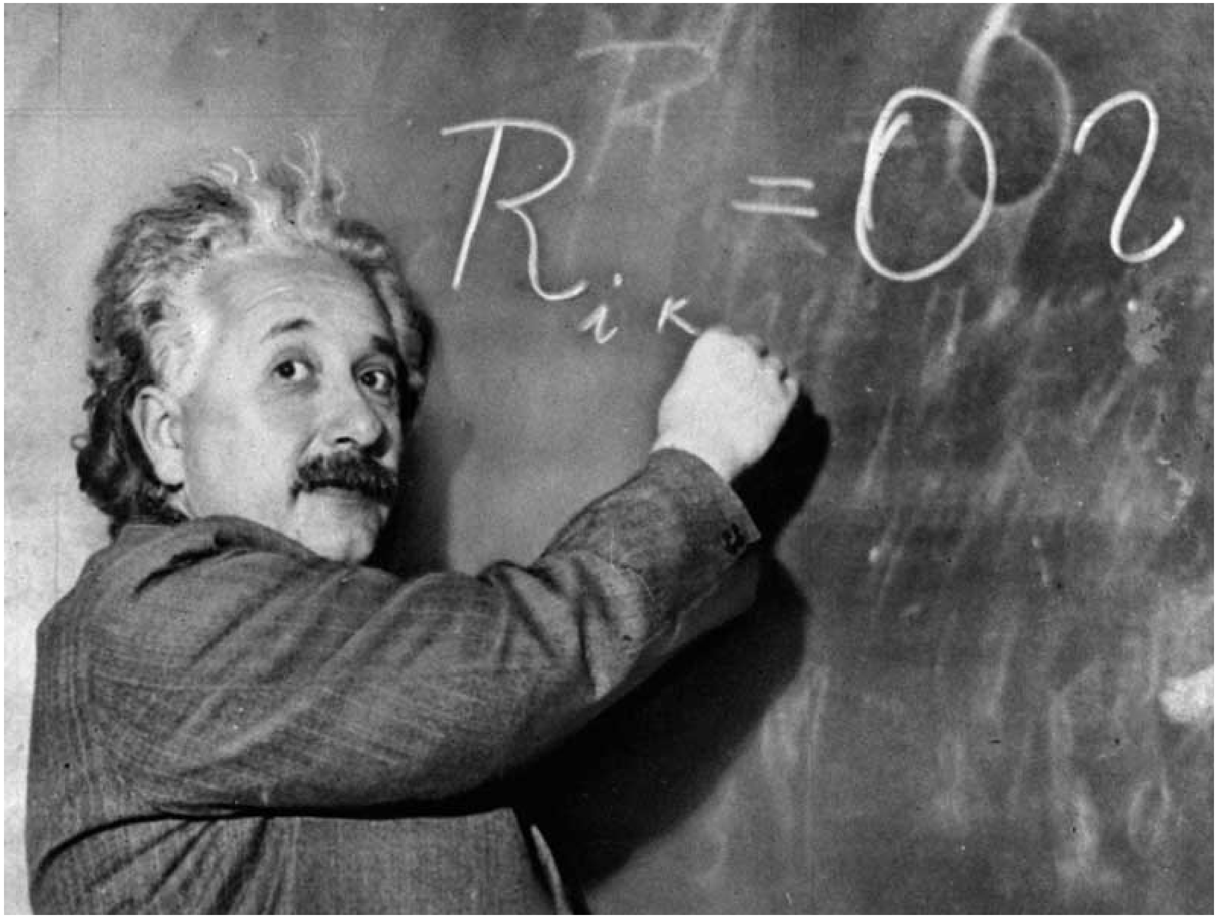
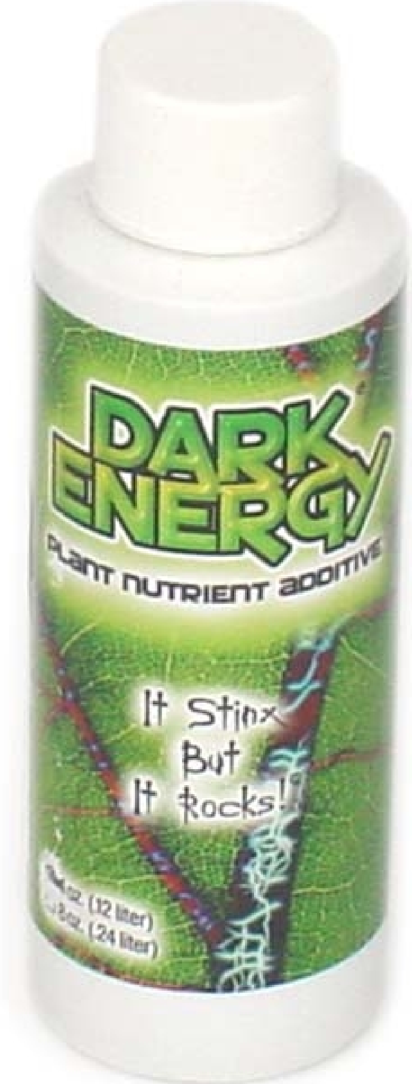
PARTICLE ASTROPHYSICS LECTURE 7

Dark Energy





The Cosmological Constant Problem

The Einstein field equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

We could, arbitrarily, choose to set Λ equal to zero.

