Modified Gravity

MOND

- Suggestion that gravity is not Newtonian at large scales/small accelerations
- Remove need for dark matter by changing dynamics of rotating galaxies and clusters
- Constant galaxy rotation curves obtained from most prominent MOND model

Modified Newtonian Dynamics

- MOND suggests that for very small accelerations

$$F = ma \text{ tends to } F = m\left(\frac{a}{a_0}\right)a$$

$$\text{with } a_0 \approx 10^{-10} \text{ ms}^{-2}$$

$$\frac{GMm}{r^2} = m\frac{a^2}{a_0}$$

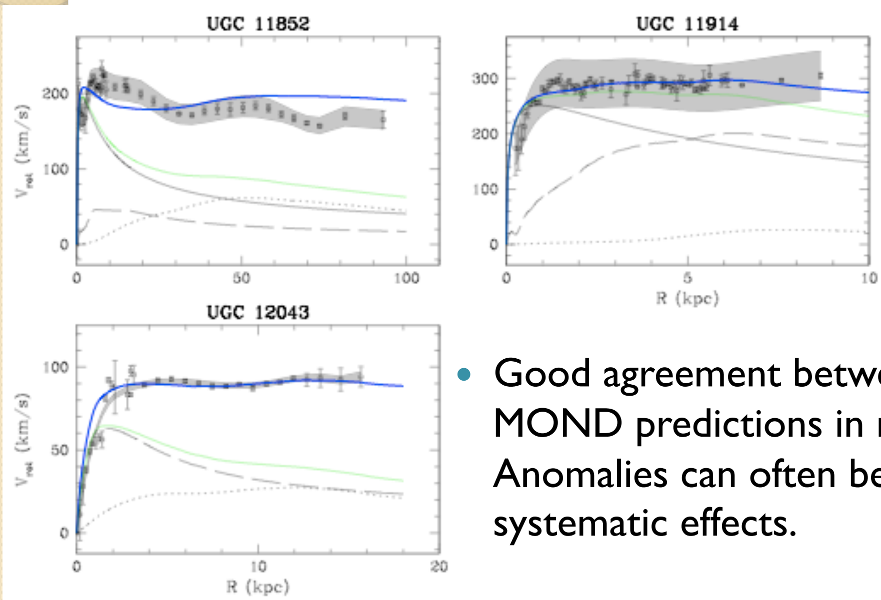
$$a = \frac{v^2}{r} = \frac{\sqrt{GMa_0}}{r}$$

This gives constant v for rotation curve for galaxies at large distances.

Astrophysical Journal 270: 365–370

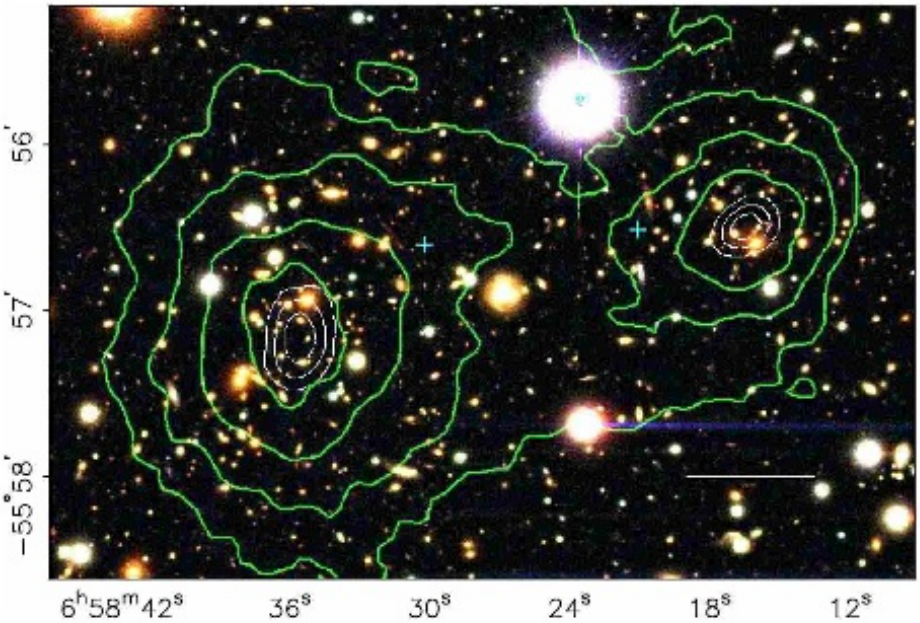
MOND rotation curves

Use observed mass distribution from visible matter and a_0 to predict rotation curves.



