

Structure and Evolution of Stars

Lecture 2: Observational Properties

- Distance measurement
- Space velocities
- Apparent magnitudes and colours
- Absolute magnitudes and luminosities
- Blackbodies and temperatures of stars
- Wien's Law
- Stars as blackbodies – relating L , T and R

Direct Distance Determination

- Trigonometric parallax - the apparent change in the direction of an object due to change in the position of the observer is the only generally applicable method for direct distance determination (familiar in everyday life from binocular vision)
- Diameter of Earth's orbit can be used as the baseline change for the observer (observations made approximately 6 months apart)
- Average radius of Earth's orbit defines the astronomical unit = $AU = 1.496 \times 10^{11}m$

Stellar Parallax

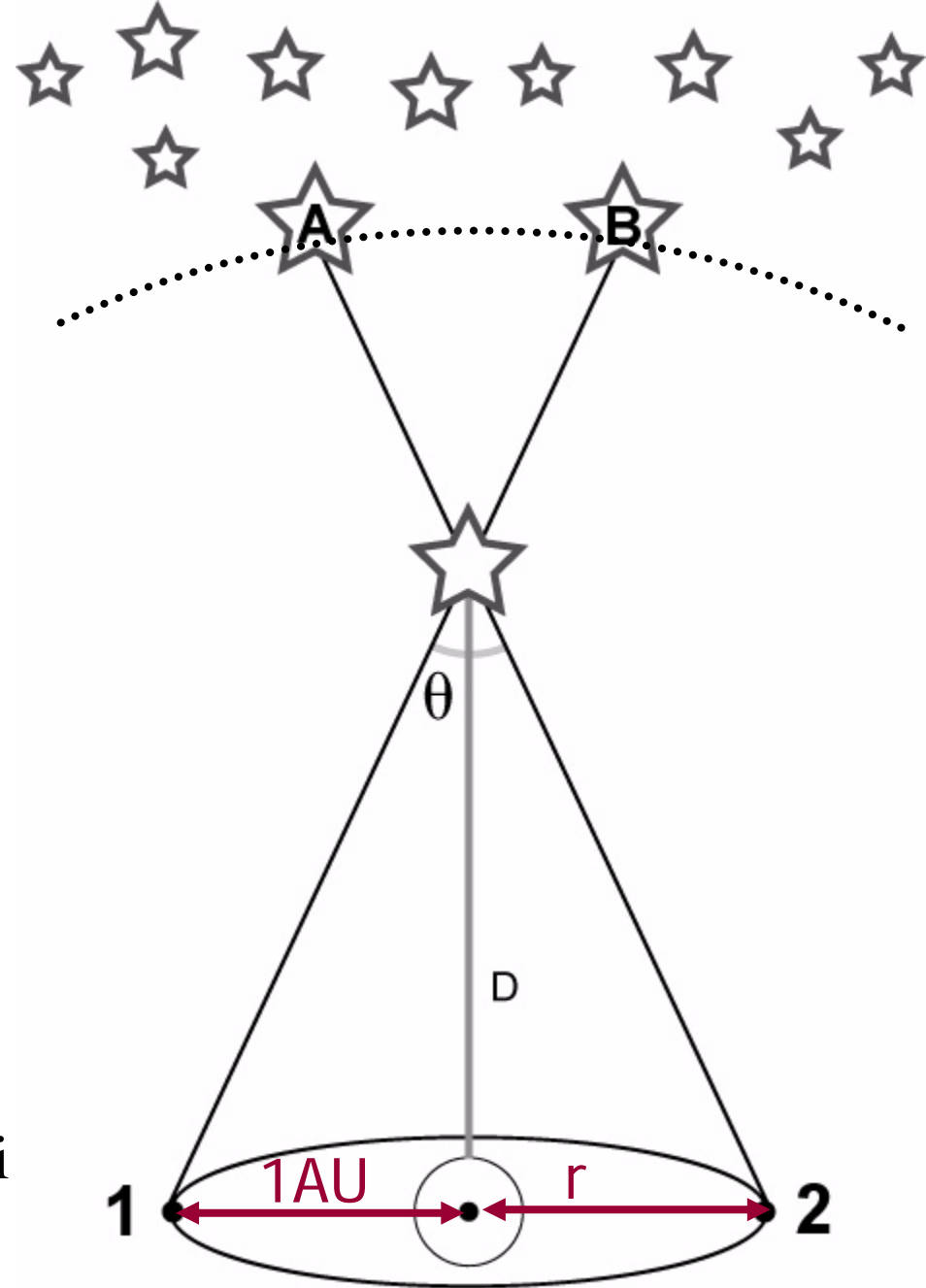
- View a star over a year to measure θ (or 2θ)
- Distance $D \gg 1\text{AU}$

$$\tan \theta = \frac{r}{D}$$

$$\tan \theta \approx \sin \theta \approx \theta \approx \frac{r}{D}$$

$$D = \frac{r}{\theta}$$

- Nearest star Proxima Centauri
 $p=0.764''$



Trigonometric Parallax

- $D=r/\theta$ where θ is in radians and if r taken as 1AU then D is also in AU.
- In practice, θ is small, even for the nearest stars and is usually expressed in arcseconds (1 degree = 60 minutes = $60' = 3600$ arcseconds = $3600''$).
- The value of the distance D when $\theta=1''$ defines the “parsec”, abbreviated “pc”, and $1\text{pc}= 3.09 \times 10^{16}\text{m}$. The parsec is the standard measure of distance in astronomy ($1\text{pc} = 206265\text{AU}$).
- Example: star at $D=50\text{pc}$ has parallax = $0.02''$

Table 3.1 The ten nearest stars after the Sun.

Name	Parallax /arcsec	Distance		Proper motion /arcsec yr ⁻¹	Comment ^a
		/pc	/ly		
Proxima Centauri	0.772	1.30	4.22	3.9	} triple system
α Centauri A	0.747	1.34	4.36	3.7	
α Centauri B	0.747	1.34	4.36	3.7	
Barnard's Star	0.547	1.83	5.95	10.4	
Wolf 359	0.419	2.39	7.77	4.7	
Lalande 21185	0.393	2.54	8.28	4.8	
Sirius A	0.380	2.63	8.57	1.3	} binary system
Sirius B	0.380	2.63	8.57	1.3	
L-726-8A	0.373	2.68	8.73	3.4	} binary system
L-726-8B	0.373	2.68	8.73	3.4	

- Parallax measurement difficult
 - “background stars” do not define “fixed” reference for measurement
 - apparent motion of stars on the sky, “proper motion”, must also be determined
 - Atmospheric seeing is at best 0.5” and refraction within the atmosphere further complicates precise determination of stellar positions
- Star with parallax measure, combined with angular position on the sky (right ascension and declination), gives location of star in 3-dimensions.
- Limit to accuracy of parallax measures from the Earth is $\sim 0.01''$ (ie $D < 100\text{pc}$), satellites, notably HIPPARCOS, reached $\sim 0.001''$ ($D < 1000\text{pc}$). Small compared to size of the Galaxy – only small sphere with reliable distance determinations. GAIA satellite will give $\sim 0.000010''$