But in quantum field theory, the zero-point vacuum energy of each field is of order:

$$\langle \rho_{\text{vac}} \rangle = \int_0^\infty \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2} \sim \frac{k_{\text{max}}^4}{16\pi^2}.$$

This makes a Lorentz invariant contribution to the energy momentum tensor

$$T_{\mu\nu} = -\langle \rho_{\text{vac}} \rangle g_{\mu\nu},$$

i.e.

$$\Lambda_{\text{vac}} \sim \frac{k_{\text{max}}^4}{16\pi^2} 8\pi G.$$

So, setting $k_{\text{max}} \sim M_{Pl}$, $8\pi G = 1/M_{Pl}^2$,

$$\Lambda_{\text{vac}} \sim M_{pl}^2.$$

However, observationally we know that:

$$\Lambda \lesssim H_0^2 \sim (10^{-41}\,\text{GeV})^2,$$

Hence

$$\Lambda \lesssim 10^{-120} M_{Pl}^2.$$

What cancels Λ_{vac} to such extraordinarily high precision? *There is no known physical mechanism that can do this!*

How can we test a cosmological constant? This is very difficult to do.

- Standard 'clock': Measure the ages of objects in the Universe or the growth rate of fluctuations.

- Standard 'rod': Measure the angular diameter distance (e.g. using the CMB).

- Standard 'candle': Measure the luminosity distance.

The major breakthrough came in 1998 with the application of the last of these methods to Type 1a supernovae.

Type Ia Supernovae

Supernovae are classified into types according to their spectra:

