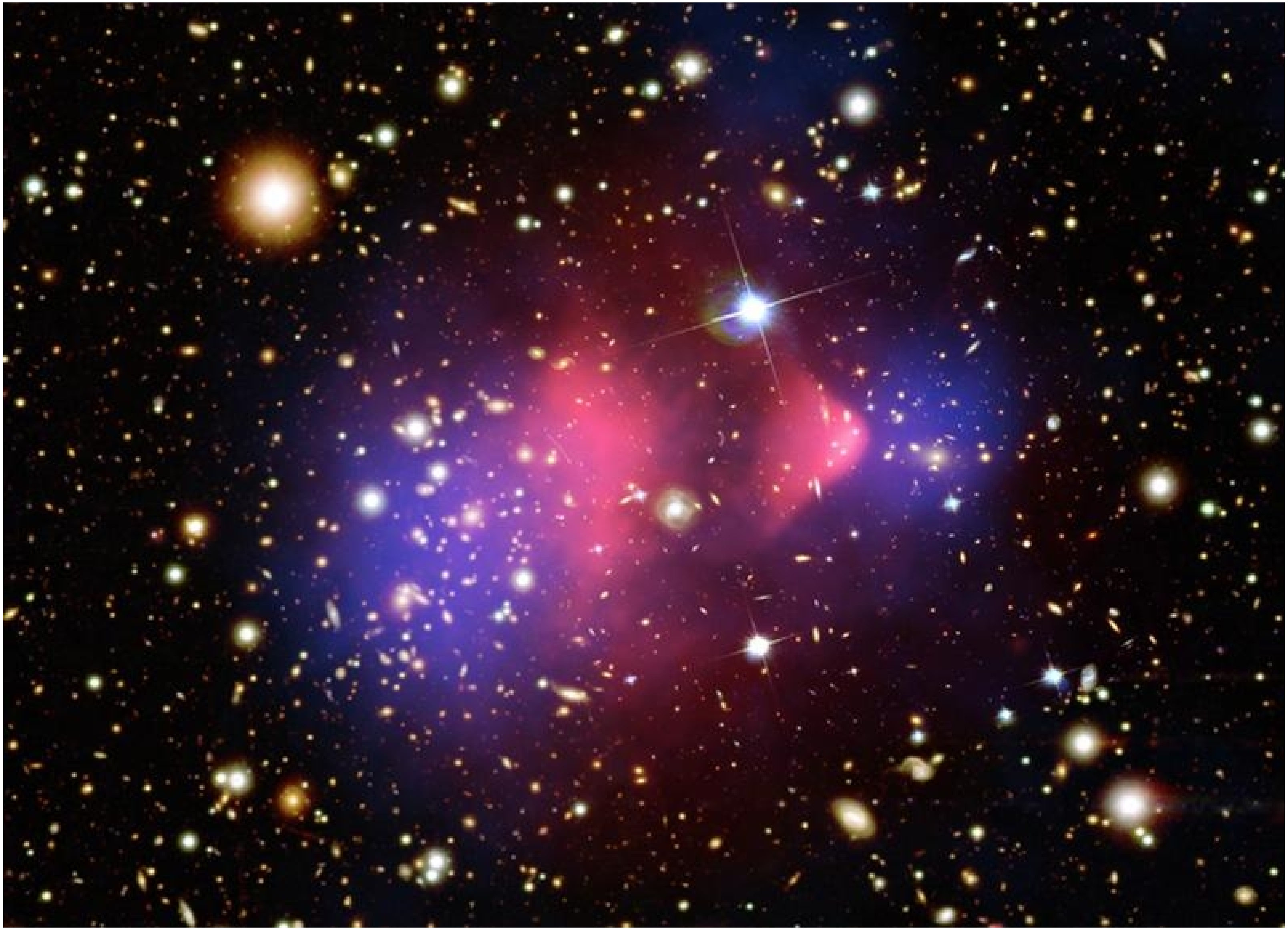
PARTICLE ASTROPHYSICS LECTURE 5

Dark Matter in the Universe



Experimental Evidence for Dark Matter

There are two ways of measuring masses of objects in the Universe:

- Dynamic measurements

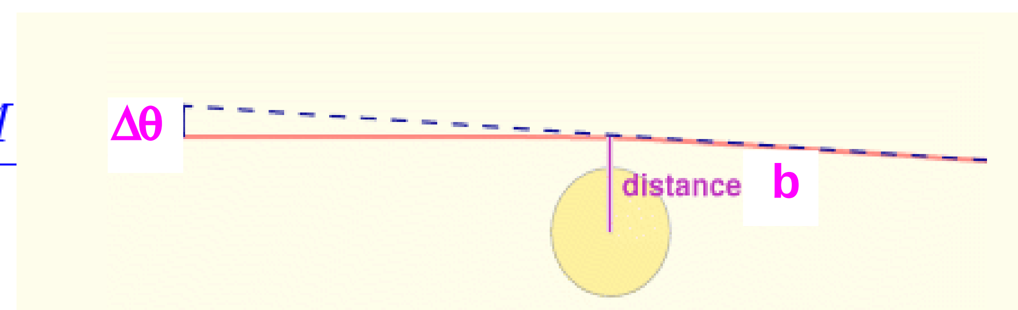
$$\text{Virial Theorem}: \quad 2T + W = 0$$

$$\text{Galaxy rotation curves}: \quad V_{\text{rot}}^2(r) = \frac{GM(r)}{r}$$

$$\text{Hydrostatic equilibrium}: \quad M(<r) = -\frac{r^2}{G\rho(r)}\frac{dP}{dr}$$

- Gravitational lensing

$$\text{Einstein bend angle}: \quad \Delta\theta = \frac{4GM}{bc^2}$$



The need for dark matter

From galaxy redshift surveys we can measure the *galaxy luminosity function* (mean number density of galaxies with luminosities in the range $\mathcal{L} \to \mathcal{L} + d\mathcal{L}$). It is well approximated by a *Schechter function:*

$$\phi(\mathcal{L})d\mathcal{L} = \phi^* \left(\frac{\mathcal{L}}{\mathcal{L}^*}\right)^\alpha \exp\left(-\frac{\mathcal{L}}{\mathcal{L}^*}\right) \frac{d\mathcal{L}}{\mathcal{L}^*},$$

with parameters:

