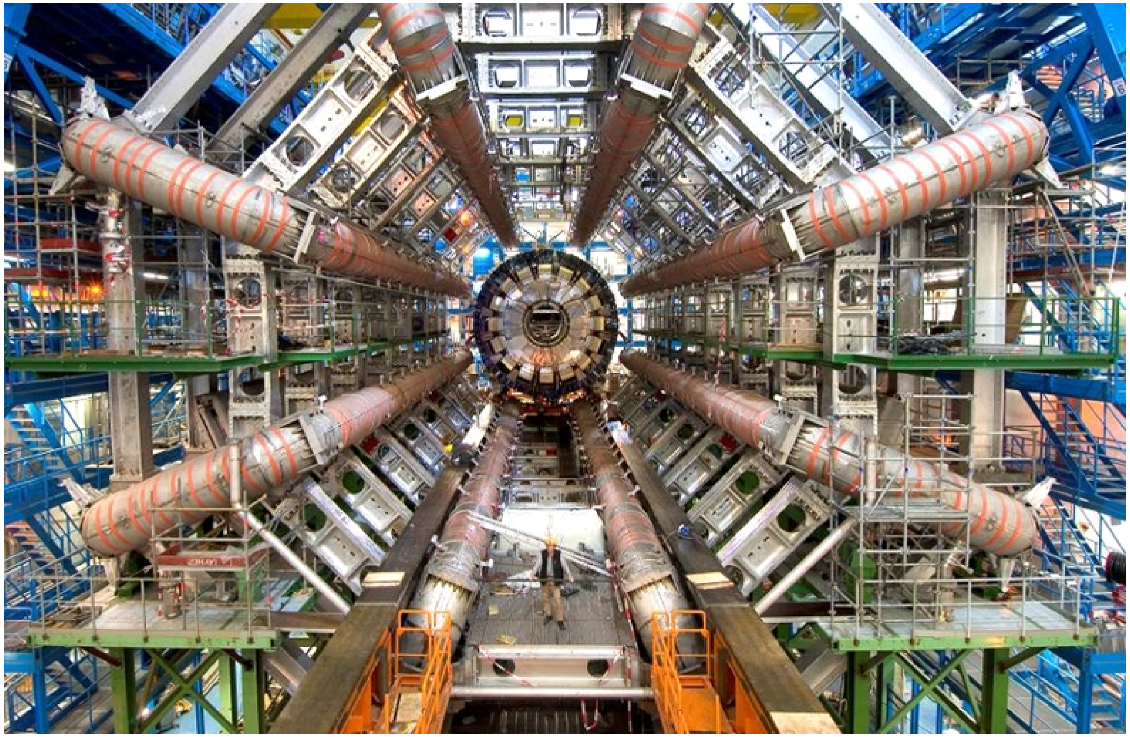
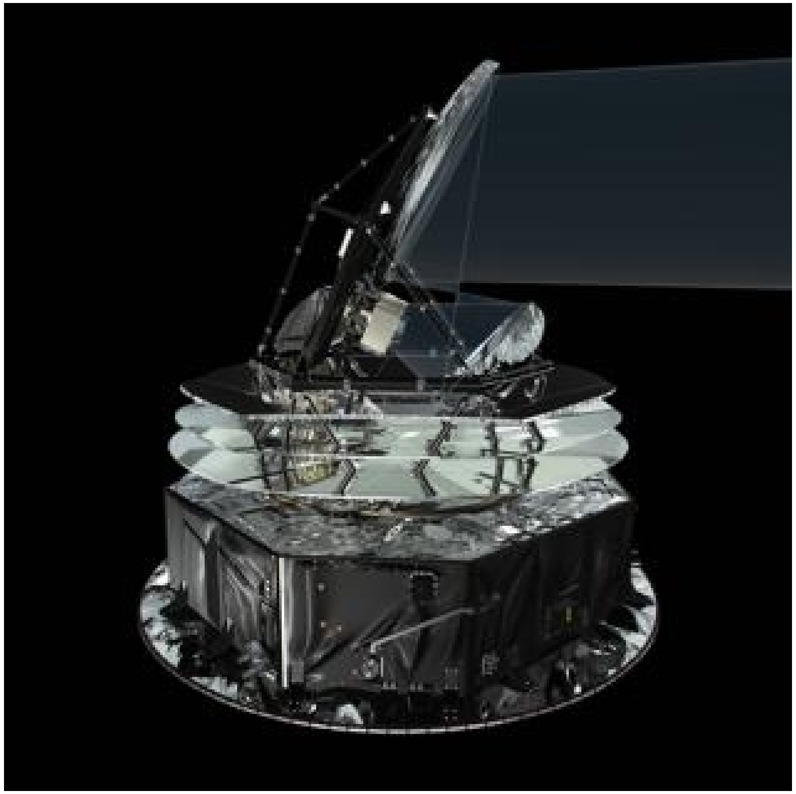
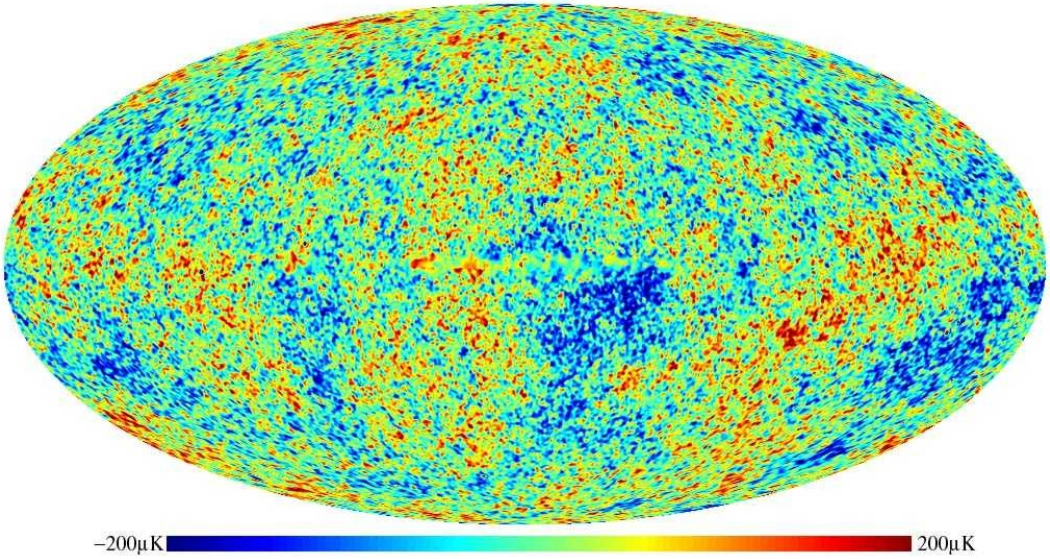
PARTICLE ASTROPHYSICS LECTURE 2







Planck Units

$$\hbar, \quad c, \quad G$$

$$\text{Planck length}: \quad \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.6 \times 10^{-35} \text{ metres}$$

$$\text{Planck mass}: \quad \left(\frac{\hbar c}{G}\right)^{1/2} = 2.1 \times 10^{-8} \text{ kgrams}$$

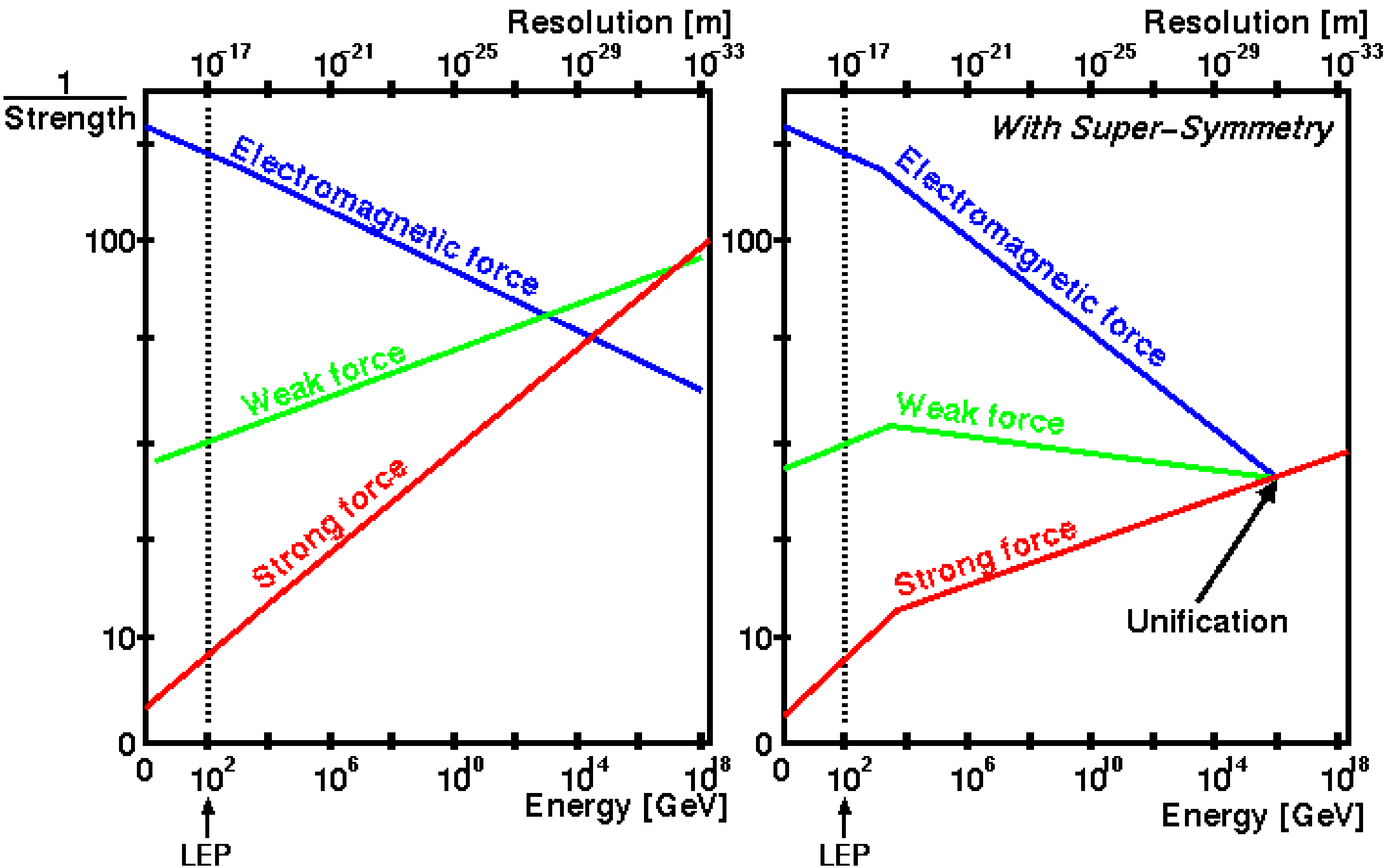
$$\text{Planck time}: \quad \left(\frac{\hbar G}{c^5}\right)^{1/2} = 5.4 \times 10^{-44} \text{ seconds}$$

$$\text{Planck energy}: \quad \left(\frac{\hbar c^5}{G}\right)^{1/2} = 1.2 \times 10^{19} \text{ GeV}$$

