Supersymmetry at the LHC

(with review of gauge theories)

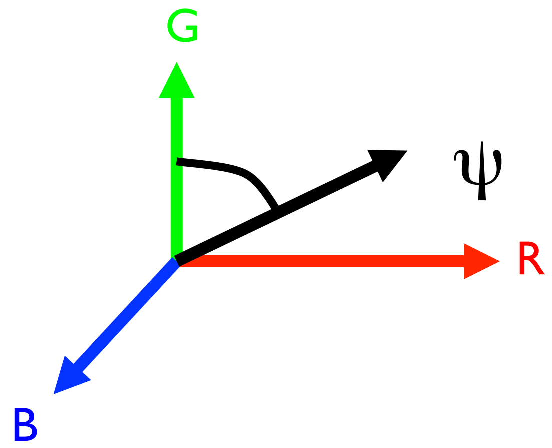
Gauge theory recipe

1. Write down free particle Lagrangian for particles (scalar, fermion, vector boson....) to include.
2. Choose a gauge group
3. Impose invariance under group transformations
4. Read off the new fields and interactions needed
5. Note conserved quantum numbers
6. Write down Feynman rules.

Gauge theories are guaranteed to be renormalisable, so all calculations will give finite results.

QCD

1. QCD applies to quarks: Dirac particles
2. Each quark comes in one of 3 colour states (R, G, B). Rotations in RGB space give group SU(3)
3. Require that Lagrangian is invariant under SU(3)



4. There are 8 possible combinations of R,G,B without net colour -SU(3) generators- 8 massless coloured gluons carry around changes in colour.
5. Colour quantum number must be conserved.

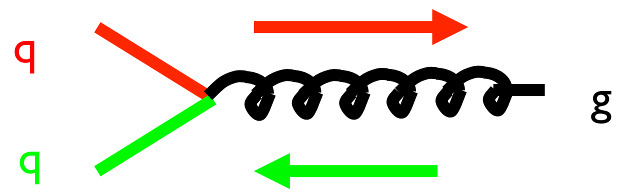
SU(3) generators

There are 8 3x3 matrices which can transform one colour into another.
 - carry away one colour, deliver another: colour/anticolour combinations

These matrices do not commute (each has its own colour state).

$$r\bar{g} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \text{Quark state} = \begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \text{Red} \rightarrow \text{green}$$



SU(2)$_L$

Group particles into left handed doublets:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \text{Weak isospin}: \begin{pmatrix} \tfrac{1}{2} \\ -\tfrac{1}{2} \end{pmatrix}$$

Weak interaction rotates in weak isospin space:

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$W^+ + e^- \to \nu_e \qquad W^- + \nu_e \to e^-$$

But SU(2) also allows for $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$W^0 + \nu_e \to \nu_e$ Weak neutral current

Invariance under SU(2): 3 massless gauge bosons: W^+, W^0, W^-

SU(3)xSU(2)xU(1)

Single family of the SM is spread across 5 different multiplets:

$$\begin{pmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \end{pmatrix}_L^{\frac{1}{3}}$$

$$\begin{pmatrix} u_r & u_g & u_b \end{pmatrix}_R^{\frac{4}{3}} \qquad \begin{pmatrix} d_r & d_g & d_b \end{pmatrix}_R^{-\frac{2}{3}}$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L^{-1} \qquad \left(e^-\right)_R^{-2}$$

Superscript = weak hypercharge Y, the natural quantum number of electroweak theory. $Y=2(Q-I_{3L})$

Q=electric charge, I_{3L}=weak isospin

Eg for left-handed u-quarks, $I_{3L}=1/2$, $Q=2/3 \rightarrow Y=1/3$

For right-handed electron $I_{3L}=0$, $Q=-1 \rightarrow Y=-2$

This pattern looks unnatural

The SU(5) model

SU(5) is the smallest group which can contain $SU(2)_L x(U1)xSU(3)$

It is possible to get all the SM states into an SU(5) 10-plet and 5-plet, using only left-handed states:

$$\begin{bmatrix} \bar{d}_r \\ \bar{d}_g \\ \bar{d}_b \\ e^- \\ \nu_e \end{bmatrix} \begin{array}{c} \\ g \\ \\ X \\ W \end{array} \qquad \begin{bmatrix} 0 & \bar{u}_b & \bar{u}_g & u_r & d_r \\ -\bar{u}_b & 0 & \bar{u}_r & u_g & d_g \\ -\bar{u}_g & -\bar{u}_r & 0 & u_b & d_b \\ -u_r & -u_g & -u_b & 0 & e^+ \\ -d_r & -d_g & -d_b & -e^+ & 0 \end{bmatrix}$$

Apply local SU(5) gauge symmetry - get 24 gauge bosons: γ, W^+, W^-, Z, 8 gluons and 12 X,Y bosons mediating quark-lepton transitions.

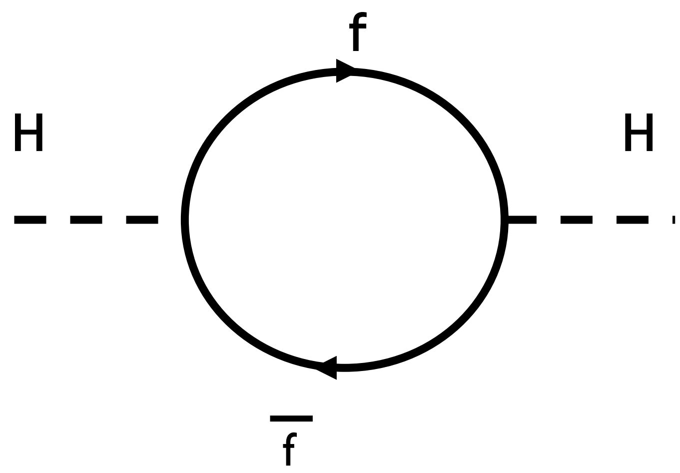
These violate baryon and lepton number.

