Astroparticle Physics Lent Term 2010

Question sheet 1

1. From Lecture 1.

a) Consider a scalar field ϕ with a potential $V = -\exp(-\alpha^2\phi^2)$ where α is a real constant.

What is the mass of the scalar, and what are its couplings?

b) the Proca Lagrangian for a spin-1 field A is

$$L = -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{8}\left(\frac{mc^2}{\hbar}\right)A^{\nu}A_{\nu}$$

$$\text{where } F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$

Show that, under the local gauge transformation $A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\lambda(x)$, this Lagrangian is only invariant for the case m=0.

What is the significance of this result?

2. From Lecture 1.

The Patti-Salam GUT model suggests that leptons are related to quarks. The leptons have a "white" colour quantum number and so do not feel the strong interaction. The model therefore replaces the SU(3) colour group with SU(4), with red, green, blue and white colour charges. All of the left-handed particle states in each standard model family are in a single multiplet of the form $SU(4)xSU(2)_L$, and all the right-handed states are in the conjugate multiplet $SU(4)xSU(2)_R$.

The left-handed multiplet is

$$\begin{bmatrix} u_r & u_g & u_b & \nu_e \\ d_r & d_g & d_b & e \end{bmatrix}_L$$

where the subscripts r, g, b refer the red, green and blue colour states.

a) Identify the gauge bosons which allow transitions around the multiplet, including those already in the SM and the new ones.
b) For the new gauge bosons (not in the standard model), write down their charge, colour and SU(2) quantum numbers.
c) Consider the right-handed multiplet. Write down the quantum numbers of the neutrino state. What interactions will it have?
d) Does this model allow the proton to decay?

3. From Lecture 4.

The figure below shows data from the NA31 CP-violation experiment at CERN (Physics Letters B, 237(1990) 303) . The main plots show the rate of neutral kaon decays to pairs of pions, as a function of time, measured in units of the short-lived kaon lifetime (0.9 x 10^{-10} s). The inset plot shows the interference between the short- and long-lived states in the region from 4 to 18 lifetimes. The neutral kaon states have a time-dependence of the form

$$\left|K_S^0\right\rangle = \exp i\left(\omega_S t + \frac{i}{2\tau_S} + \phi_S\right) \qquad \omega = mc^2/\hbar \quad \phi = \text{initial phase}$$

$$\left|K_L^0\right\rangle = \exp i\left(\omega_L t + \frac{i}{2\tau_L} + \phi_L\right) \qquad \tau = \text{lifetime}$$

The magnitude of CP-violation in the decay to two pions is given by the ratio of decay rates:

$$\frac{\left\langle \pi\pi \middle| K_L^0 \right\rangle}{\left\langle \pi\pi \middle| K_S^0 \right\rangle} = \eta$$

If the beam is created with an equal number of K_L and K_S, find an expression for the number of kaons present as a function of time. Use the data to extract values of η , the phase difference at production, and the mass difference between the two kaons (note that η<<1) .

