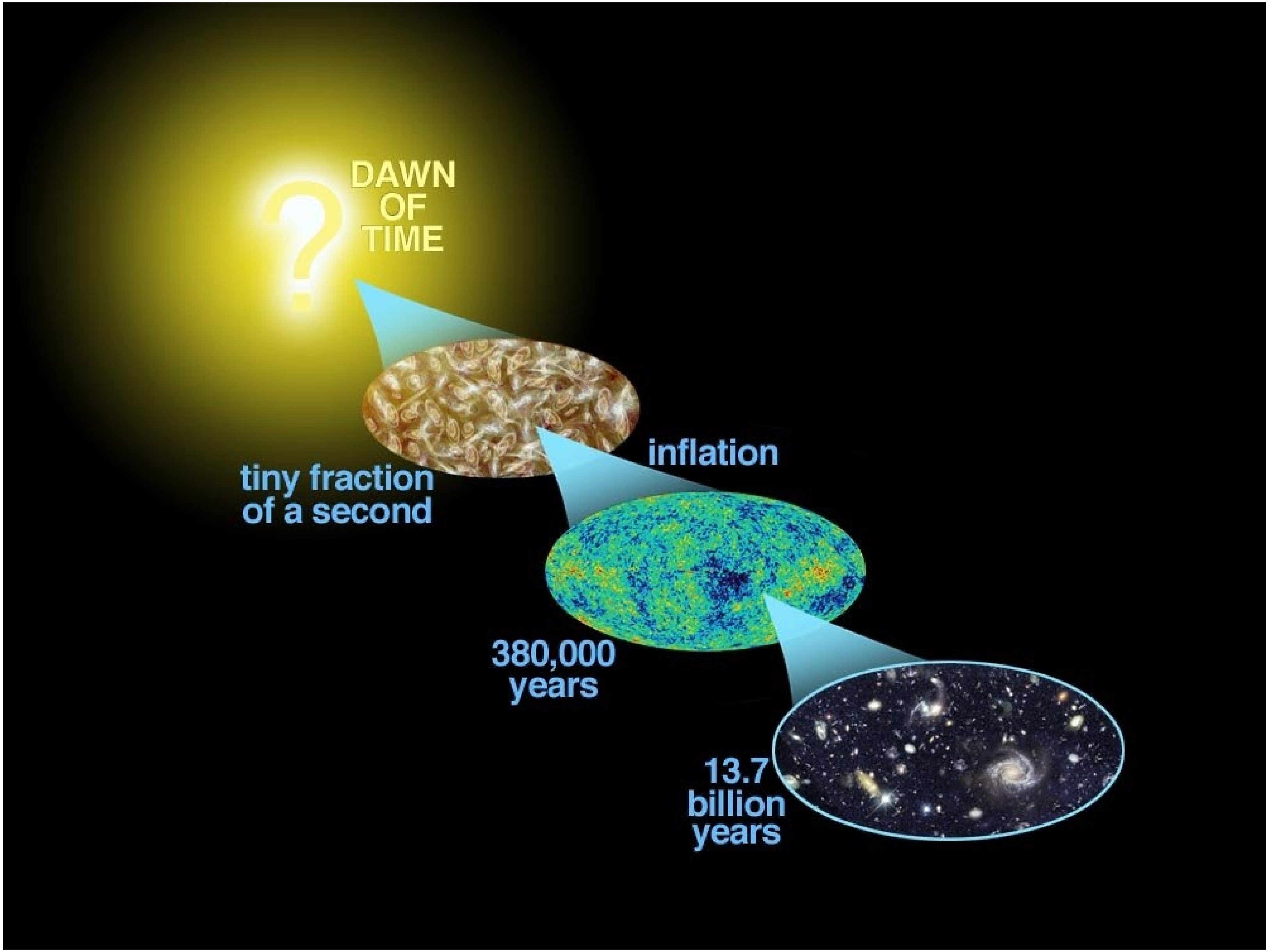
PARTICLE ASTROPHYSICS LECTURE 8

Inflation



Inflation

- The Horizon Problem

Starting with the FRW metric:

$$ds^2 = c^2 dt^2 - R^2(t)\left[\frac{dr^2}{(1-Kr^2)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right],$$

imagine light pulse emitted at $t = 0$ that is detected at time t. This travels a coordinate distance $r_p(t)$

$$\int_0^{r_p(t)} \frac{dr}{(1-Kr^2)^{1/2}} = c\int_0^t \frac{dt'}{R(t')}.$$

The surface defined by $r_p(t)$ defines a *particle horizon* . We are in causal contact only with particles within our particle horizon.