The Hierarchy Problem revisited

Try to calculate m_H again:

Our first order prediction is $m_H = \sqrt{2}\mu\hbar / c$
But Higgs couples to fermions as

$$-\lambda_f H \bar{f} f$$



Need to compute contribution of every fermion loop diagram.

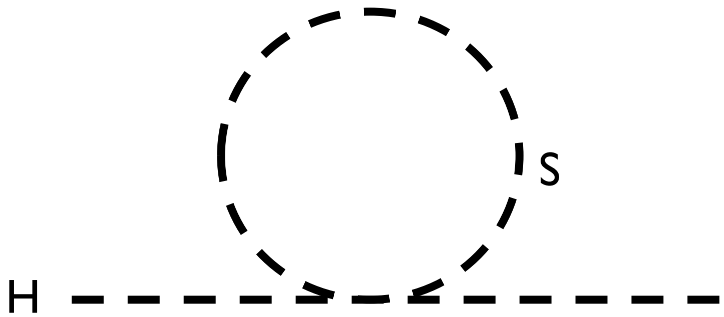
Full calculation gives

$$\Delta m_H^2 = \frac{|\lambda_f|^2}{16\pi^2}\left[-2\Lambda_{UV}^2 + 6m_f^2 \ln(\Lambda_{UV}/m_f) + \ldots\right]$$

Λ_{UV} is scale of new physics.

Reference for this Martin: hep-ph/9709356

Consider the contribution from new scalar particles:



Scalar couples to Higgs with strength λ_S

This diagram gives a contribution of

$$\Delta m_H^2 = \frac{\lambda_s}{16\pi^2}\left[\Lambda_{UV}^2 - 2m_s^2 \ln(\Lambda_{UV}/m_s) + \ldots\right]$$

The change of sign is a fundamental property of scalars vs fermions.

Hypothesise 2 scalars for every fermion, with matching couplings: cancel divergent term completely!

$$\Delta m_H^2 = \frac{\lambda}{16\pi^2}[6m_f^2 \ln(\Lambda_{UV}/m_f)$$
$$- 4m_s^2 \ln(\Lambda_{UV}/m_s) + \ldots]$$
$$\text{where } \left|\lambda_f^2\right| = \lambda_s = \lambda$$

Check this is now ok:

Take $\Lambda_{UV} = M_{PL} = 10^{19}$ GeV, $m_f = m_{top}$, and $\lambda = 1$

$$\text{Top contribution } = \frac{\lambda}{16\pi^2}[6m_f^2 \ln(\Lambda_{UV}/m_f)]$$
$$= \frac{1}{16\pi^2}[6 \times 174^2 \ln(10^{19}/174)]$$
$$\Delta m_H = 217 \text{ GeV}$$

But this will be partially cancelled by the scalar contribution - looks ok.

If we want to use this mechanism, without introducing arbitrary constants, we must impose a symmetry which naturally introduces two scalars for every fermion, and which relates the gauge couplings in the correct way:

SUPERSYMMETRY!

Supersymmetry supposes an operator exists to relate bosons to fermions:

$$Q|Boson\rangle = |Fermion\rangle$$

$$Q|Fermion\rangle = |Boson\rangle$$

Q and $Q^\dagger$ carry spin 1/2.

Chiral supermultiplets

Each fermion has two helicity states, each with 2 degrees of freedom, in a Dirac spinor with 4 components.

A complex scalar field has two degrees of freedom, so SUSY => two scalars per fermion.

Label superpartners with ~, and add s to name.

So for the electron we have

$$e_L \xrightarrow{Superpartner} \tilde{e}_L$$

$$e_R^\dagger \xrightarrow{Superpartner} \tilde{e}_R^*$$

Note that the selectrons are scalars. The L and R labels do not indicate helicity states, but show which weak couplings they have.

The gauge interactions of normal and SUSY partners are identical.

The Minimal Supersymmetric Standard Model

Multiplet name

	Multiplet name	Spin 0	Spin 1/2
Quarks/squarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)
	$\bar{u}$	$\tilde{u}_R^*$	$u_R^\dagger$
	$\bar{d}$	$\tilde{d}_R^*$	$d_R^\dagger$
Leptons/ sleptons	L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)
	$\bar{e}$	$\tilde{e}_R^*$	$e_R^\dagger$
Higgs/ higgsinos	H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$
	H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$

	Spin 1/2	Spin 1
Gluino/gluon	$\tilde{g}$	g
Winos, W's	$\tilde{W}^\pm, \tilde{W}^0$	$W^\pm, W^0$
Bino, B	$\tilde{B}^0$	B^0

The MSSM contains the minimum number of new states required to make the SM into a SUSY theory.

These states will mix to form the physical mass eigenstates, for example 5 physical Higgs states, four neutral, weakly interacting neutralinos, and 4 charginos:

$$H_u^0, H_d^0, H_u^+, H_d^- \rightarrow h^0, H^0, A^0, H^\pm \qquad \text{Higgs}$$

$$\tilde{B}^0, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0 \rightarrow \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0 \qquad \text{Neutralinos}$$

$$\tilde{W}^\pm, \tilde{H}_u^+, \tilde{H}_d^- \rightarrow \tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm \qquad \text{Charginos}$$

All of these states are generated automatically by the application of the SUSY operators. Only one new parameter is needed, to allow two higgs doublet fields instead of the one in the SM.

R-parity

We can label the states with a multiplicative quantum number called R-parity:

$$R_P = (-1)^{3(B-L)+2s}$$

B, L are baryon and lepton number and s is spin. All SM particles have $R_P = +1$ and all SUSY particles have $R_P = -1$.

