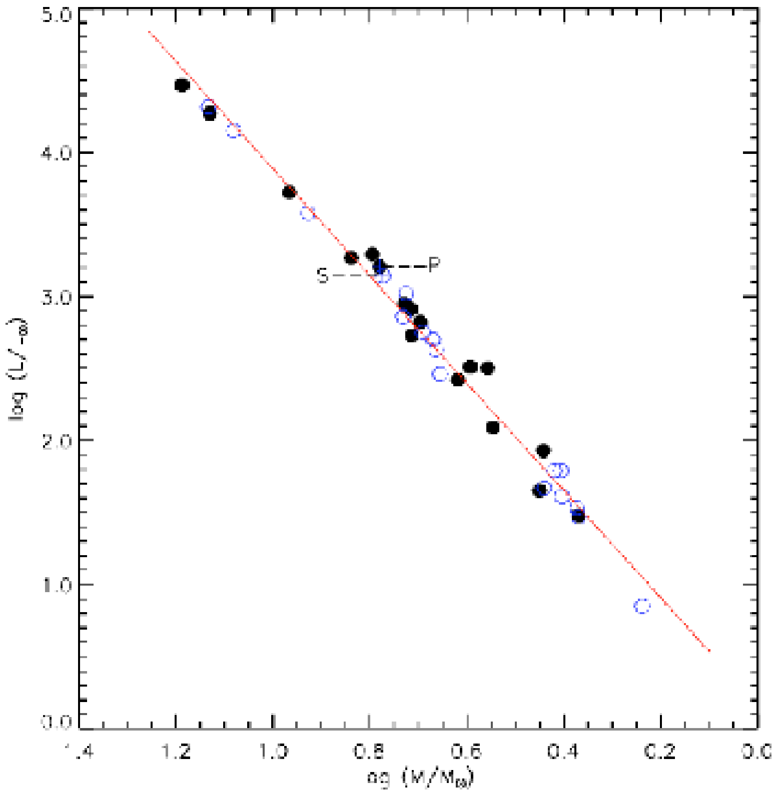


R C Smith 1983

$$\frac{L}{L_\odot} \propto \left(\frac{M}{M_\odot}\right)^{2.3} \qquad\qquad \frac{L}{L_\odot} \propto \left(\frac{M}{M_\odot}\right)^{4}$$

Masses from growing sample of binary systems allows empirical determination of the mass-luminosity relation

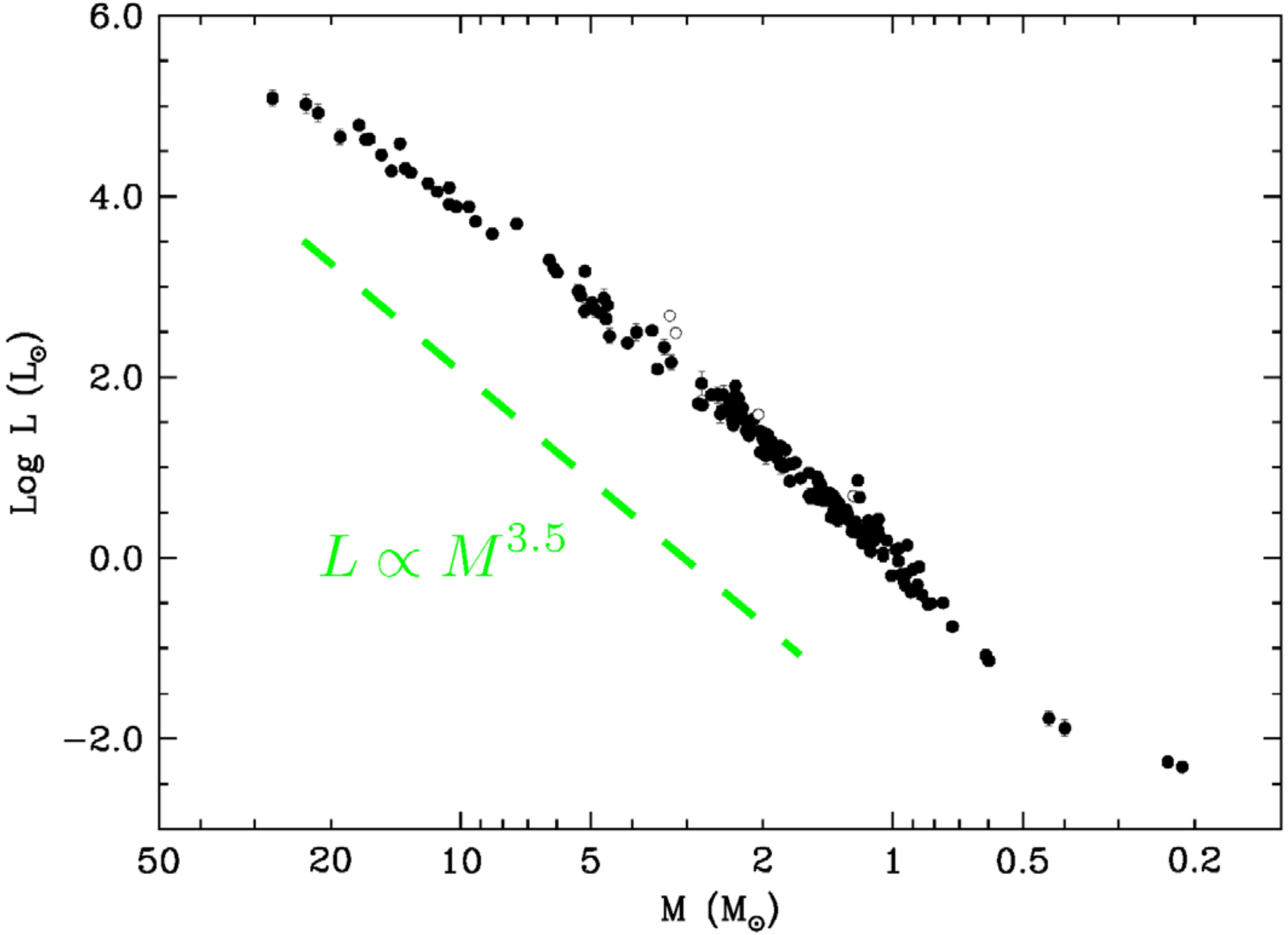
Mass-Luminosity relation for B-type eclipsing binaries. Filled circles: primary components; open circles: secondary components.



Yakut et al. 2007

$$\log\left(\frac{L}{L_\odot}\right) = 3.724\log\left(\frac{M}{M_\odot}\right) + 0.162$$