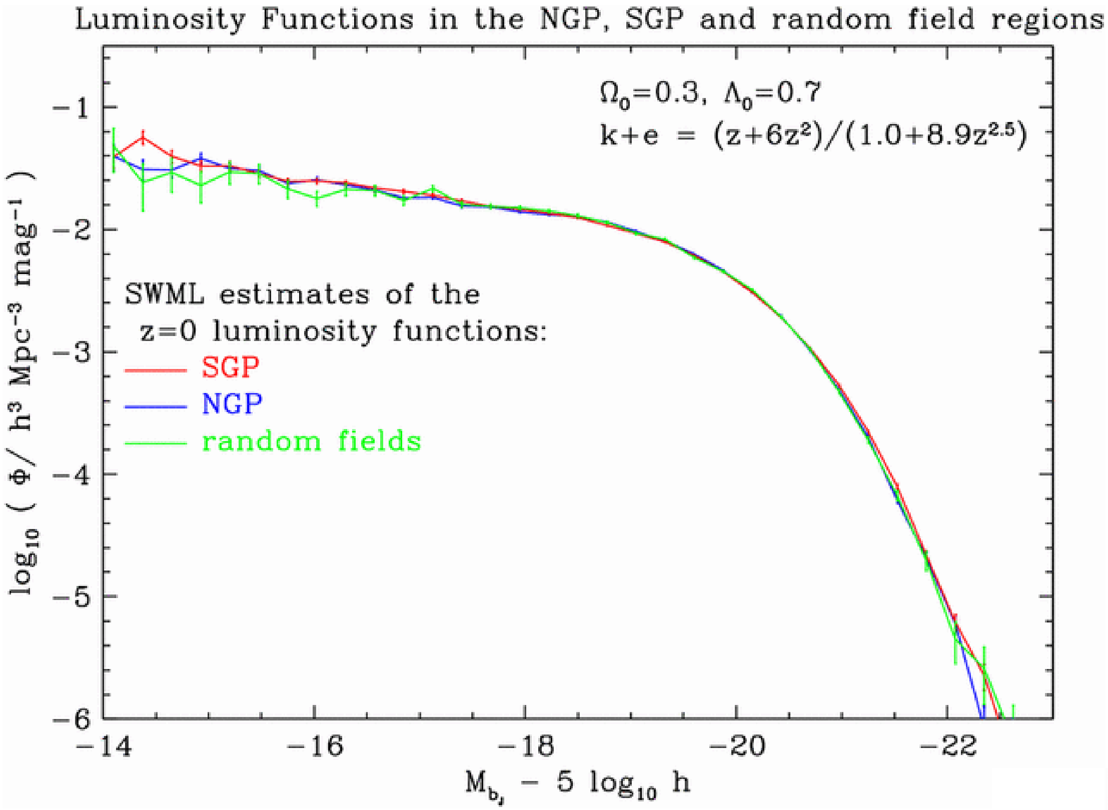
$$\phi^* = 1.4\times10^{-2}h^3\mathrm{Mpc}^{-3}, \quad \mathcal{L}^*{}_B = 1.3\times10^{10}h^{-2}L_\odot \; (\mathrm{B-band}), \quad \alpha \approx -1.$$

Hence the mean luminosity density is

$$\langle\mathcal{L}\rangle = \int_0^\infty \mathcal{L}\phi(\mathcal{L})d\mathcal{L} \approx (1.7 \pm 0.2) \times 10^8 h\mathcal{L}_\odot \ \mathrm{Mpc}^{-3},$$

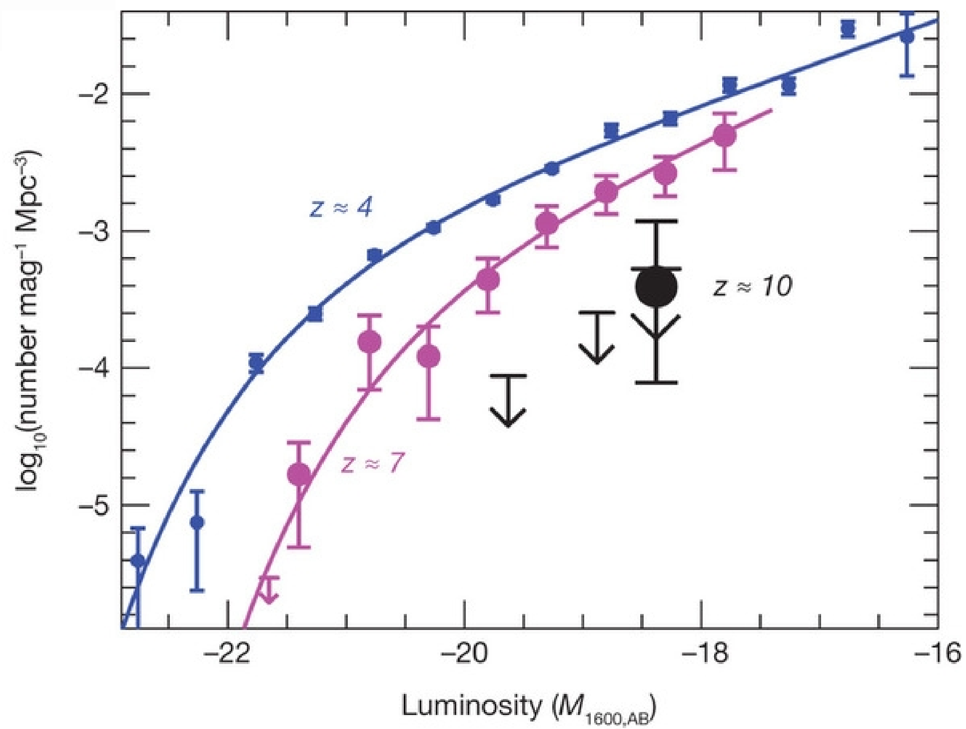
whereas the critical density of the Einstein-de Sitter model is

$$\rho_c = 2.8 \times 10^{11} h^2 M_\odot \mathrm{Mpc}^{-3}.$$



At low redshift

At high redshift



So, a critical density Universe requires

$$\left(\frac{M}{\mathcal{L}}\right)_{\rm crit} \approx (1580 \pm 190) \left(\frac{M}{\mathcal{L}}\right)_{\odot}.$$

But:

Main sequence stars : $\quad \dfrac{M}{\mathcal{L}} \approx \left(\dfrac{M_{\odot}}{M}\right)^{3} \left(\dfrac{M}{\mathcal{L}}\right)_{\odot}$

Typical stellar populations : $\quad \dfrac{M}{\mathcal{L}} \approx 2 - 10 \left(\dfrac{M}{\mathcal{L}}\right)_{\odot}$

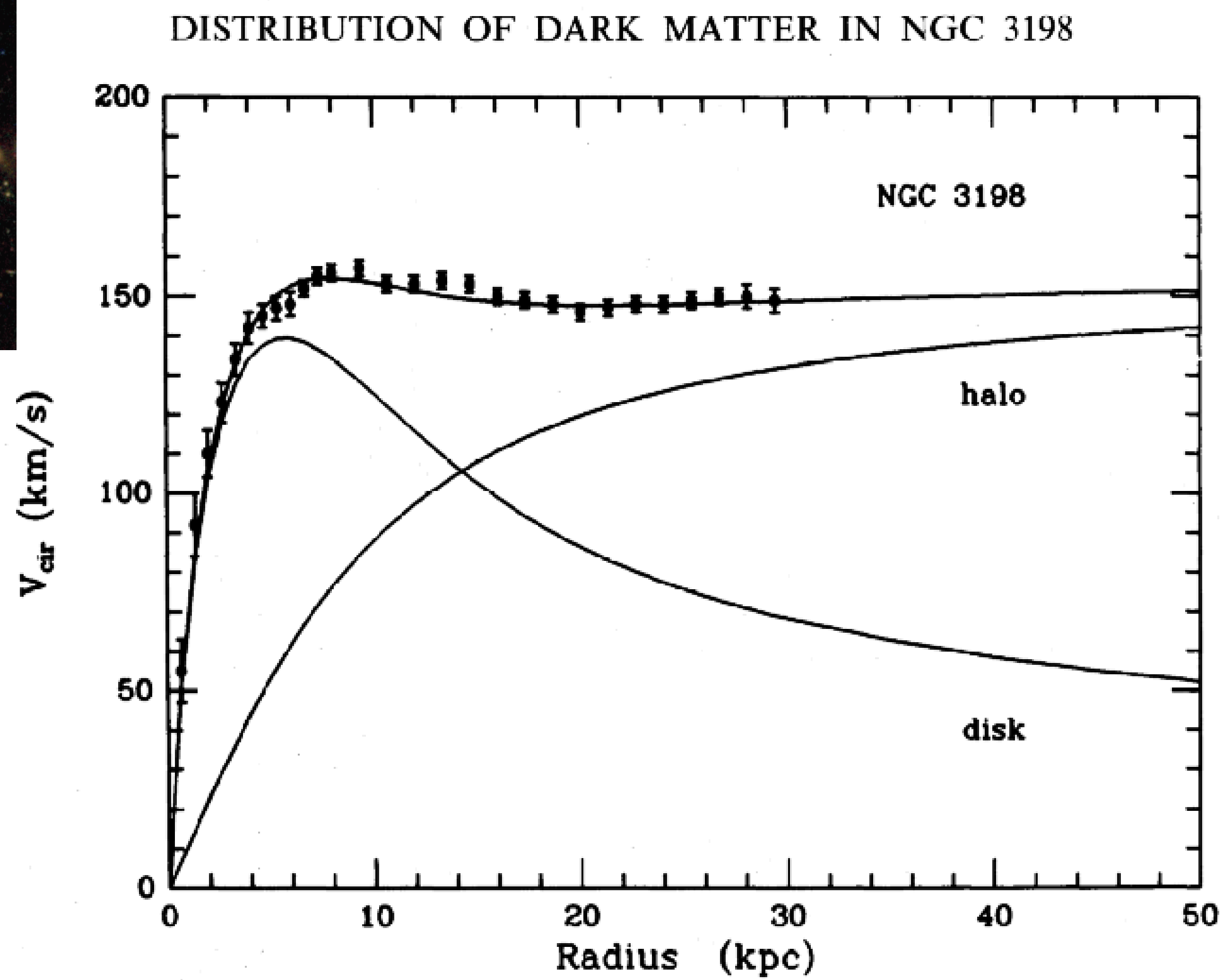
Ordinary stars in galaxies therefore contribute only:

$$\Omega_{*} \approx 0.002 - 0.003,$$

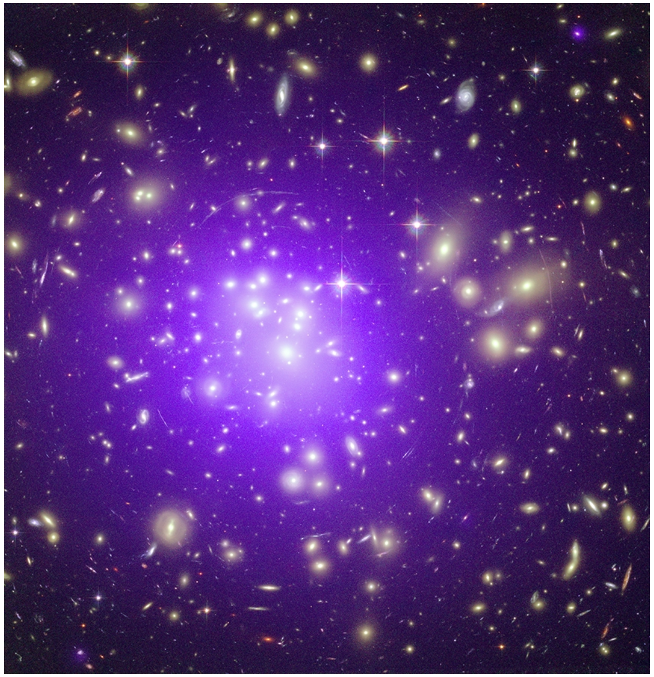
Most of the matter in the Universe must therefore be dark!