R_P conservation has the effect of suppressing proton decay to negligible levels. However models in which it is violated do exist (see later).

It also means that the SUSY and SM states cannot mix.

R-parity conservation has important consequences for experimental searches:

• any initial state must have $R_P = +1$, so SUSY particles must be produced in pairs. This requires energies of twice the SUSY mass.

• Any SUSY particle decay must be to a state with $R_P = -1$, and so each final state contains another SUSY particle.

• The lightest SUSY particle (the LSP) must be stable.

• A stable LSP (unless very heavy) must be electrically neutral and weakly interacting to have escaped detection. This is just what is required for dark matter.

SUSY breaking

The SUSY lagrangian is highly constrained:

- fermion and boson couplings linked
- only one new parameter from two Higgs doublets
- all other parameters known from SM.

If SUSY were an exact symmetry, the SUSY particles would have the same masses as their SM partners -> ruled out!

Hence, if SUSY exists in nature, it must be broken in the current vacuum ground state, giving SUSY particles masses.

But we must keep the SUSY relationships between the couplings in order to solve the hierarchy problem.

-> Use "soft" SUSY breaking:

The scale of SUSY breaking is m_{Soft} and this sets the mass of the SUSY particles.

Need $m_{Soft} < 1$ TeV:

- keep the splitting between the superpartner masses "natural"
- generate the correct SM Higgs VEV

The SM particles get their masses from electroweak symmetry breaking, with a scale < Higgs VEV = 174 GeV, while all SUSY particles get theirs from $m_{Soft} \sim 1$ TeV.

-> explains why SUSY not yet seen

One exception is the lightest higgs state h^0 -> mass related to the electroweak scale.

First approximation: $m_h < M_Z$

With loop corrections $m_h < 150$ GeV.

-> Higgs searches could soon rule SUSY out. But if a Higgs is found at <150 GeV, will need to understand if it is SM or SUSY type.

Soft SUSY breaking works by introducing terms into the Lagrangian which only involve the SUSY particles, and not their SM partners - explicitly breaking the symmetry.

Only certain terms are allowed in a renormalisable theory:

$$M_1, M_2, M_3 \quad \text{Gaugino masses } (\tilde{\mathrm{B}}^0, \tilde{W}^{\pm,0}, \tilde{g})$$

$$\mathbf{a_u}, \mathbf{a_d}, \mathbf{a_e} \quad 3 \times 3 \text{ matrices of (scalar)}^3 \text{ couplings}$$

$$\mathbf{m_Q^2}, \mathbf{m_L^2}, \mathbf{m_{\bar{u}}^2}, \mathbf{m_{\bar{d}}^2}, \mathbf{m_{\bar{e}}^2} \quad 3 \times 3 \text{ matrices of } \tilde{\mathrm{q}}, \tilde{\mathrm{l}} \text{ masses}$$

$$m_{H_u}^2, m_{H_d}^2, b \quad \text{Higgs mass terms}$$

The matrices cover the 3 families for each chiral supermultiplet.

This makes a total of 105 new parameters !!! (Masses, phases, mixing angles).

This looks like a disaster: we want to fix up the hierarchy problem -> one parameter, m_H, which is out of control.

SUSY provides a natural way to stabilise m_H, with only one new parameter.

SUSY breaking introduces a large set of apparently arbitrary parameters, with no connections to the SM .

-> need to find a mechanism for breaking SUSY in a simple way.

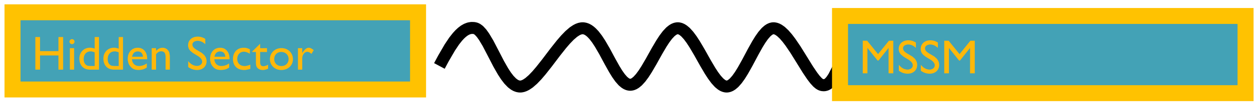
This is the main topic of papers on SUSY theory (for a lot of years now…)

However, many of these parameters must be set to simple values (eg 0) to avoid conflict with precision data on electroweak interactions, CP violation etc.

SUSY breaking mechanisms

The MSSM fields alone cannot break SUSY dynamically.

-> use new physics at a high scale.



A set of new fields interacts weakly with MSSM fields . SUSY breaking occurs in this “hidden sector” by the appearance of a VEV, which is communicated by the interactions to the MSSM SUSY particles.

Number of parameters vastly reduced if the interactions are “flavour blind”.

Maybe gravity can do the job -> SuperGravity theories…

Supergravity (SUGRA)

Hidden sector communicates with MSSM with gravitational-type interactions.

In the minimal version of these models we have

$$M_1 = M_2 = M_3 = m_{1/2}$$

$$\mathbf{a}_{\mathbf{u}}, \mathbf{a}_{\mathbf{d}}, \mathbf{a}_{\mathbf{e}} = A_0 \mathbf{y}$$

Flavour blind

$$\mathbf{m}^2_{\mathbf{Q}}, \mathbf{m}^2_{\mathbf{L}}, \mathbf{m}^2_{\bar{\mathbf{u}}}, \mathbf{m}^2_{\bar{\mathbf{d}}}, \mathbf{m}^2_{\bar{\mathbf{e}}} = m_0^2 \mathbf{1}$$

$$m^2_{H_u} = m^2_{H_d} = m_0^2$$

$$b = B_0 \mu$$

There are now only 5 parameters:

$m_{1/2}$ m_0 A_0 b μ

Conventionally replace b and μ by $\tan\beta$ and $\mathrm{sgn}(\mu)$, where β is the ratio of the H_u and H_d VEVs.

Also get a gravitino with mass $m_{3/2} \sim m_{Soft}$ which is a good dark matter candidate

Gauge mediated SUSY breaking

"Messenger" particles interact with a SUSY breaking VEV in the hidden sector, and with the MSSM particles with ordinary gauge interactions. Messengers could be states from a larger gauge group such as SU(5).