:Ways of testing fundamental physics:

- Direct tests in accelerators (e.g. creating Higgs, SUSY particles)
- Indirect tests (e.g. radiative corrections)
- Low energy experiments (e.g. direct dark matter detection, neutrino masses)
- Astrophysical accelerators (high energy cosmic rays)
- Background cosmology (tests of GR, dark energy)
- Cosmological perturbations (e.g. CMB, large-scale structure)
- Interactions (e.g. dark matter annihilation)
- Relics (light elements, baryons, dark matter, defects......)

An indirect test:



The Friedmann-Robertson-Walker Solution

The FRW metric is

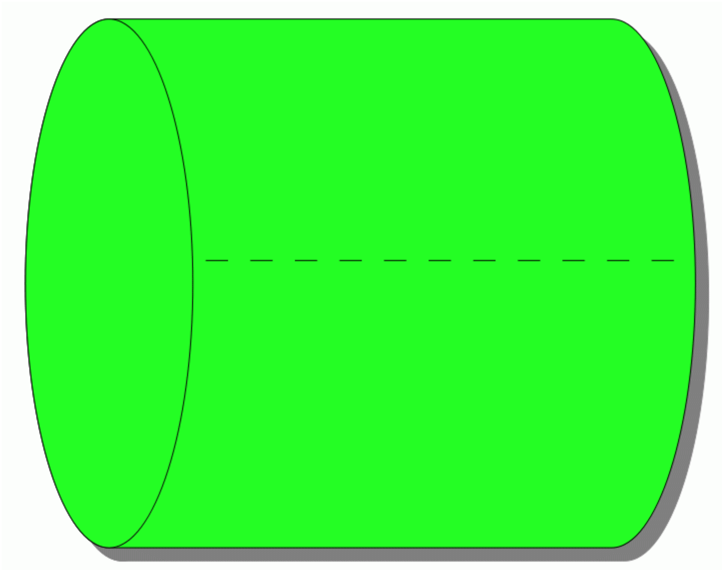
$$ds^2 = c^2 dt^2 - R^2(t)\left[\frac{dr^2}{(1-Kr^2)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

The spatial part of this metric describes a *maximally symmetric 3-space*. Its is characterised by one number, K. The curvature tensor is

$$R_{\lambda\rho\sigma\nu} = K(g_{\lambda\sigma}g_{\rho\nu} - g_{\lambda\nu}g_{\rho\sigma})$$

By repeated contraction:

$$\begin{aligned} \text{Ricci tensor :} \qquad R_{\rho\sigma} &= g^{\lambda\nu}R_{\lambda\rho\sigma\nu} = -2Kg_{\rho\sigma} \\ \text{Curvature scalar :} \qquad R &= R^{\sigma}_{\sigma} = -6K \end{aligned}$$



For a cylinder it is obvious that

$$ds^2 = dr^2 + r^2 d\theta^2,$$

describes a flat surface.

(Transform $x = r\sin\theta$, $y = r\cos\theta$).

Now consider a three-sphere embedded in four-dimensional space:

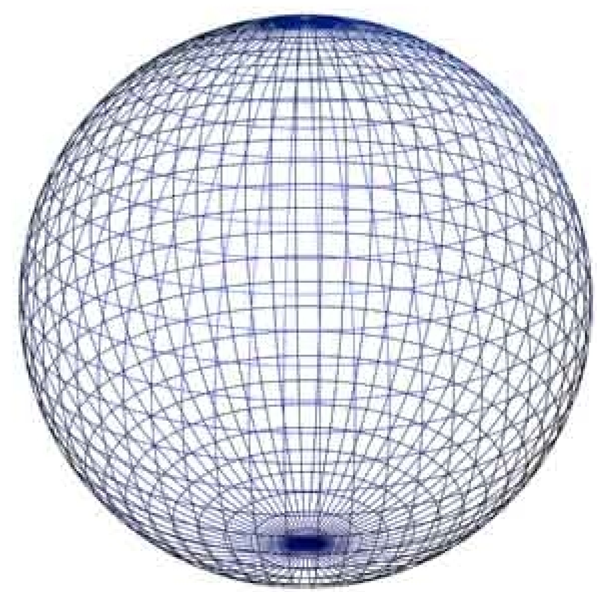
$$ds^2 = dx^2 + dy^2 + dz^2 + dw^2.$$

The three-sphere is defined by

$$x^2 + y^2 + z^2 + w^2 = a^2,$$

Hence,

$$2xdx + 2ydy + 2zdz + 2wdw = 0.$$



So the metric is

$$ds^2 = dx^2 + dy^2 + dz^2 + \frac{(xdx + ydy + zdz)^2}{[a^2 - (x^2 + y^2 + z^2)]}.$$

Now transform to spherical polar coordinates:

$$\begin{aligned} x &= r \sin\theta \sin\phi, \\ y &= r \sin\theta \cos\phi, \\ z &= r \cos\theta, \end{aligned}$$

we get the metric

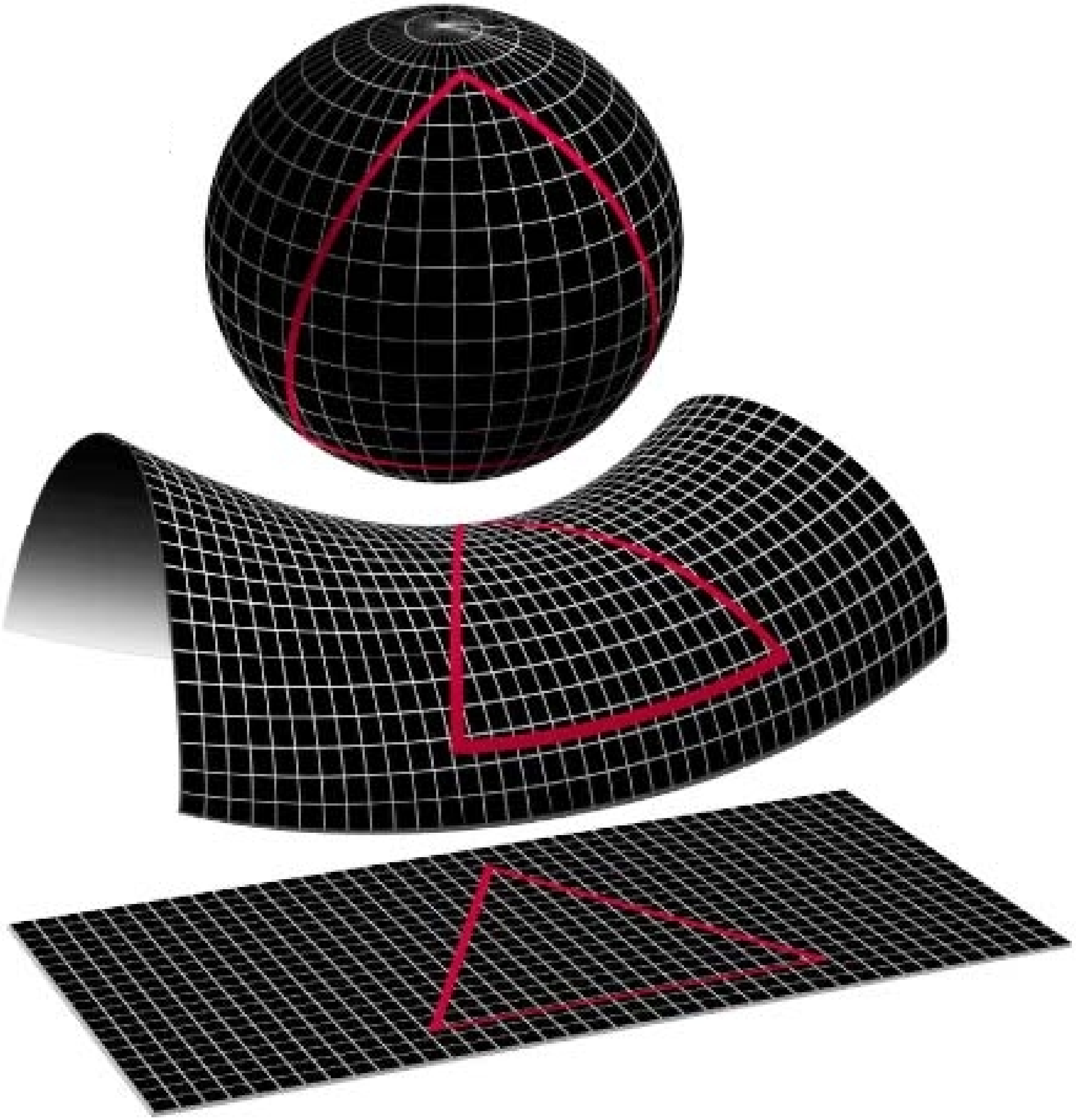
$$ds^2 = \frac{dr^2}{(1 - r^2/a^2)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2.$$

(cf. FRW metric).