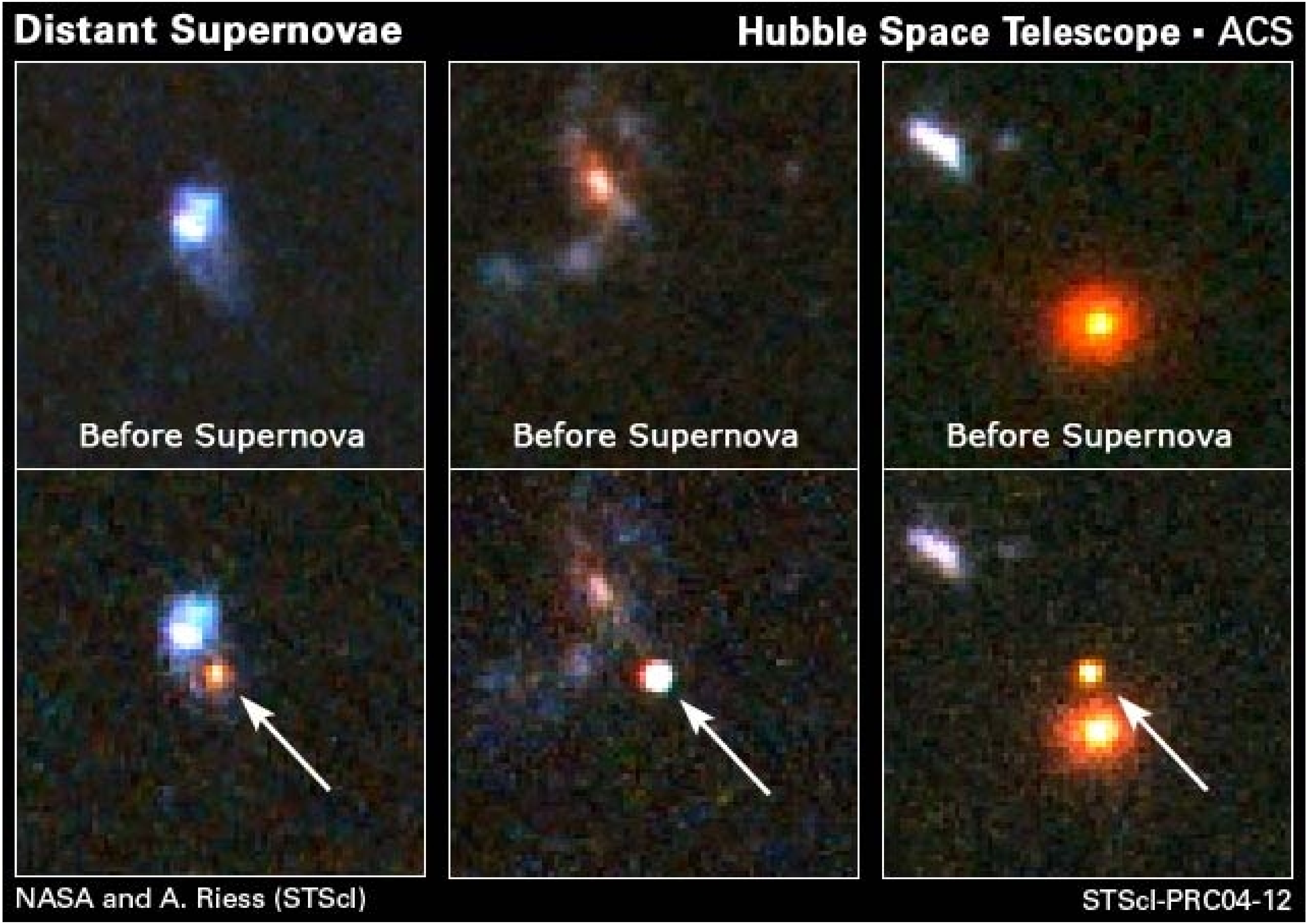
- Type I: No Hydrogen lines

Type Ia :	strong silicon features
Type Ib :	no silicon but have He lines
Type Ic :	no silicon and no He lines

- Type II: Strong Hydrogen lines

Type II SN are the core collapsed explosion of a massive star ($> 8M_\odot$) with an extended red supergiant envelope.

Type Ia supernovae are the explosions of accreting carbon-oxygen white dwarfs. *From low redshift observations they are known to be accurate standard candles.*



Distant Supernovae
Hubble Space Telescope · ACS
Before Supernova
Before Supernova
Before Supernova
NASA and A. Riess (STScI)
STScI-PRC04-12

For each supernova, we need to measure the peak magnitude m_i of a supernova at redshift z normalised to a uniform passband (this is called the *K-correction*). If the SN is a standard candle with absolute magnitude M_B, the expected apparent magnitude is

$$m_i^{\mathrm{pred}} = M_B + 25 + 5\log_{10}[d_L(z_i, \Omega_m, \Omega_\Lambda)],$$

where $d_L(z, \Omega_m, \Omega_\Lambda)$ (in Mpc) is the *luminosity distance.*

In the FRW model the luminosity distance to an object at redshift z can be written as:

$$d_L(\Omega_m, \Omega_\Lambda) = \frac{c(1+z)}{H_0|\Omega_K|^{1/2}} \sin_K\left[|\Omega_K|^{1/2} x(z, \Omega_m, \Omega_\Lambda)\right],$$

where the densities are evaluated at the present day and satisfy the constraint

$$\Omega_K = 1 - \Omega_m - \Omega_\Lambda$$

$$x(z, \Omega_m, \Omega_\Lambda) = \int_0^z \frac{dz'}{[\Omega_m(1+z')^3 + \Omega_K(1+z')^2 + \Omega_\Lambda]^{1/2}},$$

and

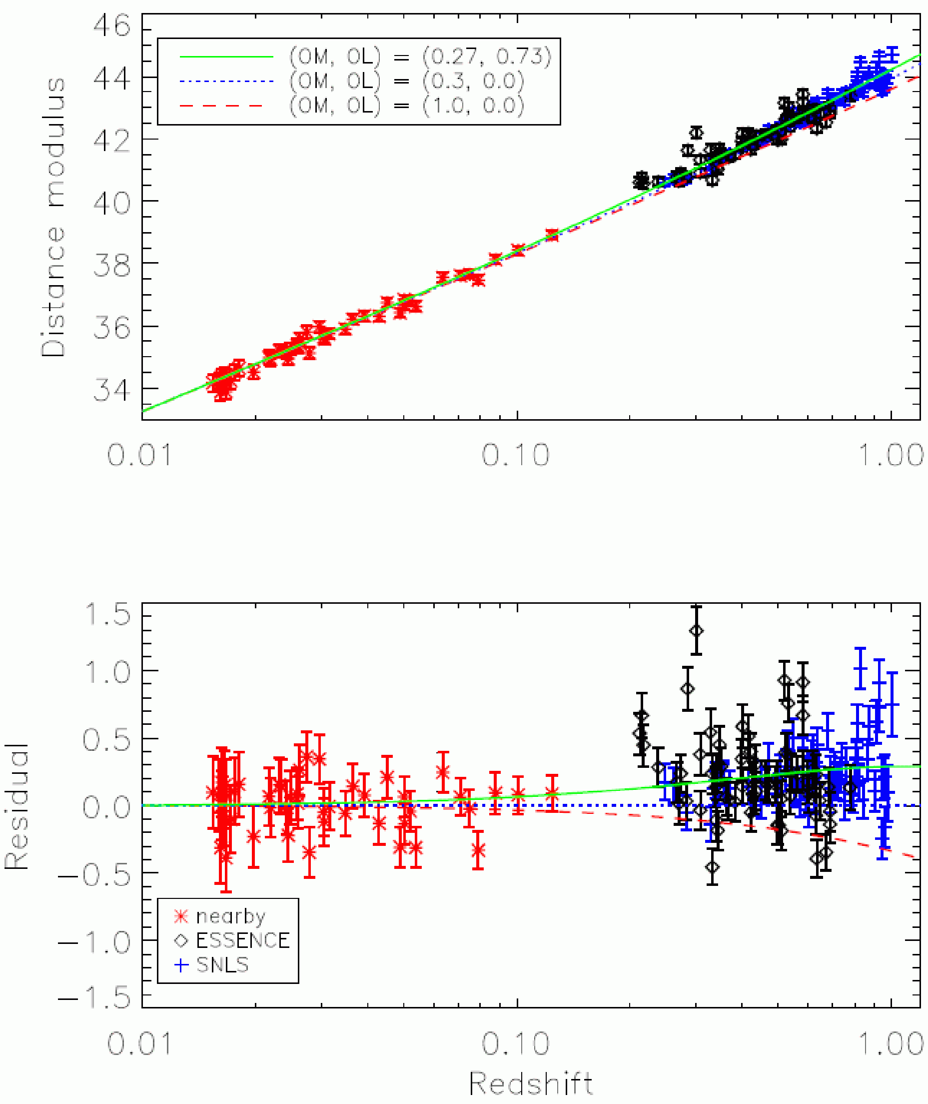
$$\mathrm{sin}_K = \begin{cases} \mathrm{sinh} & \text{if } \Omega_K > 0, \quad \text{(open universe)} \\ \mathrm{sin} & \text{if } \Omega_K < 0, \quad \text{(closed universe)} \end{cases}$$

