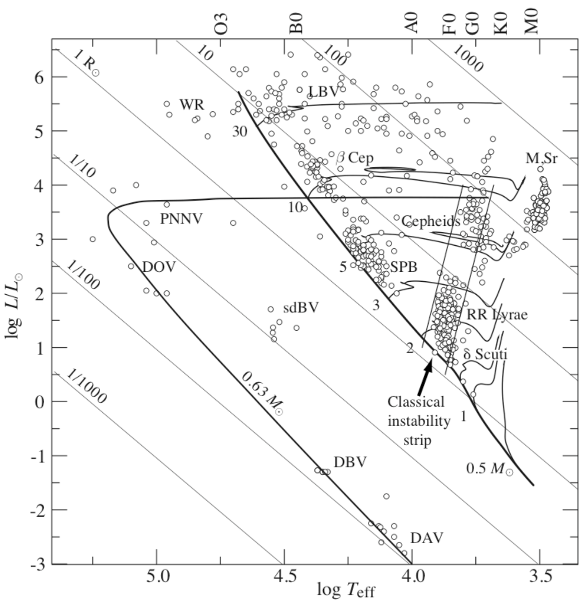
EXERCISES: Set 4 of 4

Q1: The figure below shows, among other things, the path followed in the H-R diagram by a cooling white dwarf of mass $M = 0.63M_\odot$.



(i) Deduce the slope of the straight-line track followed by the cooling WD. How does it compare with the slope in the Figure?

(ii) What is the radius (in solar units) of the white dwarf when $L/L_\odot = 0.01$?

Q2: The following three approximate relations apply to massive stars on the main sequence:

(1) The mass–luminosity relation:

$$\log\left(\frac{L}{L_\odot}\right) \approx 0.78 + 2.76 \log\left(\frac{M_\mathrm{i}}{M_\odot}\right)$$

where M_i is the initial mass;

(2) The mass-loss rate–luminosity relation:

$$\log\left(\frac{\mathrm{d}M}{\mathrm{d}t}\right) \approx -12.76 + 1.30\,\log\left(\frac{L}{L_\odot}\right)$$

where $\mathrm{d}M/\mathrm{d}t$ is in $M_\odot/\mathrm{yr}$; and

(3) The main-sequence lifetime–mass relation:

$$\log\tau_{\mathrm{MS}} \approx 7.72 - 0.66\,\log\left(\frac{M_{\mathrm{i}}}{M_\odot}\right)$$

(i) Use these relations to calculate the fraction of the initial mass that is lost by massive stars with $M_{\mathrm{i}} = 25$, 40, 60, 85, and $120\,M_\odot$ before they evolve off the main sequence.

(ii) A star with $M_{\mathrm{i}} = 85M_\odot$ has a convective core that contains 83% of the stellar mass. Calculate the time after the star appears on the main sequence at which the products of nuclear burning will appear at the surface.

Q3: (i) Show that for a spherically symmetric star the equations of mass continuity and hydrostatic equilibrium can be combined into the second-order differential equation:

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{r^2}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r}\right] = -4\pi G\rho$$

(ii) For a gas with an equation of state, $P = K\rho^\gamma$, where K is a constant, use the above equation to derive a second-order differential equation involving only density and radius. Using the dimensionless variables $r' \equiv r/R_*$ and $\rho' \equiv \rho/\rho_0$, show that the term $K\rho_0^{\gamma-2}/R_*^2$ is a dimensionless constant, and hence that $R_* \propto \rho_0^{\gamma/2-1}$.

(iii) White dwarfs obey the equation of state $P = K\rho^\gamma$, with $\gamma = 5/3$ for non-relativistic conditions and $\gamma = 4/3$ in the relativistic regime. Using the above result, $R \propto \rho_0^{\gamma/2-1}$, show that for non-relativistic white dwarfs $R \propto M^{-1/3}$, while for relativistic white dwarfs $R \neq f(M)$.

Q4: In a $10M_\odot$ star the $1M_\odot$ core collapses to produce a Type II supernova. Assume that 100% of the energy released by the collapsing core is converted to neutrinos and that 1% of the neutrinos are absorbed by the overlying envelope to power the ejection of the supernova remnant.

(i) Estimate the final radius of the stellar remnant if the energy liberated is just enough to eject the remaining $9M_\odot$ to infinity. State clearly any assumptions made.

(ii) What is the typical velocity of the ejecta, if the energy absorbed by the envelope is 10^{51} erg?

(iii) An astronomer announces the discovery of a Type II supernova in a Globular cluster, but her colleagues are skeptical. Why?

Q5: Type Ia supernovae are thought to be the explosion and complete disruption of a white dwarf in a binary system. Carbon and oxygen, the dominant constituents of white dwarfs, are burned to heavy elements, primarily Ni and Fe, during the explosion.