The proper distance on a $t =$ constant hypersurface is

$$d_p(t) = \int_0^r \sqrt{g_{rr}} dr = R(t) \int_0^r \frac{dr}{(1 - Kr^2)^{1/2}},$$

Hence the proper distance to the particle horizon is

$$d_p(t) = cR(t) \int_0^t \frac{dt'}{R(t')}.$$

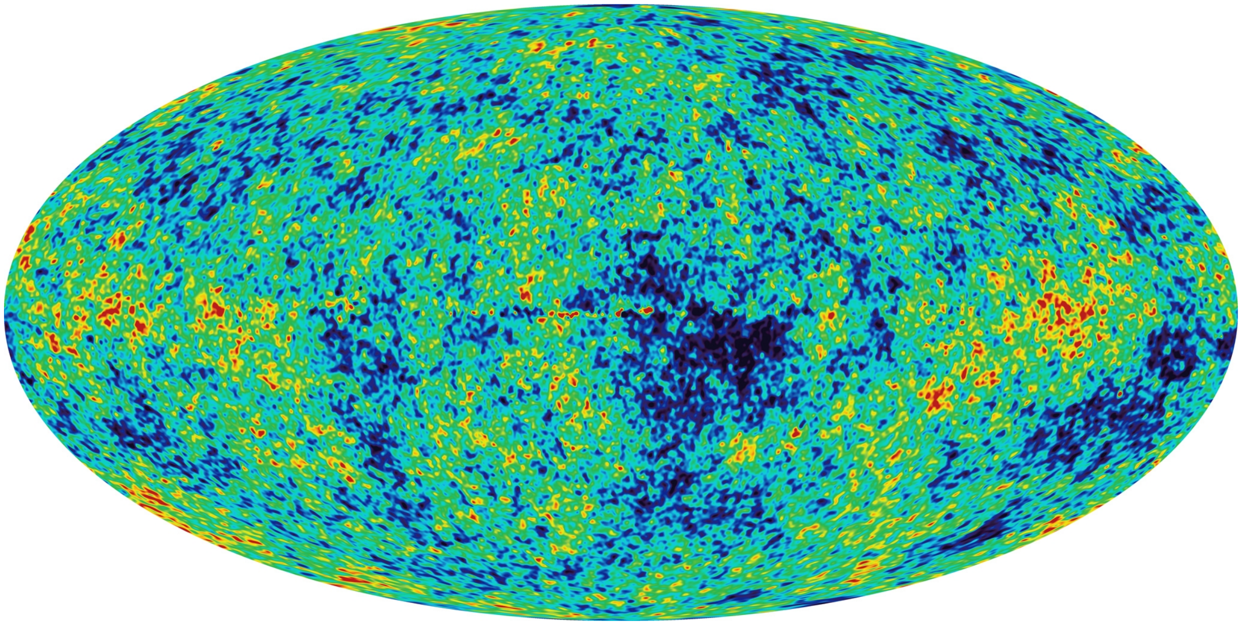
If, for example, $R(t) \propto t^\alpha$ ($R \propto t^{1/2}$ in the radiation era), then

$$d_p(t) = \frac{ct}{(1 - \alpha)}.$$

So for a conventional equation of state the particle horizon is of order ct.

This means that the region of causal contact shrinks as we go back in time.

As an example, consider the size of the horizon at the time that matter and radiation decoupled, $z \sim 1000$ ($t \approx 10^{13}$s). The particle horizon at the time of decoupling subtends an angle on the sky of $\sim 3^{\circ}$.



• Why is the Universe so nearly homogeneous and isotropic on large scales?

