## EXERCISES: Set 1 of 4

**Q1:** On the basis of the ratio of the 'characteristic' distance between two of the objects below to their 'characteristic' size (radius), which are 'closer' together, planets in the solar system, stars in the Milky Way Galaxy, or galaxies in the local Universe?

**Q2:** Suppose the space density of stars within the Galaxy is $0.1\,\mathrm{pc}^{-3}$.

(i) How many stars would you see per square degree in a direction where the Galaxy extends to: (a) 100 pc, (b) 1000 pc and (c) 10 000 pc?

At a distance of 50 pc from the Sun, there is a cluster of 3000 stars, occupying (uniformly) a sphere of radius 1.25 pc.

(ii) If in that direction the Galaxy extends to 250 pc, how many field stars will occupy the same apparent area as the cluster? How many will be in front and how many behind?

About 2% of the stars, in both the field and the cluster, can be identified spectroscopically as being virtually identical to the Sun.

(iii) What is the apparent magnitude of the solar-type stars in the cluster? And what is the apparent magnitude of (a) the brightest and (b) the faintest, of the solar-type field stars projected on the cluster?

(iv) Sketch a histogram of the cumulative apparent magnitude distribution of the solar type stars, both in the cluster and the field, putting them in bins of width 0.5 mag. For the field component, show that the number of stars in successive bins increases by very nearly a factor of two per bin.

**Q3:**

(i) Explain what astronomers mean by the term *'proper motion'*.

(ii) Two stars are at distances of 10 pc and 1 kpc from the Sun, respectively. Which of the two stars would you expect to show the higher proper motion and why?

(iii) Two solar-type stars are both at a distance of 50 pc from the Sun. One star is a member of the halo population, while the other is a disk star. Which of the two stars would you expect to show the higher proper motion? Which other physical property would you expect to be different between the two stars?

(iv) A star at a distance of 10 pc is travelling at 5 km $s^{-1}$ along a path perpendicular to our line of sight. What is its proper motion, in seconds of arc per century?

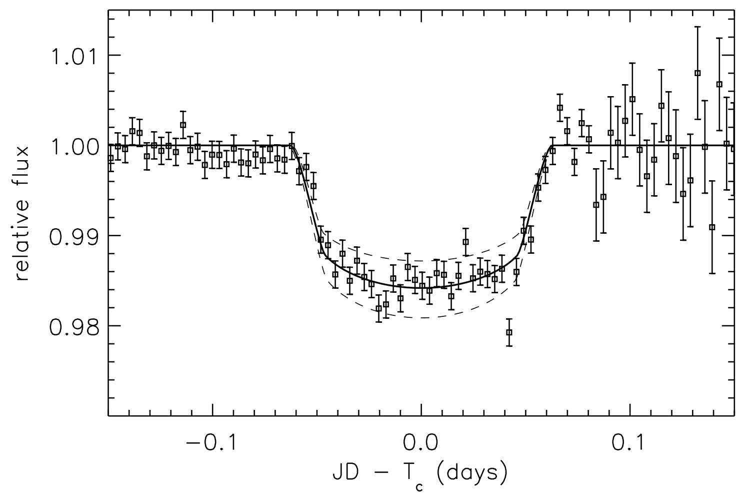
(v) Suppose the Galaxy is rotating rigidly with a period of $10^8$ years. Find the proper motion of any star in the plane of the Galaxy, as measured relatively to a fixed background of extragalactic objects.

(vi) An astronomer measures the following positions for a bright star at yearly intervals:

| Date | RA | Dec |
|---|---|---|
| 1 Jan 2000 | 12h 30m 15s.1 | $+51^\circ\,20'\,15''.0$ |
| 1 Jan 2001 | 12h 30m 16s.9 | $+51^\circ\,20'\,32''.3$ |
| 1 Jan 2002 | 12h 30m 18s.0 | $+51^\circ\,20'\,42''.5$ |
| 1 Jan 2003 | 12h 30m 18s.2 | $+51^\circ\,20'\,44''.5$ |
| 1 Jan 2004 | 12h 30m 19s.0 | $+51^\circ\,20'\,52''.1$ |
| 1 Jan 2005 | 12h 30m 20s.4 | $+51^\circ\,21'\,05''.7$ |

From these data, derive the proper motion of the star. What can you conclude regarding the accuracy of the positional measurements?