In these models, the gravitino is not related to m_{Soft} and is expected to be very light - as little as 1 keV, and hence it is the LSP.

All final states would contain gravitinos.

These models have 6 parameters:

$F_m \ll (10^{10}\ \text{GeV})^2$ SUSY breaking scale
M_m Messenger scale ($\Lambda \equiv F_m / M_m$)
N_5 Number of messenger 5-plets
$\tan(\beta)$ and $\text{sgn}(\mu)$ as in SUGRA
C_{grav} Coupling for decays into gravitinos

SUSY particles decay to gravitinos, but the coupling is not strong, and so the lifetime can be quite long.

Interesting possibilities include

$$\tilde{\chi}_1^0 \rightarrow \tilde{G}\gamma$$

$$\tilde{\ell}_R \rightarrow \tilde{G}\ell$$

Neutralino decays give a signature with high energy photons and missing energy. The lifetime can be long, so the photons may point away from the primary event vertex.

Sleptons may have long lifetimes, sufficient to leave the detector and be detected as real particles.

Strategy for SUSY Searches

Search for inclusive signals, measure SUSY mass scale. Inclusive signals contain a distinct signature (ie 2 leptons, or missing energy > 500 GeV) which can be produced by many processes.

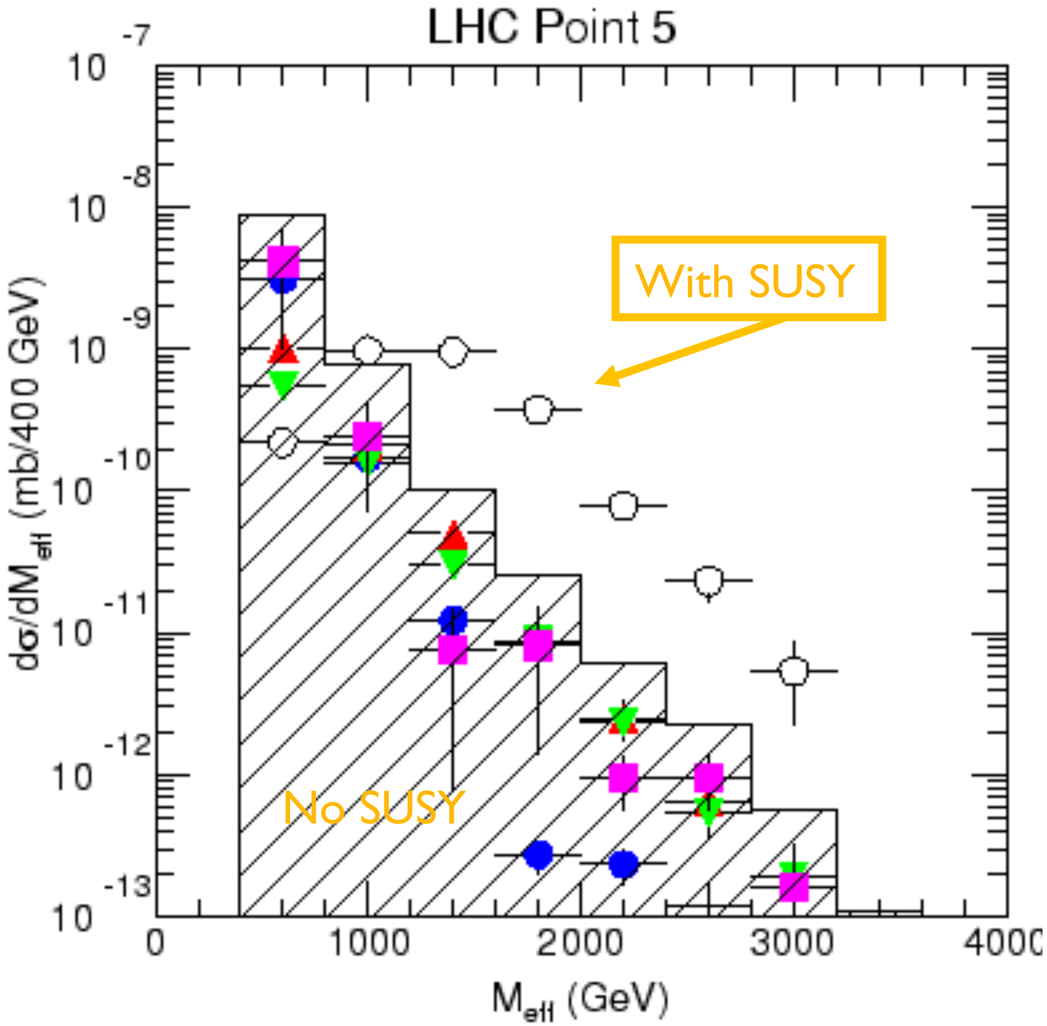
Make detailed measurements of exclusive modes, extract kinematic end-points and combinations of masses, in as model independent way as possible. Exclusive modes are particular defined decay chains, where every particle is specified.

Input all information into global fits to extract model parameters.

Discovery of SUSY is simple: understanding is not!

http://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/ for latest information.