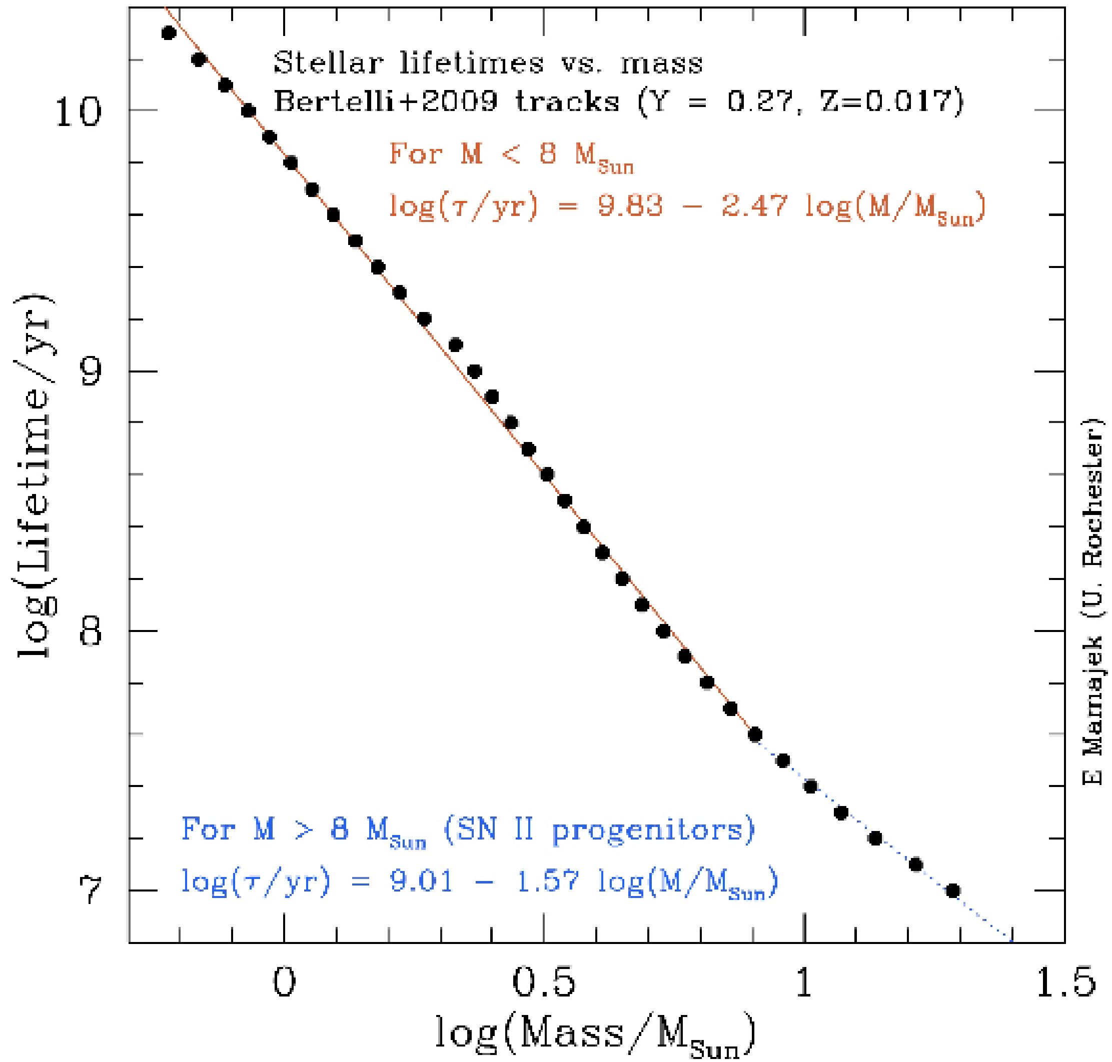
Mass-Luminosity relation for 190 stars in 95 detached binary systems whose radii and masses are known to better than 3%



Torres et al. 2010

Associate the mass-luminosity relation with the "main sequence" on the HR-diagram.

Form of the mass-luminosity relation leads to very large spread in the lifetime of stars on the main sequence and hence the total lifetime of stars as a function of mass



Mass-Luminosity Relation, Main Sequence and Stellar Lifetimes

- Mass-Luminosity relation provides natural explanation for existence of prominent "main sequence" in HR-diagram
- Stars forming from gas cloud begin life on main sequence with location determined by their mass. Stars do not evolve along the main sequence as might be a possibility without knowledge of M-L relation
- Very approximately the M-L relation shows that $L \propto M^{3.5}$
- If fuel source depends linearly on stellar mass then, for objects with ~constant L, easily seen that lifetime, t, behaves as $t \propto M^{-2.5}$.
- If Sun has $t \sim 10^{10}$yr then 30 solar mass star has $t < 10^{7}$yr
- Massive luminous stars are very short lived

Table 6.1 Selected properties of main sequence stars of various masses.

Mass/$M_\odot$	Luminosity/$L_\odot$	Surface temperature/K	Main sequence lifetime/yr
0.50	0.03	3800	2×10^{11}
0.75	0.3	5000	3×10^{10}
1.0	1	6000	1×10^{10}
1.5	5	7000	2×10^{9}
3	60	11000	2×10^{8}
5	600	17000	7×10^{7}
9	4000	23000	2×10^{7}
15	17000	28000	1×10^{7}
25	80000	35000	7×10^{6}

Lecture 4 - now able to associate masses with HR-diagram (L,T,R)

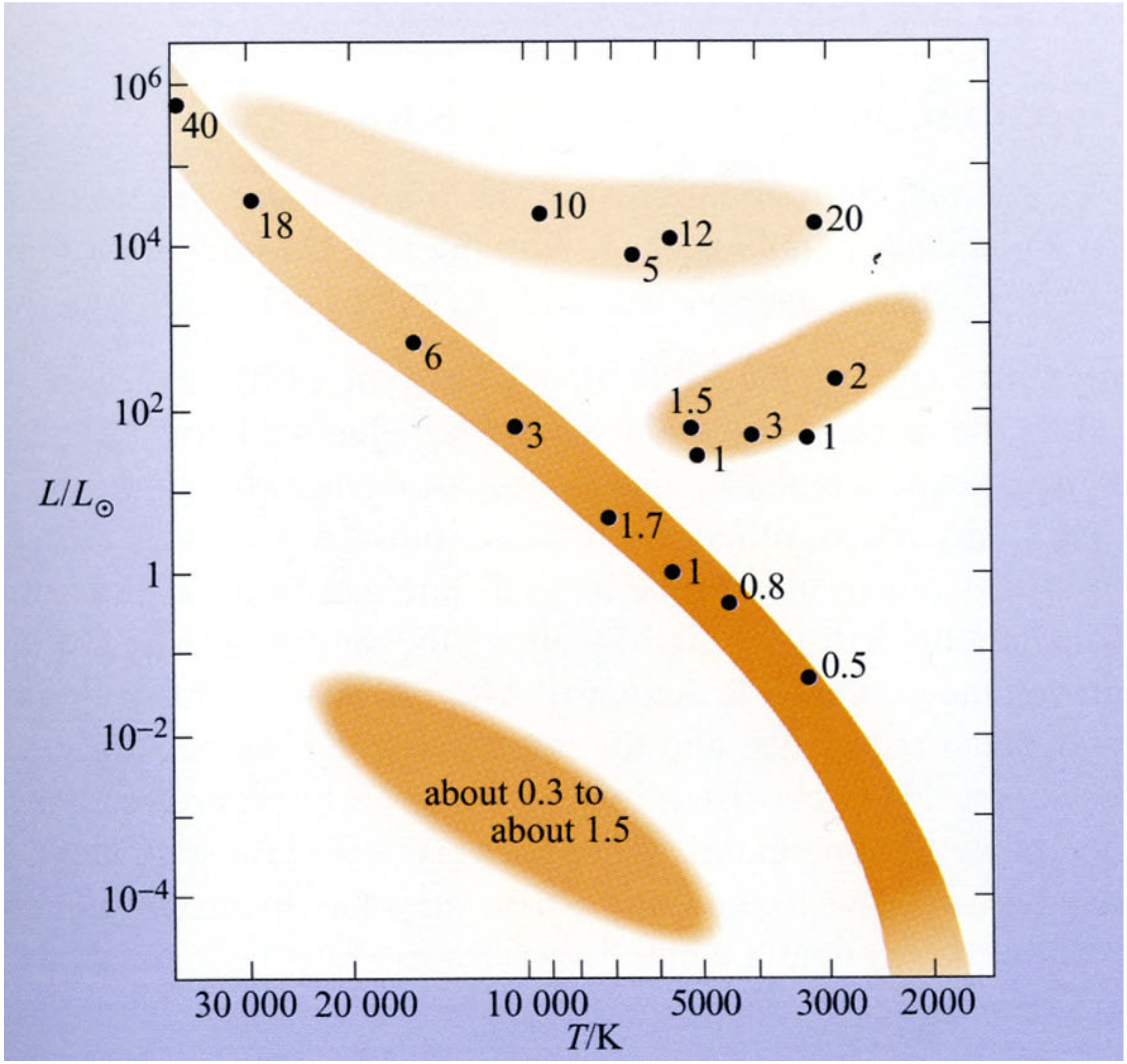


Figure 4.8 Stellar mass and the H–R diagram. Masses are given in multiples of $M_\odot$.