(i) Calculate the nuclear energy released by a Type Ia SN assuming that: (a) the exploding white dwarf has a mass of $1.4\,M_\odot$ consisting of ^{12}C and ^{16}O in equal proportions by number, and (b) all of the C and O are burned to ^{56}Ni in the explosion. The mass of a given nucleus of atomic number Z and mass number A is given by the formula:

$$m(A, Z) = Am_{\rm u} + m_{\rm ex}(A, Z)$$

where one atomic mass unit $m_{\rm u} = 931.5\,{\rm MeV/c^2}$, and the mass excess $m_{\rm ex} = 0$, -4.7, and $-53.9\,{\rm MeV/c^2}$ for ^{12}C, ^{16}O, and ^{56}Ni respectively. [Note: $1\,{\rm MeV} = 1.6 \times 10^{-13}\,{\rm J}$.]

(ii) Given that the white dwarf had a gravitational binding energy $E_{\rm g} = 5 \times 10^{43}\,{\rm J}$, obtain a simple estimate of the average velocity of the ejected matter in the explosion, if all of the energy released is transformed into kinetic energy.

Q6: Assume that a star evolves homologously and that angular momentum is not lost via a wind.

(i) How will the rotation speed of the star depend on its radius?

(ii) The Sun will ultimately evolve into a white dwarf, with radius $R_{\mathrm{WD}} = 10^7\,\mathrm{m}$. Given that the Sun has a rotation period of 28 days, obtain an estimate of the rotation period of the white dwarf it will eventually become. Comment on whether this is likely to be an upper or lower limit to the actual value.

(iii) Neutron stars are believed to be formed in Type II supernovae as the core of a massive star collapses. If the core had an initial radius $R_{\mathrm{c}} = 10^7\,\mathrm{m}$ and a rotation period of 28 days, estimate the rotation period of the neutron star, assuming $R_{\mathrm{ns}} = 10\,\mathrm{km}$. Compute the minimum rotation period possible for a neutron star. How do the two numbers compare?

Q7: Consider an accreting High Mass X-ray Binary with a circular orbit in which the donor is a $15.0M_{\odot}$ star filling its Roche lobe and the recipient is a neutron star of mass $M_{\mathrm{ns}} = 1.4M_{\odot}$ and radius $R_{\mathrm{ns}} = 10^4\,\mathrm{m}$.

(i) Comment on how the changing separation between the two stars affects the accretion.

(ii) If 2/3 of the donor mass were to accrete onto the neutron star in a continuous stream over $\sim 10^4$ years, what is the expected luminosity (primarily in the X-ray regime)? How does it compare with the Eddington luminosity? Where do you think that $10M_{\odot}$ of donated gas would actually end up?

Q8: In a distant galaxy, a burst of star formation forms stars with a Salpeter initial mass function between $M_{\rm min} = 0.1 M_\odot$ and $M_{\rm max} = 60 M_\odot$. For our purposes, the burst can be considered to be instantaneous. At the end of their lives, stars with initial mass $M_{\rm i} \leq 8 M_\odot$ leave a compact remnant with mass $M_{\rm r} = M_{\rm i}/5$, while stars with $M_{\rm i} > 8 M_\odot$ leave a remnant with mass $M_{\rm r} = 1.4 M_\odot$.

(i) Calculate the fraction of the stellar mass that is returned to the interstellar medium 10 Gyr after the burst of star formation.

(ii) Comment briefly on the result.

Q9: At the end of its life, two physical processes remove pressure support from the core of a massive star, precipitating core collapse on the timescale of a few seconds: photodisintegration and neutronisation.

(i) In photodisintegration, each iron nucleus can absorb 124.4 MeV of energy in the process $\gamma + ^{56}_{26}\text{Fe} \longrightarrow 13\,^{4}_{2}\text{He} + 4\text{n}$. If 3/4 of the core mass $M_{\rm c} = 1.4 M_\odot$ is dissociated in this way, calculate the total energy absorbed by this process.

(ii) Neutronisation is the inverse process to beta decay: $\text{p} + \text{e}^- \longrightarrow \text{n} + \nu_{\rm e}$. The conversion of protons (in nuclei) to neutrons by electron capture is possible if the gas is sufficiently dense for degenerate electrons to have an energy above the 1.3 MeV mass-energy excess of neutrons compared to protons. If each $\nu_{\rm e}$ produced by the above reaction carries away 10 MeV of energy, how much energy is removed from the core if the entire $M_{\rm c} = 1.4 M_\odot$ undergoes neutronisation?

(iii) Compare the combined energy loss by photodisintegration and neutronisation to the luminosity of a main sequence star with mass $M = 12 M_\odot$, and comment on the result.