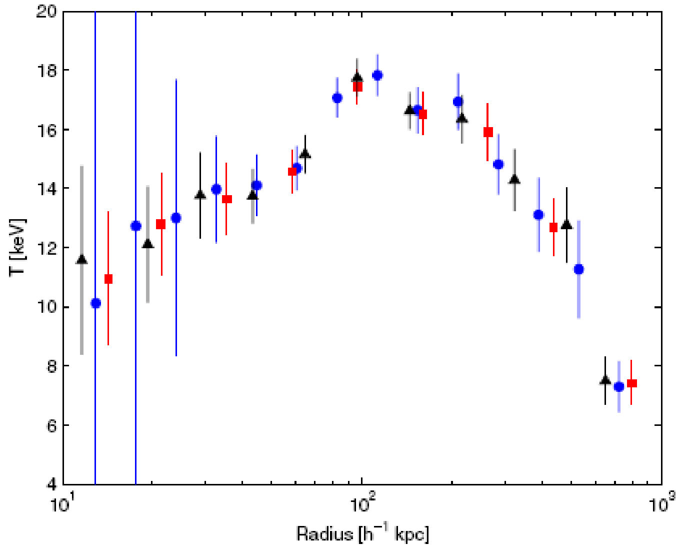
Spiral Galaxy Rotation Curve

$$V_{\rm rot}^2(r) = \frac{GM(r)}{r}$$



Galaxy Cluster Dynamics





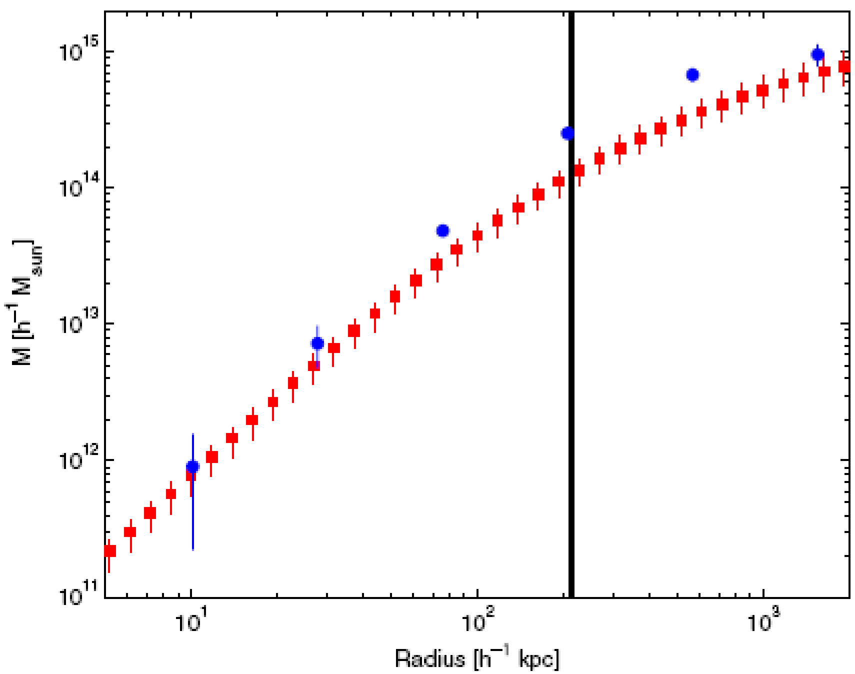
A1689

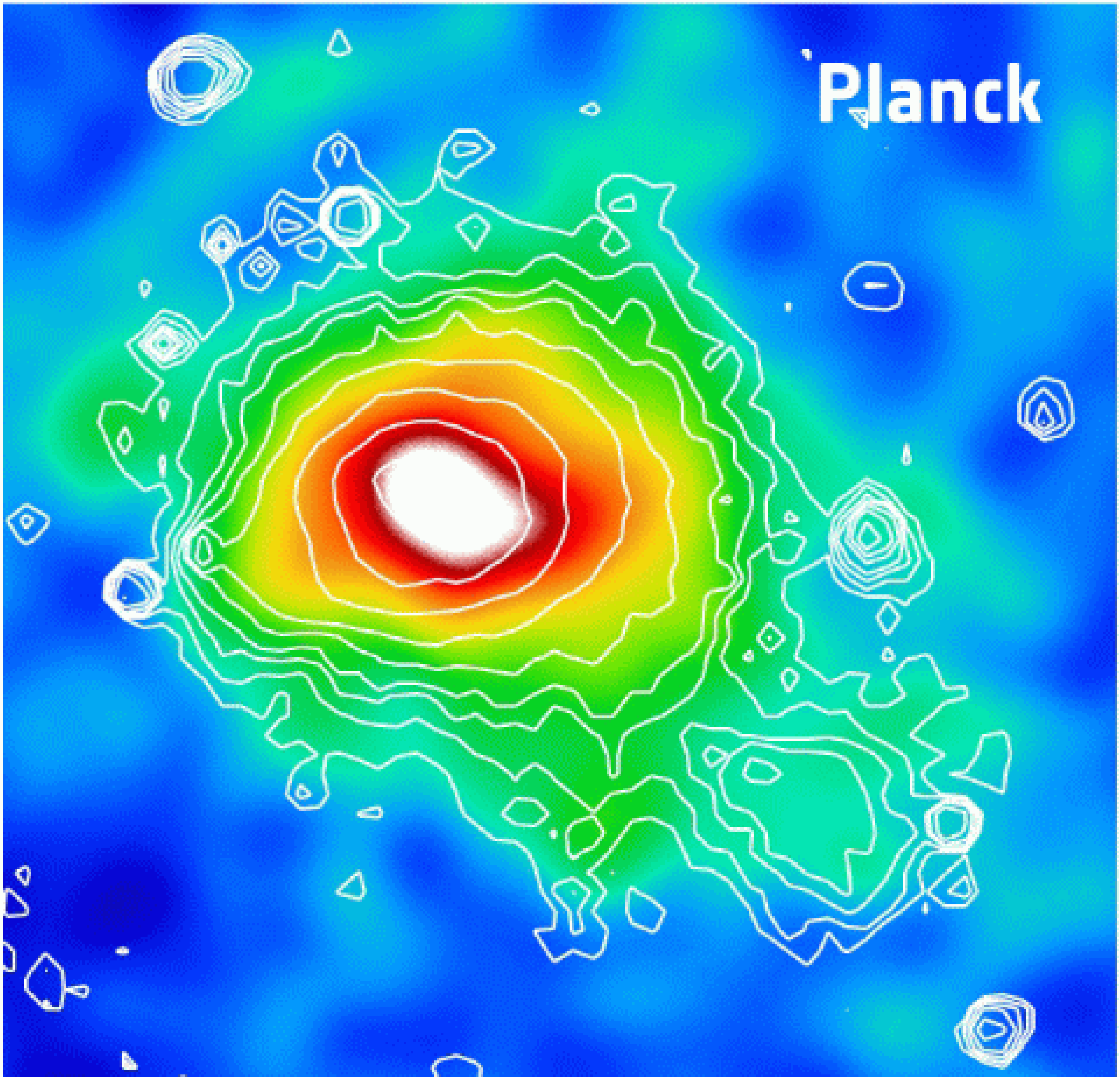
$$\langle v^2 \rangle \sim 1000\ \mathrm{kms}^{-1}$$

$$R \sim 1h^{-1}\mathrm{Mpc}$$

$$kT_e \sim 5-10\ \mathrm{keV}$$

$$\left(\frac{M}{\mathcal{L}}\right) \sim 350h\left(\frac{M}{\mathcal{L}}\right)_{\odot}$$

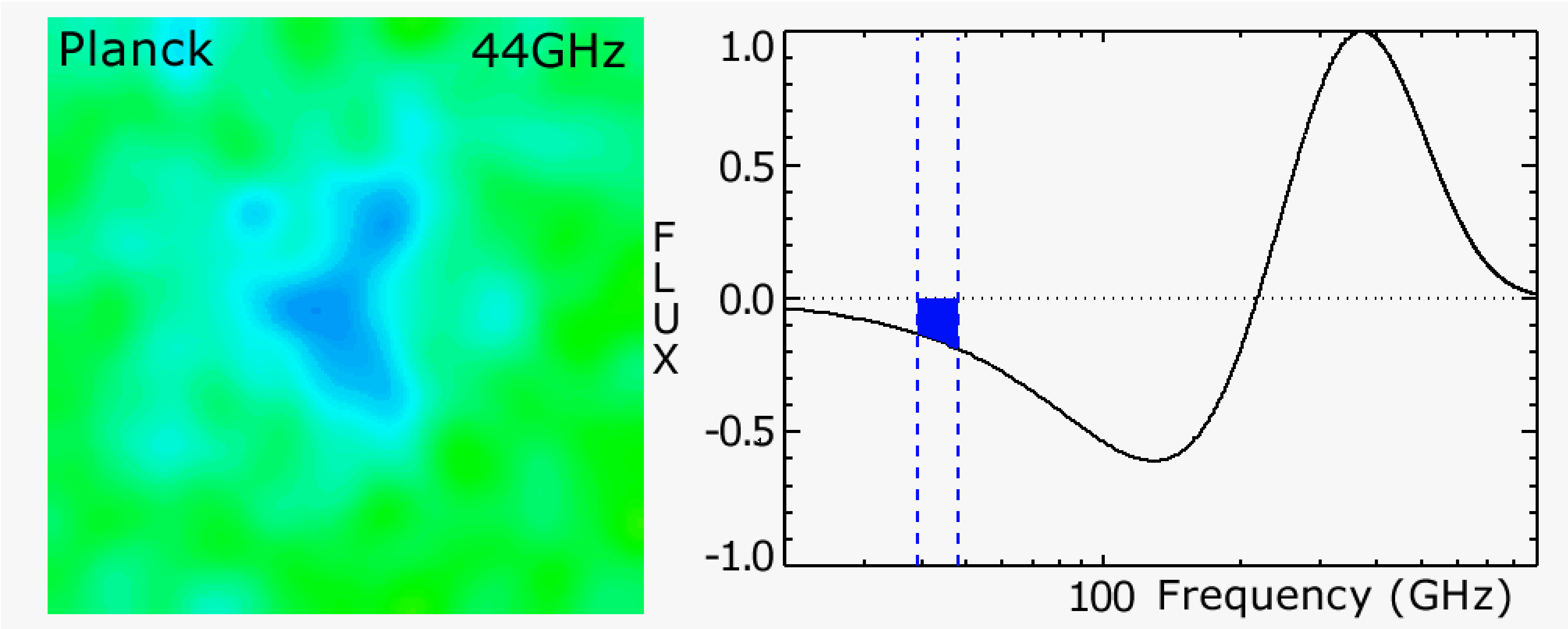



Planck

Thermal Sunyaev-Zeldovich Effect

$$\frac{\Delta T}{T}(\theta) = f(x)\frac{\sigma_T}{m_e c^2}\int n_e(r)T_e(r)\,dl_{\rm los}$$

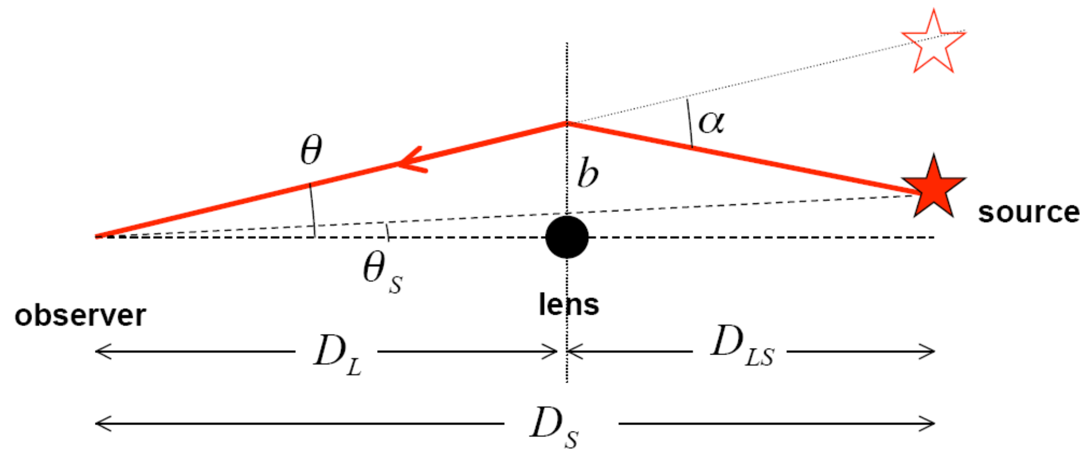
$$f(x) = x\frac{(e^x+1)}{(e^x-1)} - 4, \qquad x = \left(\frac{h\nu}{kT_\gamma}\right)$$



Massive Compact Halo Objects (MACHOS)

e.g. primordial black holes, PopIII stars, 'jupiters' ($M < 0.08 M_\odot$).