Proper Motion

- The Sun and stars in the Galaxy possess large relative space velocities
 - Rotation of Sun around Galaxy $\sim 220 \text{ kms}^{-1}$
 - Typical stellar velocities within Galactic disc $\sim 20 \text{ kms}^{-1}$
- Over time, component of relative space velocity perpendicular to the line-of-sight produces apparent motion of star on the sky – proper motion
- Proper motion for all but the nearest stars is small and values are usually quoted in units of arcseconds per year, e.g. $0.46''\text{yr}^{-1}$. For star with known distance, proper motion gives the transverse velocity (in kms^{-1})

Proper Motion

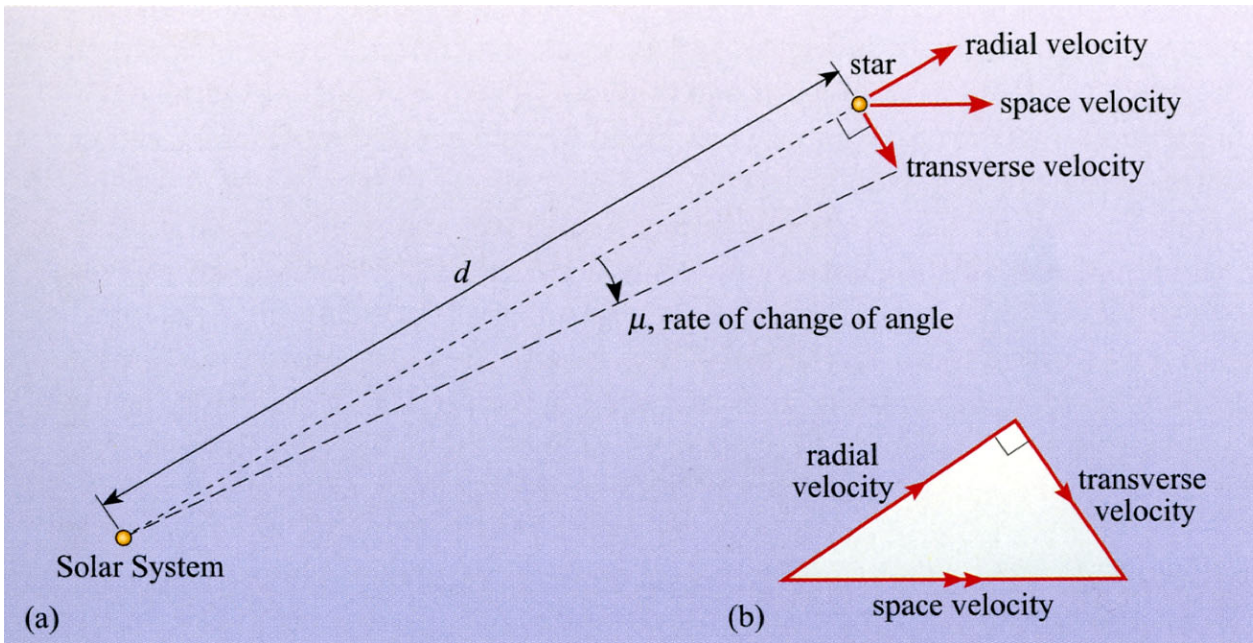


Figure 3.3 (a) A star's motion through space, relative to the Sun. (b) The overall velocity through space, the space velocity, has two components: the radial velocity in the observer's line of sight and the transverse velocity in the plane of the sky. (Space velocity will be described below.)

For star with measured distance, determination of proper motion in both right ascension and declination provides space velocity in 2 dimensions

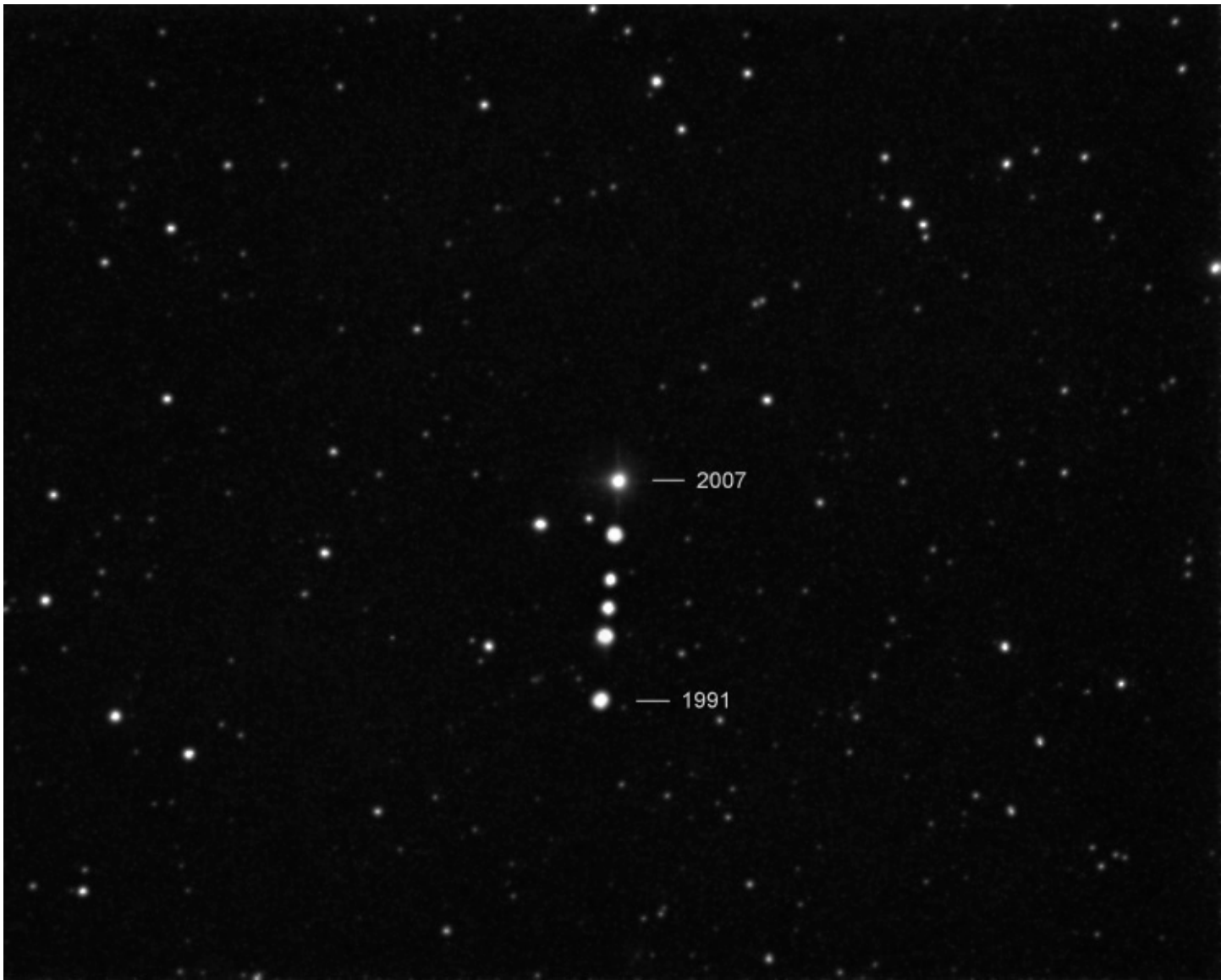
Doppler Shift

- Determining component of space velocity parallel to the line-of-sight provides final component to specify velocity in 3 dimensions
- Spectroscopic observation of star containing feature(s) of known rest-frame wavelength, λ_R . Determine the observed wavelength of the feature(s), λ_O , and calculate radial-velocity, or Doppler shift, v_r :