How to find SUSY…



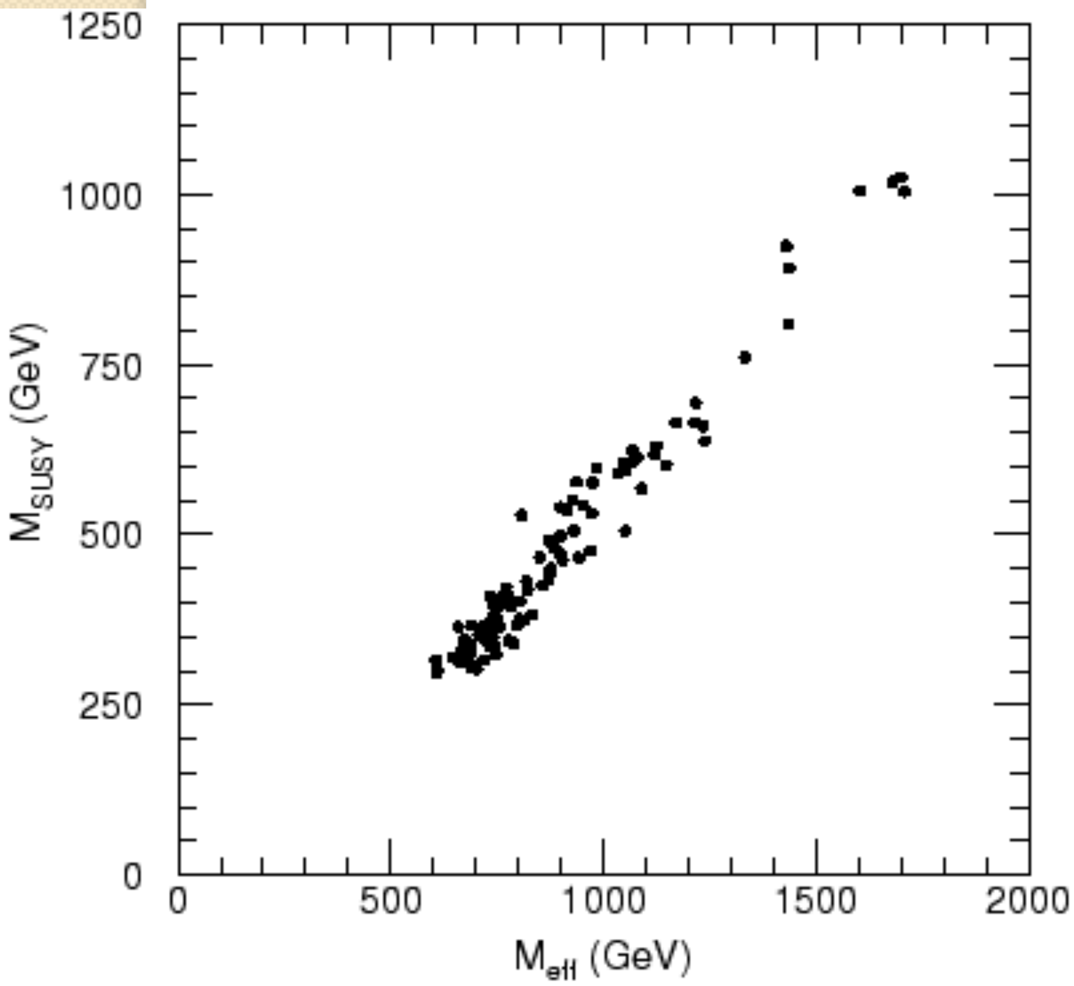
Define an effective mass for inclusive processes:

$$M_{eff} = E_T^{miss} + \sum_{i=1,4} p_T^i$$

Scalar sum over 4 highest p_T jets or leptons and include missing transverse energy.

10x more events at high M_{eff} with SUSY

All simulation, before experiment ran



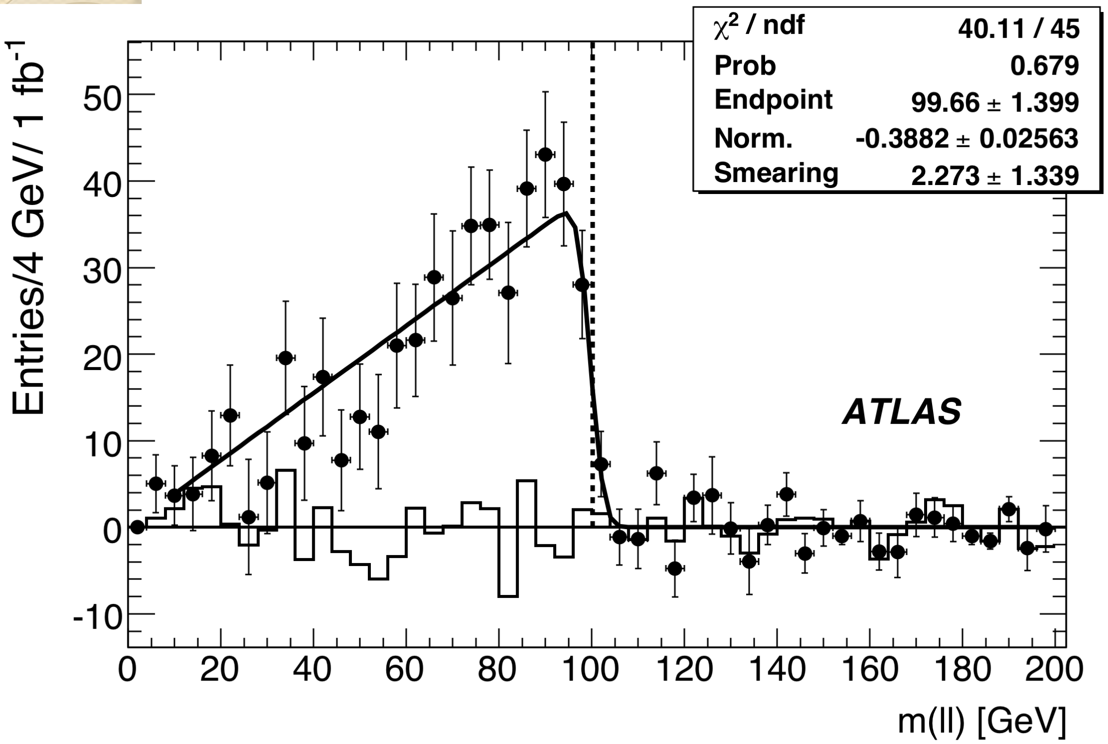
Peak of M_{eff} distribution vs the SUSY mass scale for a set of 100 minimal SUGRA model parameters.