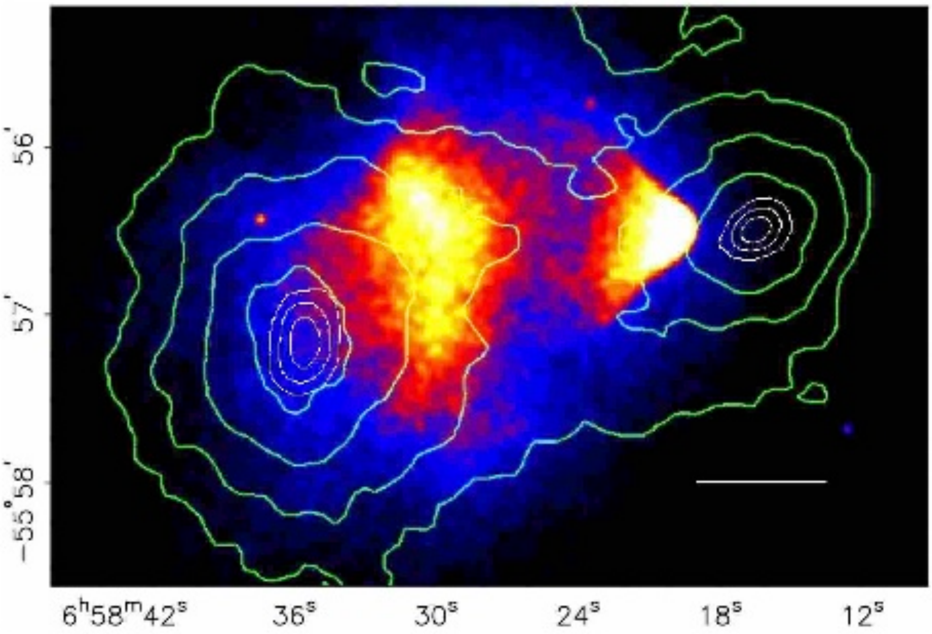
- Good agreement between data and MOND predictions in many cases. Anomalies can often be explained by systematic effects.

Bullet cluster galaxies and lensing



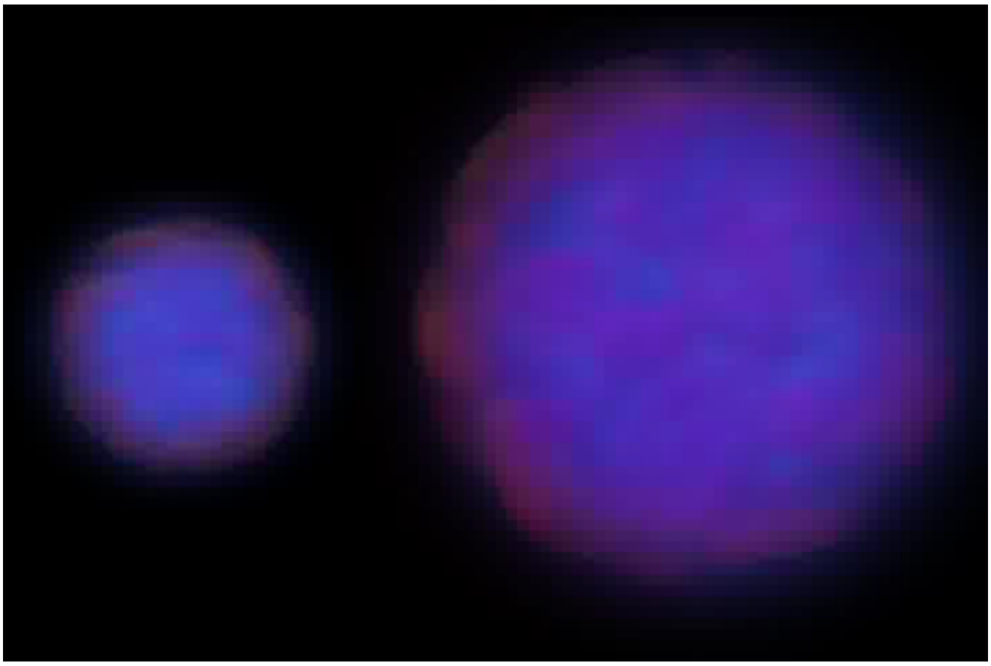
Lensing data shows well separated potential wells

Bullet cluster X-ray data



X-ray data shows position of plasma is well separated from gravitational wells: Markevitch et al. (2004) and Clowe et al. (2004)

Dark matter and plasma separation



Simple DM model can reproduce data: MOND struggles?

MOND vs DM

- MOND works well for galaxy rotation curves, using simple model tied to observed mass, but less well for data at cluster scales.
- MOND proponents agree that some undetected matter is needed in large clusters,
- ΛCDM predicts high concentration of DM at galaxy centres, which is not observed (the cuspy halo problem)
- DM has more parameters than MOND
- Of the 4% baryonic matter in ΛCDM cosmology, most has not been directly detected. Could standard matter could explain the apparent dark component at cluster scales eg Black holes, dwarf stars, heavy neutrinos?
- Consensus is against MOND models, but proponents have not given up. Ultimately must discover dark matter directly to prove that it is the correct explanation.

Recent review: arXiv:1001.3876v1

TeVeS gravity

- MOND cannot be full answer – not consistent with relativity (or even momentum conservation).
- Tensor-vector-scalar gravity attempts to make a fully relativistic MOND theory.
- Adds vector and scalar components to standard tensor gravitational field.
- Attempts to deal with issues like the CMB fluctuations
- Not fully worked out, and not generally accepted, but considered by some to be no less ugly than dark matter and dark energy.