• Furthermore, we see large-scale fluctuations at the time of decoupling!

- The Flatness Problem

If we examine the Friedmann equations:

$$\begin{aligned}\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} &= \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}. \\ 3\frac{\ddot{R}}{R} - \Lambda &= -4\pi G(\rho + 3P),\end{aligned}$$

we can see from the first of these equations that at early times the curvature term is dynamically neglible (recall that the density of matter and radiation vary as R^{-3} and R^{-4} respectively). To end up a dynamically significant curvature today requires extreme fine tuning at early times. This is the *flatness problem*.

If we ignore the cosmological constant, we can rewrite these equations as

$$\begin{aligned} \frac{K}{\mathcal{H}^2} &= \Omega - 1, \\ 2\frac{d\mathcal{H}}{d\tau} &= -(1+3w)(\mathcal{H}^2 + K), \end{aligned}$$

where τ is the conformal time, $d\tau = dt/R$, $\mathcal{H} = R^{-1}dR/d\tau$, $\Omega = \rho/\rho_c$ and w is the equation of state parameter $w = P/\rho$. Combining these equations, we find

$$\frac{d\Omega}{d\tau} = (1+3w)\mathcal{H}\Omega(\Omega - 1). \tag{1}$$

Since $\mathcal{H} > 0$, this equation tells us that provided $(1 + 3w)$ is positive, the solutions tend to $\Omega = 1$ as $\tau \to 0$. (*i.e.* $\Omega = 1$ is an attractor as $\tau \to 0$). To end up with $\Omega \sim 0.2$ at the present day requires Ω to differ from unity by about 1 part in 10^{60} at the Planck time. There is no known mechanism that can explain such fine tuning.

Inflation

Now, in the previous lecture, we showed that a homogeneous scalar field with potential $V(\phi)$ has density and pressure

$$\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + V(\phi), \\ P_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi). \end{aligned}$$

If the scalar field is moving slowly $\dot{\phi} \ll V(\phi)$, the equation of state is

$$w = \frac{P_\phi}{\rho_\phi} \approx -1.$$

This solves both the horizon and flatness problems!

The solution to the Friedmann equations gives

$$R(t) \propto \mathrm{e}^{(Ht)}, \qquad H \approx \text{constant},$$

and so the particle horizon is

$$d_p(t) = cR(t)\int_0^t \frac{dt'}{R(t')} \approx \frac{c}{H}\left(\mathrm{e}^{(\mathrm{Ht})} - 1\right).$$

This can end up to be many orders of magnitude bigger than the observable Universe ($\sim c/H_0$) today.

The exponential expansion rapidly dilutes any curvature term in the Friedmann equation, so we would expect the Universe to be spatially flat at the present day. We can see this clearly from equation (1):

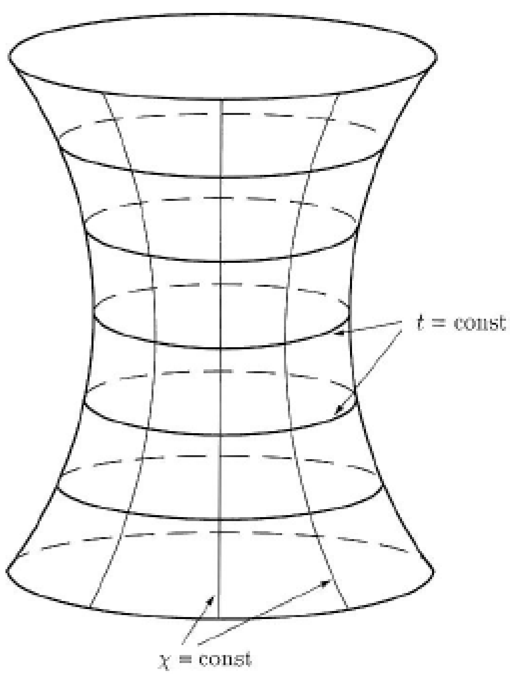
$$\frac{d\Omega}{d\tau} = (1+3w)\mathcal{H}\Omega(\Omega - 1).$$

If $(1+3w) < 0$, $\Omega = 1$ is now an attractor solution as $\tau \to \infty$.