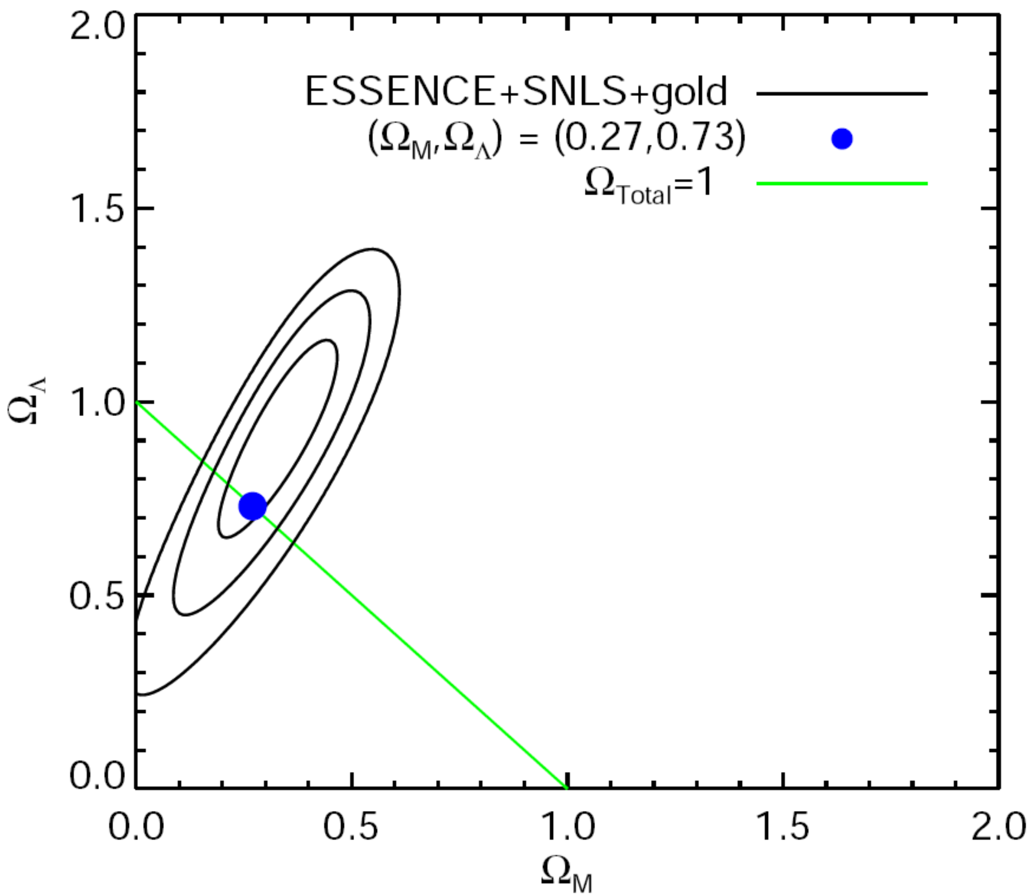
So, given a set of supernovae we can form a likelihood function

$$\mathcal{L} = \prod_i \frac{1}{(2\pi\sigma_i^2)^{1/2}} \mathrm{exp}\left\{ -\frac{(m_i - m_i^{\mathrm{pred}})^2}{2\sigma_i^2} \right\},$$