$$v_r = c \times (\lambda_O - \lambda_R) / \lambda_R = c \times \Delta\lambda / \lambda$$

where c is the velocity of light = 299792 kms⁻¹

- In ideal case, have angular position, distance (gives 3-D spatial position), proper motion and radial velocity (gives velocity in 3-D) – location and velocity of star completely specified
- Rare to have all six quantities – angular position and radial velocity relatively easy to obtain but distance and proper motion (for distant objects) often hard or impossible. GAIA will again make a dramatic increase in the accessible volume around Sun where data is available



Barnard's Star: $d=1.83\text{pc}$; proper-motion $\mu=10.4\text{ arcsec/year}$
Radial velocity= -110km/s , tangential velocity? Image $\sim 14\text{ arcmin}$ across.

Sagittarius Star Cloud



HST image of
star field
towards centre
of the Galaxy

Hubble
Heritage

The Stellar Magnitude Scale

- $L = 4\pi d^2 f$ – where L is the intrinsic luminosity, d is the distance and f is the flux measured by the observer.
[assumes that the source radiates isotropically – good approximation for most stars]
- For historical reasons the apparent brightness (= measured flux) of sources is specified using a magnitude scale
- Definition:

$$m_\lambda = -2.5 \times \log_{10} f + \text{constant}$$

where m_λ is the “apparent magnitude”. The flux, f , is measured within a well-defined magnitude range, with standard “passbands” indicated by “ λ ” or similar, e.g. $m_B = m(B) = B = B$ all used for B-band magnitude

$$m_{\lambda} = -2.5 \times \log_{10} f + \text{constant}$$

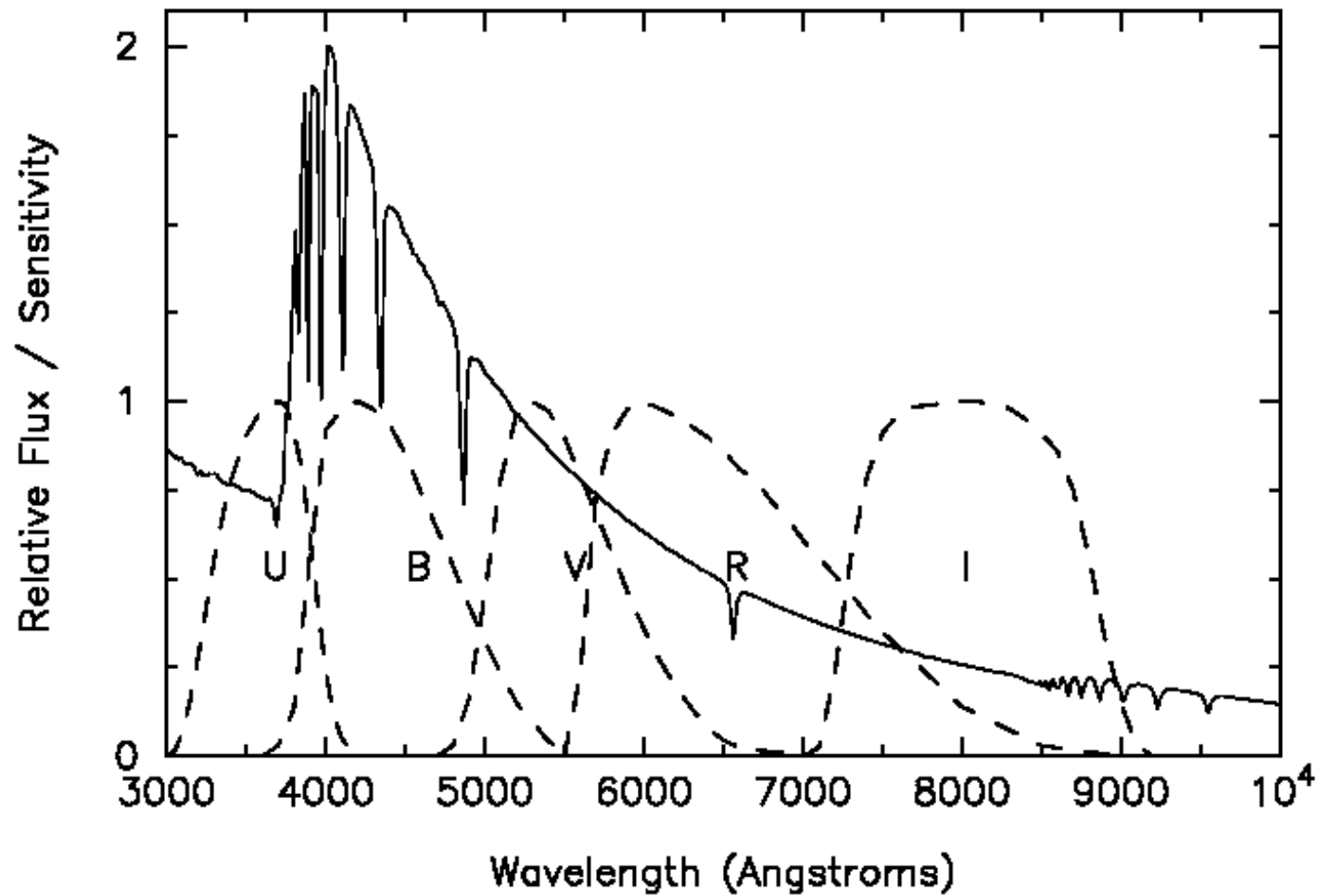
- The minus sign, “-”, is important – fainter objects have large values of m
- The constant that defines the zero-point of the scale was defined historically such that the bright visible star Vega (α Lyr) has zero-magnitude in all passbands. Thus, for Vega

$$m_{\text{B}} = m_{\text{V}} = m_{\text{R}} = \dots = m_{\text{K}} = 0.0$$

where the subscripts indicate passbands that span the optical through near-infrared region (400-2500nm)

Astronomers use of the “magnitude” system causes disbelief in many a physicist(!) but even today the system is essentially unchanged from that set up originally