Zoo of Variable Stars

- Binary stars, critical for determination of masses and radii, represent one example of "variable stars".
- Other examples arise where the properties of single stars vary with time, resulting in a change in brightness with time, i.e. variability:
  - Anisotropic surface features, e.g. star-spots, combined with rotation can lead to semi-periodic behaviour. (A problem for planet-finding)
  - Pulsating stars, where the interplay between the rate of energy generation and the effect on the stellar atmosphere can lead to pulsating behaviour, in some cases the pulsations have well-defined periodic behaviour
  - Eruptive variables, sudden brightness changes, usually due to mass gain/loss. Supernovae are an extreme case

Variable Stars in the Hertzsprung-Russell Diagram

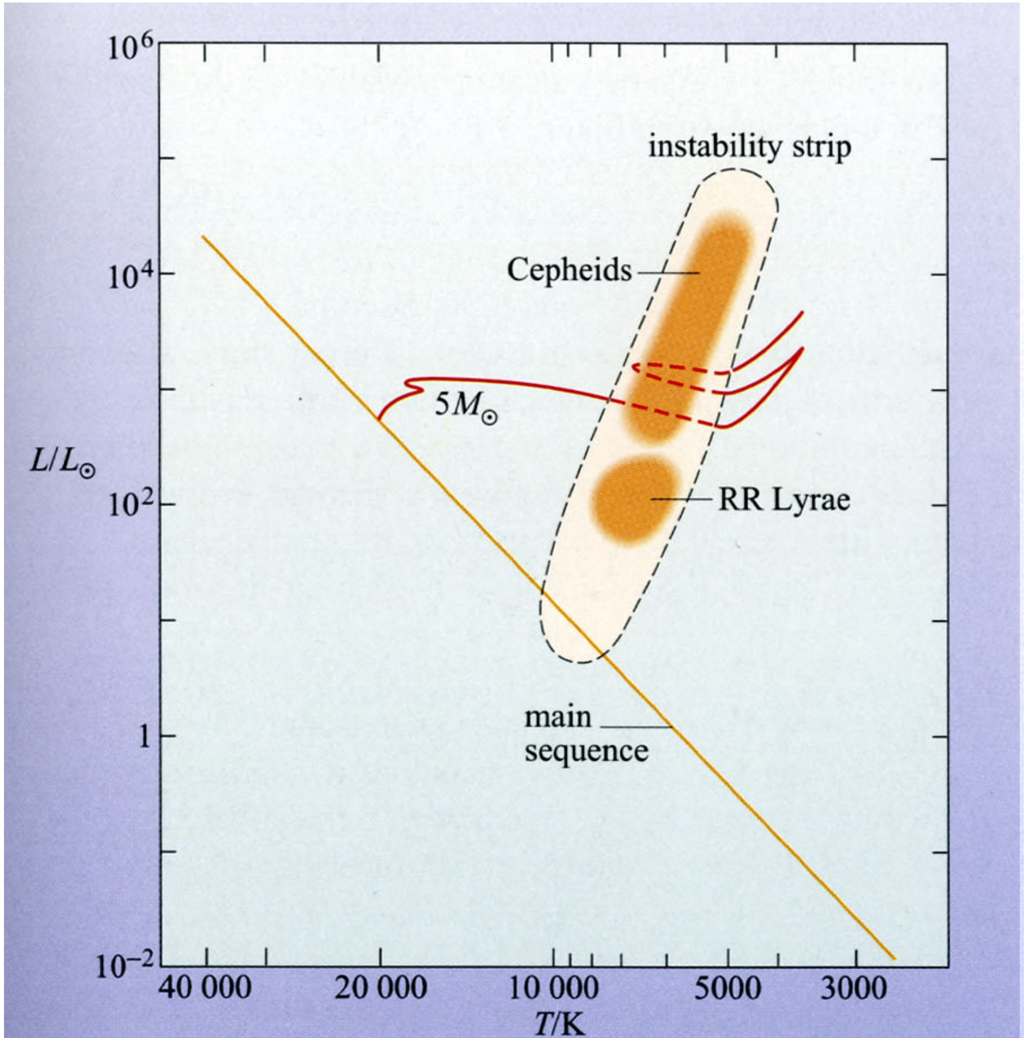


Figure 7.6 The position of the instability strip on the H–R diagram. The evolutionary track of a $5M_\odot$ star is shown dashed where it is exhibiting pulsations. At this time its position on the diagram will oscillate due to its changes in temperature and luminosity.

The Small Magellanic Cloud

Variable Stars and the HR-diagram

- Relatively cool pulsating stars occupy well-defined region above the main-sequence termed the “instability strip”
- At relatively(!) low luminosity, $M_V \approx 0.5$, RR-Lyrae stars occur in the “horizontal branch” and can provide useful “standard candle” for distance determination
- At high luminosities, $M_V \approx -3$ to -7, Cepheid variables exhibit a tight relationship between pulsation period and luminosity – provides standard candles to estimate distances out to tens of megaparsecs. Hubble Space Telescope Key Project in 1990s to determine the Hubble Constant, H_0, using Cepheid variables.
- Discovery of Cepheid “Period-Luminosity” relation by Henrietta Leavitt in late 19^{th} century illustrates power of studying system, Small Magellanic Cloud in this case, at fixed, but unknown distance