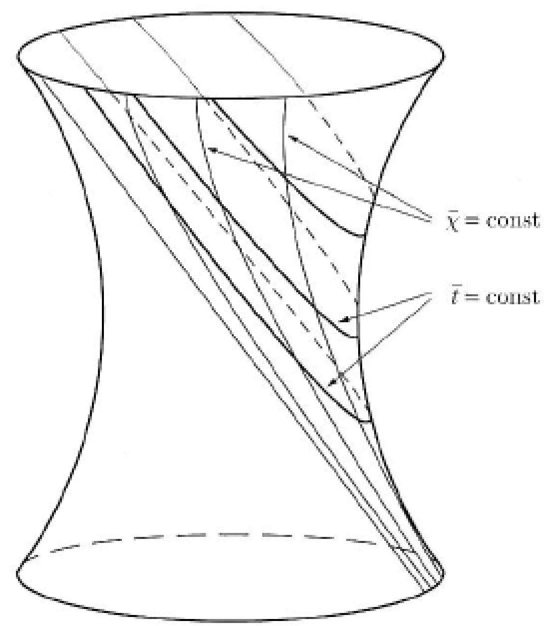
The (homogeneous and isotropic) empty space-time with $H =$ constant is known as *de-Sitter space*.

We can consider de-Sitter space as a hyperboloid in Minkowski space. The entire space-time is covered by coordinates with closed spatial curvature as shown in (a).

Or we can choose a different time-slicing covering part of de-Sitter space giving spatially flat coordinates as in (b).

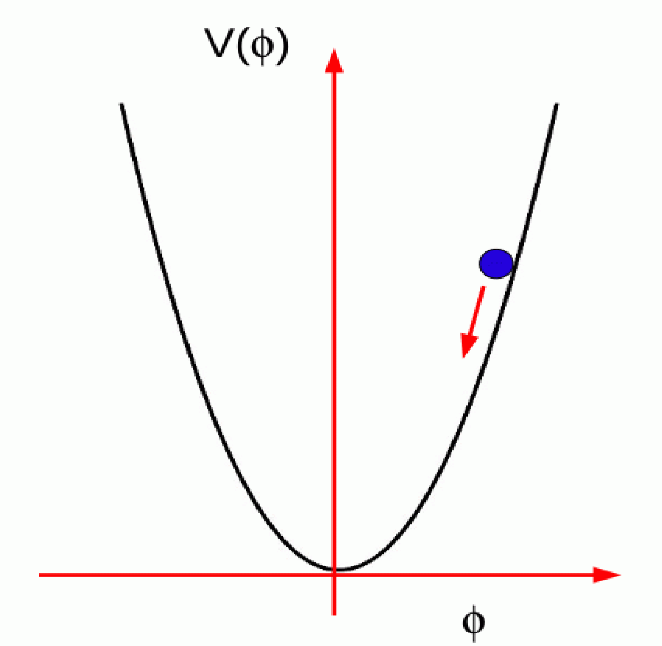


(a)



(b)

Specific Example: $V(\phi) = \frac{1}{2}m^2\phi^2$



The equation of motion of the scalar field (spatial gradients are assumed small – if inflation starts they are rapidly smoothed away) is

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi), \quad V(\phi) = \frac{1}{2}m^2\phi^2.$$

where the Hubble parameter is

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) = \frac{1}{6}(\dot{\phi}^2 + m^2\phi^2).$$

Throughout this Section, I will use Planck units with ($8\pi G = 1/M_P^2 = 1$). The field ϕ is therefore expressed in units of the (reduced) Planck mass M_{P}.