- M_{SUSY} is mass of the lighter of gluino or squark.
- Strong correlation -> spread of only 10%

M_{eff} is a good measure of the SUSY scale

If there is a signal, then look for direct evidence of type of particles....

$$\tilde{\chi}_2^0 \rightarrow \tilde{\ell}_R^{\pm} \ell^{\mp} \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^-$$



The position of the edge depends on the masses of the slepton and the two neutralinos.

$$M_{ll}^{max} = M(\tilde{\chi}_2^0)\sqrt{1-\frac{M^2(\tilde{l}_R)}{M^2(\tilde{\chi}_2^0)}}\sqrt{1-\frac{M^2(\tilde{\chi}_1^0)}{M^2(\tilde{l}_R)}}$$

Plot invariant mass of lepton pairs

The leptons in the signal must be of opposite sign and from the same family, since the slepton carries the family information down the chain. Most background comes from processes with unrelated leptons.

eg: WW, or chargino pairs create equal numbers of $\mu\mu$, ee, eμ and μe events.

Hence most background can be subtracted using opposite sign eμ pairs.

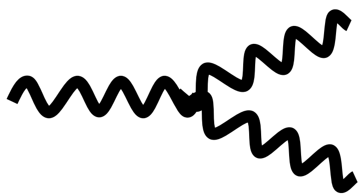
Searches for Gauge Mediated SUSY Breaking

In these models the gravitino is the lightest SUSY particle (LSP). The available signatures depend on which particle is the next lightest (NLSP), and the strength of decays to the gravitino.

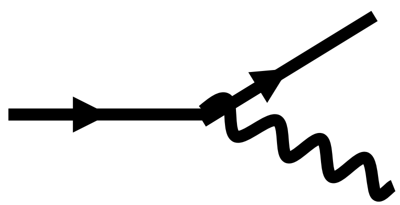
4 cases: NLSP either neutralino or slepton, and $C_{grav}=1$, giving a fast decay, or $C_{grav}>>1$, giving a slow decay of the NLSP to gravitinos.

The decay modes are:

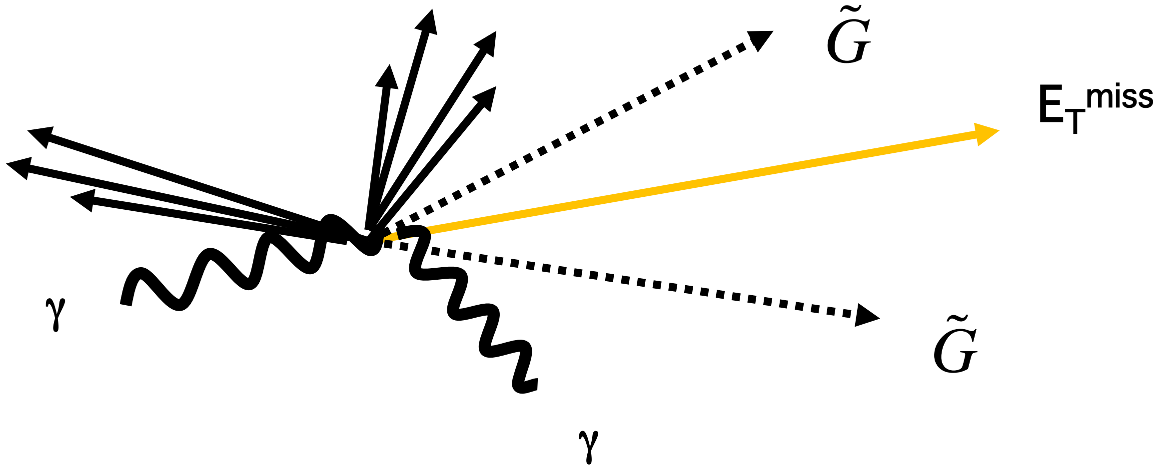
$$\tilde{\chi}_1^0 \rightarrow \tilde{G}\gamma$$



$$\tilde{\ell}_R \rightarrow \tilde{G}\ell$$



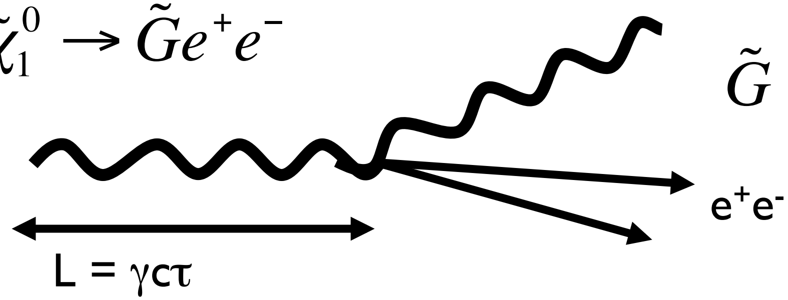
Case 1: C_{grav}=1 Fast decay $\tilde{\chi}_1^0 \to \tilde{G}\gamma$



2 high energy photons in every event, with jets, leptons and missing energy - very small SM background -> easy to discover!