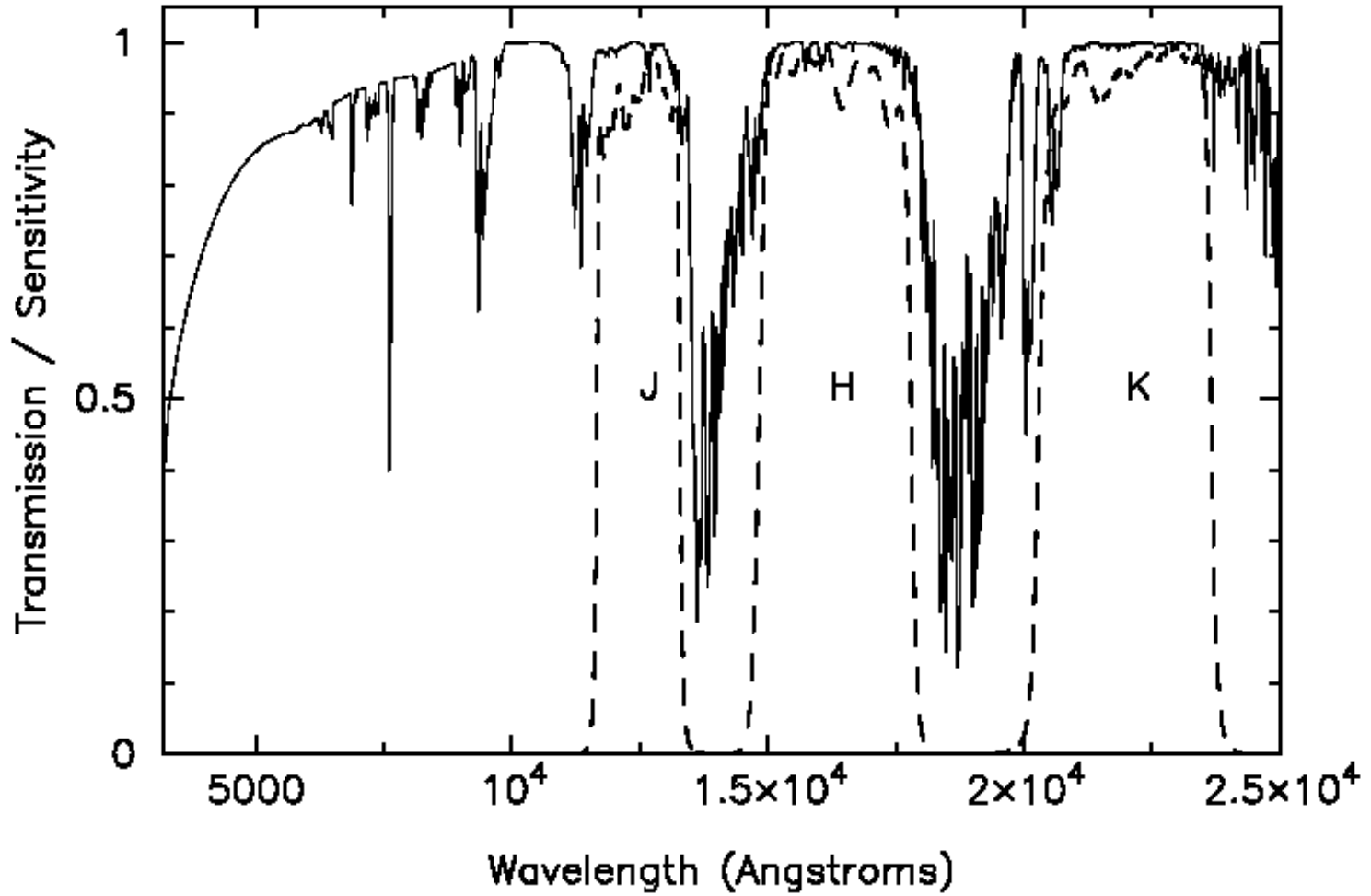
Vega Spectrum with Johnson–Cousins UBVRI Passbands



Standard Passbands in the Visible and Near-infrared

Passband	λ_{eff} (nm)	Comment
U	360	Shortest possible from ground
B	440	visible
V	550	visible
R	650	visible
I	750	
Z	850	
J	1200	near-infrared
H	1600	near-infrared
K	2200	limited by thermal emission

Atmospheric Transmission + JHK Passbands



Apparent Magnitudes

Object	m_V
Sun	-26.7
Venus (brightest)	-4.5
Sirius (brightest star)	-1.4
Vega	0.0
Faintest star (from Cambridge street)	3.5
M31 (brightest galaxy)	3.5
Faintest star (dark site, dark-adapted)	6.0
Brightest quasar	12.0
Dark night sky (surface brightness)	21.5 \square''
Faintest object detected	30.0

Absolute Magnitudes and Distance

- Absolute magnitude is defined as the magnitude an object would have if observed at a distance of 10pc
- Absolute magnitudes are indicated using capital letters e.g. M_V is the V-band absolute magnitude
- Absolute magnitude is astronomers' scheme for quantifying the luminosity of objects, $L = 4\pi d^2 f$, and provides a convenient measure of the distance of an object via the “distance modulus” (DM)

$$\begin{aligned} \text{DM} &= m - M = -2.5 \log f_d + 2.5 \log f_{10} \\ &= -2.5 \log (f_d / f_{10}) \end{aligned}$$

From inverse-square law, $f \propto 1/d^2$, and

$$f_d / f_{10} = (10/d)^2$$

and $m - M = -2.5 \log (10/d)^2$

$$DM = m - M = 5 \log d - 5$$

$$\text{or } d = 10^{(m-M+5)/5} \text{ pc}$$

If absolute magnitude for an object known then can measure apparent magnitude, m , and calculate distance, d .

[Will drop explicit base for “log”s, “log” = “log₁₀”]

Blackbody Radiation

Spectrum of blackbody as a function of temperature, T:

$$B_{\lambda}(T) = B(\lambda, T) = \frac{2hc^2 / \lambda^5}{e^{hc / \lambda kT} - 1}$$