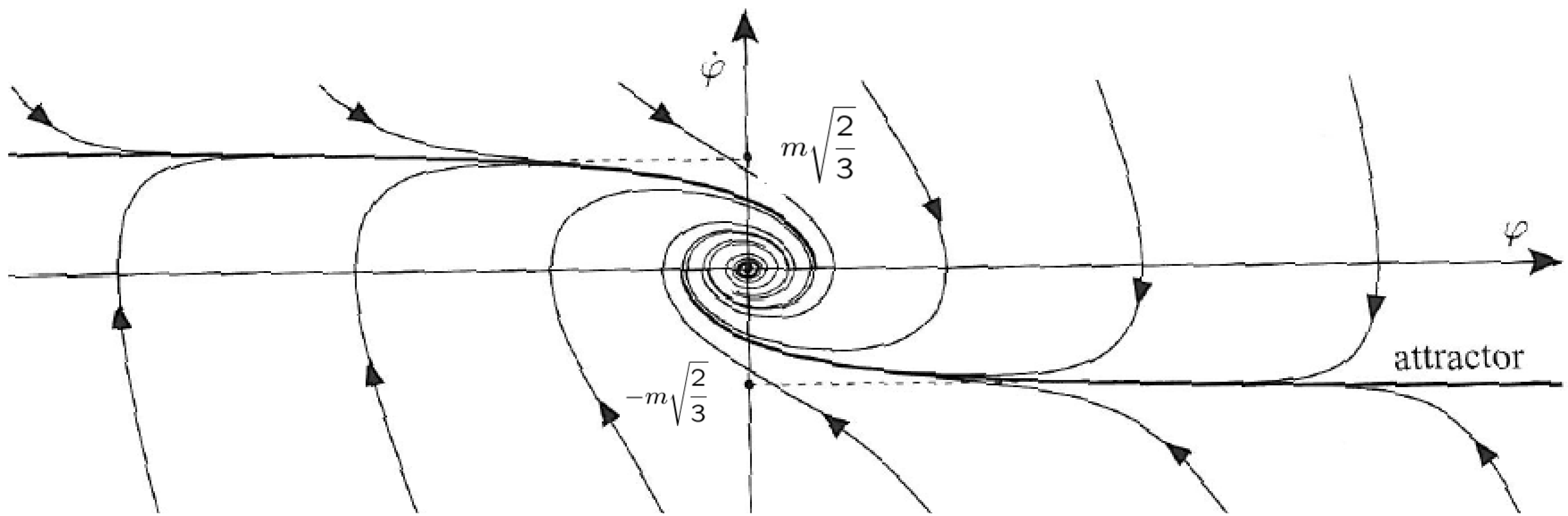
Combining the above equations:

$$\ddot{\phi} + \sqrt{\frac{3}{2}}(\dot{\phi}^2 + m^2\phi^2)^{1/2}\dot{\phi} + m^2\phi = 0,$$

If we make the substitution $\ddot{\phi} = \dot{\phi}(d\dot{\phi}/d\phi)$, the equation of motion can be written as a non-linear first order differential equation:

$$\frac{d\dot{\phi}}{d\phi} = \frac{-\sqrt{\frac{3}{2}}(\dot{\phi}^2 + m^2\phi^2)^{1/2}\dot{\phi} + m^2\phi}{\dot{\phi}}.$$

This equation has attractor solutions.



If the field starts off with a value $\phi \gg 1$, it will find the attractor

$$\dot{\phi} = \pm m\sqrt{\frac{2}{3}}, \qquad \frac{d\dot{\phi}}{d\phi} \approx 0,$$

and *inflation will begin* with an equation of state

$$P = -\rho + \dot{\phi}^2 = -\rho + \frac{2}{3}m^2 \approx -\rho.$$

In this model, inflation ends when the field drops to $\phi \sim 1$, since then

$$V(\phi) = \frac{1}{2}m^2\phi^2 \sim \dot{\phi}^2.$$

This is an example of a *high field* inflation model, since $\phi > M_P$ during inflation. But note that

$$V < M_P^4 \quad \text{if} \quad \phi \lesssim \sqrt{2}\frac{M_\mathrm{P}^2}{m},$$

so the energy density can remain well below the Planck scale for high field values.

The end of inflation and reheating

When $\phi \sim 1$, inflation ends and the field oscillates around minimum of the potential. If we could ignore Hubble expansion, the equation of motion gives SHM with angular frequency m. However, the Hubble expansion acts as a friction term and damps the oscillation.