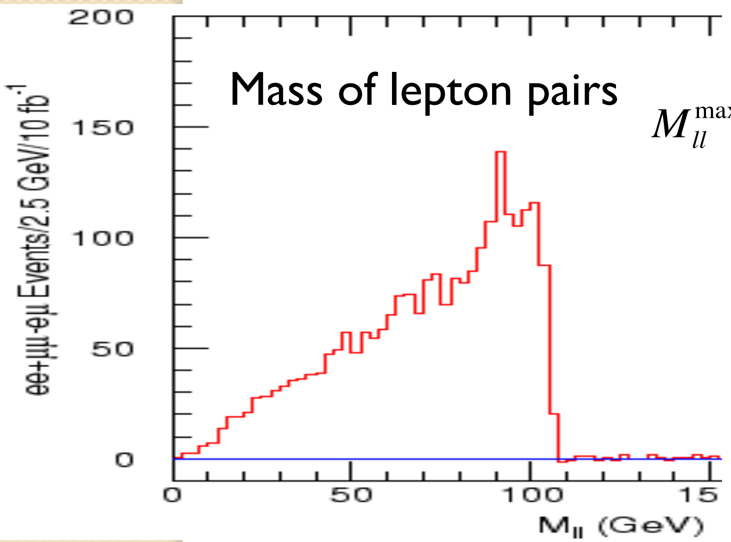
2% of decays are

$\tilde{\chi}_1^0 \to \tilde{G}e^+e^-$



Measure lifetime and hence C_{grav}

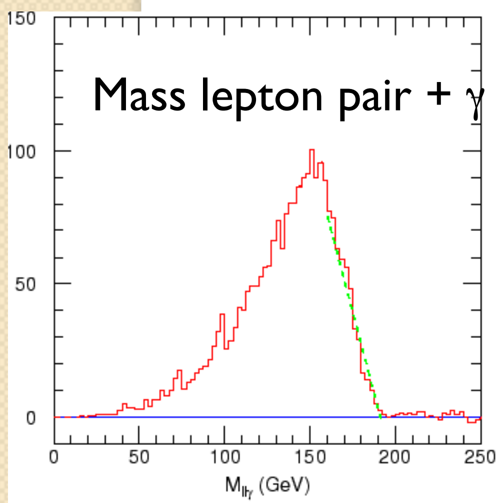
Use decay chains to get more information: $\tilde{\chi}_2^0 \rightarrow \tilde{\ell}^{\pm}\ell^{\mp} \rightarrow \tilde{\chi}_1^0 \ell^+\ell^- \rightarrow \tilde{G}\ell^+\ell^-\gamma$



$$M_{ll}^{\max} = M(\tilde{\chi}_2^0)\sqrt{1-\frac{M^2(\tilde{l}_R)}{M^2(\tilde{\chi}_2^0)}}\sqrt{1-\frac{M^2(\tilde{\chi}_1^0)}{M^2(\tilde{l}_R)}}$$

$$M_{\ell\gamma}^{(1)} = \sqrt{M_{\tilde{\ell}_R}^2 - M_{\tilde{\chi}_1^0}^2} = 112.7\,\mathrm{GeV}$$

$$M_{\ell\gamma}^{(2)} = \sqrt{M_{\tilde{\chi}_2^0}^2 - M_{\tilde{\ell}_R}^2} = 152.6\,\mathrm{GeV}$$



$$M_{ll\gamma}^{\max} = \sqrt{M_{\tilde{\chi}_2^0}^2 - M_{\tilde{\chi}_1^0}^2}$$

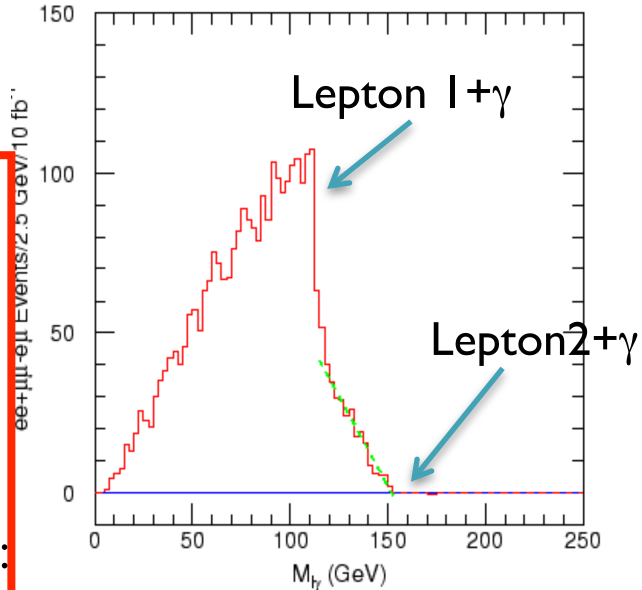
$$M_{\tilde{\ell}_R}^2 = \frac{(M_{\ell\gamma}^{(1)})^2 (M_{\ell\gamma}^{(2)})^2}{(M_{\ell\ell}^{\max})^2}$$