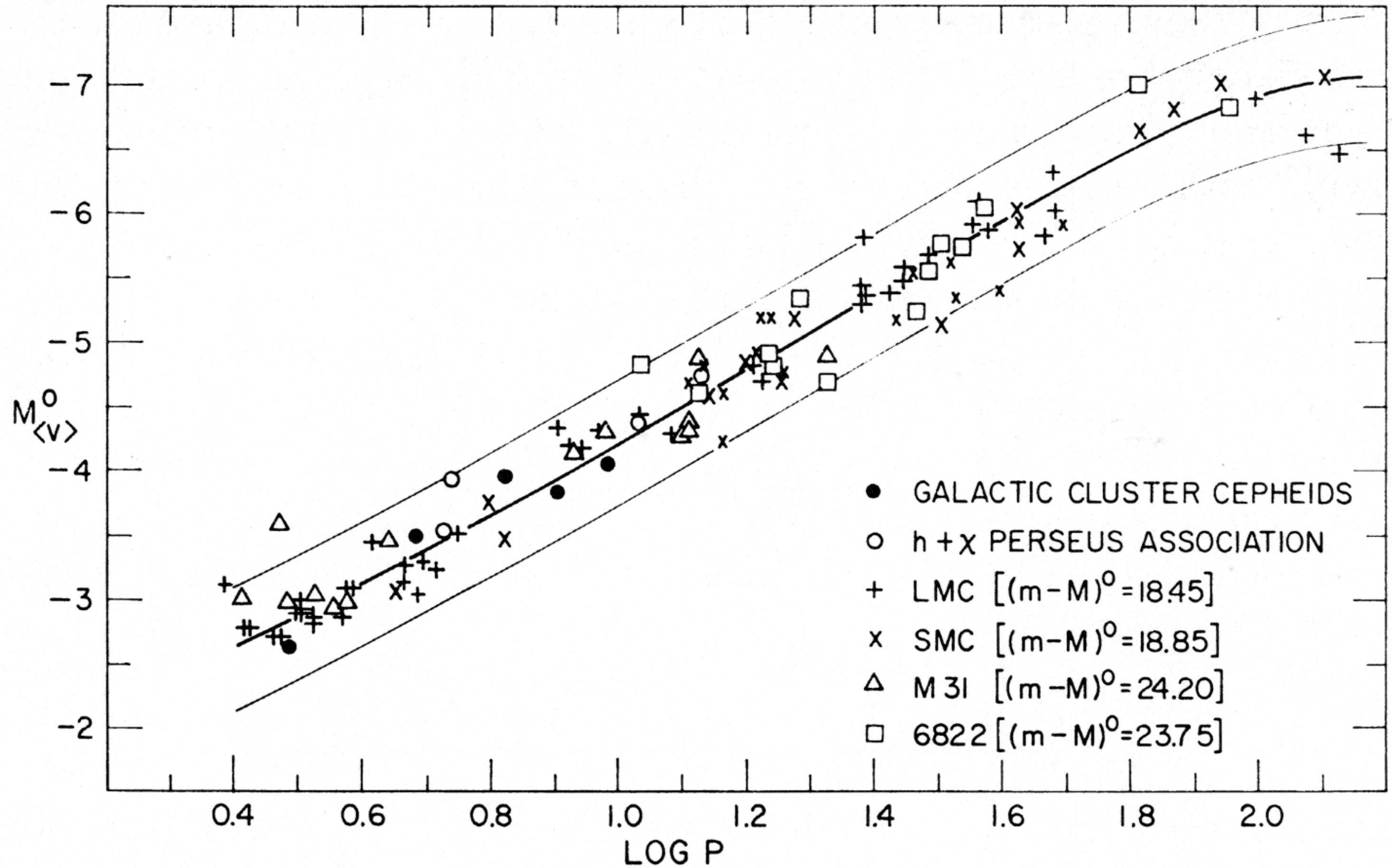
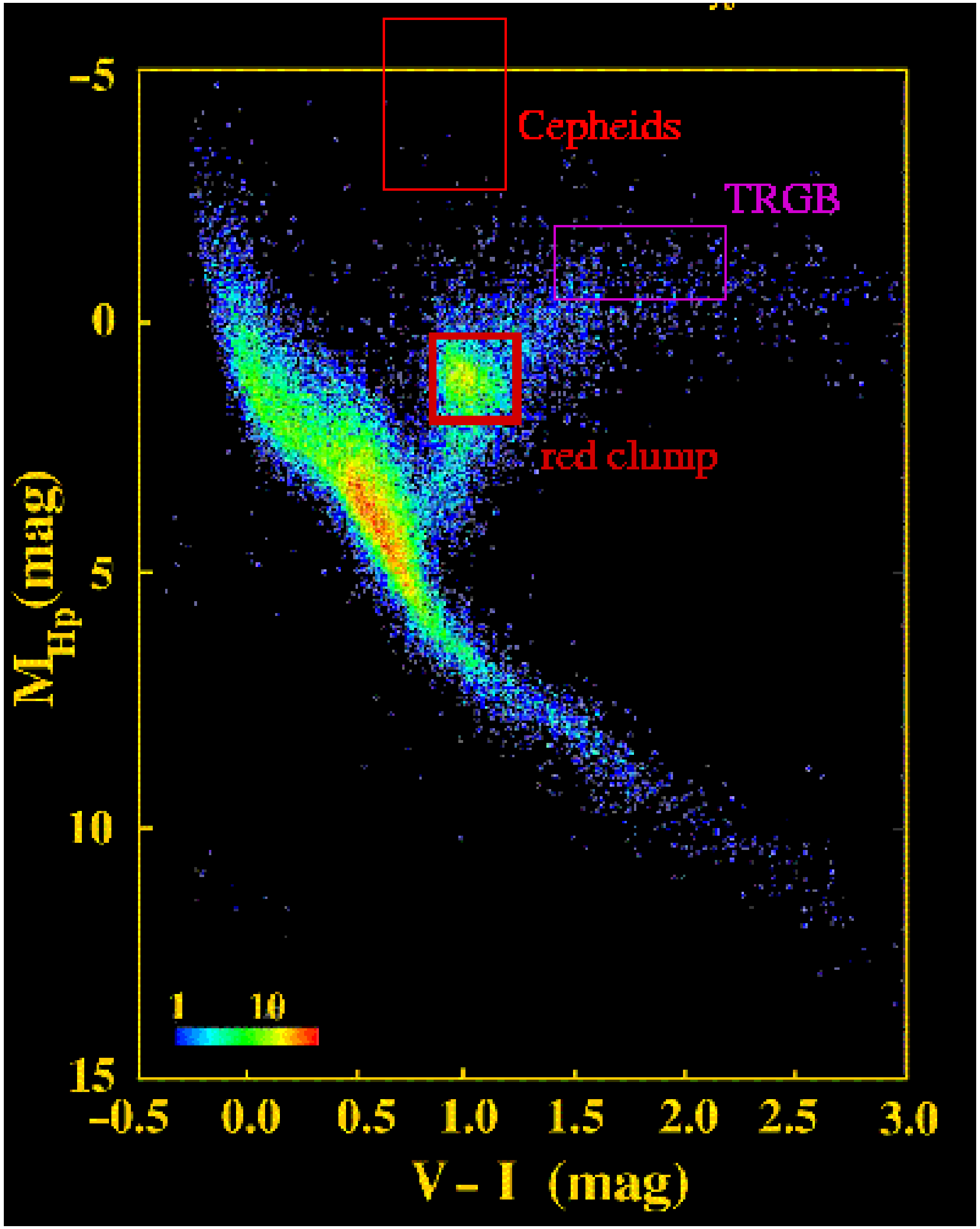


Figure 14.4 The period–luminosity relation for classical Cepheids. (Figure from Sandage and Tammann, *Ap. J.*, *151*, 531, 1968.)

The Cepheid P-L relation is well established but still debate about absolute luminosity at the 10%, or 0.1 magnitude, level due to lack of significant numbers of Cepheids with trigonometric distance measures – note sparsely populated region in the HR-diagram of all the Hipparcos stars.

GAIA results will transform situation.



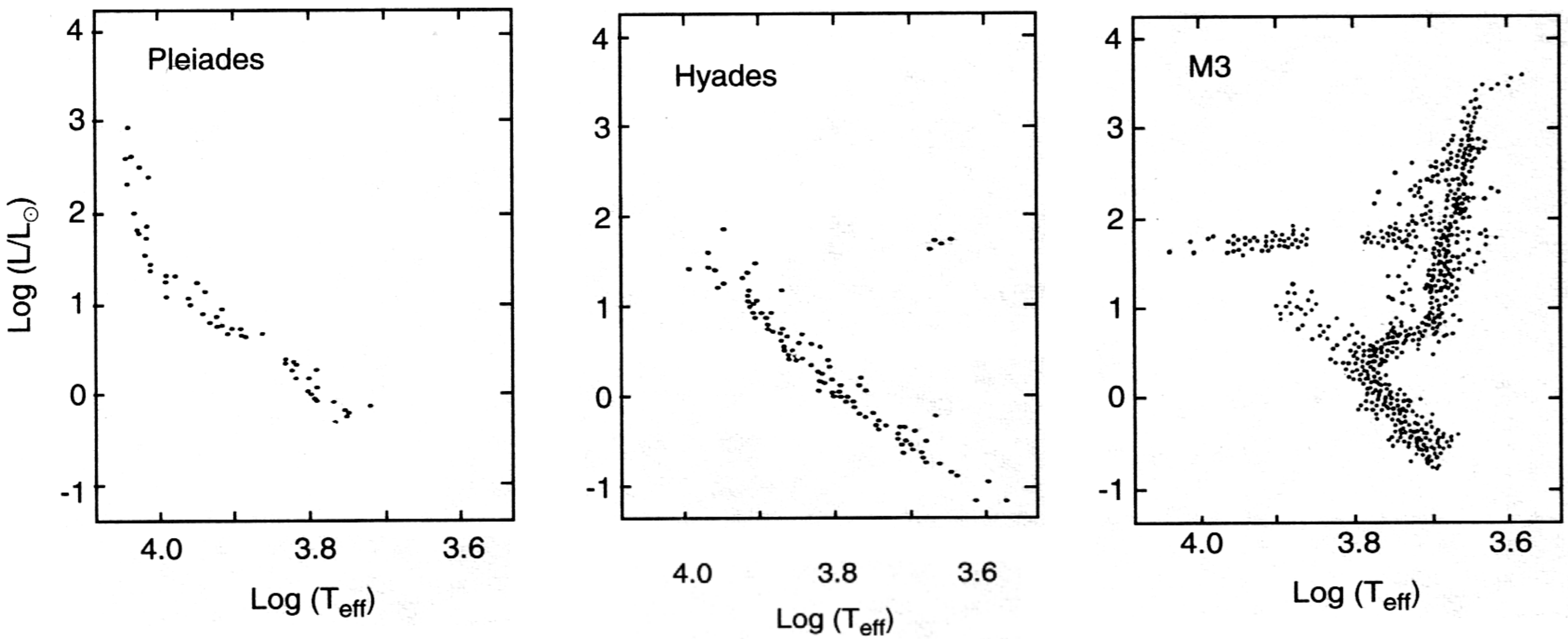
Photometric Distance Determination

- In Lecture 2, showed that from the definition of absolute magnitude, M, the distance of an object could be calculated if its absolute magnitude is known and its apparent magnitude measured

$$d = 10^{(m-M+5)/5} \text{ pc}$$

- Distances determined thus are termed “photometric distances” and the use of standard candles like Cepheids is very common
- With some confidence in the origin of the main sequence in the HR-diagram, can use “main sequence fitting” – measure m-M for main sequence of a star cluster (simply the vertical displacement in magnitudes) to calculate distance to the cluster