A perturbative solution of the equations of motion gives:

$$\phi(t) = \sqrt{\frac{8}{3}}\frac{\cos mt}{mt}\left[1 + \frac{\sin 2mt}{2mt}\right] + \mathcal{O}\left(\frac{1}{(mt)^3}\right).$$

To produce the hot Big Bang, the *inflaton ϕ must* be coupled to other matter fields. Particle production as the field oscillates causes the Universe to *reheat*.

We can model reheating phenomenologically by adding another friction term $\Gamma\dot{\phi}$, to the equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) - \Gamma\dot{\phi}.$$

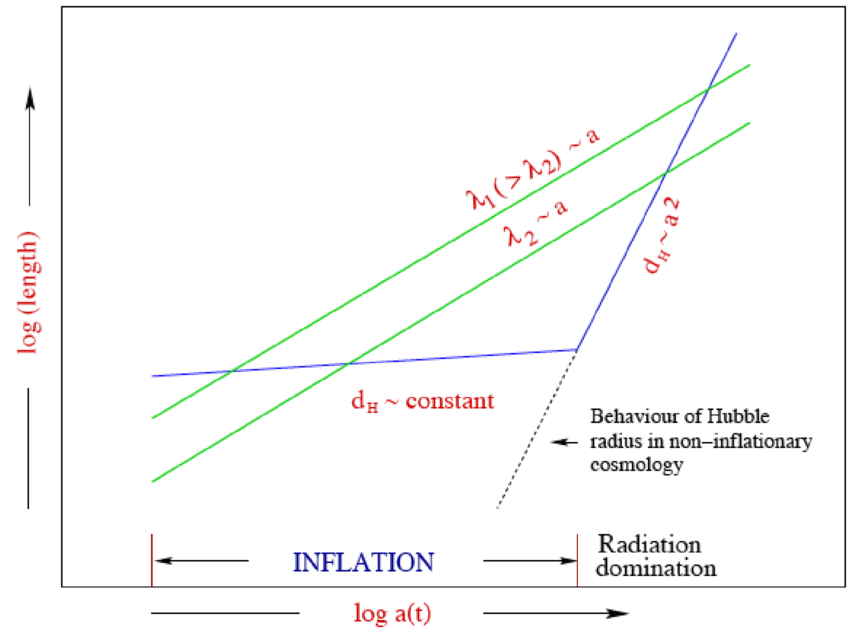
representing particle production.

The physics of reheating is complicated because Bose condensation can become important introducing parametric resonances.

Generally, the inflaton will decay within a few oscillations. The Universe is then filled with radiation of temperature T_{RH} and evolves as described in earlier lectures with $R(t) \propto t^{1/2}$ until it becomes matter dominated.

Provided $T_{RH} < T_{\mathrm{GUT}}$, we avoid producing unwanted relics such as GUT monopoles.

How many e-folds of inflation do we need?



$$N(t) = \ln\left(\frac{R(t_{\rm end})}{R(t)}\right).$$

We need inflation to produce a Universe *at least a big as the present Hubble radius*, $k_H = R_0 H_0 / c$ (k is the *comoving* wavenumber). We calculate the number of e-foldings from the time that k_H crosses the Hubble radius ($k_H = R_H H_{\rm Inf}/{\rm c}$) and the end of inflation:

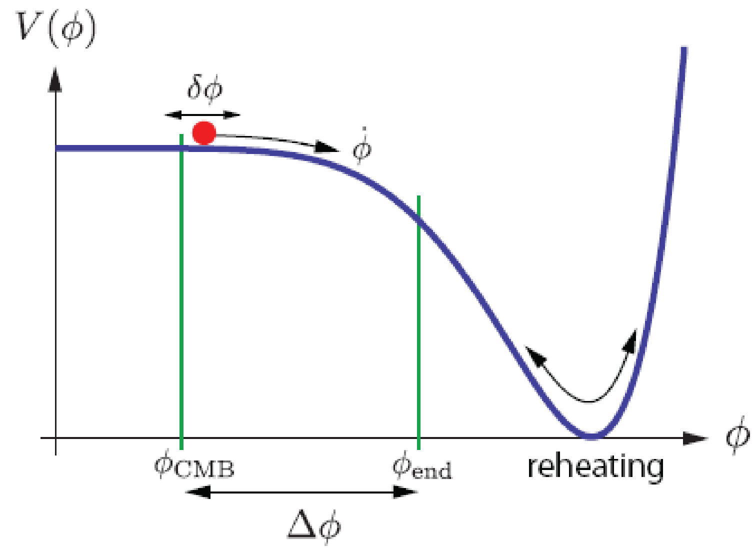
$$\frac{k_H c}{R_0 H_0} = \left(\frac{R_H}{R_{\rm End}}\right)\left(\frac{R_{\rm End}}{R_{\rm Reh}}\right)\left(\frac{R_{\rm Reh}}{R_0}\right)\left(\frac{H_{\rm Inf}}{H_0}\right).$$