Recent review: arXiv:1001.3876v1

Supersymmetry vs Extra Dimensions

Conventional method to fix Higgs mass in a Quantum Field Theory approach:

Invoke SUSY

Double the number of states in model

Invoke SUSY breaking

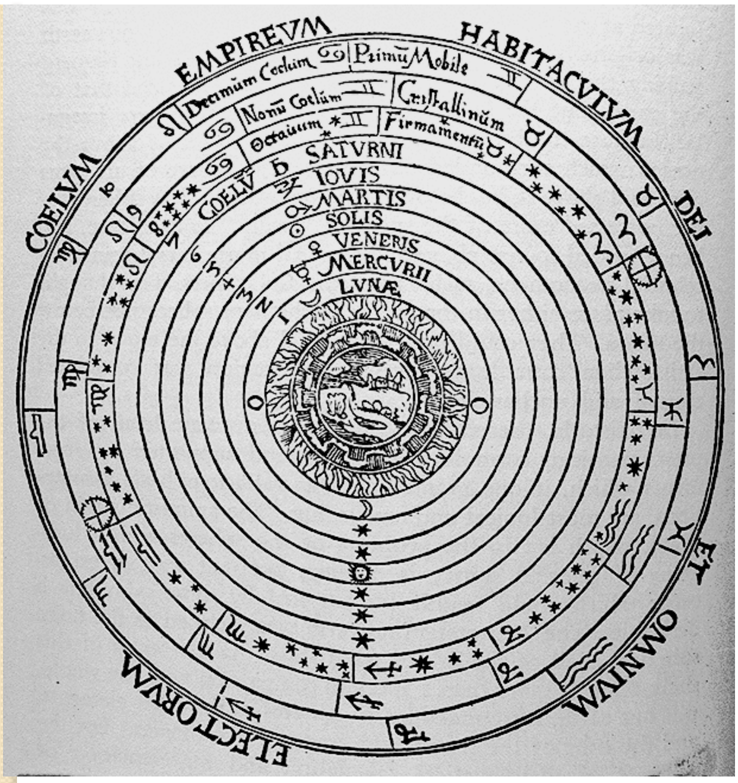
Fermion/boson loops cancel (GIM)

Higgs mass stabilised!

105 new parameters

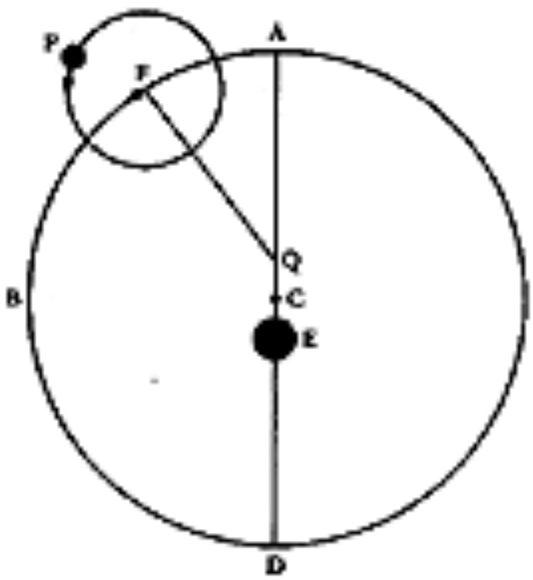
+48 more free parameters if R_P not conserved

=> SUSY is a good pension plan for experimentalists!



Supersymmetry....hidden perfection

Epicycles



From Michael J. Crowe, Theories of the World from Antiquity to the Copernican Revolution.

Typical Ptolemaic planetary model

Symmetry is assumed: all orbits are based on circles - but symmetry is broken

The Earth (E) is not at the centre of the circle C (*the eccentric*)

The planet P moves on an *epicycle F*

The epicycle moves around the *equant Q*

Extra Dimensions

Hypothesize that there are extra space dimensions

Volume of bulk space >> volume of 3-D space

Hypothesize that gravity operates throughout the bulk

Then get "diluted" gravity, as seen in 3-D

If we choose n-D gravity scale=weak scale then...

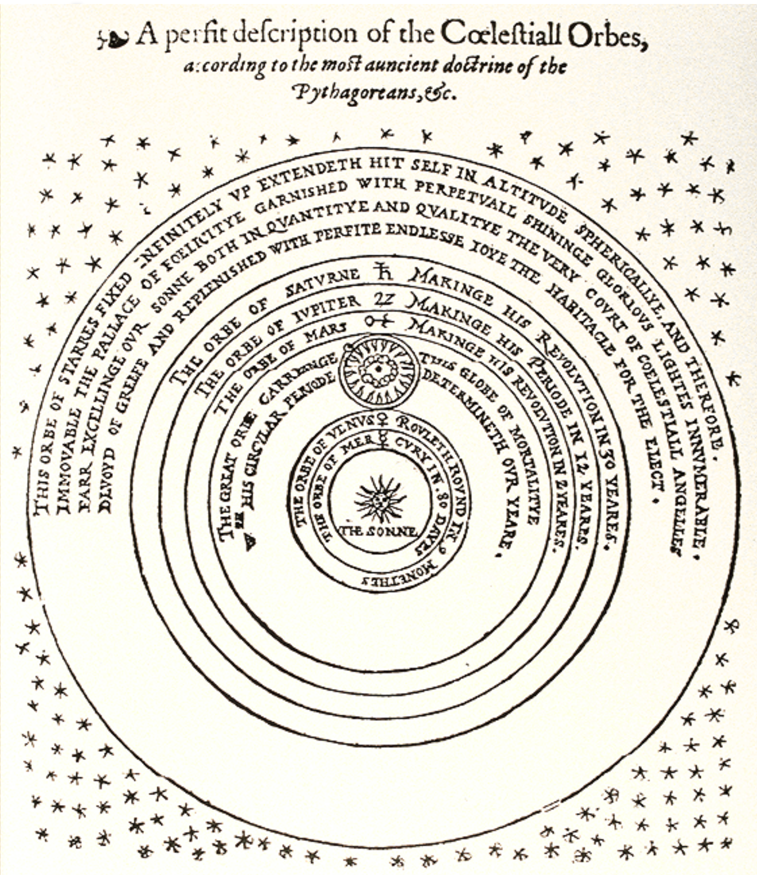
Only one scale -> no hierarchy problem!