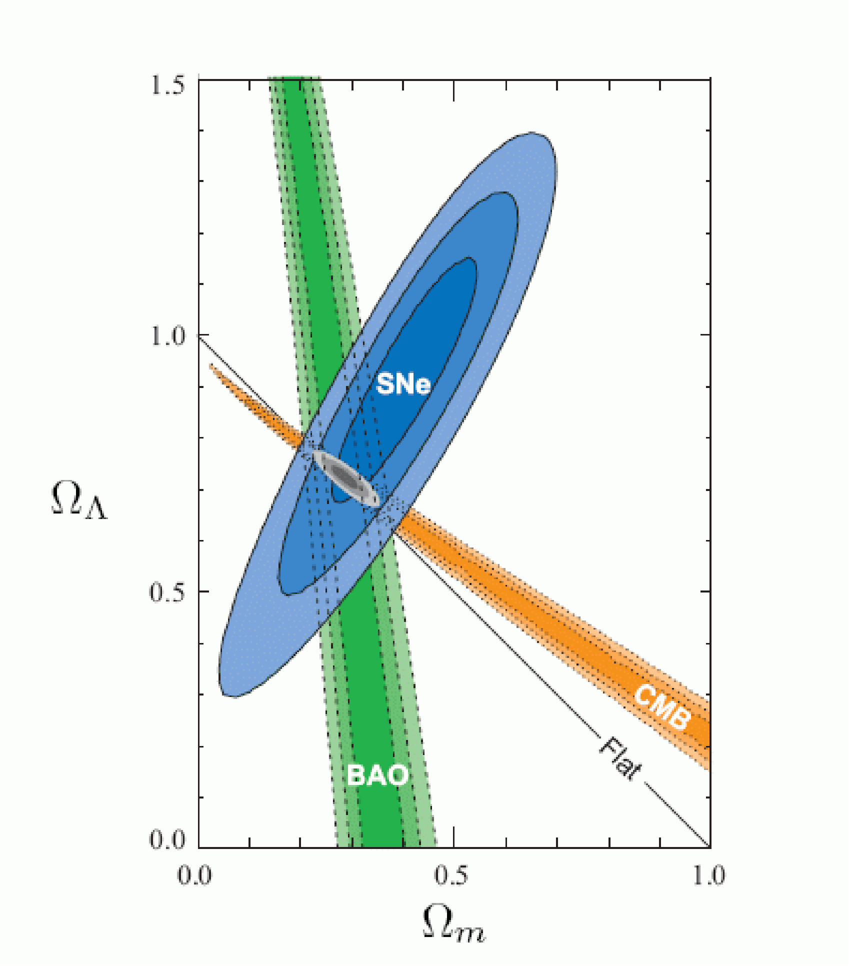
and maximise with respect to the three free parameters $\mathcal{M}_B = M_B - 5\log_{10} H_0 + 25$, Ω_m and Ω_Λ.

From Wood-Vasey *etal* arXiv:astro-ph/0701043





The SN results are highly degenerate in the $\Omega_m - \Omega_\Lambda$ plane. But the degeneracy can be broken by other measurements, in particular, observations of the anisotropies of the CMB which constrain the *angular diameter* to the last scattering surface ($z \sim 1000$):



There is therefore strong evidence that the Universe is accelerating!

Scalar Fields in Cosmology

The Lagrangian of a scalar field ϕ with potential $V(\phi)$ is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi),$$

and the action is

$$S = \int d^4x\sqrt{-g}\mathcal{L}.$$

The variation

$$\delta S = \delta \int d^4x\sqrt{-g}\ \mathcal{L} = 0,$$

with respect to the field ϕ gives the Euler-Lagrange equations of motion

$$(\partial^\mu\phi)_{;\nu} = -\frac{\partial V}{\partial\phi}.$$

The covariant derivative is

$$(\partial^\mu \phi)_{;\nu} = \partial_\nu \partial^\mu \phi + \Gamma^\mu_{\nu\kappa} \partial^\kappa \phi,$$

so, if we ignore spatial gradients in ϕ, and recalling that $\Gamma^i_{j0} = (\dot{R}/R)\delta^i_j$ in the FRW model, then the equation of motion of ϕ is

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0,}$$

where $H = \dot{R}/R$ and primes denote differentiation with respect to ϕ.

The variation of the action with respect to the metric *defines* the energy-momentum tensor:

$$\delta S = \int \frac{1}{2} d^4 x \delta g^{\mu\nu} \sqrt{-g} T_{\mu\nu} = 0.$$

(forcing the conservation law $T^{\mu\nu}_{;\nu} = 0$.)

This gives

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial^\kappa\phi\partial_\kappa\phi + g_{\mu\nu}V(\phi).$$

If we ignore spatial gradients, then

$$\begin{aligned} \rho_\phi &= T_{00} = \frac{1}{2}\dot{\phi}^2 + V(\phi), \\ P_\phi &= \frac{1}{3}T_{ii} = \frac{1}{2}\dot{\phi}^2 - V(\phi). \end{aligned}$$

Hence, the *equation of state* of the scalar field is

$$w = \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}.$$

If the field is moving slowly $\dot{\phi} \ll V(\phi)$, then

$$w \approx -1.$$

Energy conservation then requires

$$\frac{d(\rho R^3)}{dR} = -3PR^2 \approx 3\rho R^2,$$

i.e. $\rho \approx$ constant, and so the scalar field behaves *just like a cosmological constant* of magnitude

$$\Lambda = 8\pi G V(\phi).$$

We can therefore construct phenomenological *dynamical* modes of dark energy that have a *time varying* equation of state that differs from $w = -1$.