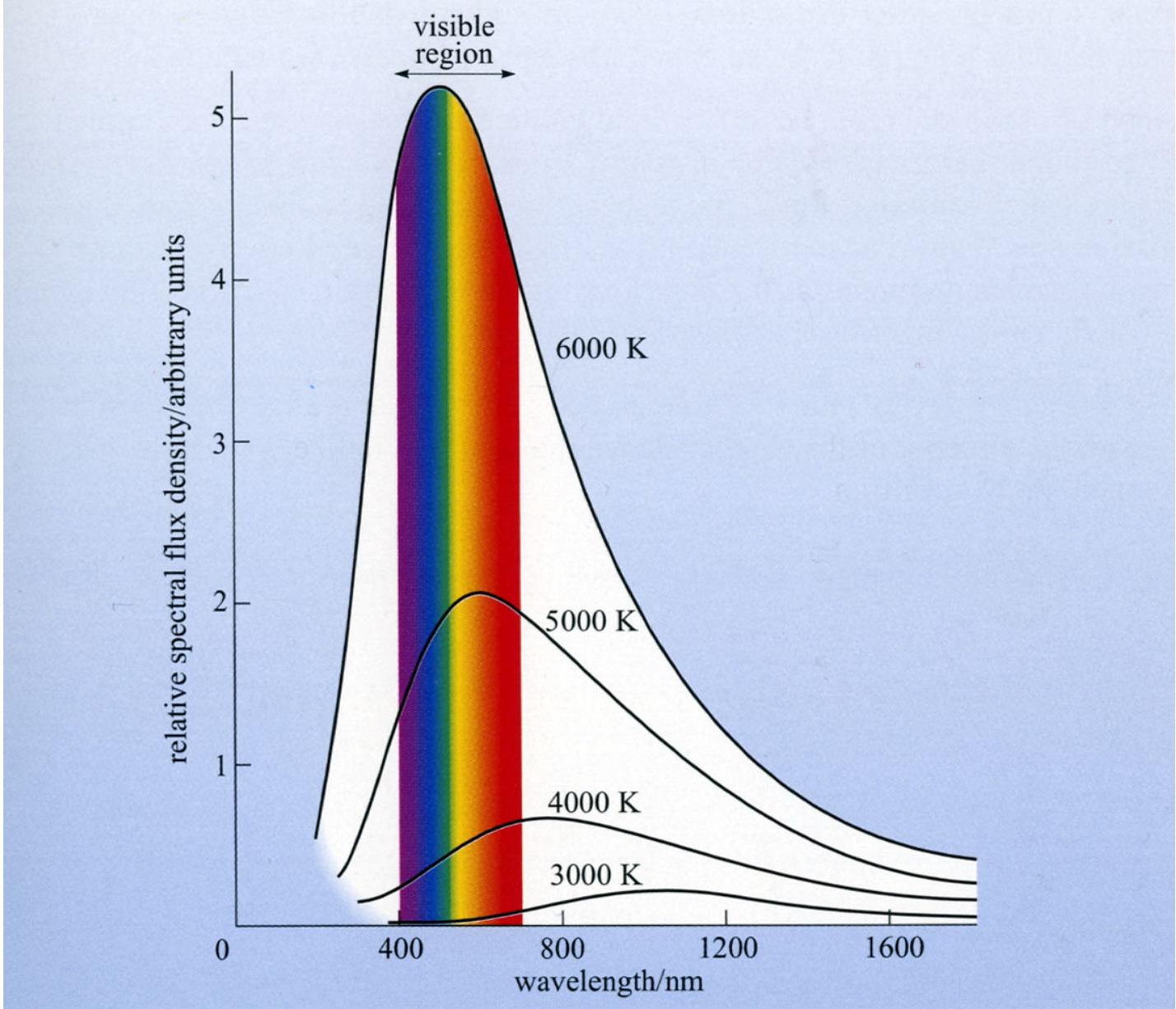
where h is Planck's constant and c is the velocity of light with common units in astronomy, $\text{ergs cm}^{-2} \text{A}^{-1} \text{sr}^{-1} \text{s}^{-1}$ or $\text{Watts m}^{-2} \text{nm}^{-1} \text{sr}^{-1}$ Alternatively, in terms of frequency, ν :

$$B_{\nu}(T) = B(\nu, T) = \frac{2h\nu^3 / c^2}{e^{h\nu / kT} - 1}$$

units, $\text{ergs cm}^{-2} \text{Hz}^{-1} \text{sr}^{-1} \text{s}^{-1}$ or $\text{Watts m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$

The blackbody spectrum is strongly peaked – most energy emitted within narrow wavelength (frequency) range

Blackbody curves for different T



Wien's Displacement Law

From the blackbody distribution can show that the wavelength at which the function peaks and the Temperature of the blackbody are simply related by:

$$\lambda_{\max} \times T = 0.290 \quad (\text{cm K})$$

Wien's Law is extraordinarily useful and worth remembering

Wien's Displacement Law

T (K)	λ (μm)	Source
20	145	Molecular cloud
293	9.9	Room temperature
1000	2.9	Brown dwarf
3400	0.85	Red giant
5800	0.50	Sun
40000	0.07	O star

Ratio of fluxes in two passbands uniquely determines the temperature of a blackbody

In terms of the magnitude system, the difference between magnitudes in two passbands gives the “colour index” or “colour” of an object, e.g. measures in the B and V band

$$B = -2.5 \log f_B + \text{const}$$

$$V = -2.5 \log f_V + \text{const}$$

$$B - V = -2.5 \log (f_B/f_V)$$

and the B-V colour thus measures T

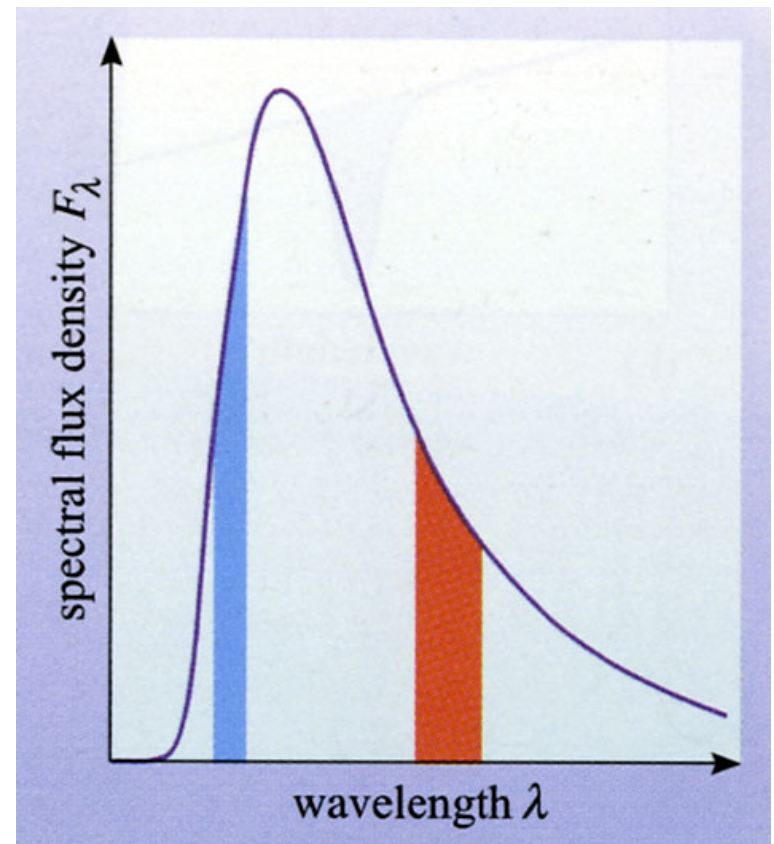


Figure 3.19 The photometric method of obtaining photospheric temperatures. The ratio of the amount of energy measured in two different wavelength regions (shaded) is uniquely defined by the temperature if the object emits like a black body.