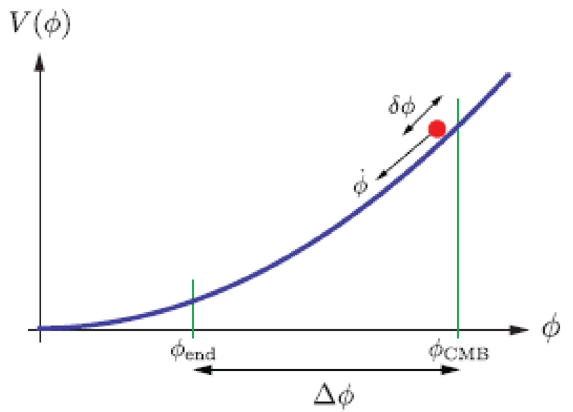
The first term on the rhs is the required number of e-foldings. The remaining terms depend on the timescale of reheating and the energy scales of inflation and reheating. If reheating is assumed to be instantaneous at the end of inflation $R_{\rm Reh} \approx R_{\rm End}$, this gives

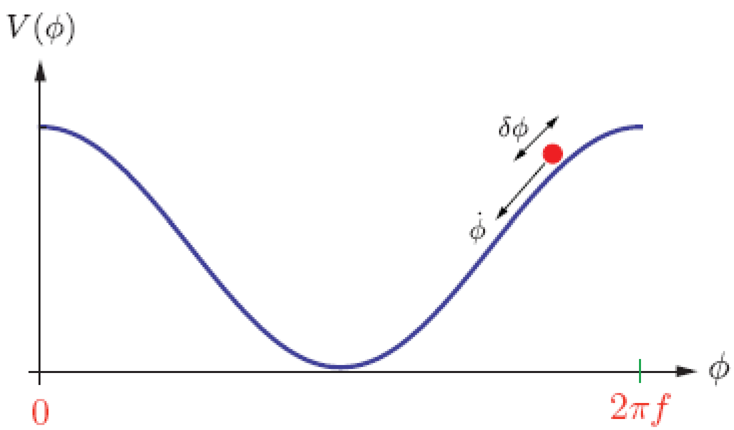
$$N \approx 60.4 - \ln\left(\frac{k_H c}{R_0 H_0}\right) + \frac{1}{2}\ln\left(\frac{V_{\rm Inf}}{10^{16}{\rm GeV}}\right) - \ln\left(\frac{T_{\rm Reh}}{10^{16}{\rm GeV}}\right).$$