Such models are sometimes called *'quintessence'* models.

These models come in different types:

• Parameterised $w(z)$, *e.g.*

$$w(z) = w_0 = \text{constant}.$$

• 'Thawing' models: ϕ stays constant until late times and then starts to evolve, *e.g.*

$$V(\phi) = V_0 + V'\phi.$$

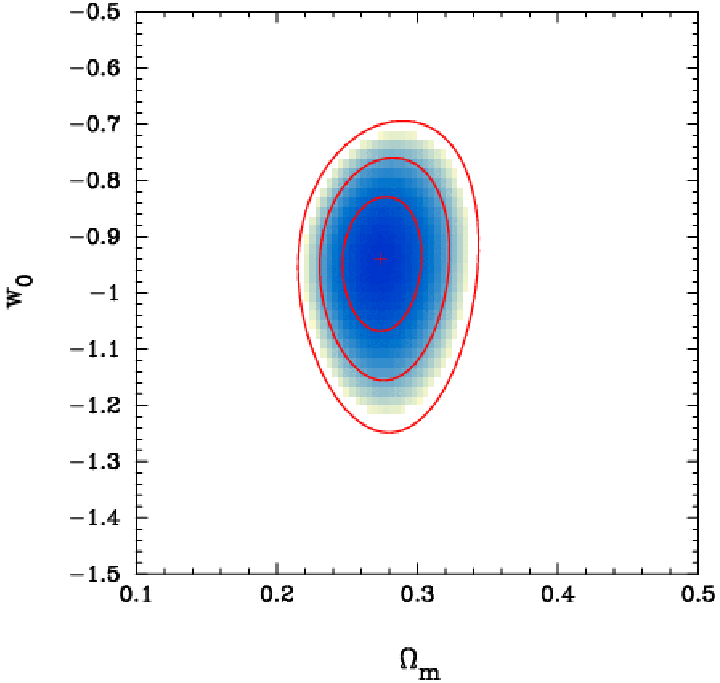
• 'Freezing' models: ϕ evolves at early times but slows at late times, *e.g.*

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha}.$$

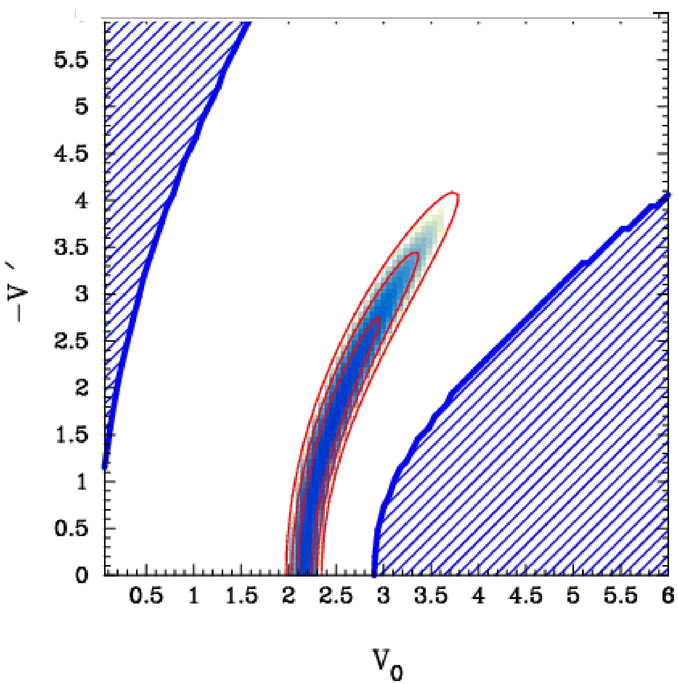
Observational constraints on each of these models are shown in the next figure:

From Efstathiou arXiv:astro-ph/0802.3185

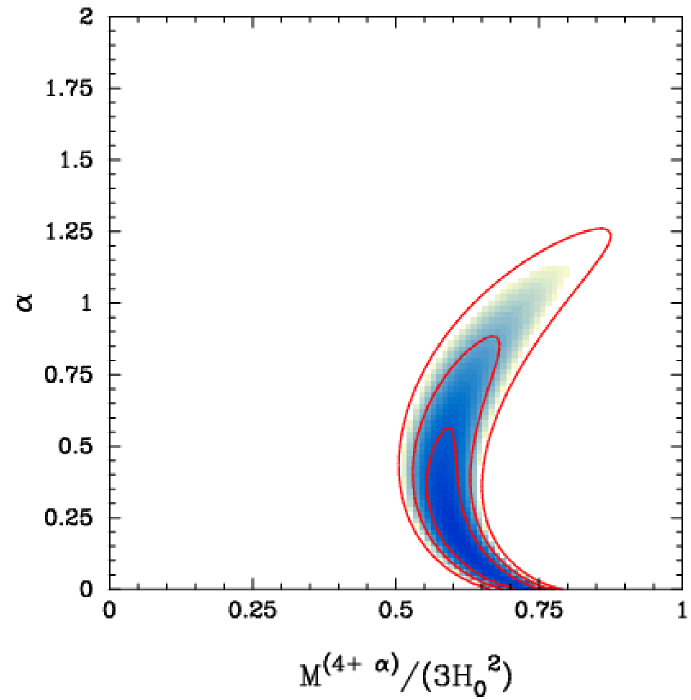
$w(z) = w_0 = \text{constant}.$



$V(\phi) = V_0 + V'\phi.$



$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha}.$



Why I don't like quintessence (see Efstathiou arXiv:astro-ph/0712.1513)