Colours of Stars

It follows from the definition of the zero-point for the magnitude system that for Vega all colours have zero values, i.e. $U-B = B-V = V-R = V-I = \dots = J-K = 0.0$

Conventionally, all colours are specified in sense blue-red

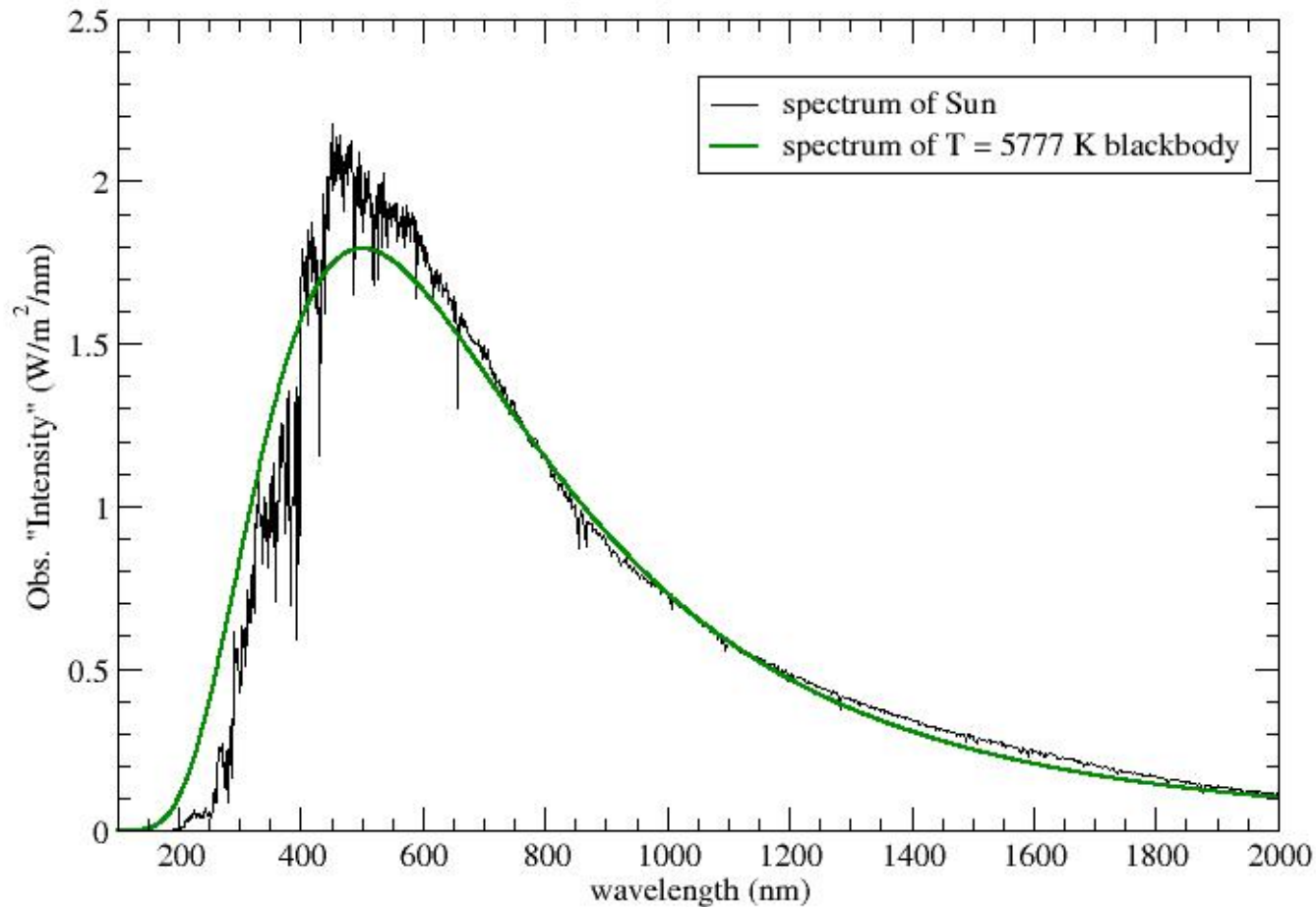
Thus, objects with redder spectral energy distributions than Vega have positive colours, e.g. for the Sun, $B-V=0.6$

Objects with bluer spectral energy distributions than Vega have negative colours, e.g. for an O star, $B-V \approx -1.0$

B-V colours most common for historical reasons (photographic emulsions) but longer baseline often better

Sun's Spectrum vs. Thermal Radiator

of a single temperature $T = 5777 \text{ K}$



Most stellar spectra are well approximated by blackbodies

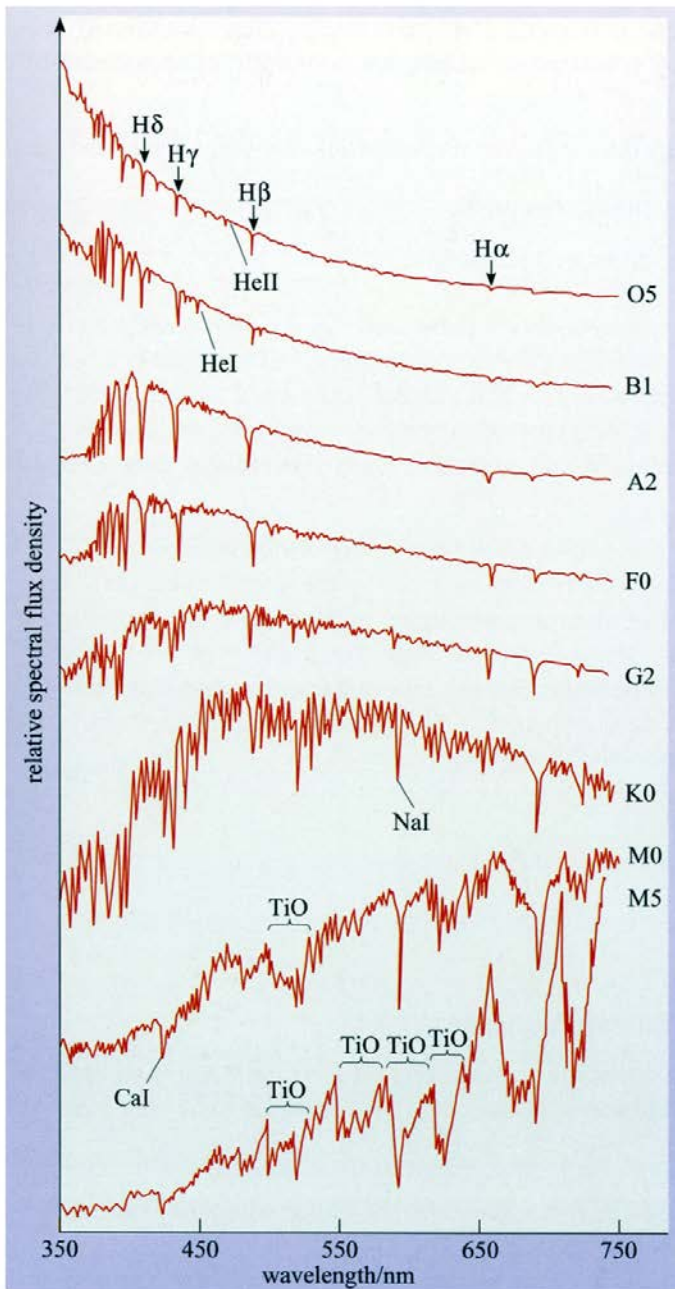


Figure 3.26 The stellar absorption spectra given in Figure 3.25 are more usually presented as graphs of relative flux density versus wavelength for ease of identification of the prominent absorption lines. The spectra have been plotted without spectral flux density scales and displaced vertically for clarity. (Kaufmann and Freedman, 1998)