Can experimentally access quantum gravity!

But extra dimension is different scale from "normal" ones

-> new scale to explain

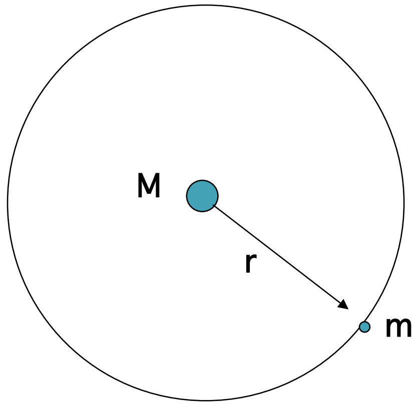
Extra dimensions are more of a lottery bet than a pension plan!



Extra dimensions....

different scales

Gravity in 3-D space



$$(F/m)\,4\pi r^2 = 4\pi GM$$

$$\boxed{F = GMm/r^2}$$

Note that gravity is weak because of the low coupling of the field to the mass.

Gauss's theorem:

Field at r given by

$$\oint \vec{F}/m\, d\vec{S} = 4\pi GM$$

The Planck Mass

Planck mass is point at which gravity becomes strong:

Consider two masses M_{PL} separated by their reduced Compton wavelength

$$\lambda\!\!\!^{-} = \frac{\hbar}{M_{PL}c}$$

Set their PE equal to their rest mass:

$$M_{PL}c^2 = \frac{GM_{PL}^2}{\lambda\!\!\!^{-}} = \frac{GM_{PL}^3 c}{\hbar}$$

$$M_{PL} = \sqrt{\frac{\hbar c}{G}}$$

Define $\overline{M}_{PL} = \frac{M_{PL}}{\sqrt{8\pi}}$

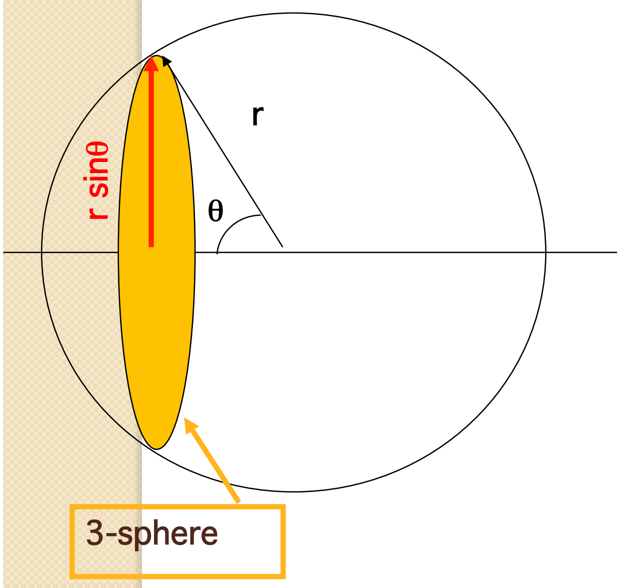
$G=6.67\times10^{-11}\ m^3\,kg^{-1}\,s^{-2}$

$M_{PL}=2.2\times10^{-8}$ kg = 1.2×10^{19} GeV/c^2

$\overline{M}_{PL}=2.4\times10^{18}$ GeV/c^2

Gravity in 4-D space

4-sphere



Compute volume of 4-sphere

$$V_4(r) = \int_0^{\pi} V_3(r\sin\theta)\, r\sin\theta\, d\theta$$

$$= \int_0^{\pi} \tfrac{4\pi}{3} r^4 \sin^4\theta\, d\theta$$

$$= \tfrac{1}{2}\pi^2 r^4$$

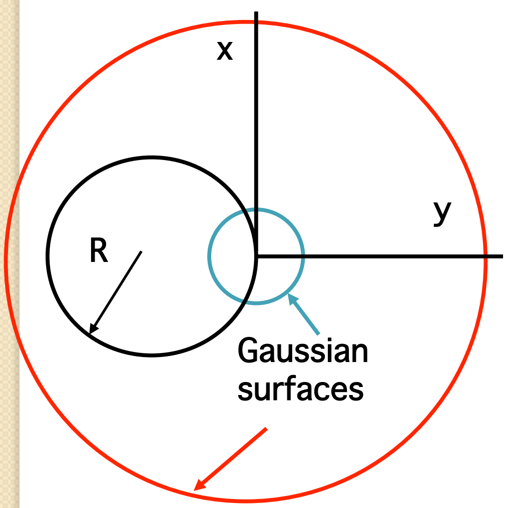
$$S_4 = \frac{d}{dr} V_4 = 2\pi^2 r^3$$

$$(F/m) S_4 = 4\pi G_4 M$$

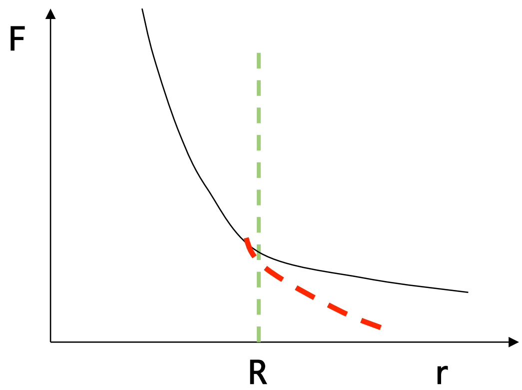
$$F = \frac{2G_4 Mm}{\pi r^3}$$

Get surface of 4-sphere and hence force law

No extra dimensions have been observed, so hypothesize that they are curled up to a radius R.