- We have already seen that the cosmological constant requires a very small number:

$$V(\phi) \sim 3H_0^2 G \sim (10^{-3}\text{eV})^4.$$

- If the field is to show interesting dynamical behaviour, it must be nearly massless:

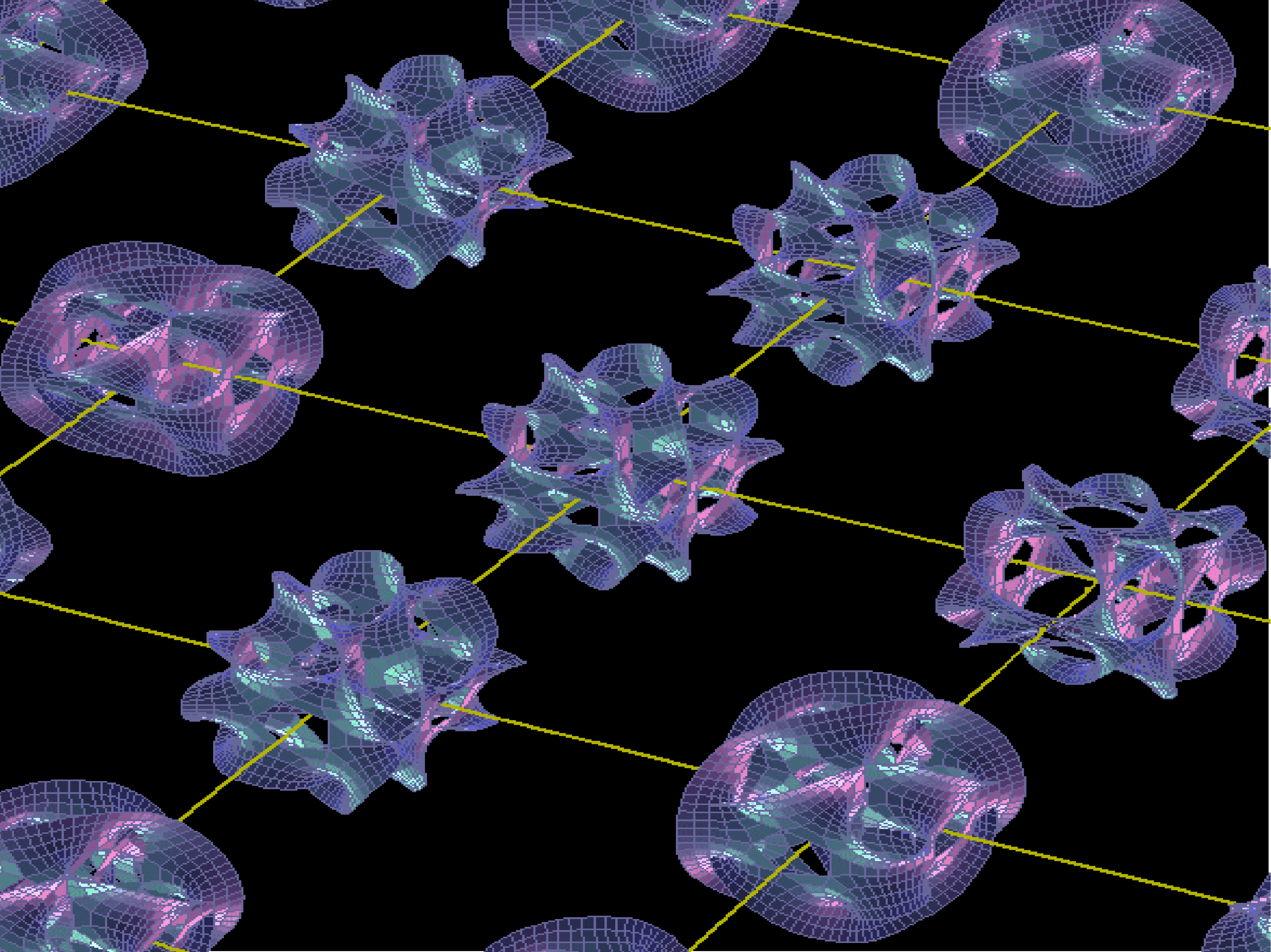
$$m_\phi \sim \left(\frac{V''}{2}\right)^{1/2} \sim H_0 \sim (10^{-33}\text{eV}).$$