$K > 0$

$K < 0$

$K = 0$



The Einstein field equations are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

where (for a perfect fluid) the energy momentum tensor is:

$$T^{\mu\nu} = \left(\rho + \frac{P}{c^2}\right) u^\mu u^\nu - P g^{\mu\nu}.$$

We want to solve these equations for a homogeneous, isotropic, cosmology, for which:

$$\rho(t), \quad P(t), \quad u^\mu = \frac{dx^\mu}{d\tau}, \quad u^0 = u_0 = 1, \quad u^i = u_i = 0.$$

(See the appendix for mathematical details).

Summary:

Friedman equations:

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}.$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

Energy conservation:

$$\frac{d(\rho R^3)}{dR} = -3PR^2.$$

The general character of the solutions for $\Lambda = 0$, $P = 0$, can be deduced by inspection of the Friedman equations. Energy conservation gives $\rho \propto R^{-3}$, so

$$\dot{R}^2 = \frac{8\pi G}{3}\rho_0 \frac{R_0^3}{R} - K$$

where the subscript 0 refers to the value at the present day. In terms of the Hubble constant, $H_0 = \dot{R}_0/R_0$,

$$\boxed{K = \left[\frac{8\pi G}{3}\rho_0 - H_0^2\right]}$$

and

$$\dot{R}^2 = \frac{A}{R} - K, \qquad A = \text{constant.}$$

So:

$$\begin{aligned} K &= 0, & \dot{R} &\to 0 \text{ as } R \to \infty, \\ K &< 0, & \dot{R} &\to \text{constant as } R \to \infty, \\ K &> 0, & \dot{R} &= 0 \text{ at a maximum radius } R_m = A/K. \end{aligned}$$

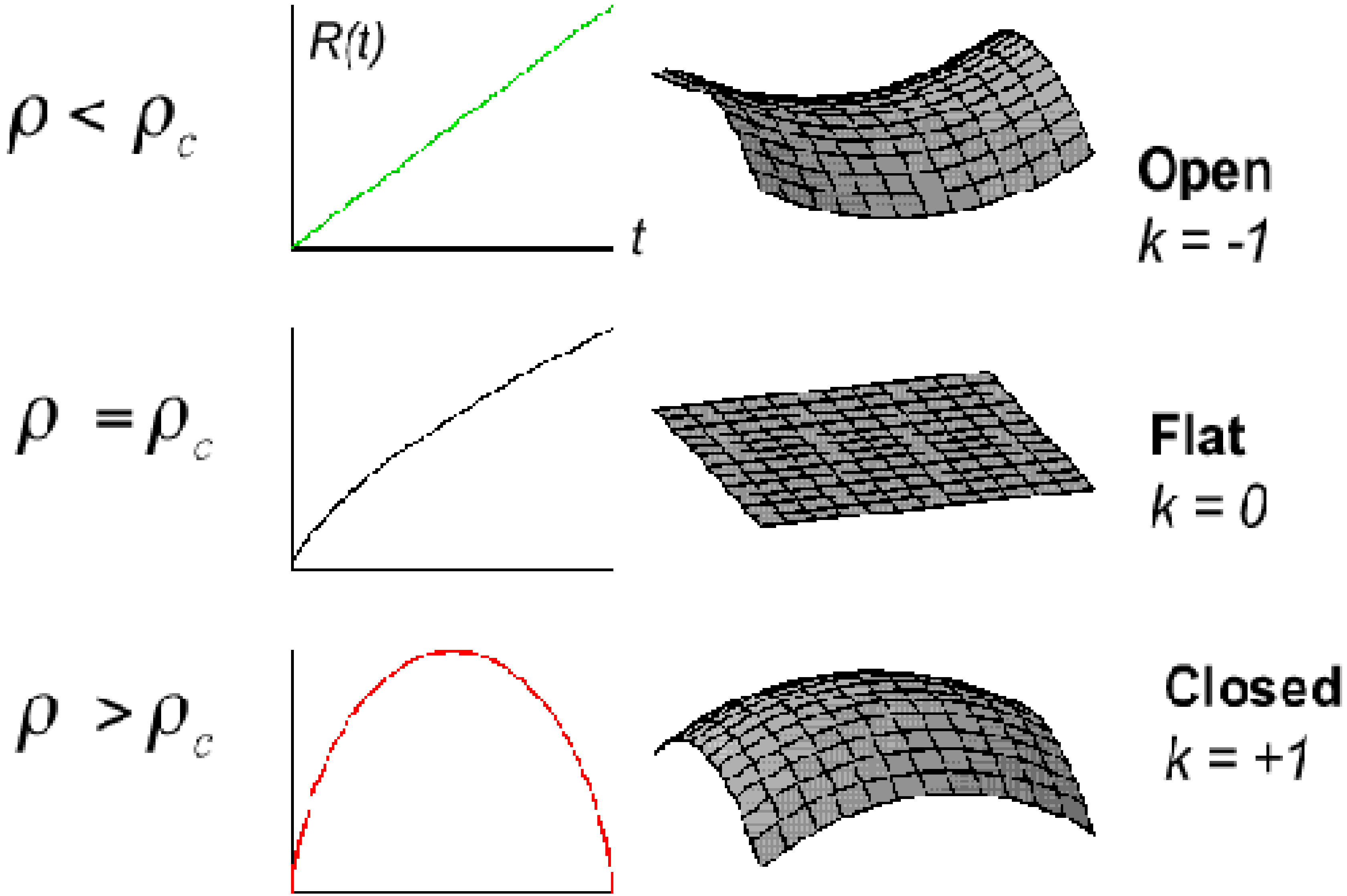
This establishes a link between the *dynamics* of the FRW solution and the *geometry* of the Universe.

$$K = 0 \quad \text{(spatially flat)} \quad \Rightarrow \rho_c = \frac{3H_0^2}{8\pi G}.$$

The density ρ_c is known as the *critical density* and means that we can define a dimensionless density parameter

$$\boxed{\Omega = \rho/\rho_c.}$$

Link between dynamics and geometry



$$\begin{aligned}\Omega &< 1 \Rightarrow K < 0 \quad \text{(negative curvature)},\\ \Omega &= 1 \Rightarrow K = 0 \quad \text{(spatially flat)},\\ \Omega &> 1 \Rightarrow K > 0 \quad \text{(positive curvature)},\end{aligned}$$

Note that since

$$3\frac{\ddot{R}}{R} = -4\pi G(\rho + 3P) + \Lambda$$

a positive *cosmological constant* causes the Universe to accelerate (as does matter with an equation of state $P < -\rho/3$).

Cosmological Redshift

Suppose we have two observers A and B separated by coordinate distance r_{AB}. A emits a light pulse at time $t = t_1$ which reaches B at time t_0. Since light rays are null, $ds = 0$ along the path of a photon. Hence

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_0^{r_{AB}} \frac{dr}{(1 - Kr^2)^{1/2}}.$$

A emits a second pulse at time $t_1 + \delta t_1$ which arrives at B at time $t_0 + \delta t_0$. Hence

$$\int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{R(t)} = \int_0^{r_{AB}} \frac{dr}{(1 - Kr^2)^{1/2}} = \int_{t_1}^{t_0} \frac{dt}{R(t)}.$$

Thus

$$\frac{\delta t_0}{R(t_0)} = \frac{\delta t_1}{R(t_1)},$$

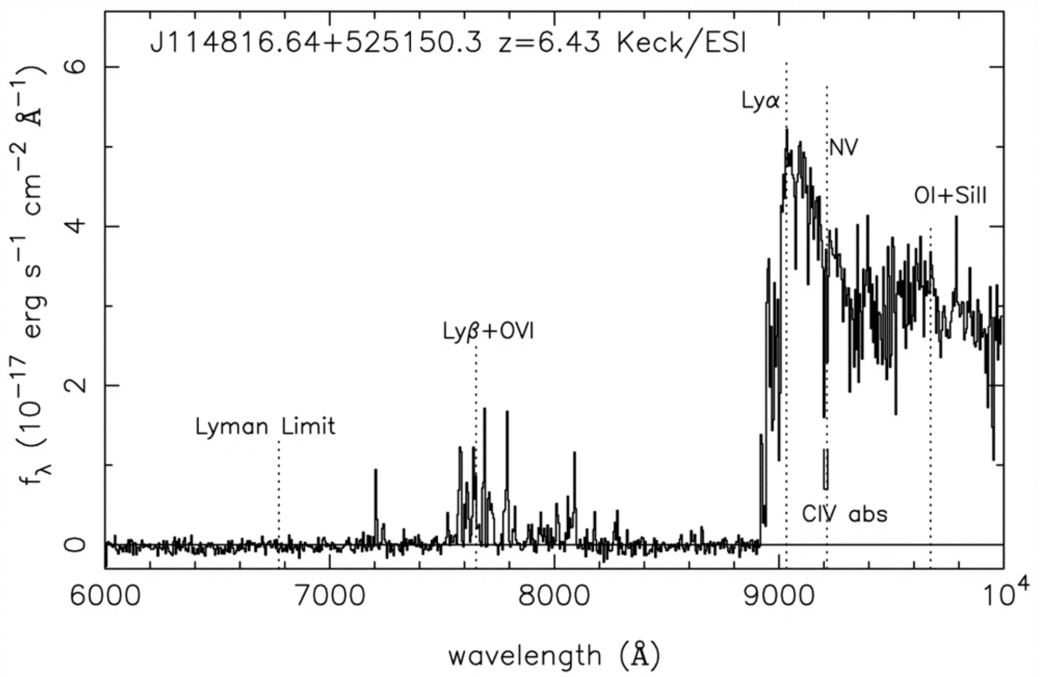
and so light of frequency ν_e at A will be received at frequency ν_0 at B, where

$$\frac{\nu_0}{\nu_e} = \frac{\delta t_1}{\delta t_0} = \frac{R(t_1)}{R(t_0)}.$$

Thus the light received by B is *redshifted*:

$$1 + z = \frac{R(t_0)}{R(t_1)}, \qquad z = \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{\nu_e - \nu_0}{\nu_0}.$$