HR-diagrams for 3 stellar clusters of very different ages



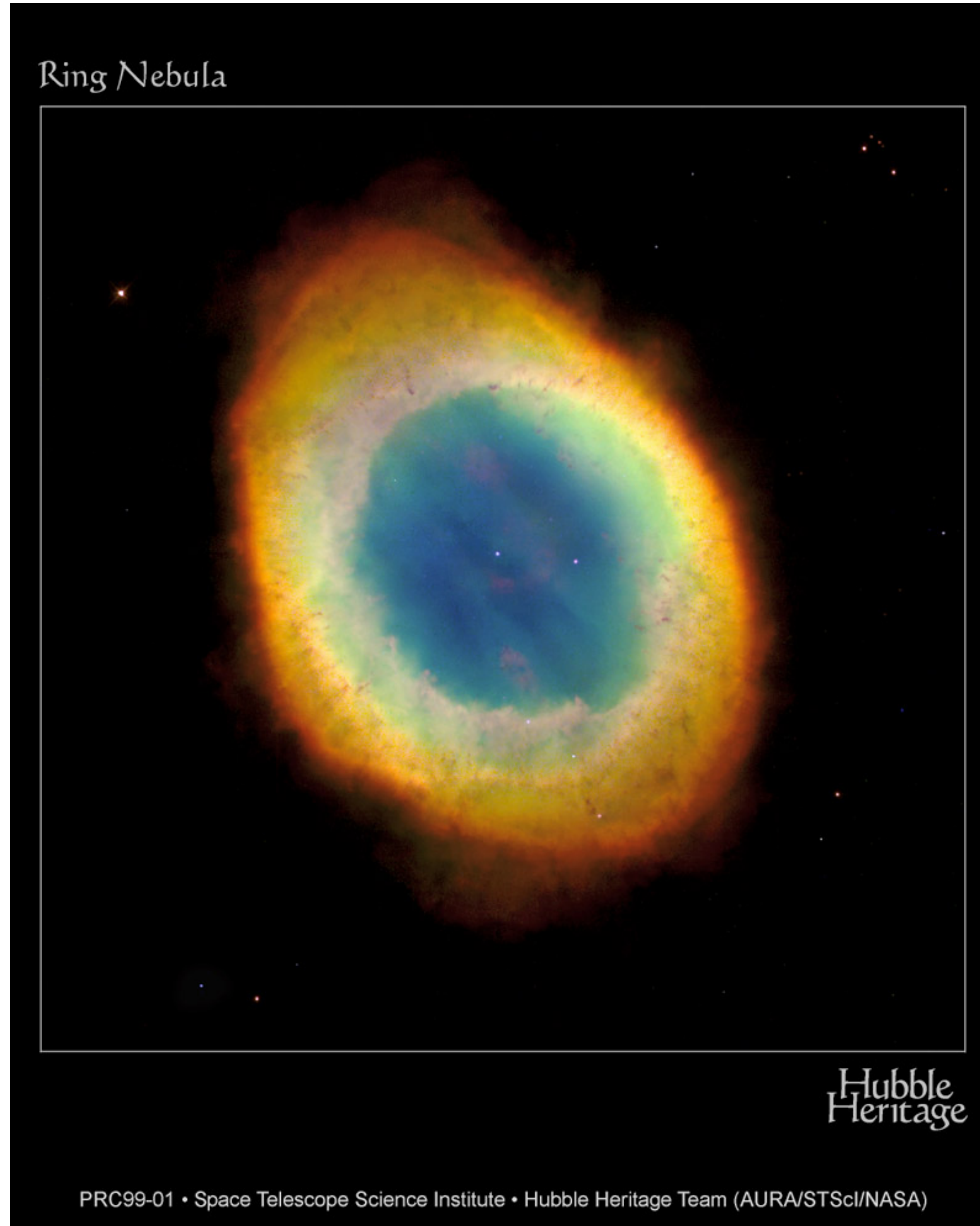
Radically different morphology of HR-diagrams as a function of age. An "isochrone" describes location of stars of fixed age in HR-diagram. (M3 $\approx 11 \times 10^9$yr; Hyades $\approx 6 \times 10^8$yr)

Planetary nebulae represent remnant of extreme form of eruptive variable star.

Cataclysmic variability often provides clue to important stage in the evolution of a star.

How do you interpret the existence of “empty” regions of the HR-diagram?

Two classes of explanation - what are they?



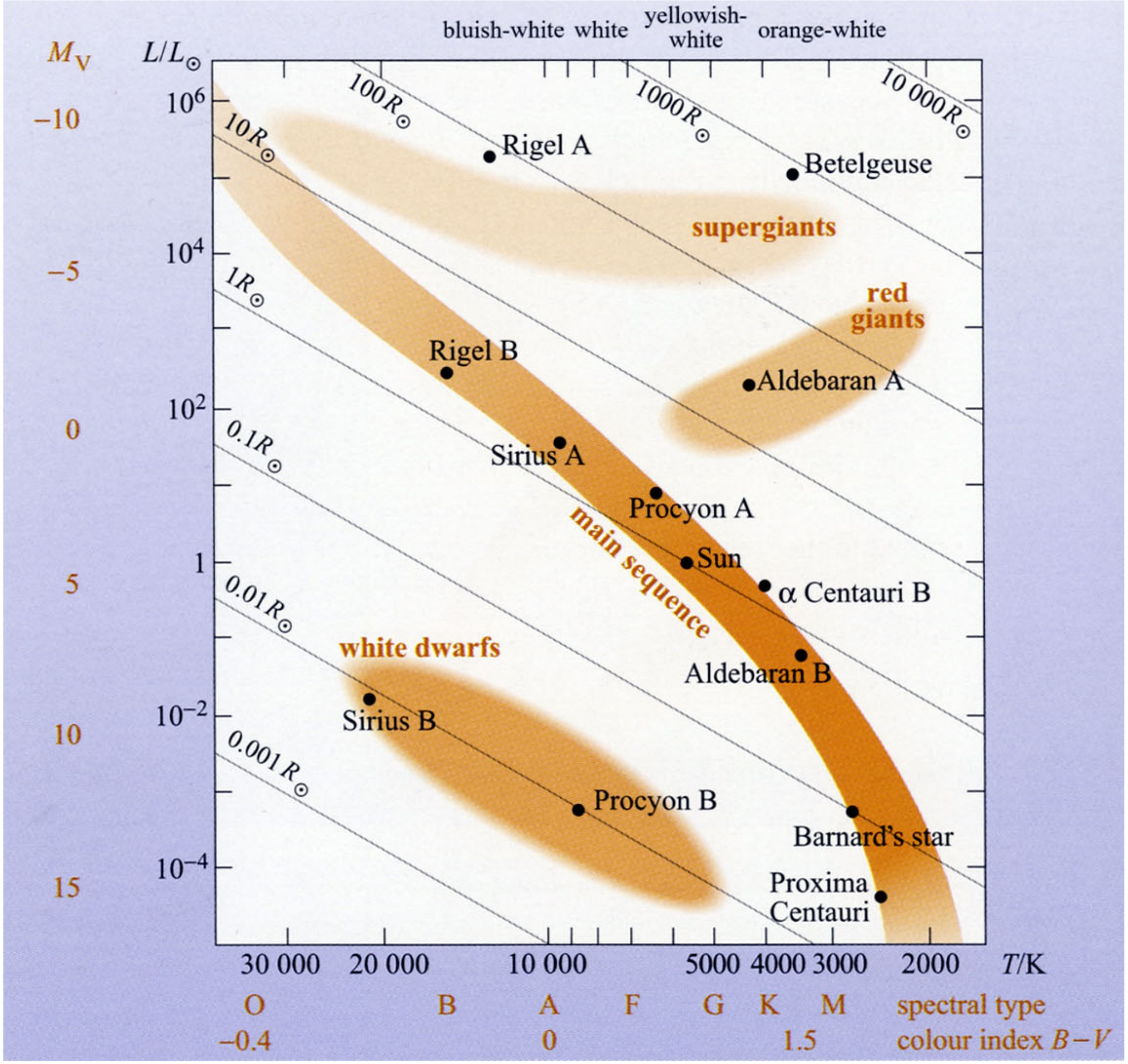


Figure 4.5 The H–R diagram in Figure 4.3, with the addition of stellar radii, and other information. (Adapted from Seeds, 1984)

