Astroparticle Physics Lent Term 2010

Question sheet 2

1. From Lecture 10.
   Show that the expression given in the lecture for the field in the region of an infinite planar domain wall, $\phi = \frac{\mu}{\lambda}\tanh\left(\frac{x}{\Delta}\right)$, is the solution of the equation of motion of the field, and find Δ. Find an expression for the potential energy associated with the wall as a function of x. Using the first order prediction for the Higgs mass from the Higgs potential, explain how Δ relates to the properties of the Higgs.

2. From Lecture 10.
   Show that the temperature T in a radiation-dominated era is related to time by

$$t = \left(\frac{1.3\ \text{MeVs}^{1/2}}{kT}\right)^2$$

   The equilibrium number density of a particle of mass M is given by

$$n_{eq} = g\left(\frac{MT}{2\pi}\right)^{3/2}\exp\left(-\frac{M}{T}\right)$$

   (in natural units) where g is the number of spin states for the particle.

   Assume that the WIMP annihilation rate Γ is given by

$$\Gamma = n_{eq} < \sigma v >$$

   where $<\sigma v> = G_F^2 M^2/\pi$ (in natural units.
   Plot the expansion time (1/H(t)) and the collision time ($1/\Gamma$) as a function of time.
   Hence find the ratio M/T at freezeout, for a WIMP mass of 500 GeV.

3. From Lecture 12.
   Show that the spinor relations on pages 6 and 7 of Lecture 12 are correct. Show that for m=0, U_L, U_R, V_L and V_R are helicity eigenstates and find the eigenvalues.

   Show that the neutrino mass matrix leads to the physical neutrino masses as stated in the lecture. What would these masses be if the RH mass is related to the Planck scale?