High-z quasar spectrum →



Blackbody Radiation

$$\rho_\gamma(\nu,t)d\nu = \frac{8\pi h\nu^3 d\nu}{\left[\exp\left(\frac{h\nu}{kT_\gamma}\right) - 1\right]}.$$

Hence

$$\rho_\gamma = aT_\gamma^4, \quad P_\gamma = \frac{1}{3}\rho_\gamma c^2, \quad a = \frac{8\pi^5 k^4}{15h^3c^3}.$$

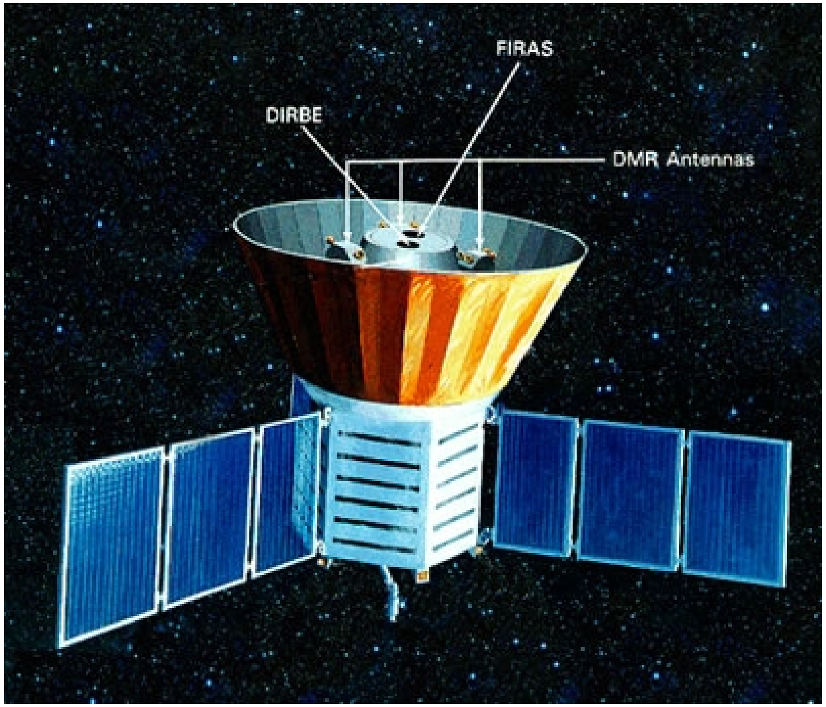
Note that energy conservation

$$\frac{d(\rho R^3)}{dR} = -3PR^2,$$

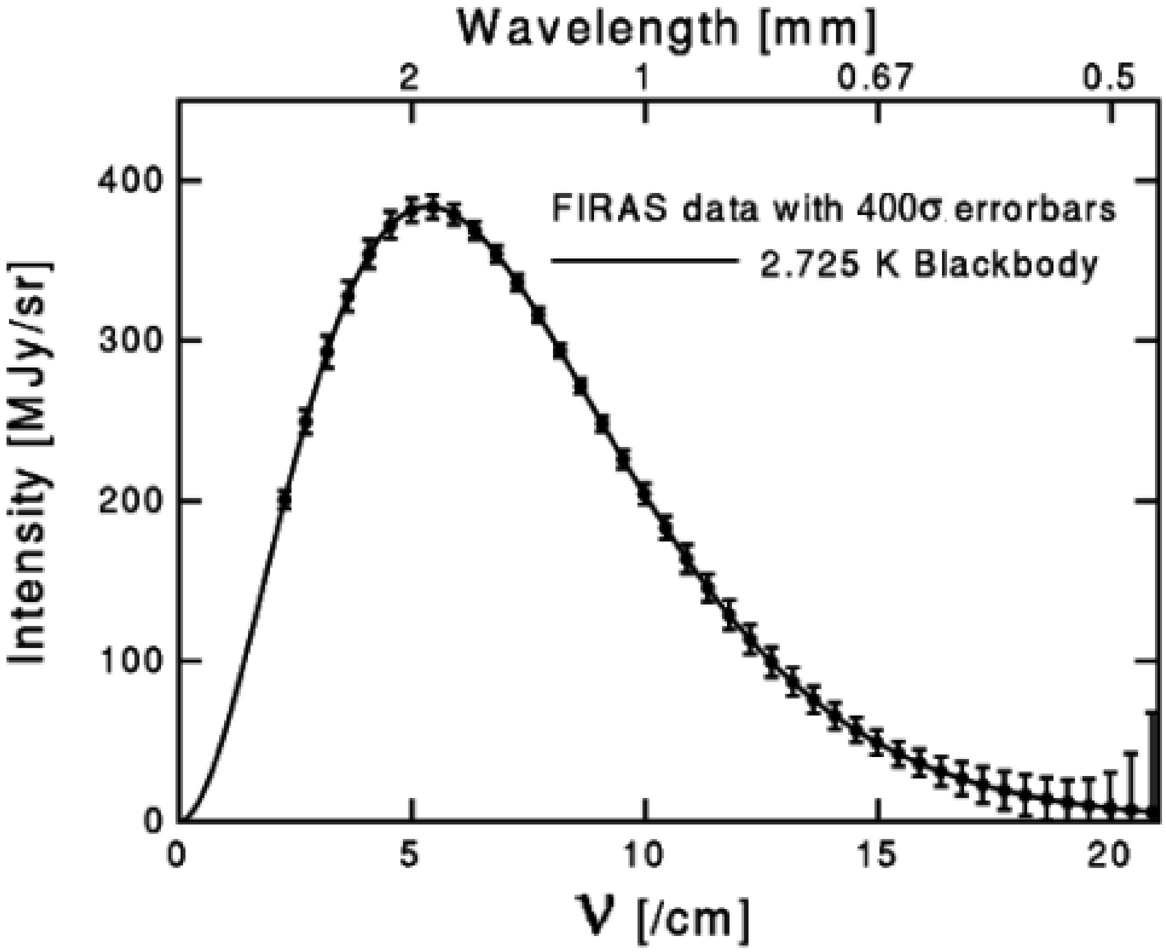
requires requires

$$\rho_\gamma \propto 1/R^4 \Rightarrow T_\gamma \propto 1/R,$$

and since $\nu \propto 1/R$, the blackbody radiation *retains its blackbody shape as the Universe expands* (Liouville's theorem).



COBE satellite



The photon entropy per unit volume is (see next lecture)

$$S = \frac{4aT^3}{3},$$

and the photon 'entropy per baryon' is

$$\frac{S}{kn_b} = \frac{4aT^3}{3kn_b} \sim \frac{n_\gamma}{n_b} \sim 10^8 (\Omega_b h^2)^{-1},$$

- a large dimensionless number that needs explaining. Other useful numbers:

$$\Omega_\gamma(0) = \frac{\rho_\gamma}{\rho_c} = 2.4 \times 10^{-5} h^{-2}.$$

Matter and radiation have equal densities are a redshift:

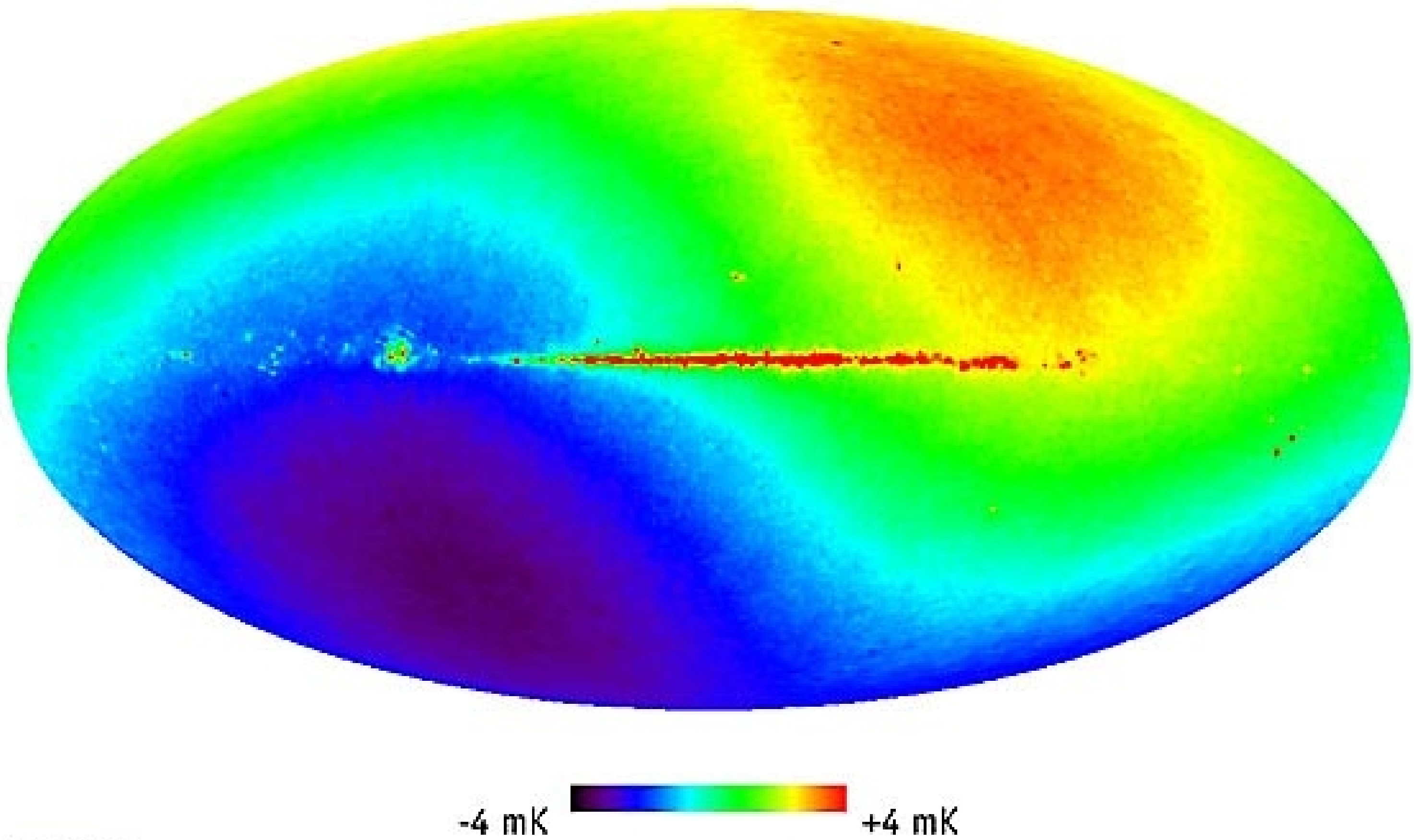
$$(1 + z_{\mathrm{equ}}) = 42,000\ \Omega_m h^2,$$

when the photon temperature was

$$T_{\mathrm{equ}} = 1.1 \times 10^5 (\Omega_m h^2)\ \mathrm{K} \approx 10(\Omega_m h^2)\ \mathrm{eV}.$$