r > R Get 3-D result
r < R Get 4-D result



We can relate the gravitational coupling in 3D to that in 4D:

$$F = \frac{2G_4 Mm}{\pi r^3}$$

Separation can't exceed R in ED

$$V(r) = \frac{G_4 Mm}{\pi r^2} \qquad r < R$$

$$V(r) = \frac{G_4 Mm}{\pi R r} \qquad r > R$$

$$\frac{GMm}{r} = \frac{G_4 Mm}{\pi R r}$$

$$G = \frac{G_4}{\pi R}$$

$$\overline{M}_{Pl}^2 = M_4^3 R$$

Generalise to n extra dimensions

$$\overline{M}_{Pl}^2 = R^n M_*^{(2+n)}$$

Use this as Defn of M*

where M_* is the bulk Planck mass in n+4 dimensions, and R is the radius of the extra dimensions.

Now solve the hierarchy problem:

Set $M_*=M_{weak}$ - this gives gravity the same strength (in the bulk) as the other forces.

Try to get the correct strength of gravity in 3D

If n=1 need

$$R = \frac{M_{Pl}^2}{M_{Weak}^3} = \frac{(10^{18})^2}{(10^3)^3} = 10^{27}\ \text{GeV}^{-1}$$

$R=10^{27}$ x 0.2 GeV fm = $2\text{x}10^{26}$ fm = $2\text{x}10^{11}$m

This is much too large: deviations from Newtonian gravity would be observed in the solar system.

Try n=2:

$$R^2 = \frac{M_{Pl}^2}{M_{Weak}^4} = \frac{\left(10^{18}\right)^2}{\left(10^3\right)^4} = 10^{24}\ \text{GeV}^{-2}$$

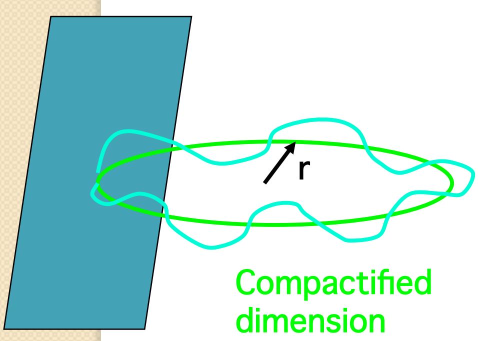
$$R = 10^{12}\ \text{GeV}^{-1} = 2 \times 10^{-4}\ m$$

R is now 200 microns. This is very large on a quantum scale, but small on the scale used to test gravity in experiments – close to current limit of 100 microns

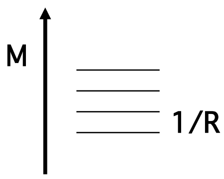
Conclude that we can set the bulk Planck scale equal to the electroweak scale in any model with n ≥ 2, and obtain a size for the extra dimensions compatible with current data on gravity.

Kaluza Klein modes

4-D brane



Compactified dimension



Tower of Kaluza-Klein modes

$$p = \hbar / \lambda, \qquad \hbar c = 0.2\ \text{GeVfm}$$

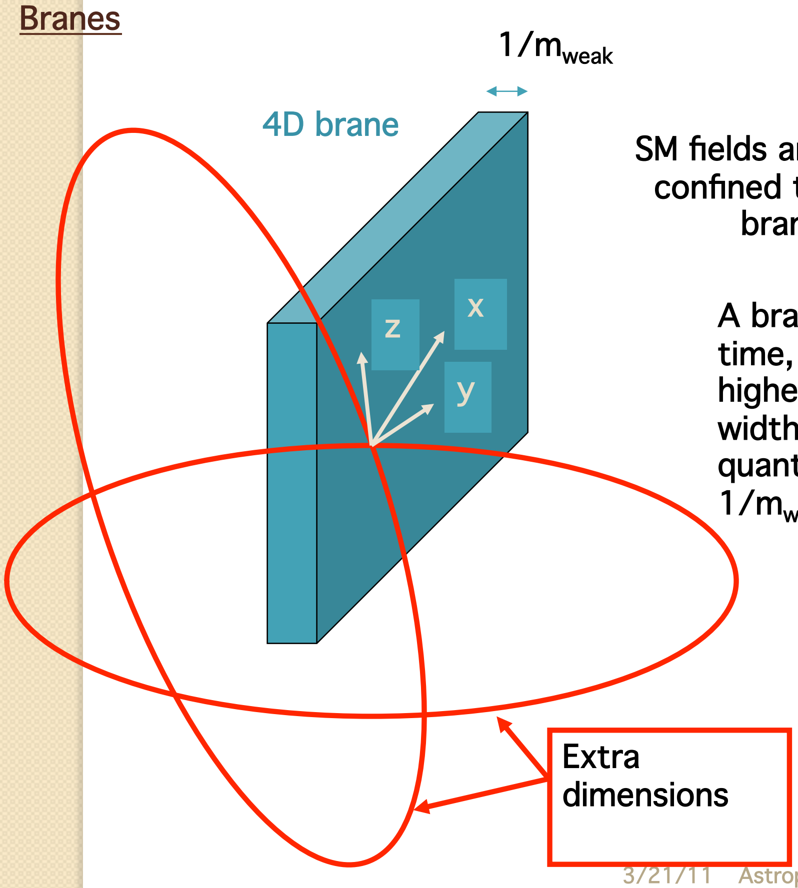
$$\lambda = 1\ \text{mm}, \qquad p = 0.2/10^{12} = 2.10^{-13}\ \text{GeV}$$

Particles in compact extra dimension: wavelength set by periodic boundary condition. States will be evenly spaced in mass.