Gravitational lensing:



$$\boldsymbol{\alpha} D_{LS} + \boldsymbol{\theta}_s D_S = \boldsymbol{\theta} D_S,$$

giving the lensing equation:

$$\boldsymbol{\alpha}(\theta D_{LS}) = \frac{D_S}{D_{LS}}(\boldsymbol{\theta} - \boldsymbol{\theta}_s).$$

The distances D_s, D_{LS}, D_S are *angular diameter* distances, *e.g.*

$$D_{LS} = \frac{R_0(r_S - r_L)}{(1 + z_S)}$$

where r_s, r_L are the comoving coordinate distances of the source and lens.

For a point mass lens, we can define the *Einstein angle*

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_S D_L}},$$

and so we can write the lens equation as

$$\theta_E^2 \frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|^2} = (\boldsymbol{\theta} - \boldsymbol{\theta}_S).$$

So, defining

$$\mathbf{y} = \frac{\boldsymbol{\theta}_S}{\theta_E}, \quad \mathbf{x} = \frac{\boldsymbol{\theta}}{\theta_E},$$

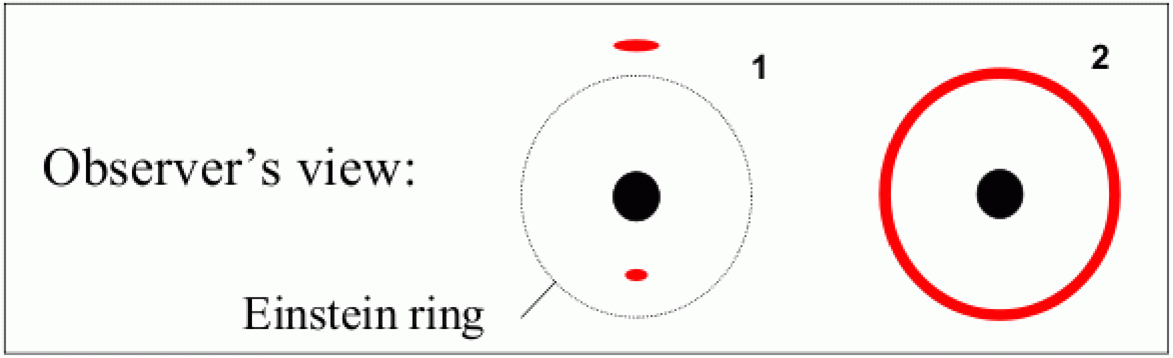
the lensing equation is evidently a quadratic equation

$$\mathbf{y} = \mathbf{x} - \frac{\mathbf{x}}{|\mathbf{x}|^2},$$

with solutions

$$\mathbf{x} = \frac{1}{2}(|\mathbf{y}| \pm \sqrt{4 + |\mathbf{y}|^2})\frac{\mathbf{y}}{|\mathbf{y}|}.$$

If $\mathbf{y} = 0$, the solution is $|\mathbf{x}| = 1$, *i.e.* an *Einstein Ring*, $|\boldsymbol{\theta}| = \theta_E$.



Lensing preserves surface brightness, and so the images are *magnified*:

$$\mu = \left| \frac{\partial \boldsymbol{\theta}_S}{\partial \boldsymbol{\theta}} \right|^{-1} .$$

For a point mass lens and a 'small' source

$$\mu_{\pm} = \frac{1}{4} \left(\frac{y}{\sqrt{y^2 + 4}} + \frac{\sqrt{y^2 + 4}}{y} \pm 2 \right) .$$

$\mu+ > 1$ for all source positions, whereas μ_- can be greater or less than unity depending on the position of the source.

Consider a star in our Galaxy:

$$\theta_E = 0.9 \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D_L}{10\text{kpc}}\right)^{-1/2} \left(1 - \frac{D_L}{D_S}\right)^{1/2} \text{mas.}$$

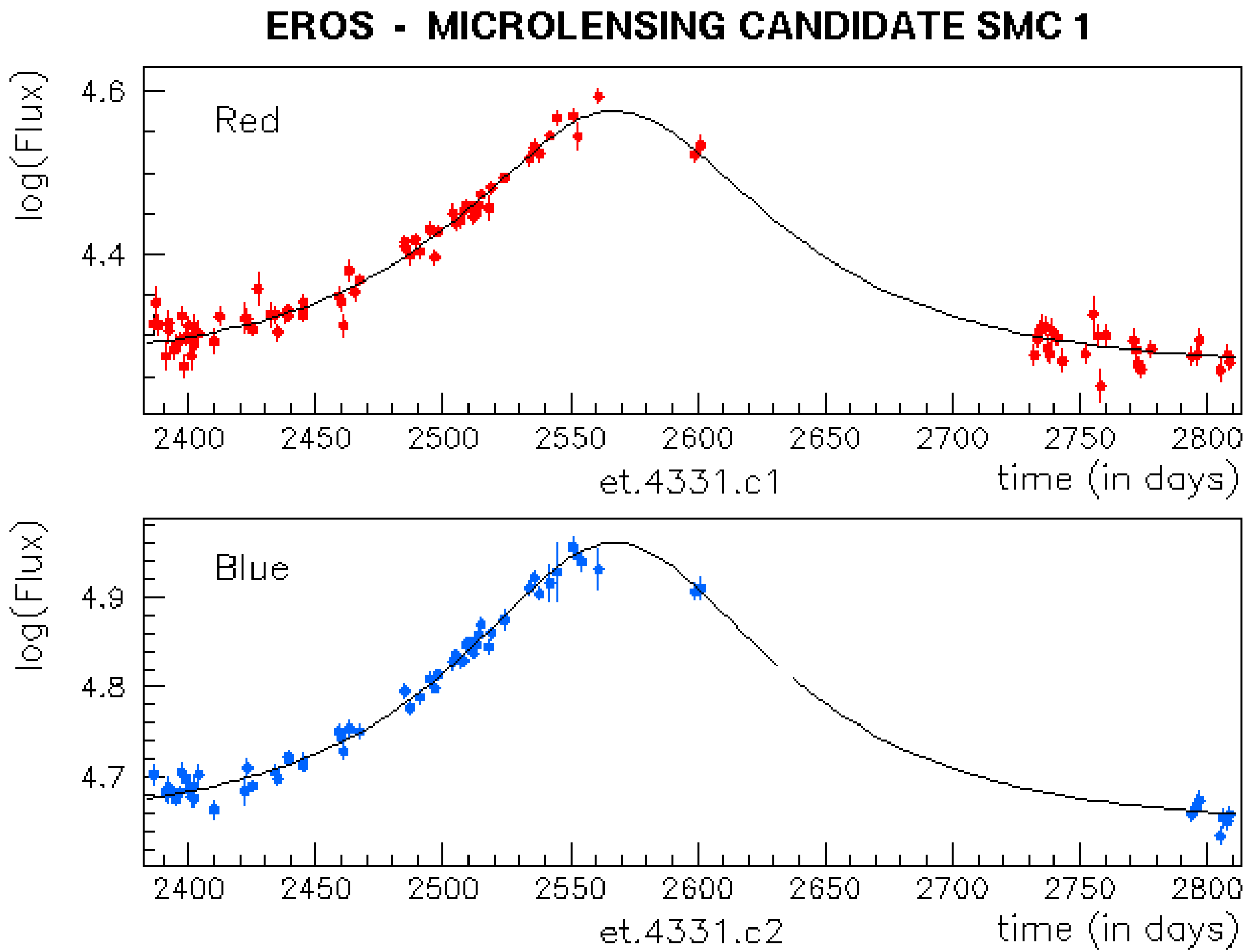
Although the image separation is unobservably small, the magnification *is* observable. If a point mass has a transverse velocity:

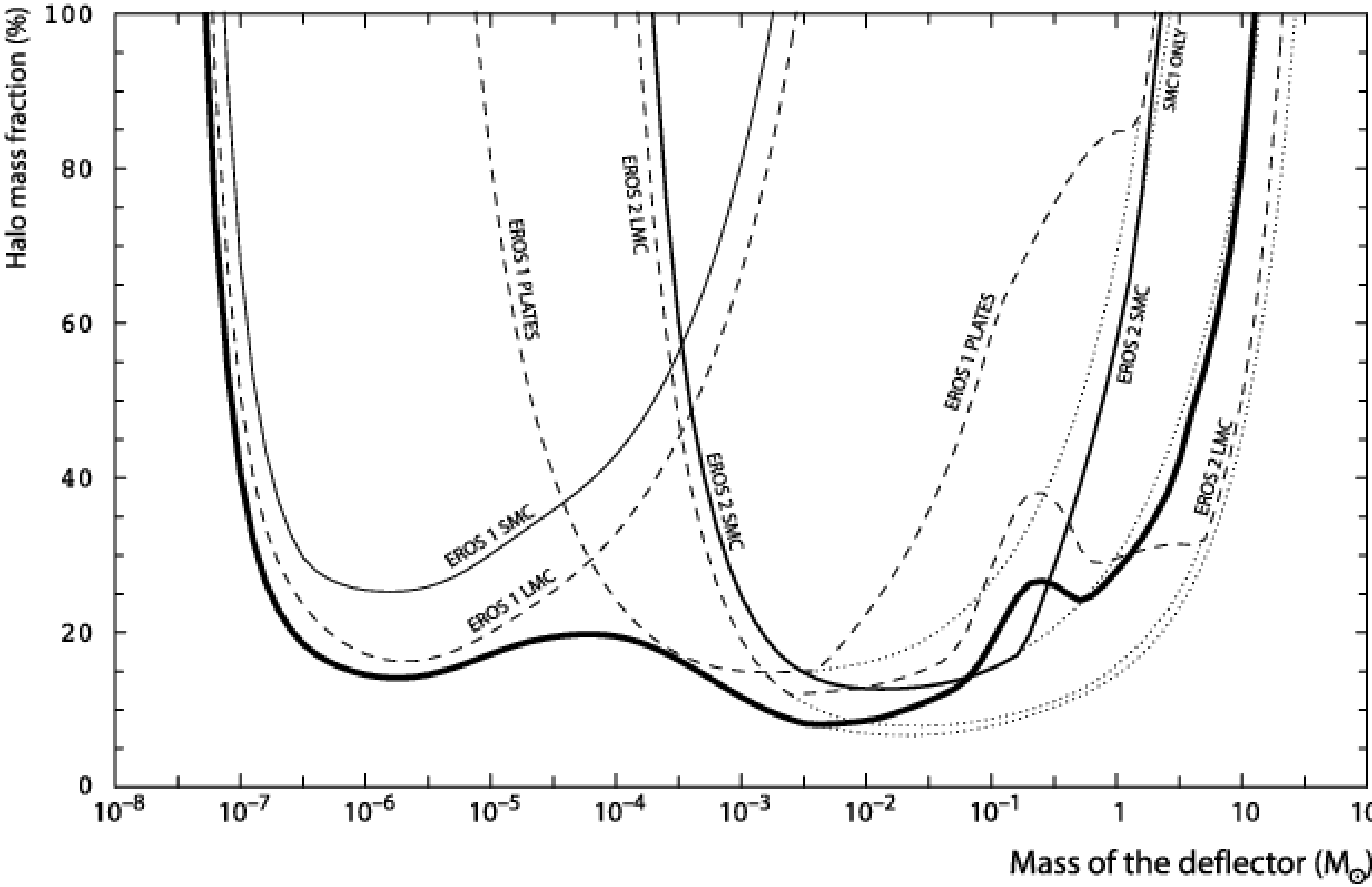
$$\frac{v}{D_L} = \dot{\theta} = 4.2\text{mas/y} \left(\frac{v}{200\text{kms}^{-1}}\right) \left(\frac{D_L}{10\text{kpc}}\right)^{-1},$$

then the lensed background star will vary on a timescale:

$$t_E = \frac{\theta_E}{\dot{\theta}} = 0.2\text{y} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D_L}{10\text{kpc}}\right)^{1/2} \left(1 - \frac{D_L}{D_S}\right)^{1/2} \left(\frac{v}{200\text{kms}^{-1}}\right)^{-1}.$$

We can therefore search for *microlensing events* by imaging dense star fields. (MACHO and EROS experiments imaged the Magellanic Clouds). Frequency and duration of microlensing events constrains MACHOS in the Galactic halo.


EROS - MICROLENSING CANDIDATE SMC 1
log(Flux)
4.6
4.4
Red
2400
2450
2500
2550
2600
2650
2700
2750
2800
et.4331.c1
time (in days)
log(Flux)
4.9
4.8
4.7
Blue
2400
2450
2500
2550
2600
2650
2700
2750
2800
et.4331.c2
time (in days)



Halo mass fraction (%)
100
80
60
40
20
0
EROS 1 SMC
EROS 1 LMC
EROS 1 PLATES
EROS 2 LMC
EROS 2 SMC
EROS 1 PLATES
EROS 2 SMC
SMC1 ONLY
EROS 2 LMC
10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10 10^{2}
Mass of the deflector ($M_{\odot}$)

Massive neutrinos

The mass eigenstates $|\nu_1\rangle$, $|\nu_2\rangle$, $|\nu_3\rangle$, need not be the same as the flavour eigenstates $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$. For Dirac neutrinos, the flavour and mass eigenstates are related by a coupling matrix $|\nu_f\rangle = U|\nu_i\rangle$:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

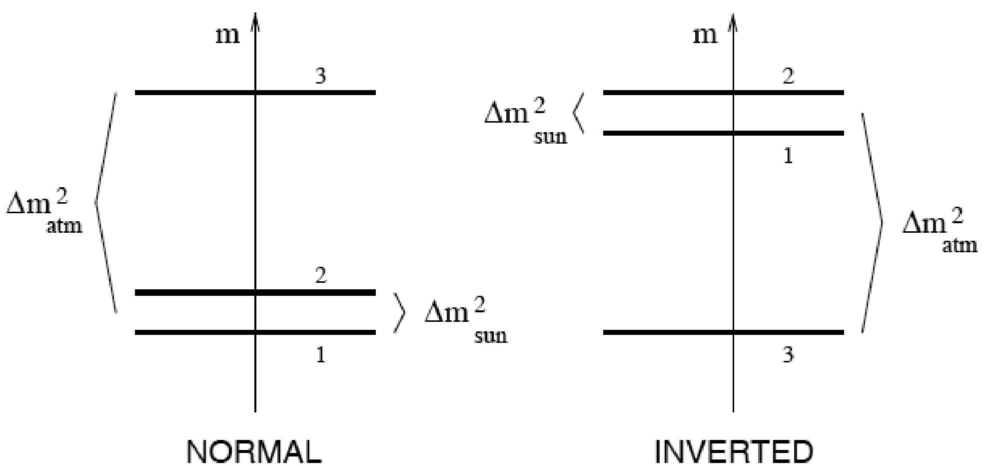
where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ and δ is a CP violating phase. Non-zero neutrino masses therefore lead to *flavour oscillations*.