Evidence for FRW Universe

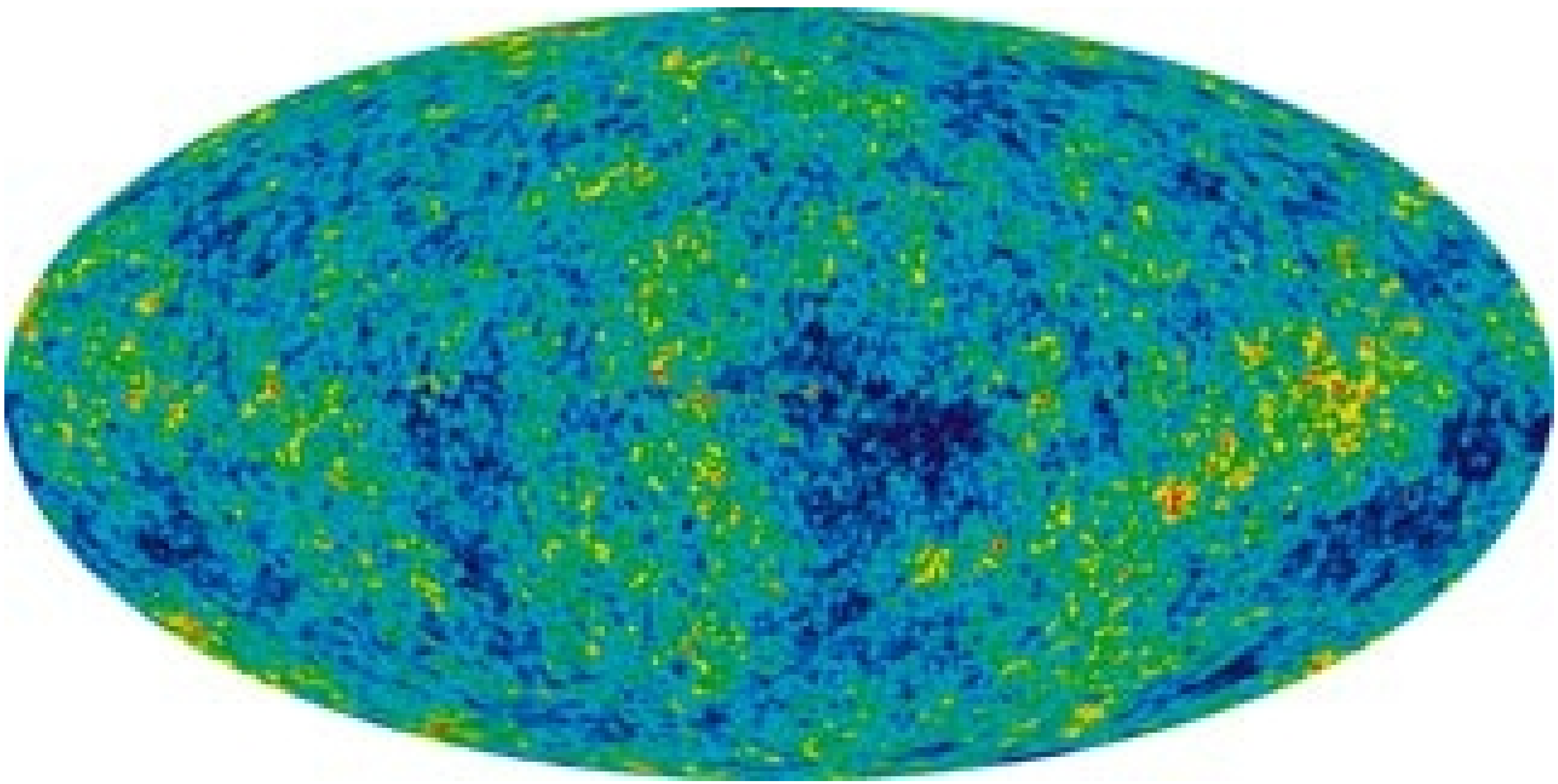
Cosmic Microwave Background Radiation:



Dipole Anisotropy:

$$T' = \frac{T}{\gamma(1 - v\cos\theta')} \approx T(1 + v\cos\theta' + \dots).$$

With dipole and Galactic emission subtracted:



Residual temperature fluctuations of 0.001%.

Large-Scale Structure:

Density perturbation $\delta(x,t)$,

$$\rho(\mathbf{x},t) = \overline{\rho}(1+\delta(\mathbf{x},t)).$$

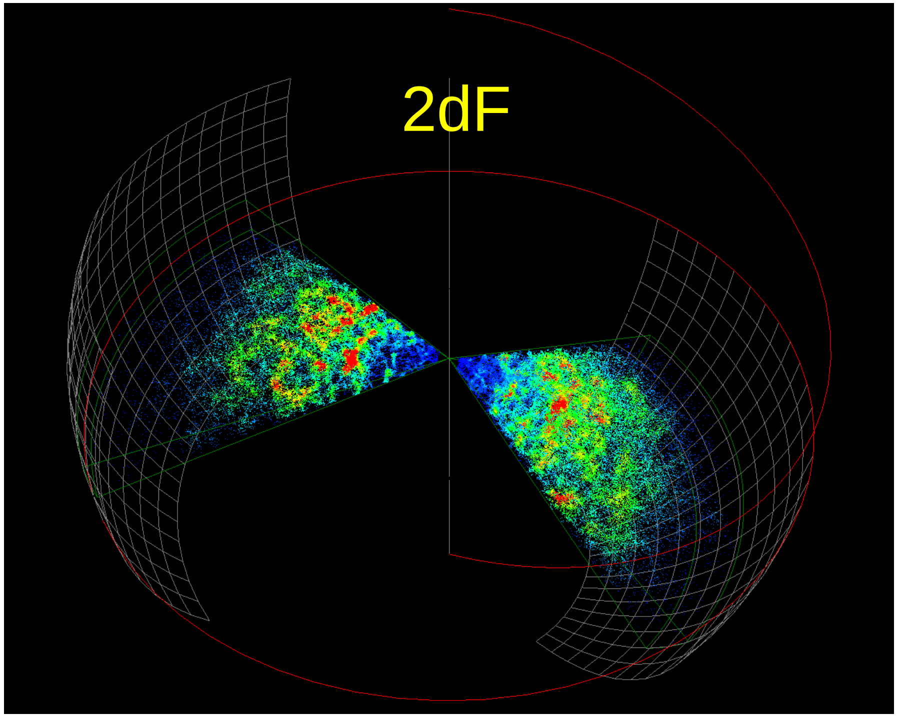
Fourier transform,

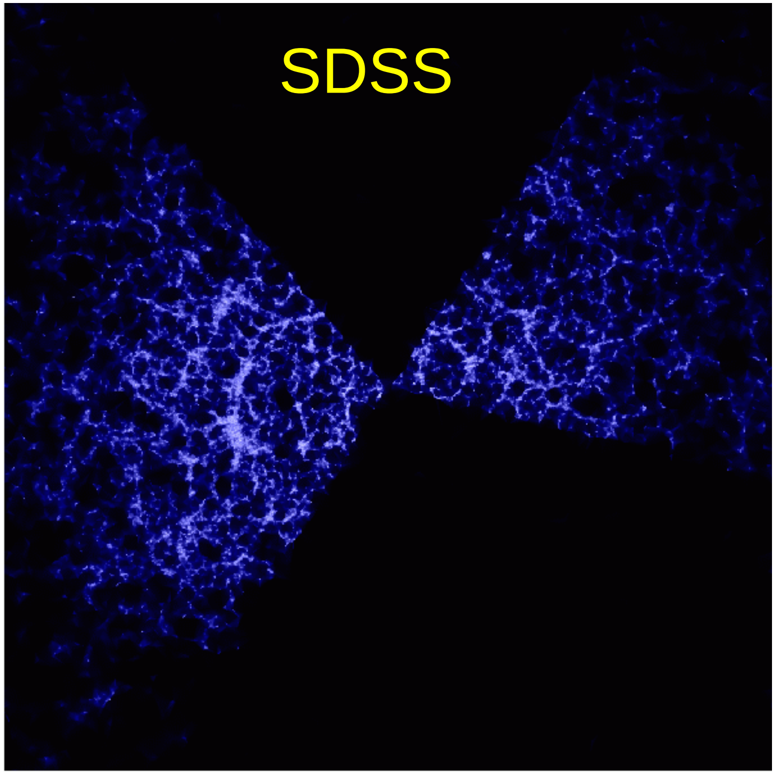
$$\delta_{\mathbf{k}} = \frac{1}{V}\int \delta(\mathbf{x},t)d^3\mathbf{x},$$