**Q4:** A gas planet orbits a $1M_\odot$ solar-type star and transits in front of the star once every 3.5 days. The light curve of the transit is shown below:




(i) From the depth of the eclipse, calculate the radius of the planet, assuming that it is completely dark.

(ii) What other information might be inferred from the shape of the light curve?

(iii) Suppose the brightness of the star is measured with a CCD (charge-coupled device) detector in which each photon generates one measurable electron (or 'count'). How many counts are needed to get the same accuracy as shown in the plot (i.e. errors of $\sim 0.002$ on the relative flux, assuming Poisson statistics)? How does that compare to the maximum counts in a CCD of $\sim 60\,000$ counts per pixel?

(iv) If the mass of the planet is $0.001M_\odot$ (approximately one Jupiter mass), calculate the radial velocity amplitude of the star due to the orbiting planet.

(v) What wavelength shift would that give to an absorption line in the star's spectrum at 5000 Å? How does that compare to the typical resolution of 0.1 Å of the spectrographs normally used for these kinds of observations?

(vi) Another, similar, planet, which was in a wider orbit around the star, survives during the late stages of stellar evolution and ends up orbiting the stellar remnant—a white dwarf. With a (future) very sensitive instrument the orbit of the planet around the white dwarf can be followed. The planet's period is 244 yr, and the mass of the white dwarf is $0.6\,M_\odot$. What is the semi-major axis of the white dwarf–planet system?

(vii) As projected onto the sky, the orbit of the planet around the white dwarf appears to be a perfect circle of radius $1''$, but the white dwarf, instead of being in the centre of the circle, is 60% of the way to the edge. Show that the true orbit is an ellipse, with eccentricity $e = 3/5$, and calculate the size of the semi-minor axis and the distance to the system.

**Q5a:** Starting from the Planck function for blackbody radiation, derive:

(i) the Stefan-Boltzmann law

$$\int_0^\infty \pi B_\nu \mathrm{d}\nu = \sigma T^4$$

and

(ii) Wien's law

$$\lambda_{\mathrm{max}} T = 0.290\,\mathrm{cm\ K}$$

**Q5b:** A person is standing in a room at a temperature of 20 C. The average human body has a temperature of 36 C and a surface area of skin of $1.4\,\mathrm{m}^2$. Assuming that the average person absorbs and emits radiation according to the Planck formula for blackbody radiation:

(i) Calculate the energy per second radiated by an average person.

(ii) Determine the peak wavelength of the blackbody radiation emitted by the average person. In what region of the electromagnetic spectrum is this wavelength found?

(iii) Calculate the energy per second absorbed by the average person from the room.

(iv) Calculate the net energy difference between (i) and (iii), and compare its value with the power consumption of an everyday household item. What is the source of this energy?

**Q6a:** A star has an apparent magnitude $m_V = 2.5$. You measure its parallax to be $\theta = 0.002$ arcseconds.

(i) What is the star's absolute magnitude in $V$?

(ii) Given that a main-sequence A0 star has $M_V = +0.6$, what can you deduce about the nature of the star whose parallax you have measured?

**Q6b:** The star explodes as a supernova, increasing its luminosity by a factor of 50 000. What are the new values of apparent and absolute magnitude?

**Q6c:** The supernova remnant, in the shape of a ring, is expanding with a speed $v = 10,000$ km s$^{-1}$. After one day, would it be possible to resolve the ring with a ground-based telescope? And with the *Hubble Space Telescope*?

**Q7**:

(i) X-ray observations have shown that the outer atmosphere of the Sun (the corona) reaches a temperature of nearly $10^6$ K. Why doesn't the Sun appear as a blackbody with $T_{\rm eff} \simeq 10^6$ K?

(ii) If the temperature gradient of a star's atmosphere were reversed, so that the temperature increased *outwards*, what type of spectral line would you expect to see in the star's spectrum at wavelengths where the opacity is greatest?

(iii) Consider a star surrounded by a large hollow spherical shell of hot gas. Under what circumstances would you see this shell as a ring around the star? If you observed the ring with a spectrograph, what type of spectrum would you see?