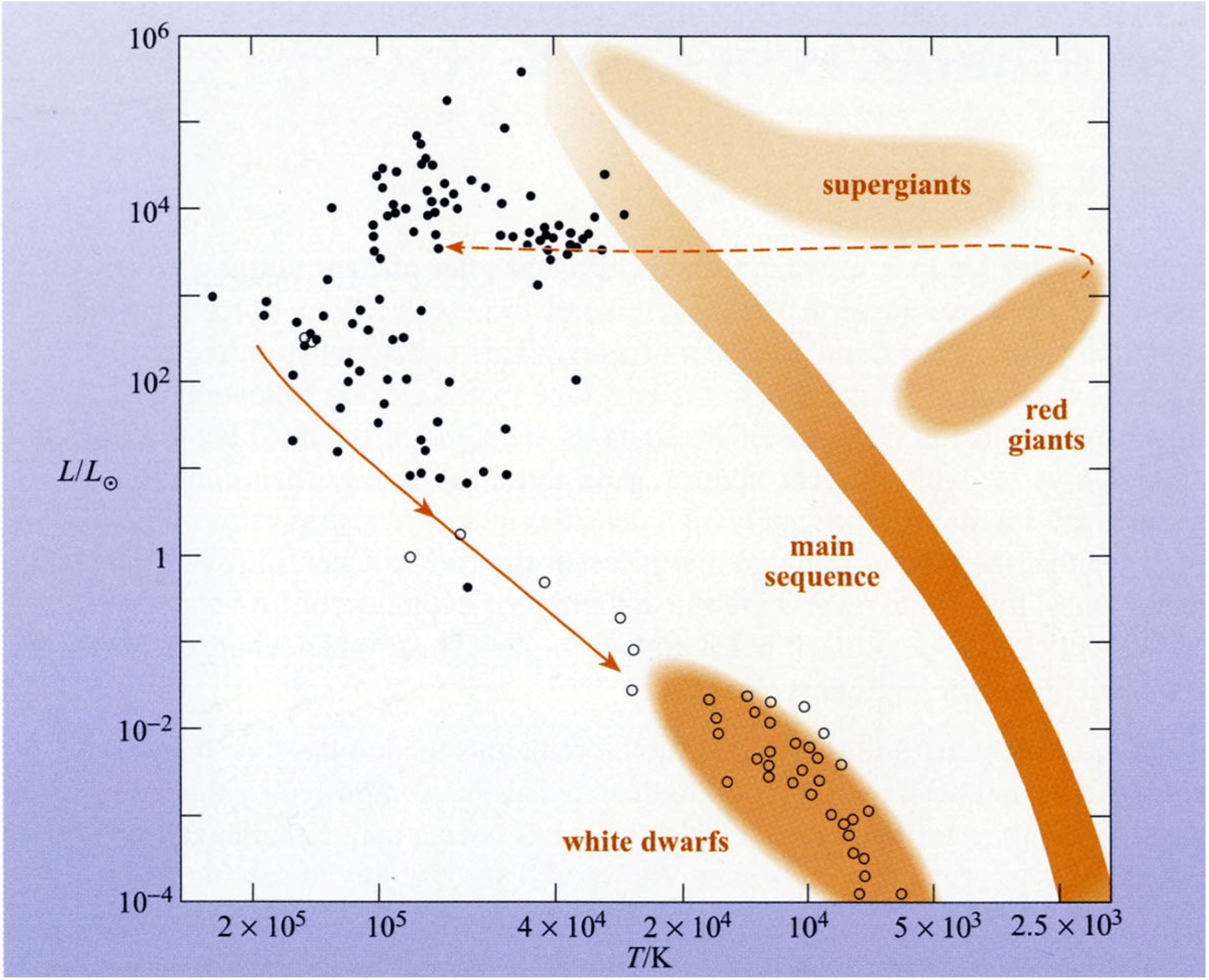


Figure 9.1 The positions of central stars associated with planetary nebulae (dots) and of white dwarfs (open circles) on the H–R diagram. Also shown (solid line) is the evolutionary track that would be followed by a star of constant radius as it cools and (dashed line) a schematic evolutionary track between the regions occupied by AGB stars (Section 8.2.1) and by the central stars of planetary nebulae.

What is a Star?

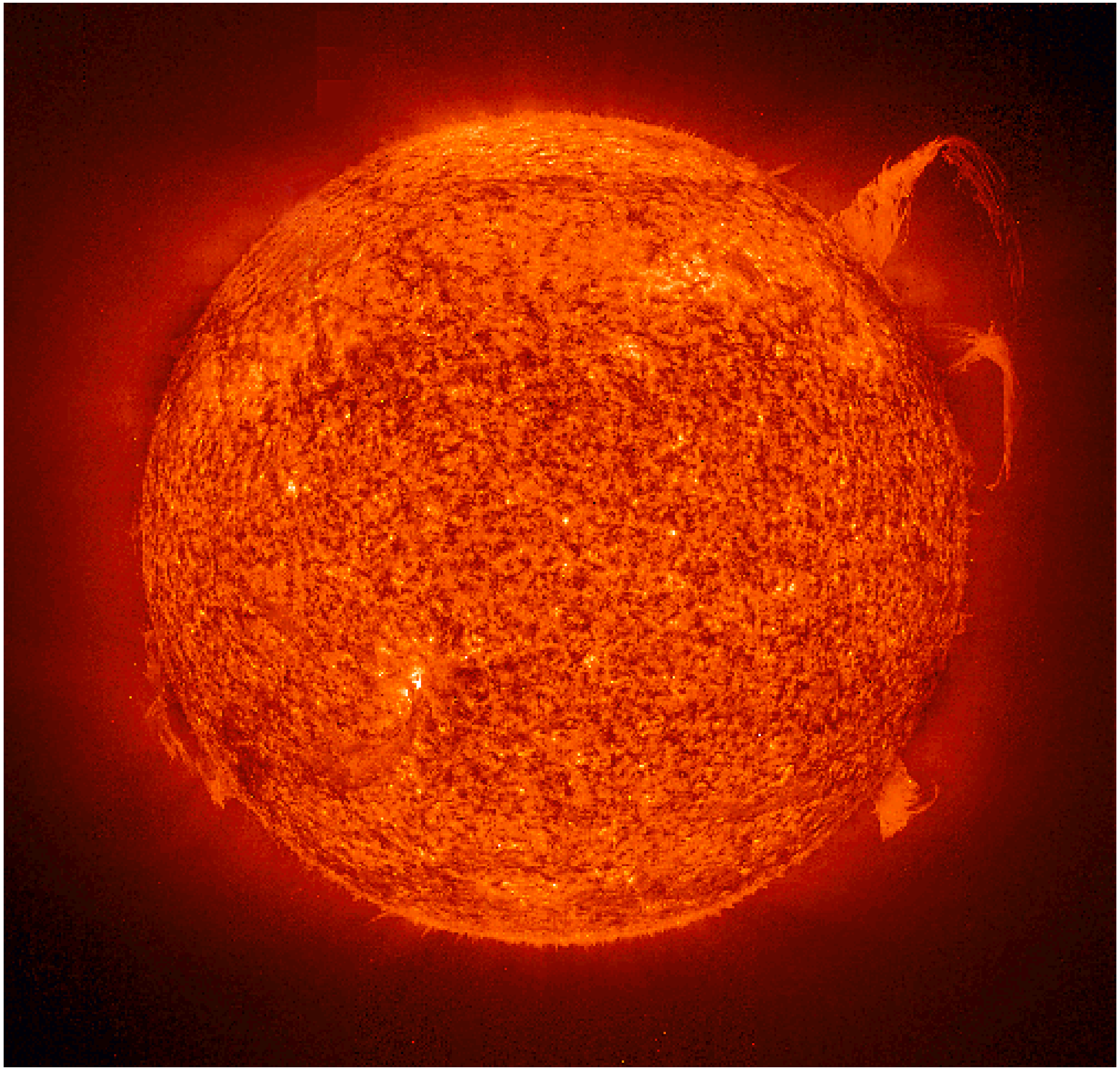
- So far, have avoided definition of a “star”!
- Key conditions:
  1. Held together by self-gravity
  2. Radiates due to an internal energy source
- Self-gravity requires system is spherical, or very close to spherical
- Generation and radiation of energy requires that the system evolves as either the structure and/or the composition of the star changes
- Will confine ourselves to isolated objects, i.e. no significant interaction with, or perturbations from, neighbouring objects (e.g. close binaries)
- Definition excludes planets as stars

Key Timescales for Stellar Evolution

- There are three important timescales associated with stellar structure and evolution:
  - Dynamical
  - Thermal
  - Nuclear
- Start by considering the dynamical timescale, τ_{dyn}
- How long would the Sun take to collapse if gravity alone acted?