Spacing depends on scale of ED

- For large ED (order of mm) spacing is very small - use density of states
- For small ED, spacing can be very large.

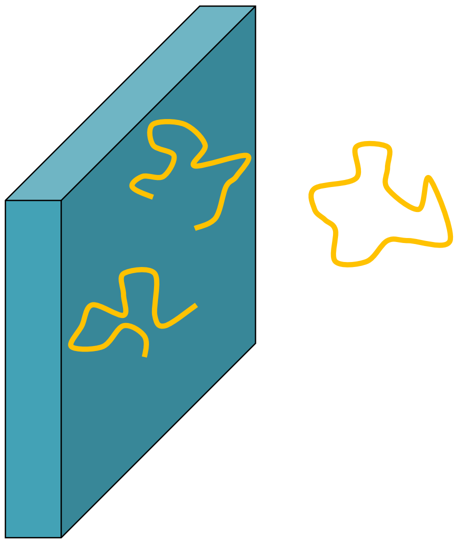
Branes



SM fields are confined to brane

A brane is a slab of 4D space time, embedded in a "bulk" higher dimensional space. The width of the brane is given by quantum fluctations of order $1/m_{weak}$

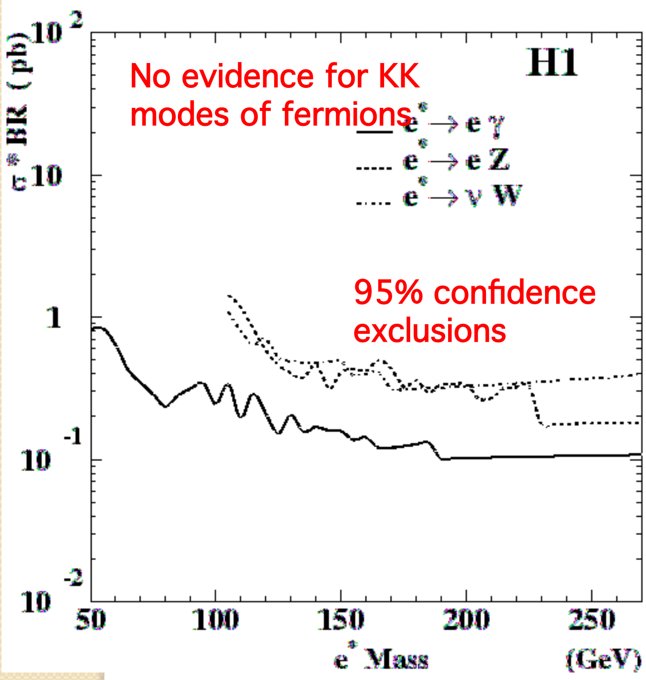
Standard Model fields on branes



Gravitons are closed loops, and so can propagate in the bulk.

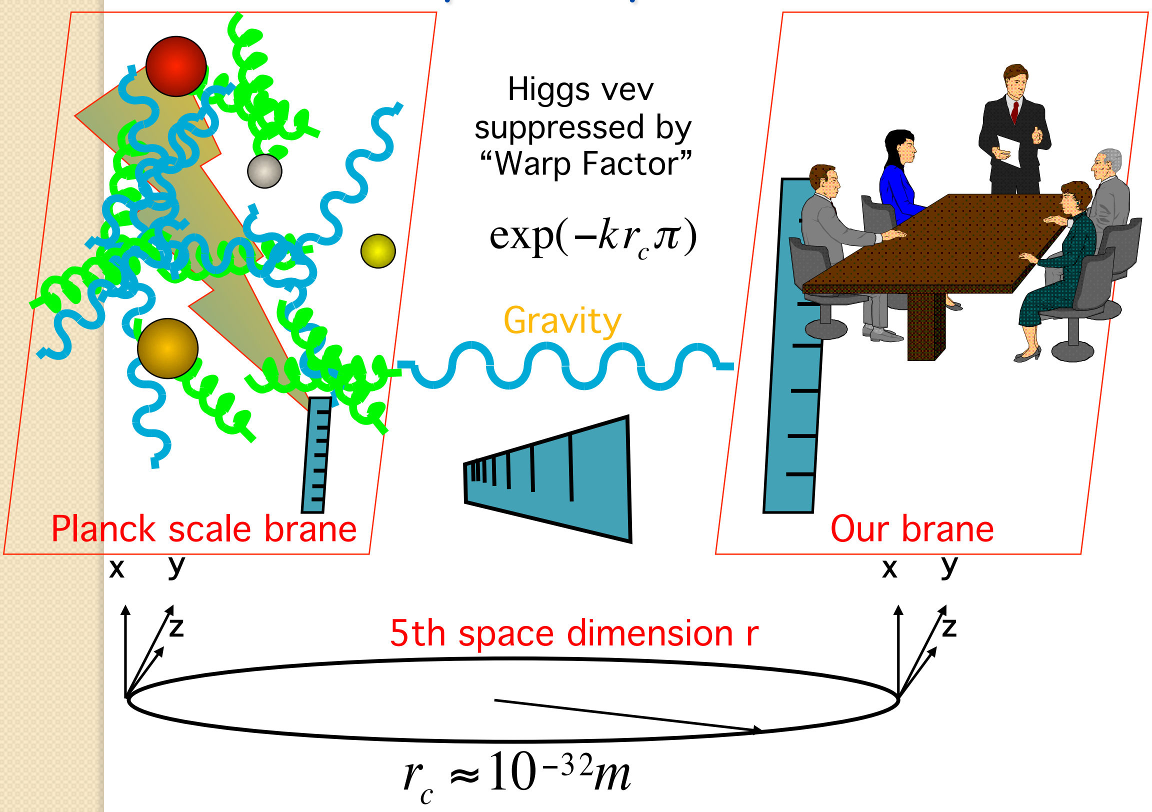
In superstring theories, SM fields (fermions and bosons) are open ended strings. The brane can be defined as the surface on which the strings end. Its width is given by the wavelength of the fields.