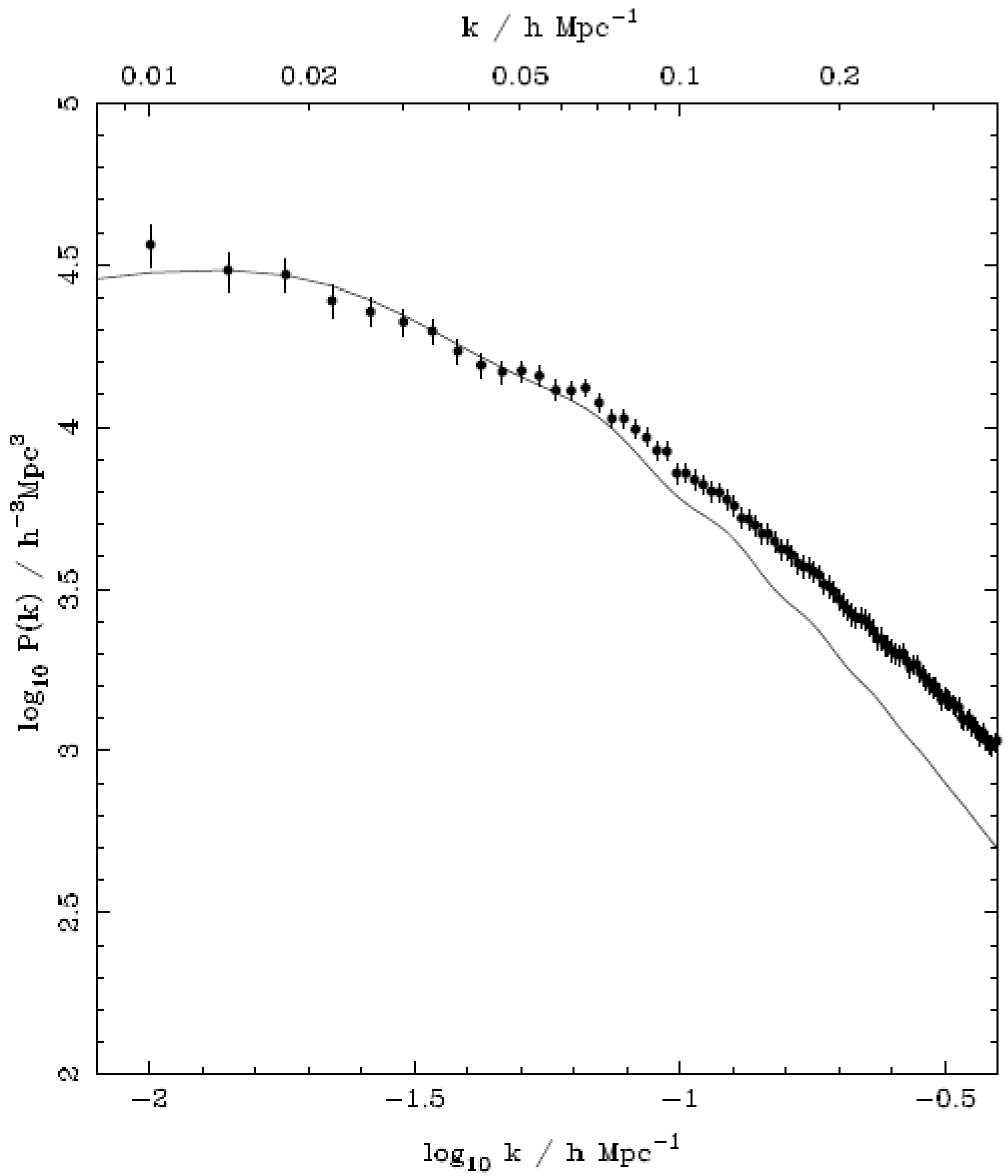
allowing us to form the power spectrum

$$P(\mathbf{k}) = |\delta(\mathbf{k})|^2,$$

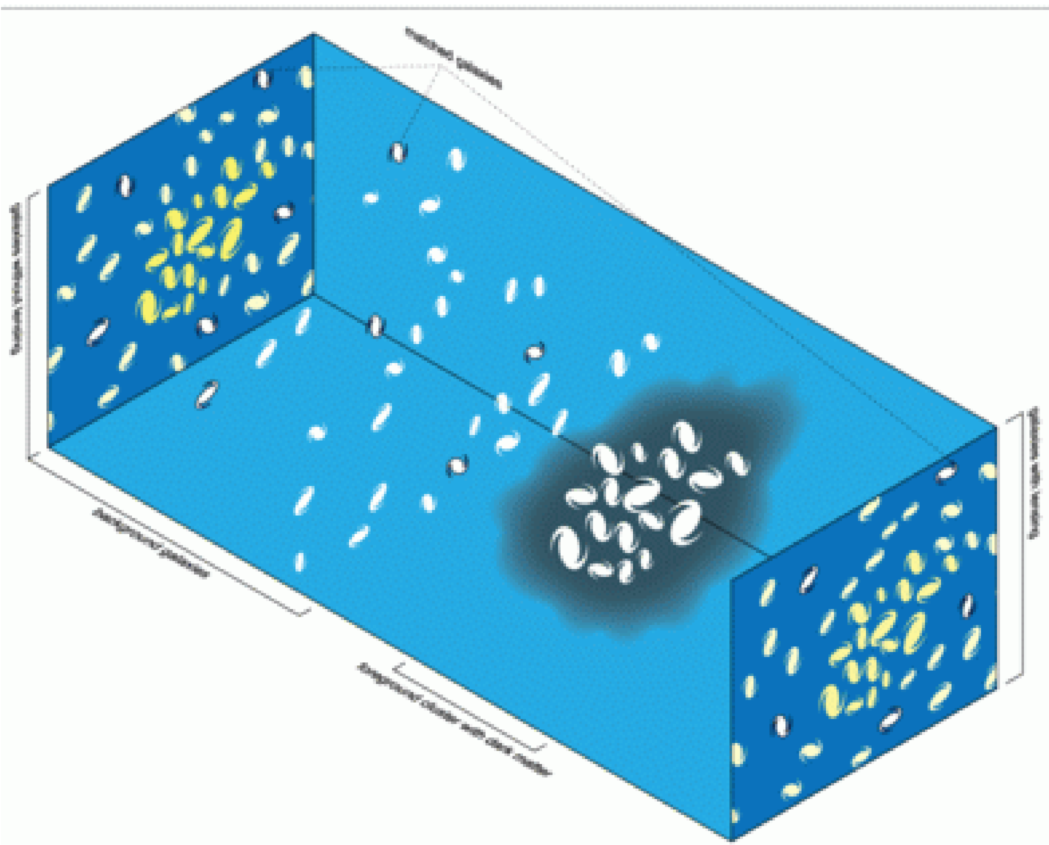
which we can measure from galaxy redshift surveys such as 2dF, SDSS.