Sun

Mass = 2 × 10^{30}kg
Radius = 7 × 10^{8}m
Lum = 3.9 × 10^{26}W
Temp = 5800K



SOHO image at 304A

For particles at radius $r = R$;

$$\frac{1}{2}v^2 = \frac{GM}{R} \quad \Rightarrow \quad v = \sqrt{2GM / R}$$

$$\tau_{dyn} \approx \frac{R}{v} = \sqrt{\frac{R^3}{2GM}}$$

noting that average density $\overline{\rho} = 3M / 4\pi R^3$

$$\tau_{dyn} \approx \frac{1}{3\sqrt{G\overline{\rho}}}$$

Free-fall collapse time for the Sun is approximately 1100s

Something holds the Sun up against gravity!

Do real objects collapse outside-in, inside-out or neither?

Would the Sun sink or float in a bucket of water?

Thermal Timescale

- Suppose that the energy source of the Sun is due only to the energy available from gravitational contraction (from infinity)
- For a star in equilibrium, with luminosity, L≈constant

$$\tau_{therm} \approx \frac{E_{Grav}}{L} = \frac{GM^2}{RL}$$

which, for the Sun, gives

$$\tau_{therm} \approx 10^{15}\,\mathrm{s} \approx 3\times10^{7}\,\mathrm{yr}$$

Also known as the Kelvin-Helmholtz timescale

Thermal Timescale

- Late in the 19th century the mismatch between the thermal timescale for the Sun and the time for evolution on the Earth (Darwin) was recognised as a major problem.
- Subsequent age dating of terrestial and lunar rock samples via radioactive decay shows that the Solar System is $\sim 4.5\times10^{9}$yr old, producing a two order of magnitude discrepancy
- Less specific but also powerful as an argument is the age of the Universe, known for decades to be $\sim 10^{10}$yr (present estimate 13.8×10^{9}yr) which would make existence of many stars with lifetimes $\sim 10^{7}$yr now extremely unlikely
- Eddington (1920) predicted that the energy source would be sub-atomic in nature, confirmed by developments in quantum and nuclear physics

Nuclear Timescale

Einstein's relation $E=mc^2$ applies when nuclear fusion takes place and a small fraction of the rest-mass energy is liberated as the binding energy per nucleon increases. For the conversion of H into He, the fraction of the rest-mass liberated is ≈0.007, i.e. ~1%
For a star in equilibrium, with luminosity, L≈constant, and

$$\tau_{nuc} \approx \frac{0.007 Mc^2}{L}$$

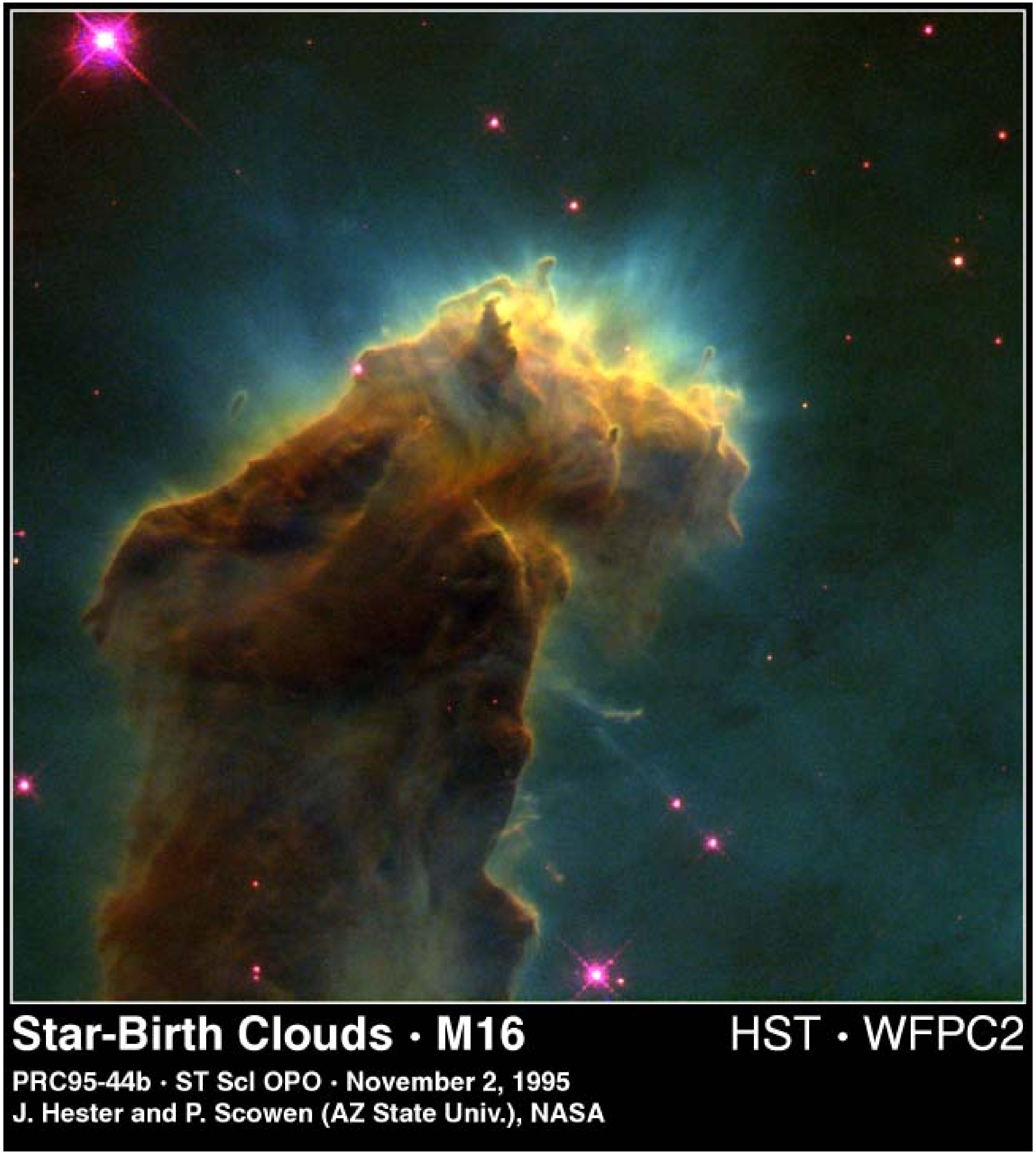
which, for the Sun, gives

$$\tau_{nuc} \approx 3\times10^{18}\,\mathrm{s} \approx 10^{11}\,\mathrm{yr}$$

a lifetime well in excess if the age of the Universe

Timescales and Stellar Evolution

- Can you think of examples of when, in the lifetime of stars, evolution is governed by:
  - Thermal timescale
  - Nuclear timescale
  - Dynamical timescale


Star-Birth Clouds · M16
HST · WFPC2
PRC95-44b · ST ScI OPO · November 2, 1995
J. Hester and P. Scowen (AZ State Univ.), NASA