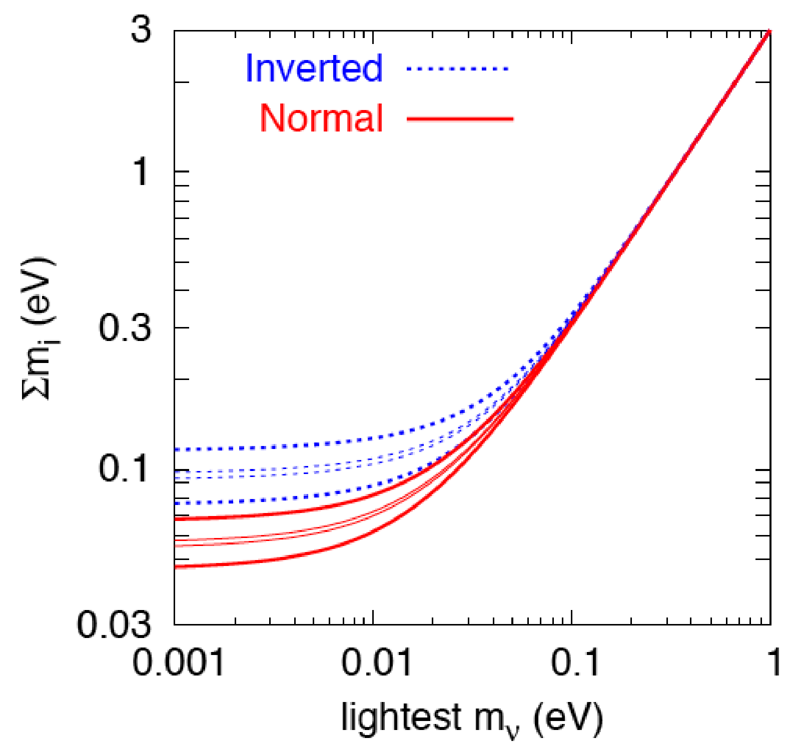
From solar neutrino experiments (Δm^2_{21}) and atmospheric neutrino experiments (Δm^2_{31}), the following (3σ)) constraints have been derived:

$$\Delta m^2_{21} = (7.9^{+1.0}_{-0.8}) \times 10^{-5}\ \text{eV}^2, \quad \Delta m^2_{31} = (2.2^{+1.1}_{-0.8}) \times 10^{-3}\ \text{eV}^2,$$

$$s^2_{21} = 0.3^{+0.10}_{-0.06}, \quad s^2_{23} = 0.50^{+0.18}_{-0.16}, \quad s^2_{13} \leq 0.43.$$

Note that tritium β-decay experiments limit $m_{\nu_e} < 2.2$ eV. These observations lead to the following possibilities:





Recall that neutrinos decouple at $kT \sim 3$ MeV $\gg m_\nu$, so neutrinos were *relativistic* at the time of decoupling. But collisionless particles satisfy the Boltzmann equation

$$\frac{\partial f}{\partial t} - \frac{\dot{R}}{R} p \frac{\partial f}{\partial p} = 0,$$

which we can write as

$$\left(\frac{\partial f}{\partial t} \right)_q = 0,$$

in terms of a comoving momentum $q = pR$. The neutrino distribution function therefore retains its relativistic shape

$$f(p,t) = \frac{1}{\left[\exp\left(\frac{pc}{kT_\nu}\right) + 1\right]},$$

with $T_\nu \propto 1/R$, *even if neutrinos have a finite rest mass*.

Note also that because e^+e^- annihilate after neutrino decoupling,

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma.$$

The neutrino and photon number densities are therefore:

$$\begin{aligned} n_\nu &= \frac{4\pi g_\nu}{h^3}\int \frac{p^2 dp}{\left[\exp\left(\frac{pc}{kT_\nu}\right)+1\right]} = \frac{4\pi g_\nu}{h^3}\left(\frac{kT_\nu}{c}\right)^3 \frac{3}{4}\Gamma(3)\zeta(3), \\ n_\gamma &= \frac{4\pi g_\nu}{h^3}\int \frac{p^2 dp}{\left[\exp\left(\frac{pc}{kT_\gamma}\right)-1\right]} = \frac{4\pi g_\nu}{h^3}\left(\frac{kT_\gamma}{c}\right)^3 \Gamma(3)\zeta(3), \end{aligned}$$

Hence

$$n_\nu = \frac{3}{4}\left(\frac{T_\nu}{T_\gamma}\right)^3, \quad n_\nu = \frac{3}{11}n_\gamma, \quad n_\gamma \approx 409\text{cm}^{-3},$$

and if neutrinos have mass

$$\rho_\nu = m_\nu c^2 n_\nu.$$

So:

$$\Omega_\nu h^2 = 1.02 \left(\frac{m_\nu}{100 \text{ eV}}\right).$$

and if $m_\nu \approx 0.06$ eV (most likely possibility, unless neutrino masses are degenerate), $\Omega_\nu = 0.0006h^{-2}$. If this is true, then curiously, $\Omega_\nu \approx \Omega_*$!

Note also that if neutrinos are massive, there is a characterstic length scale

$$\lambda_\nu \sim ct_{\text{nr}}, \quad kT_\nu(t_{\text{nr}}) = m_\nu c^2.$$

i.e. the Hubble radius at the time that neutrinos become non-relativistic. Neutrino fluctuations are *damped by free-streaming* on scales ($\lambda \lesssim \lambda_\nu$). (Note that velocities decay as $1/R$ at $t > t_{\rm nr}$, so most of the damping occurs when the neutrinos are relativistic). The present physical scale of the neutrino damping length is

$$\lambda_\nu \sim c t_{\rm nr} \left(\frac{R_0}{R(t_{\rm nr})} \right) \sim 14 \left(\frac{\Omega_\nu h^2}{0.3} \right)^{-1} \mathrm{Mpc}.$$

If neutrinos dominated the dark matter density, *galaxies and clusters could not form.*

WIMPs: Weakly Interacting Massive Particles

If the dark matter particles were *cold* (*i.e.* random motions of the particles negligible) and *weakly interacting*, the damping scale would be negligibly small.