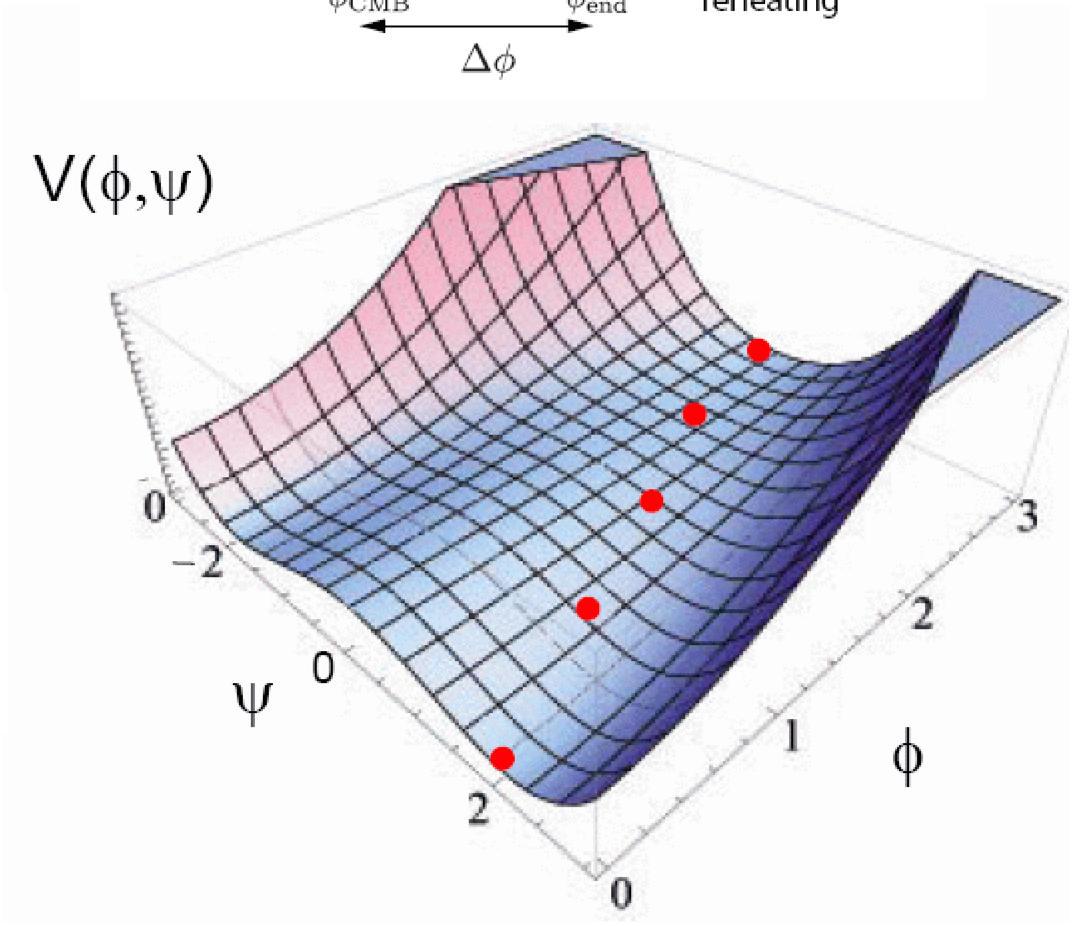
So, if inflation happens at $\sim 10^{16}$GeV, we need about 60 e-folds of inflation to produce the observable Universe. Note that a galaxy scale $\sim$ 1Mpc crossed the Hubble radius only $\sim$ 9 e-folds after k_H. The cosmological structure that we see in the Universe today crossed the Hubble scale during a brief period well before the end of the inflationary phase.

Inflation can be realised in many different ways, and can involve more than one field:









In fact there are many, many, models of inflation...... @ Paul Shellard

S-dimensional assisted inflation
assisted brane inflation
anomoly-induced inflation
assisted inflation
assisted chaotic inflation
boundary inflation
brane inflation
brane-assisted inflation
brane gas inflation
brane-antibrane inflation
braneworld inflation
Brans-Dicke chaotic inflation
Brans-Dicke inflation
bulky brane inflation
chaotic inflation
chaotic hybrid inflation
chaotic new inflation
D-brane inflation
D-term inflation
dilaton-driven inflation
dilaton-driven brane inflation
double inflation
double D-term inflation

dual inflation
dynamical inflation
dynamical SUSY inflation
eternal inflation
extended inflation
extended open inflation
extended warm inflation
extra dimensional inflation
F-term inflation
F-term hybrid inflation
false-vacuum inflation
false-vacuum chaotic inflation
fast-roll inflation
first-order inflation
gauged inflation
Hagedorn inflation
higher-curvature inflation
hybrid inflation
hyperextended inflation
induced gravity inflation
intermediate inflation
inverted hybrid inflation
isocurvature inflation.....................