Sun

Mass = 2×10^{30}kg
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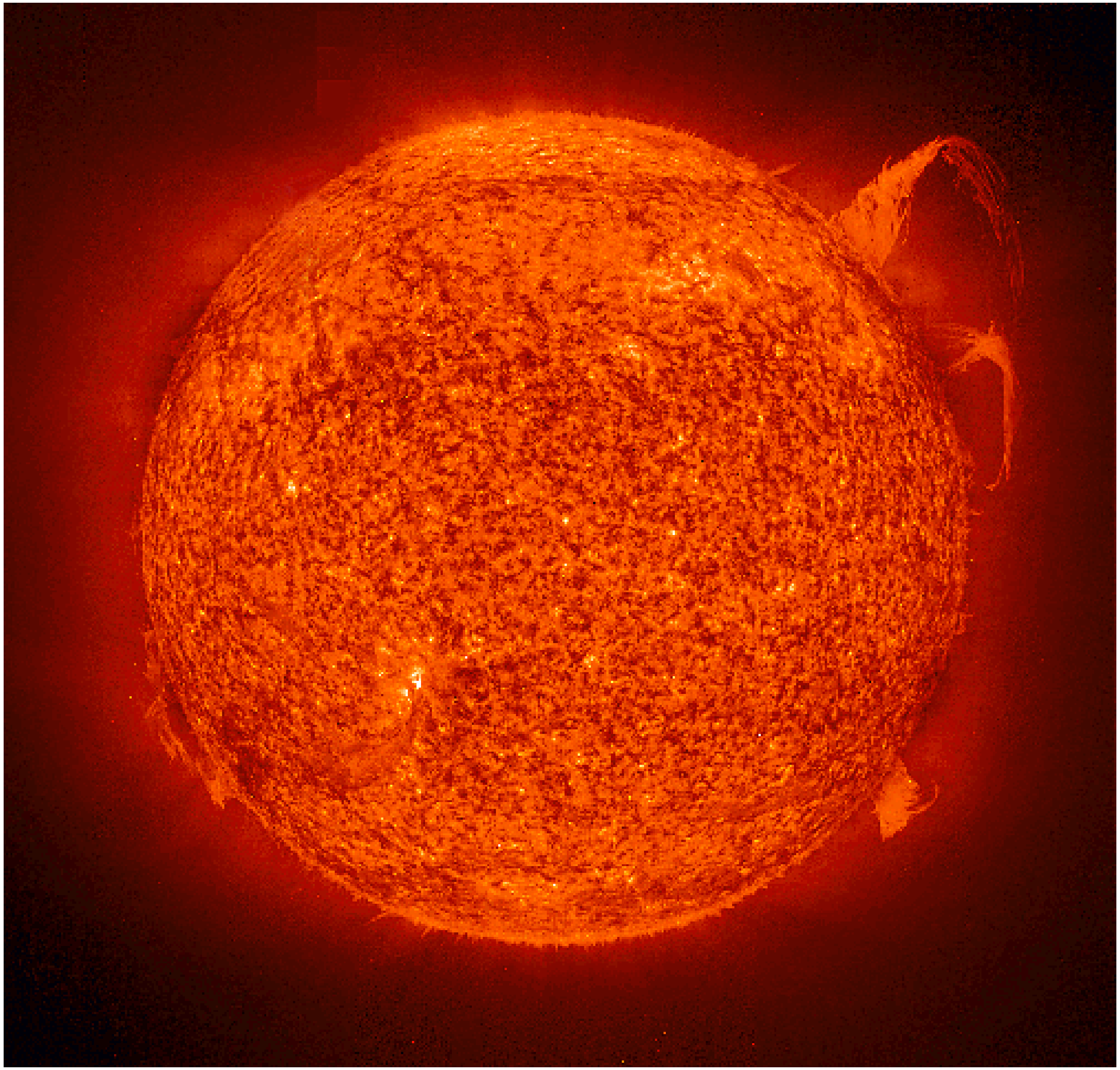
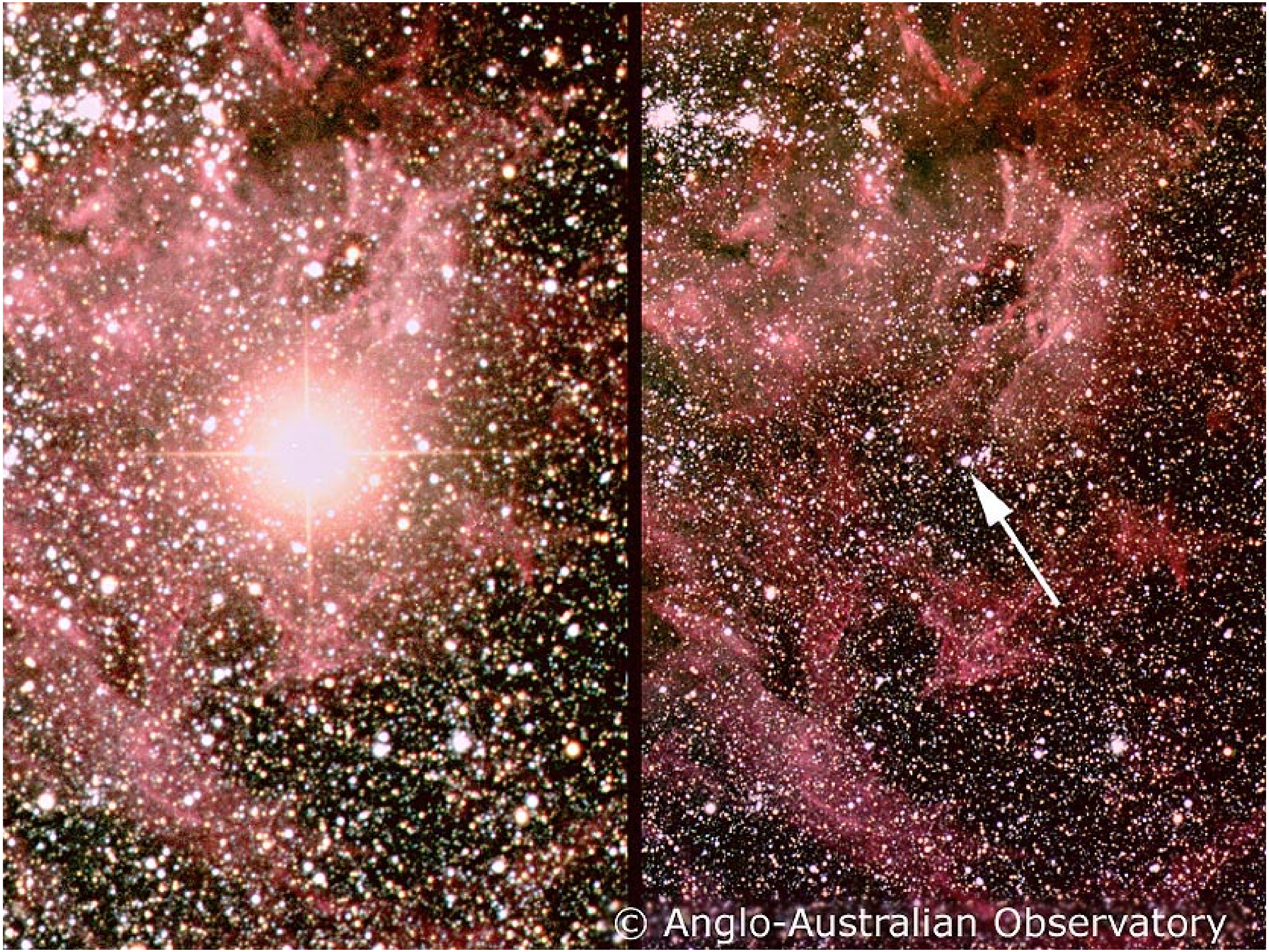
SOHO image at 304A

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Lecture 5: Summary

- Empirical Mass-Luminosity relation "Main sequence"
- Variable stars, standard candles and photometric distances
- Sparsely populated regions of the Hertzsprung-Russell diagram. You should understand the significance of the density of stars along an isochrone and in the HR-diagram as a whole
- Operational definition of a star – gravity is critical
- Key timescales for stellar evolution:
  - Dynamical
  - Thermal
  - Nuclear

  You should understand the implications of the very different timescales and be prepared to link the appropriate timescales to phases of stellar evolution

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