Hence SM fields are confined to the brane (at least in this class of string theory).



Interactions of SM fields measured to very high precision at scales of 10^{-18} m
If gauge forces acted in bulk, deviations would have been measured
KK modes would exist for SM particles
For large ED, mass splitting would be small.

Warped 5-d spacetime



Warped Extra dimensions

Consider Randall and Sundrum type models as test case
Gravity propagates in a 5-D non-factorizable geometry
Hierarchy between M_{Planck} and M_{Weak} generated by "warp factor"
Need $kr_c \approx 10$: no fine tuning

Gravitons have KK excitations with scale

$$\Lambda_\pi = \overline{M}_{Pl} \exp(-kr_c\pi)$$

This gives a spectrum of graviton excitations which can be detected as resonances at colliders.

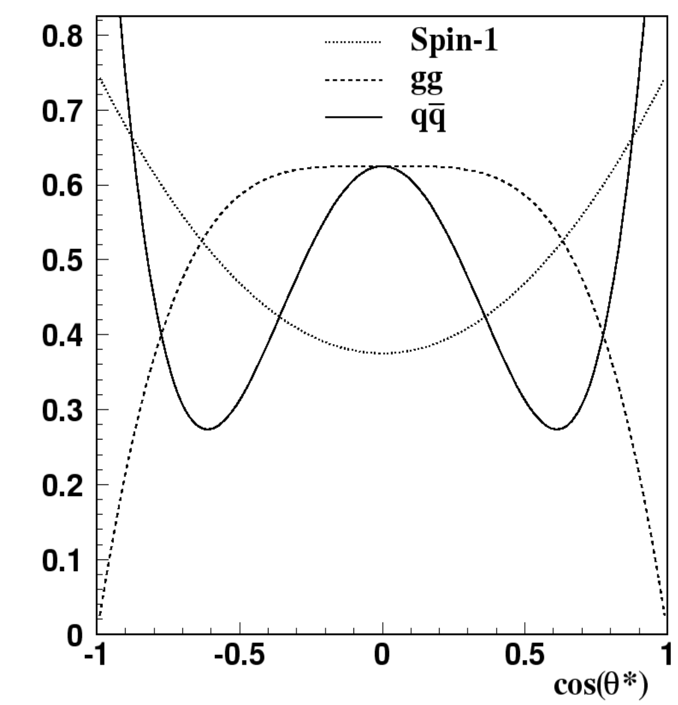
First excitation is a$m_1 = kx_1 \exp(-kr_c\pi) = 3.83 \frac{k}{\overline{M}_{Pl}} \Lambda_\pi$

where $0.01 \le \frac{k}{\overline{M}_{Pl}} \le 1$

Analysis is model independent: this model used for illustration

Allanach et al, JHEP 0009:019, 2000/0212:039, 2002

Angular distributions of e^+e^- in graviton frame

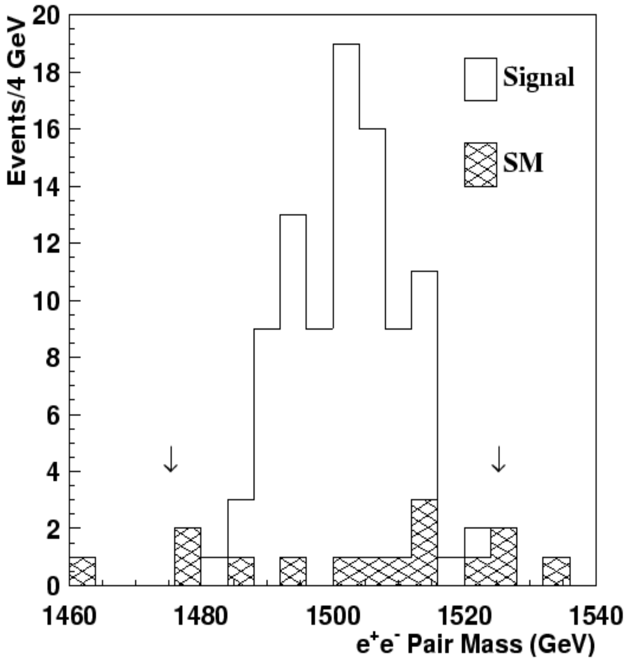


- Angular distributions are very different depending on the spin of the resonance and the production mechanism.