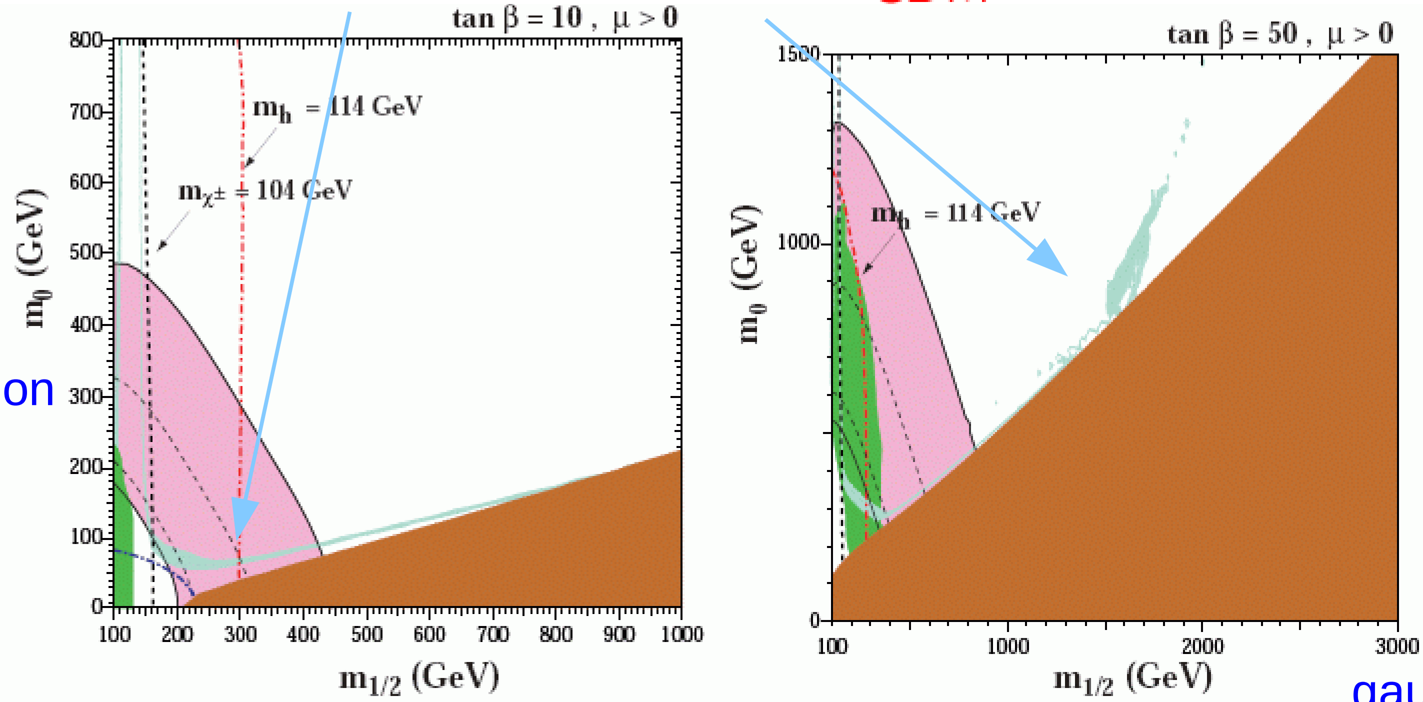
The most attractive possibility is a *supersymmetric* particle. SUSY particles are expected to be created in pairs with opposite values of R-parity. Heavier SUSY particles decay to lighter ones in R-conserving processes ending up with a lightest stable SUSY particle (LSP). To explain the dark matter, the LSP must be electrically neutral.

'Best' candidate is a *neutralino* . In MSSM, there are four neutralino states χ_n^0 that are linear combinations of the bino, zino and two higgsinos. The mass matrix depends on θ_W and m_Z of the SM and on four parameters of the C(onstrained)MSSM (gaugino mass $m_{1/2}$; squark/slepton mass m_0; ratio of vacuum expectation values of the higgsinos $tan\beta$; mass parameter of higgsinos μ).

From J. Ellis 'Prospects for Discovering Supersymmetry at the LHC':arXiv:0810.1178 [hep-ph]

allowed by the WMAP constraint $\Omega_{CDM}h^2 = 0.1099 \pm 0.0062$

squark/slepton mass



gaugino mass

Fig. 1. *The CMSSM* $(m_{1/2}, m_0)$ *planes for (a)* $\tan\beta = 10$ *and (b)* $\tan\beta = 50$*, assuming* $\mu > 0$*,* $A_0 = 0$*,* $m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV [22]. The near-vertical (red) dot-dashed lines are the contours for* $m_h = 114$ *GeV, and the near-vertical (black) dashed line is the contour* $m_{\chi^\pm} = 104$ *GeV. Also shown by the dot-dashed curve in the lower left is the region excluded by the LEP bound* $m_{\tilde{e}} > 99$ *GeV. The medium (dark green) shaded region is excluded by* $b \to s\gamma$*, and the light (turquoise) shaded area is the cosmologically preferred region. In the dark (brick red) shaded region, the LSP is the charged* $\tilde{\tau}_1$*. The region allowed by the measurement of* $g_\mu - 2$ *at the 2-*σ *level assuming the* e^+e^- *calculation of the Standard Model contribution, is shaded (pink) and bounded by solid black lines, with dashed lines indicating the 1-*σ *ranges.*

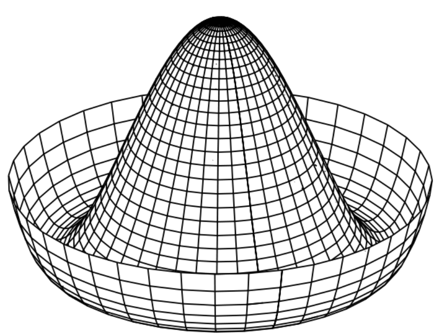
Axions

In QCD Lagrangian can contain a CP violating term

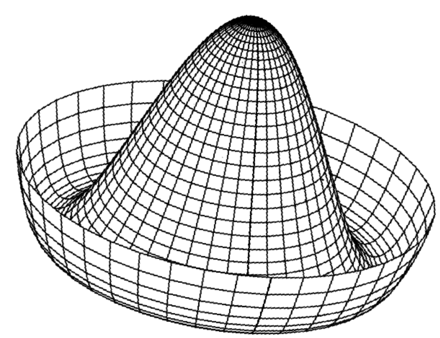
$$\mathcal{L} = \theta \frac{g^2}{16\pi} G^{\mu\nu} \tilde{G}_{\mu\nu}, \qquad (1)$$

where $G^{\mu\nu}$ is the gluon field strength. However the observed degree of CP violation on the strong interaction is small. For example, the experimental limit on the neutron magnetic moment limits $|\theta| \lesssim 10^{-9}$. Such a small number requires an explanation. This is the *strong CP problem*.

Possible solution suggested by Peccei and Quinn. Introduce a new $U(1)$ symmetry:



$t < t_{QCD}$



$t > t_{QCD}$ (schematic)

The degree of freedom around the minimum of the potential is a Goldstone boson, the *axion*. At the QCD phase transition, non-linear instanton effects cause the potential to develop a minimum (shown schematically in the figure), exactly cancelling the CP violating term (1).

After the QCD phase transion, the axion has a *small* mass:

$$m_a \sim \frac{\Lambda_{\mathrm{QCD}}^2}{f_{PQ}} \sim 0.6 \left(\frac{10^7 \mathrm{Gev}}{f_{PQ}} \right) \mathrm{eV},$$

where $\Lambda_{\mathrm{QCD}} \sim 200$ MeV is the energy scale of the quark-hardron phase transition and f_{PQ} is the Peccei-Quinn symmetry breaking scale. But unlike neutrinos, the axion is a coherently oscillating scalar field that obeys the equation of motion:

$$\ddot{\phi} + \frac{3\dot{R}}{R}\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0.$$

Approximating $V(\phi) = (m_a^2/2)\phi^2$, and neglecting the Hubble expansion (fast oscillations) the solution is SHM with angular frequency m_a. The time averaged density and pressure of the oscillating field is therefore:

$$\begin{aligned} \langle \rho \rangle &= \frac{1}{2}\langle \dot{\phi}^2 \rangle + \langle V(\phi) \rangle = \langle \dot{\phi}^2 \rangle, \\ \langle P \rangle &= \frac{1}{2}\langle \dot{\phi}^2 \rangle - \langle V(\phi) \rangle = 0. \end{aligned}$$

Thus the oscillating scalar field behaves like cold dark matter, *even if the mass, m_a, is small.*

Note that the PQ axion can oscillate into a photon of energy m_a in the presence of a magnetic field. Laboratory and various astrophysical limits strongly constrain PQ axions. But axion-like fields seem to be prevelant in string theory.