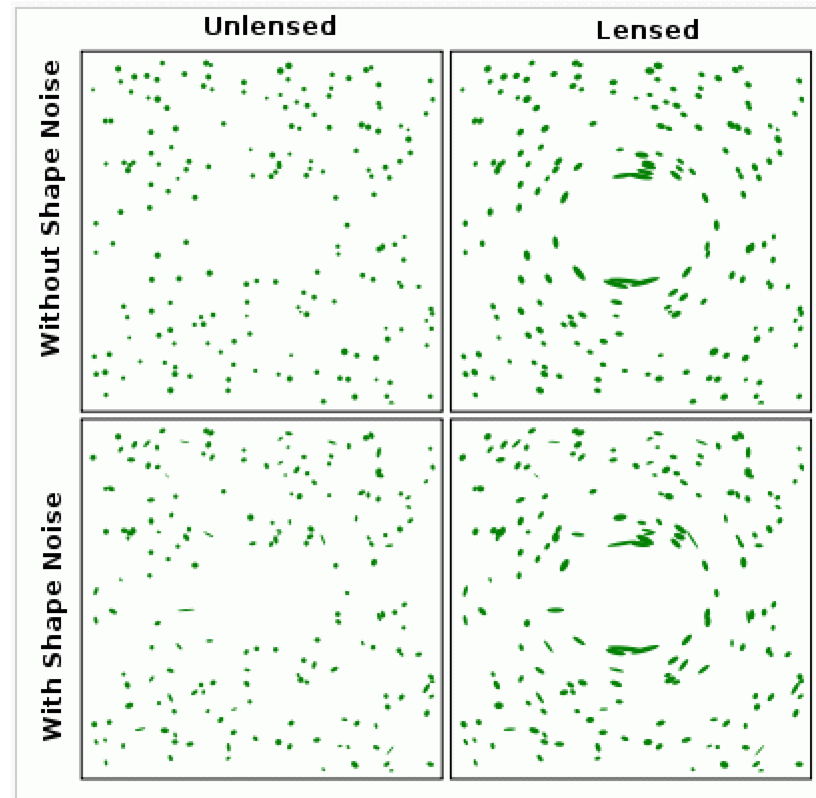

2dF

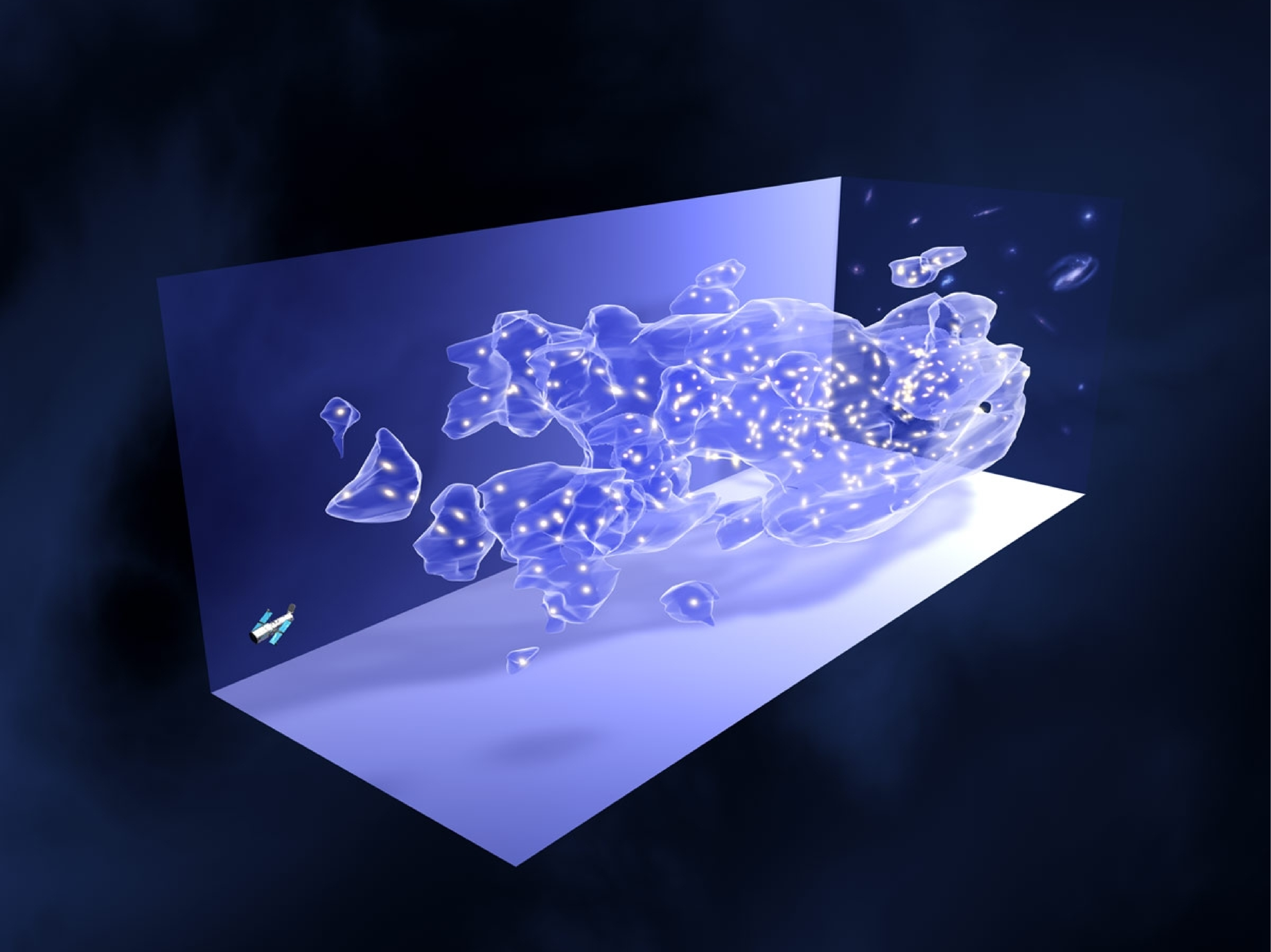

SDSS

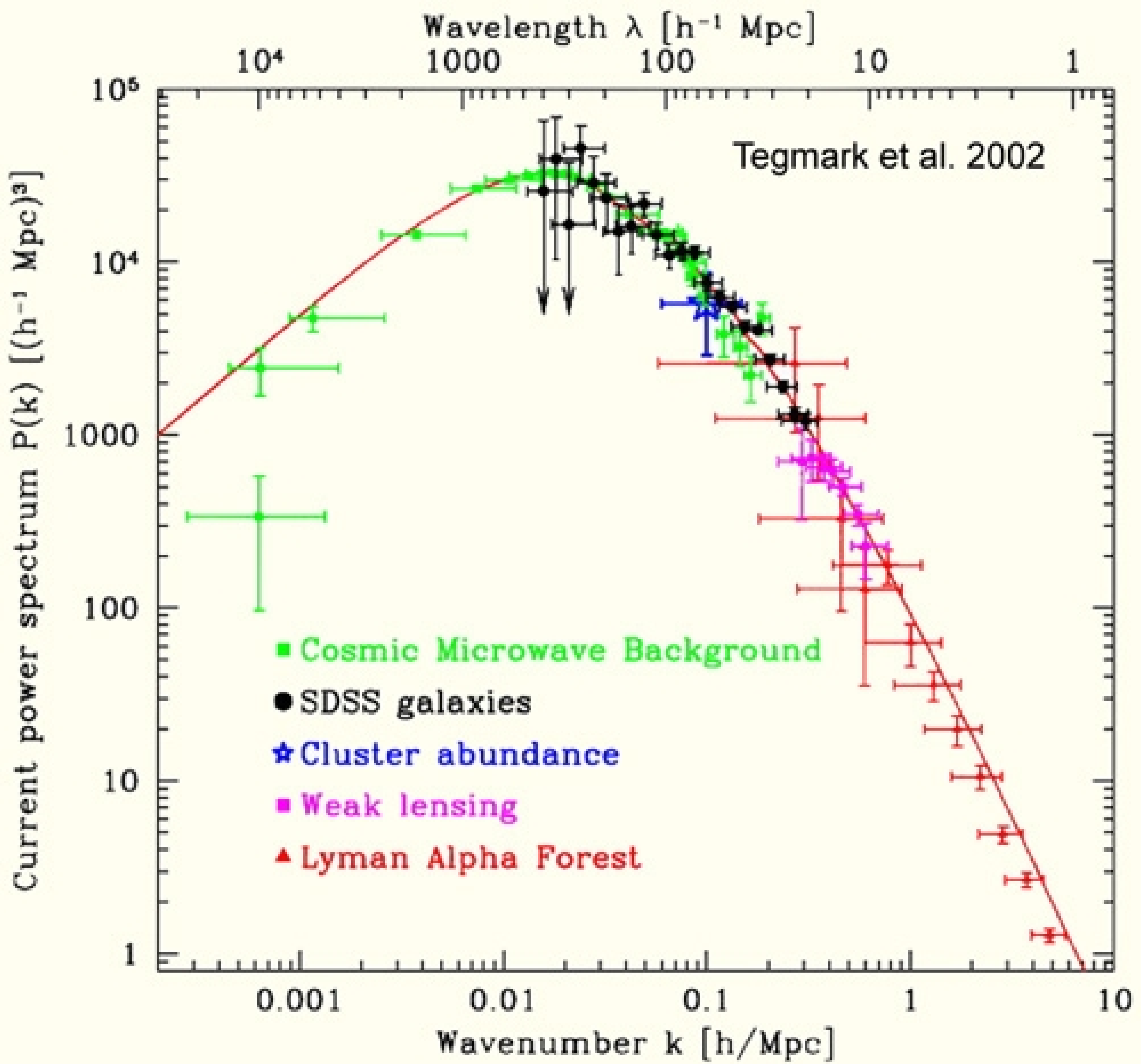

k / h Mpc^{-1}
0.01
0.02
0.05
0.1
0.2
$\log_{10}$ P(k) / h^{-3}Mpc^{3}
5
4.5
4
3.5
3
2.5
2
−2
−1.5
−1
−0.5
$\log_{10}$ k / h Mpc^{-1}

Gravitational Lensing:

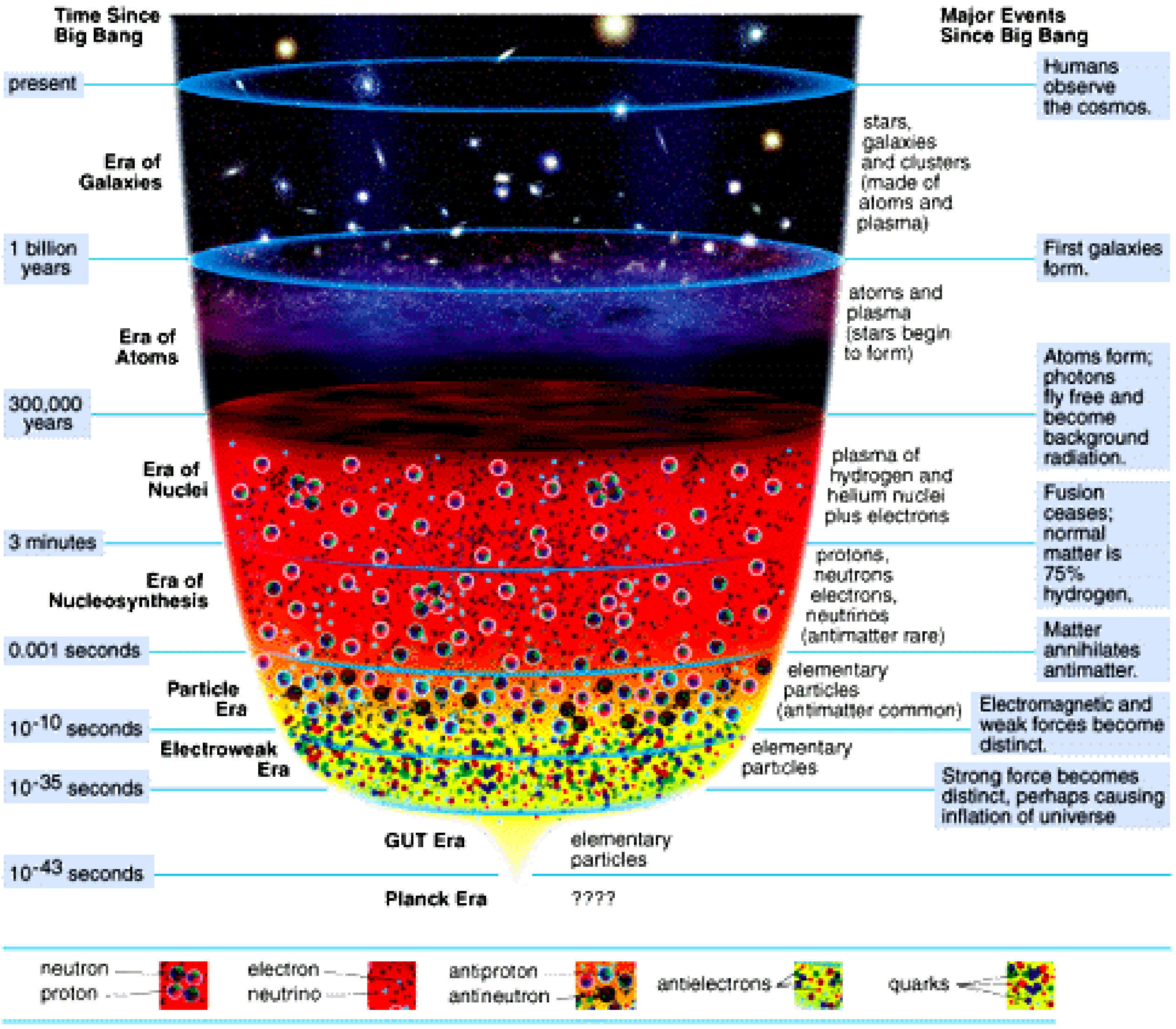








Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
Tegmark et al. 2002
Current power spectrum P(k) [(h⁻¹ Mpc)³]
10⁵
10⁴
1000
100
10
1
Cosmic Microwave Background
SDSS galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest
0.001
0.01
0.1
1
10
Wavenumber k [h/Mpc]