- =>get information on the spin and couplings of the resonance

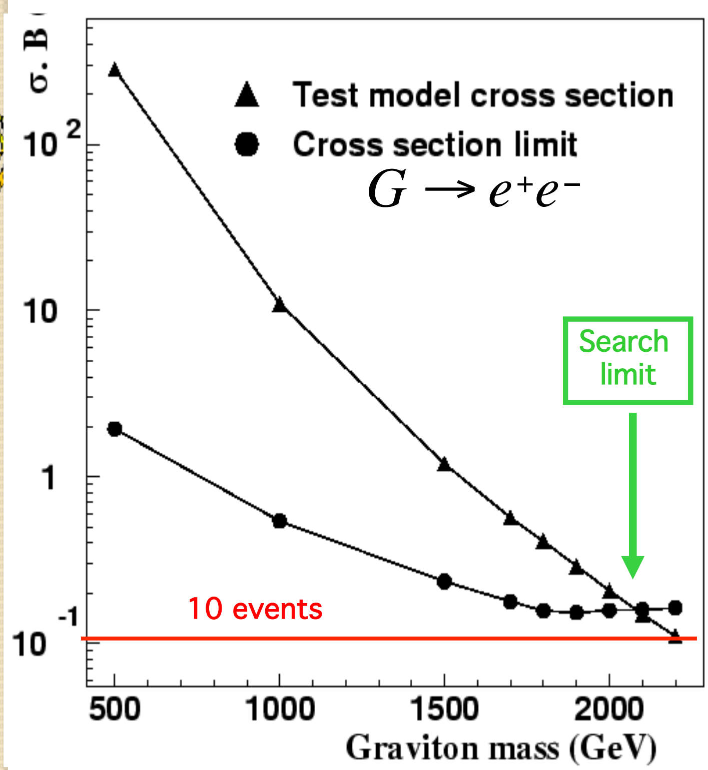
Graviton Resonance

$$G \rightarrow e^+e^-$$



- Graviton resonance is very prominent above small SM background, for $100fb^{-1}$ of integrated luminosity
- Plot shows signal for a 1.5 TeV resonance, in the test model.
- The Drell-Yan background can be measured and subtracted from the sidebands.
- Detector acceptance and efficiency included.

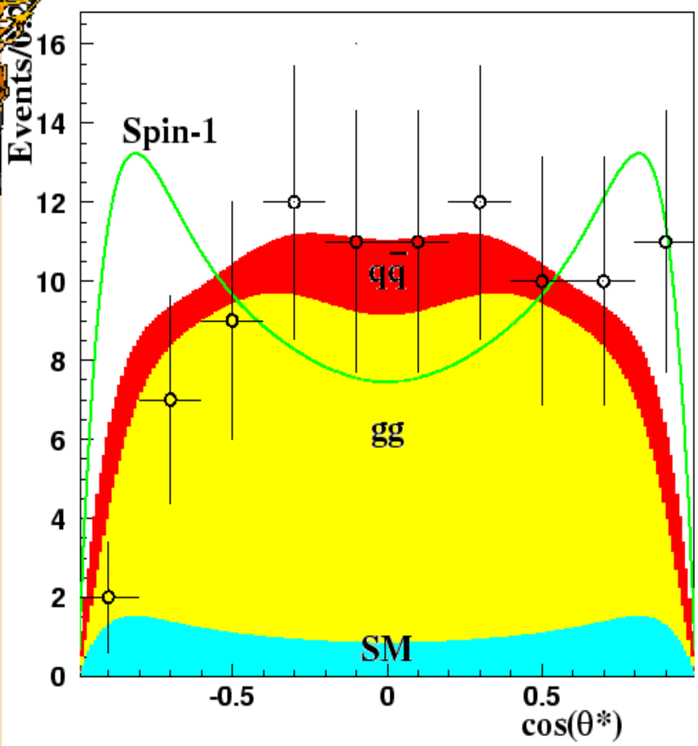
Production Cross Section



10 events produced for $100fb^{-1}$ at m_G=2.2 TeV.

With detector acceptance and efficiency, search limit is at 2080 GeV, for a signal of 10 events and $S/\sqrt{B}>5$

Angular distribution observed in ATLAS



$$G \rightarrow e^+e^-$$

- 1.5 TeV resonance mass
- Production dominantly from gluon fusion
- Statistics for $100 fb^{-1}$ of integrated luminosity (1 year at high luminosity)
- Acceptance removes events at high cos θ^*

Exploring the extra dimension

- Check that the coupling of the resonance is universal: measure rate in as many channels as possible: μμ,γγ,jj,bb,tt,WW,ZZ
- Use information from angular distribution to separate gg and qq couplings
- Estimate model parameters k and r_c from resonance mass and σ.B
- For example, in test model with M_G=1.5 TeV, get mass to ±1 GeV
- and σ.B to 14% from ee channel alone (dominated by statistics).
- Then measure

$$k = (2.43 \pm 0.17) \times 10^{16}\ GeV$$

$$r_c = (8.2 \pm 0.6) \times 10^{-32} m$$