- If $\dot{\phi}^2/V \ll 1$, the equation of motion imposes a constraint:

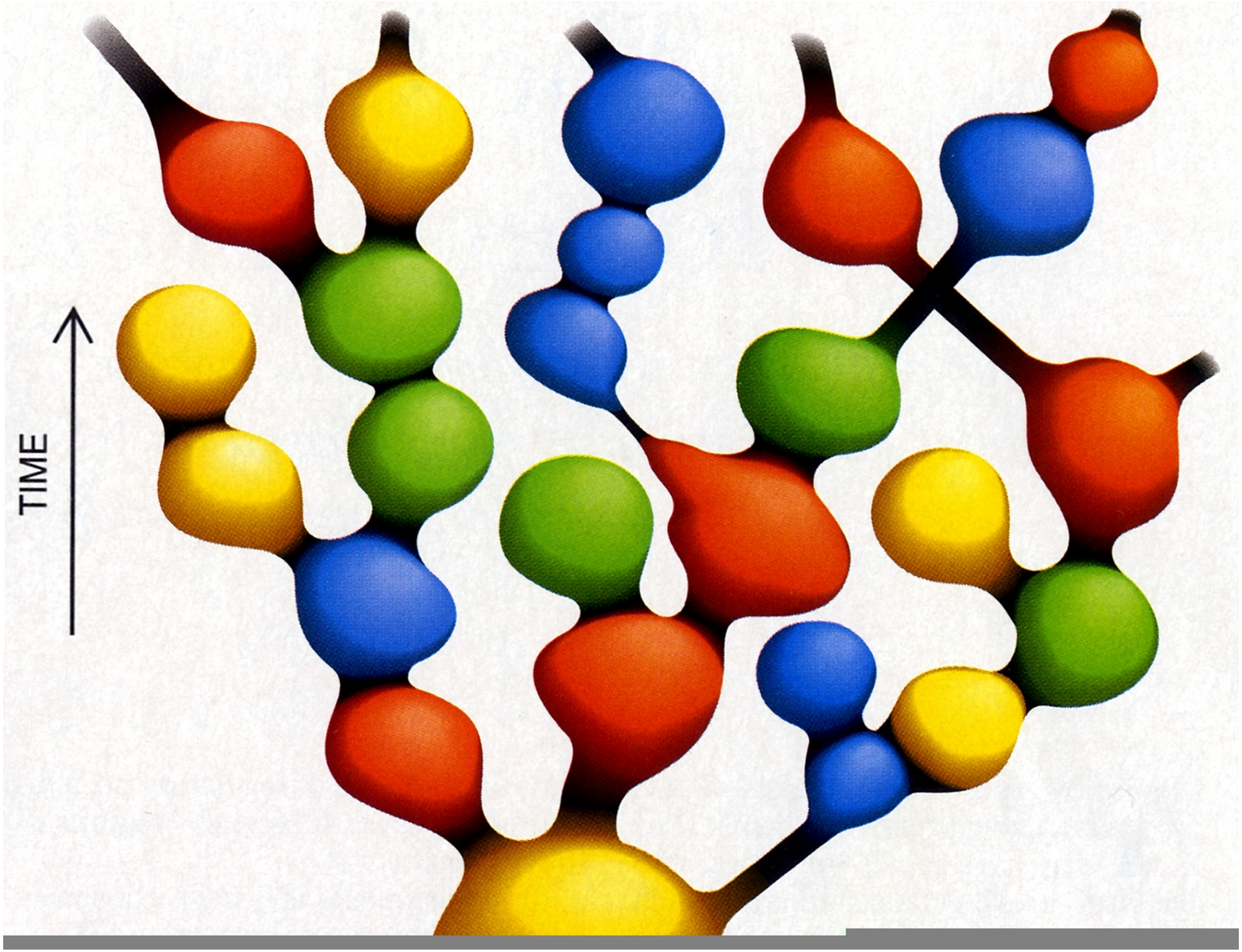
$$\left|\frac{V'}{V}\right| \approx \sqrt{3}\left(\frac{1+w_{\phi,0}}{\Omega_{\phi,0}}\right)^{1/2},$$

so the *observed* constraint $w \approx -1$ requires a small first derivative (in Planck units).

All of these conditions require unexplained fine tunings! (Whereas metastable vacua with positive Λ seem to be prevalent in string theory.)



Multiverse?



Modified Gravity?

Note that there is a large literature on modifications to GR as an explanation of dark energy. E.g. '*$f(R)$ gravity*':

The action for $f(R)$ gravity is

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}f(R) + S_m(g_{\mu\nu}, \phi_m).$$

GR is given by $f(R) = R - 2\Lambda$, but we can easily choose forms of $f(R)$ that lead to late time acceleration, *e.g.*

$$f(R) = R - \frac{\alpha}{R^n}.$$

The challenge for this type of theory is to find a model that passes the stringent local constraints on GR and that is also compatible with the CMB, growth rates of structure, etc.