Stefan Boltzmann Law: Approximating Stars as Blackbodies

- Integrate the blackbody spectrum over all wavelengths

$$\int B_{\lambda}(T)d\lambda = \sigma T^4$$

where σ is the Stefan-Boltzmann constant.

- If surface element of the photosphere of a star emits like a blackbody then

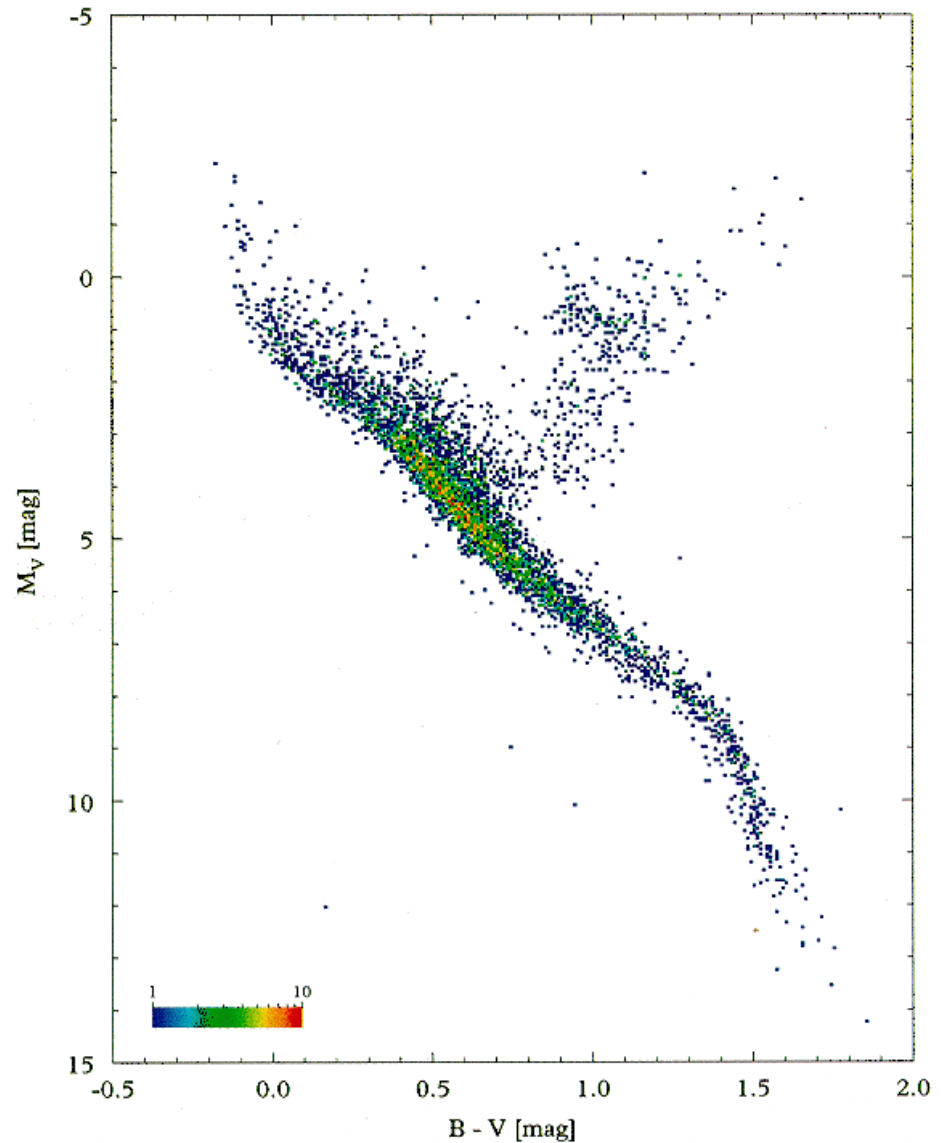
$$L = 4 \pi R^2 \sigma T_e^4$$

relates luminosity, L , radius, R , and effective temperature, T_e of the star. Stars are not in fact perfect blackbodies and T_e is the temperature the star (radius R) would have if it were a blackbody.

- $L = 4 \pi R^2 \sigma T_e^4$ is very powerful relation as now have L and T for some stars, gives first handle on radii