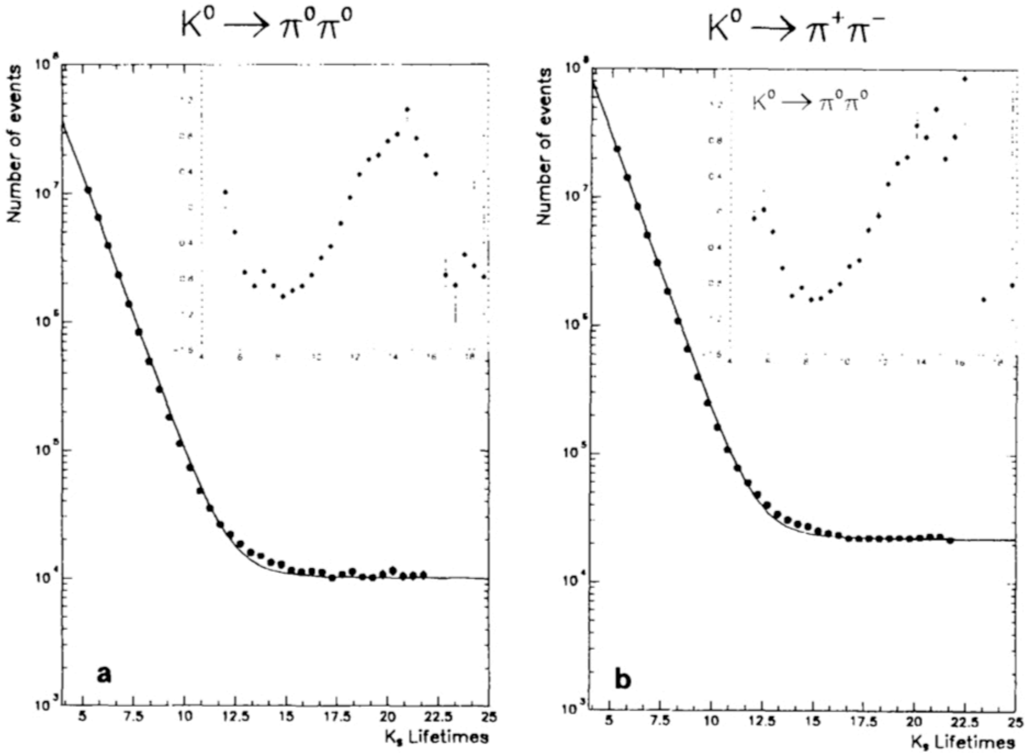

Fig. 5. The rate of decay of kaons to charged and neutral pions as a function of the K_S lifetime. Superimposed are the fitted lifetime distributions with the interference term removed. The insets show the interference terms extracted from the data.

4. From Lecture 6.

a) Show that in a two body decay $m_1 \to m_2 + m_3$, the outgoing momentum, in the rest frame of 1, is given by

$$p \approx E_3 = \frac{m_1^2 - m_2^2 + m_3^2}{2m_1}$$

where the approximation is valid for small m_3.

b) In "gauge mediated" SUSY models, the lightest SUSY particle is a gravitino, which is light, normally with a mass in the keV range. It is produced with a photon in a two-body decay of the neutralino. Consider the process

$$\tilde{\chi}_2^0 \to \tilde{\ell}^{\pm}\ell^{\mp} \to \tilde{\chi}_1^0 \ell^+ \ell^- \to \tilde{G}\ell^+\ell^-\gamma \quad .$$

Show that the invariant mass of the lepton pair cannot exceed

$$m_{\ell\ell}^{\max} = m(\tilde{\chi}_2^0)\sqrt{1 - \frac{m^2(\tilde{\ell})}{m^2(\tilde{\chi}_2^0)}}\sqrt{1 - \frac{m^2(\tilde{\chi}_1^0)}{m^2(\tilde{\ell})}}$$

(neglecting the masses of the leptons and gravitino).

c) Show that the invariant mass of the lepton pair with the photon cannot exceed

$$m_{\ell\ell\gamma}^{\max} = \sqrt{m^2(\tilde{\chi}_2^0) - m^2(\tilde{\chi}_1^0)}$$

d) Show that there are two edges in the invariant mass distribution of the lepton-gamma pairs, given by

$$m_{\ell\gamma(1)} = \sqrt{m^2(\tilde{\ell}) - m^2(\tilde{\chi}_1^0)}$$
$$m_{\ell\gamma(2)} = \sqrt{m^2(\tilde{\chi}_2^0) - m^2(\tilde{\ell})}$$

e) An experiment measures these edges at the following positions:

$$m_{\ell\ell}^{\max} = 105.1 \text{ GeV} \qquad m_{\ell\ell\gamma}^{\max} = 189.7 \text{ GeV}$$
$$m_{\ell\gamma(1)} = 112.7 \text{ GeV} \qquad m_{\ell\gamma(2)} = 152.6 \text{ GeV}$$

Extract the masses of the SUSY particles.