Time Since Big Bang
Major Events Since Big Bang
present
Humans observe the cosmos.
Era of Galaxies
stars, galaxies and clusters (made of atoms and plasma)
1 billion years
First galaxies form.
Era of Atoms
atoms and plasma (stars begin to form)
300,000 years
Atoms form; photons fly free and become background radiation.
Era of Nuclei
plasma of hydrogen and helium nuclei plus electrons
3 minutes
Fusion ceases; normal matter is 75% hydrogen.
Era of Nucleosynthesis
protons, neutrons electrons, neutrinos (antimatter rare)
0.001 seconds
Matter annihilates antimatter.
Particle Era
elementary particles (antimatter common)
10^-10 seconds
Electromagnetic and weak forces become distinct.
Electroweak Era
elementary particles
10^-35 seconds
Strong force becomes distinct, perhaps causing inflation of universe
GUT Era
elementary particles
10^-43 seconds
Planck Era
????
neutron
proton
electron
neutrino
antiproton
antineutron
antielectrons
quarks

Additional Questions

Note that we can rewrite the Friedmann equation

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}\rho_m - \frac{K}{R^2} + \frac{\Lambda}{3}$$

as

$$1 = \Omega_m + \Omega_K + \Omega_\Lambda$$

- Why is the Universe so close to being spatially flat? ($-0.0179 < \Omega_k < 0.0081$, WMAP 95% CL).

- $\Omega_{\text{baryons}} < \Omega_{\text{darkmatter}} < 1$. Why are these densities comparable and close to unity?

- Observations tell us that the Universe is accelerating today. ($\Omega_\Lambda \approx 0.74$). What is the 'dark energy'? Why is it becoming dynamically important only now?

- Photon/baryon ratio is $n_\gamma/n_b \sim 10^{10}$. What fixes this large dimensionless number? Why is there a slight matter-antimatter asymmetry in the early Universe?

Appendix

The FRW metric is diagonal with components:

$$
\begin{aligned}
g_{00} &= 1, & g^{00} &= 1, \\
g_{rr} &= -\frac{R^2}{(1-Kr^2)} = -R^2\tilde{g}_{rr}, & g^{rr} &= -\frac{(1-Kr^2)}{R^2} = -\frac{\tilde{g}^{rr}}{R^2}, \\
g_{\theta\theta} &= -R^2r^2 = -R^2\tilde{g}_{\theta\theta}, & g^{\theta\theta} &= -\frac{r^2}{R^2} = -\frac{\tilde{g}^{\theta\theta}}{R^2}, \\
g_{\phi\phi} &= -R^2r^2\sin^2\theta = -R^2\tilde{g}_{\phi\phi}, & g^{\phi\phi} &= -\frac{1}{R^2r^2\sin^2\theta} = -\frac{\tilde{g}^{\phi\phi}}{R^2}.
\end{aligned}
$$

Notice that I have written the metric as:

$$ds^2 = c^2dt^2 - R^2\tilde{g}_{ij}dx^idx^j.$$

Greek indices run from $0-3$, Latin indices run from $1-3$, and henceforth I will (usually) put $c=1$.

The affine connection is

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\kappa} \left\{ \frac{\partial g_{\kappa\gamma}}{\partial x^{\beta}} + \frac{\partial g_{\beta\kappa}}{\partial x^{\gamma}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\kappa}} \right\}$$

and the only non-zero components are:

$$\begin{aligned}
\Gamma^{0}_{ij} &= \frac{1}{2} g^{00} \left\{ \frac{\partial g_{0j}}{\partial x^{i}} + \frac{\partial g_{i0}}{\partial x^{j}} - \frac{\partial g_{ij}}{\partial x^{0}} \right\} = R\dot{R}\tilde{g}_{ij}, \\
\Gamma^{i}_{j0} &= \frac{1}{2} g^{i\kappa} \frac{\partial g_{j\kappa}}{\partial x^{0}} = \frac{\dot{R}}{R}\delta^{i}_{j}, \\
\Gamma^{i}_{jk} &= \frac{1}{2} g^{il} \left\{ \frac{\partial g_{lk}}{\partial x^{j}} + \frac{\partial g_{jl}}{\partial x^{k}} - \frac{\partial g_{jk}}{\partial x^{l}} \right\} = \tilde{\Gamma}^{i}_{jk},
\end{aligned}$$

where $\tilde{\Gamma}^{i}_{jk}$ is the affine connection of the 3-space.

The Ricci tensor is:

$$R_{\mu\kappa} = \frac{\partial \Gamma^{\lambda}_{\mu\lambda}}{\partial x^{\kappa}} - \frac{\partial \Gamma^{\lambda}_{\mu\kappa}}{\partial x^{\lambda}} + \Gamma^{\eta}_{\mu\lambda}\Gamma^{\lambda}_{\eta\kappa} - \Gamma^{\eta}_{\mu\kappa}\Gamma^{\lambda}_{\eta\lambda}$$

with components:

$$
\begin{aligned}
R_{00} &= \frac{\partial \Gamma^{\lambda}_{0\lambda}}{\partial x^0} - \frac{\partial \Gamma^{\lambda}_{00}}{\partial x^{\lambda}} + \Gamma^{\eta}_{0\lambda}\Gamma^{\lambda}_{\eta 0} - \Gamma^{\eta}_{00}\Gamma^{\lambda}_{\eta\lambda} \\
&= \frac{\partial}{\partial t}\left(3\frac{\dot{R}}{R}\right) + 3\left(\frac{\dot{R}}{R}\right) = 3\frac{\ddot{R}}{R}, \qquad (1) \\
R_{0i} &= 0, \qquad (2) \\
R_{ij} &= \frac{\partial \Gamma^{\lambda}_{i\lambda}}{\partial x^j} - \frac{\partial \Gamma^{\lambda}_{ij}}{\partial x^{\lambda}} + \Gamma^{\eta}_{i\lambda}\Gamma^{\lambda}_{\eta j} - \Gamma^{\eta}_{ij}\Gamma^{\lambda}_{\eta\lambda} \\
&= \frac{\partial \Gamma^{l}_{il}}{\partial x^j} - \frac{\partial \Gamma^{l}_{ij}}{\partial x^{l}} - \frac{\partial \Gamma^{0}_{ij}}{\partial x^{0}} + \Gamma^{0}_{il}\Gamma^{l}_{0j} + \Gamma^{p}_{il}\Gamma^{l}_{pj} + \Gamma^{l}_{i0}\Gamma^{0}_{lj} - \Gamma^{0}_{ij}\Gamma^{l}_{0l} - \Gamma^{p}_{ij}\Gamma^{l}_{pl} \\
&= \tilde{R}_{ij} - \frac{\partial(R\dot{R}\tilde{g}_{ij})}{\partial t} + R\dot{R}\tilde{g}_{il}\frac{\dot{R}}{R}\delta^{l}_{j} - 3\frac{\dot{R}}{R}R\dot{R}\tilde{g}_{ij} + \frac{\dot{R}}{R}\delta^{l}_{i}R\dot{R}\tilde{g}_{lj} \\
&= \tilde{R}_{ij} - (2R^2 + R\ddot{R})\tilde{g}_{ij}. \qquad (3)
\end{aligned}
$$

But, for a maximally symmetric 3-space $\tilde{R}_{ij} = -2K\tilde{g}_{ij}$. Hence, So:

$$S_{00} = \frac{1}{2}(\rho + P) \tag{4}$$

$$S_{0i} = 0 \tag{5}$$

$$S_{ij} = PR^2\tilde{g}_{ij} + \frac{1}{2}\tilde{g}_{ij}(\rho - 3P)R^2 = \frac{1}{2}(\rho - P)R^2\tilde{g}_{ij}. \tag{6}$$

Comparing (1)-(3) with (4)-(6) gives the Friedmann equations:

$$\begin{aligned} 3\frac{\ddot{R}}{R} - \Lambda &= -4\pi G(\rho + 3P), \\ 2K + (2\dot{R}^2 + R\ddot{R}) - \Lambda R^2 &= 4\pi G(\rho - P)R^2. \end{aligned}$$

Eliminating $\ddot{R}$ from the second of these equations gives the more familiar equation:

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}.$$

we can rewrite (3) as

$$R_{ij} = -2K\tilde{g}_{ij} - (2R^2 + R\ddot{R})\tilde{g}_{ij}.$$

Now rewrite the field equations as

$$\boxed{R_{\mu\nu} - \Lambda g_{\mu\nu} = -8\pi G\left(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}T^{\mu}_{\mu}\right).}$$

We therefore need to evaluate the tensor

$$S_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^{\mu}_{\mu},$$

where

$$T_{\mu\nu} = (\rho + P)u_{\mu}u_{\nu} - Pg_{\mu\nu}.$$

hence,

$$T^{\kappa}_{\nu} = (\rho + P)u^{\kappa}u_{\nu} - P\delta^{\kappa}_{\nu},$$
$$T^{\kappa}_{\kappa} = (\rho + P) - 4P = \rho - 3P.$$

To these we must add energy conservation:

$$T^{\mu\nu}_{;\nu} = 0,$$

which gives

$$\frac{d(\rho R^3)}{dR} = -3PR^2.$$