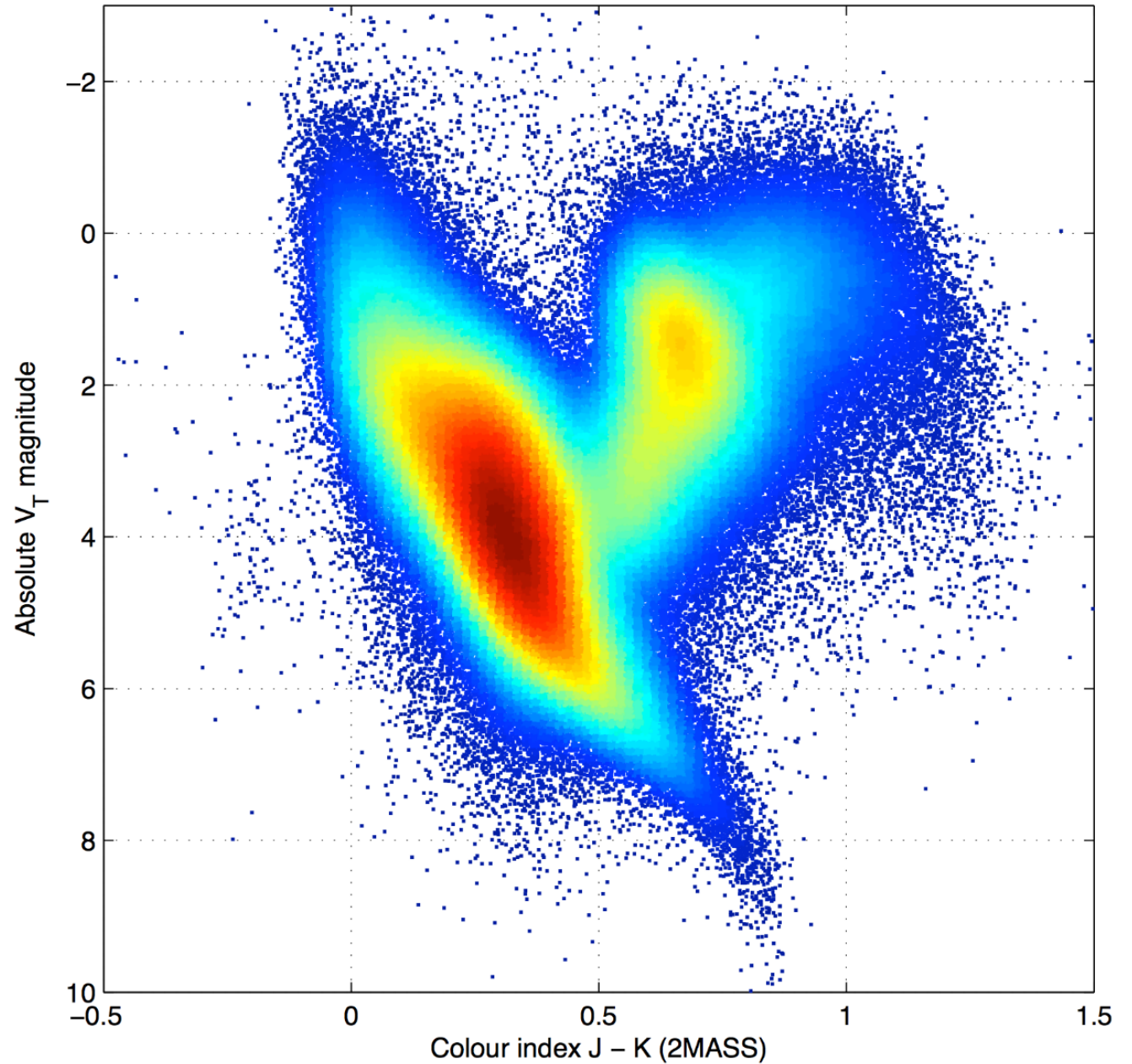
Hertzsprung-Russell diagram
from HIPPARCOS satellite –
bright, nearby stars.

What can now be deduced
from the relation
 $L = 4 \pi R^2 \sigma T_e^4$?



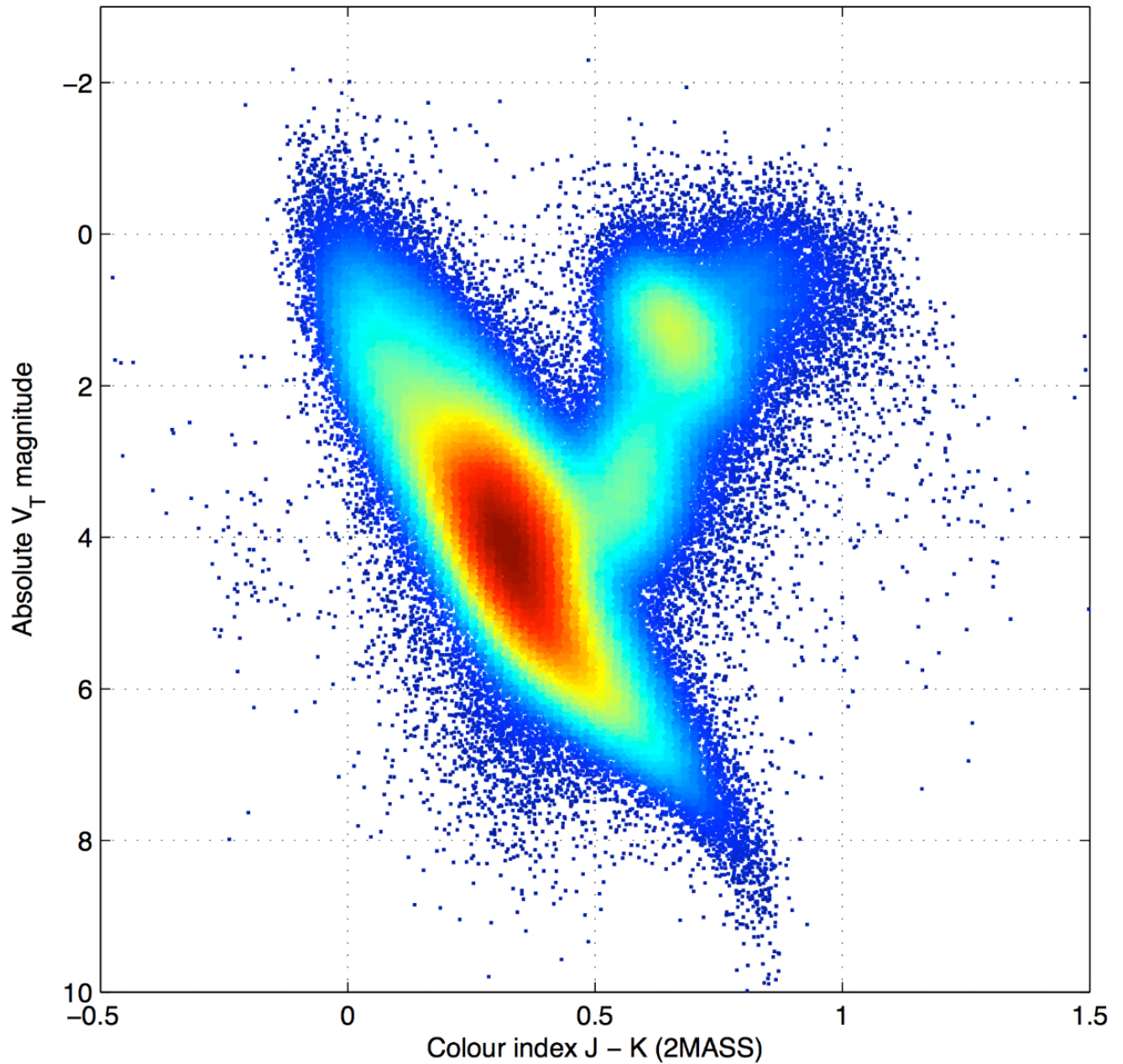
916832 non-HIP stars with $\sigma < 1.0$ mas and $\varpi / \sigma > 5.0$

GAIA DR1
HR-Diagram
S/N(dist.)>5



481147 non-HIP stars with $\sigma < 1.0$ mas and $\varpi / \sigma > 10.0$

GAIA DR1
HR-Diagram
S/N(dist.)>10



Lecture 2: Summary

Looked at how observational properties can give measurements of physical properties of interest, such as Luminosity, Temperature

You should understand the measurement of parallax, proper motion and Doppler shift and be able to calculate distances, luminosities and space velocities

You should understand the basis for the measurement of magnitudes and colours and be able to calculate relative and absolute luminosities given the magnitudes of objects

You should be able to apply Wien's Law to relate T and peak of blackbody spectrum. Approximating emission from stellar photospheres as blackbody-like provides simple but powerful relation between L , R and T_e

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