$$M_{\tilde{\chi}_1^0}^2 = M_{\tilde{\ell}_R}^2 - (M_{\ell\gamma}^{(1)})^2$$

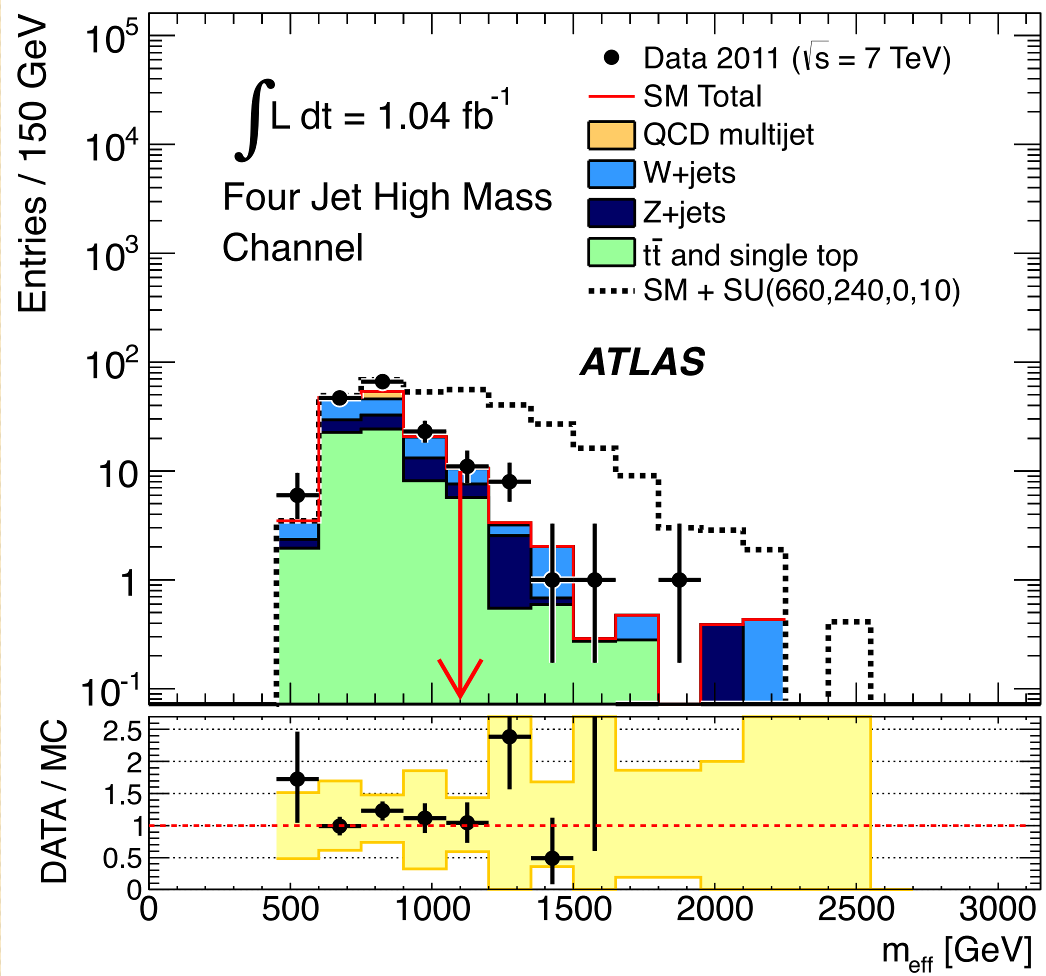
$$M_{\tilde{\chi}_2^0}^2 = M_{\tilde{\ell}_R}^2 + (M_{\ell\gamma}^{(2)})^2$$

where we have the constraint :

$$(M_{\ell\ell\gamma}^{\max})^2 = (M_{\ell\gamma}^{(1)})^2 + (M_{\ell\gamma}^{(2)})^2$$

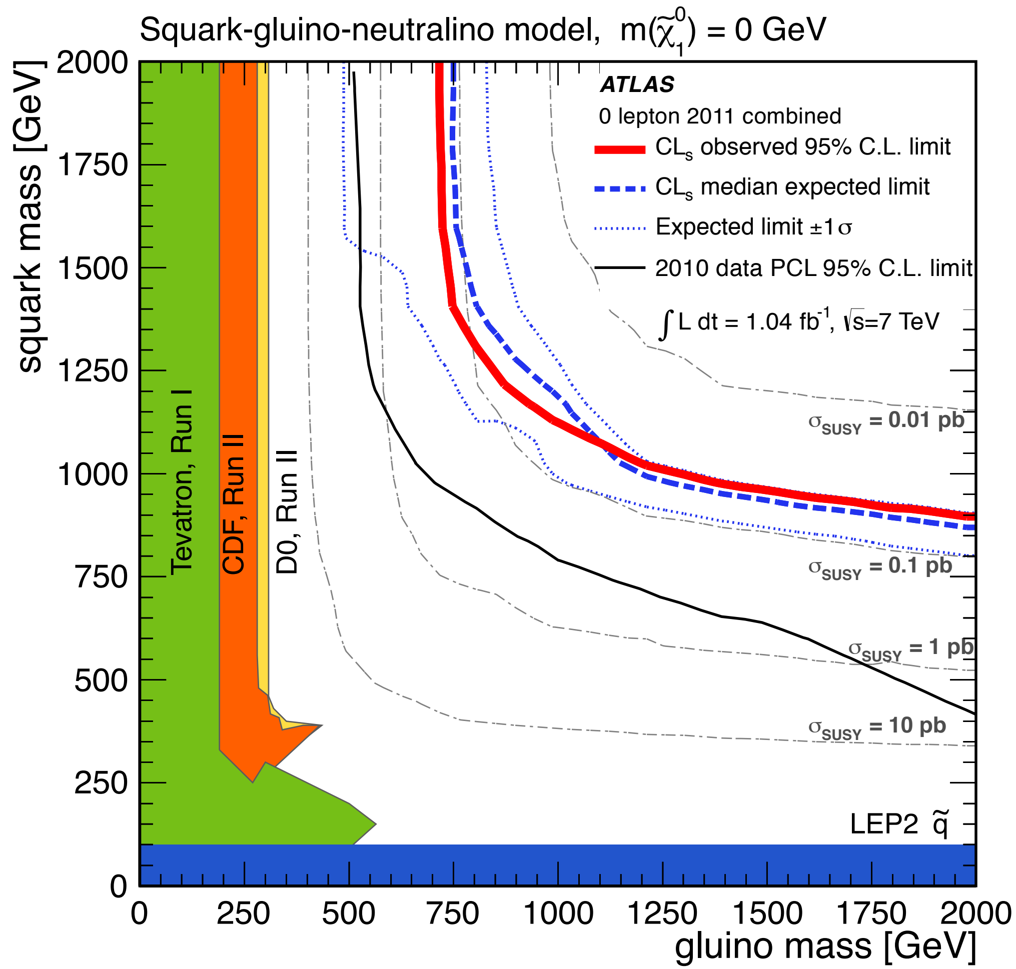


Latest ATLAS results on SUSY



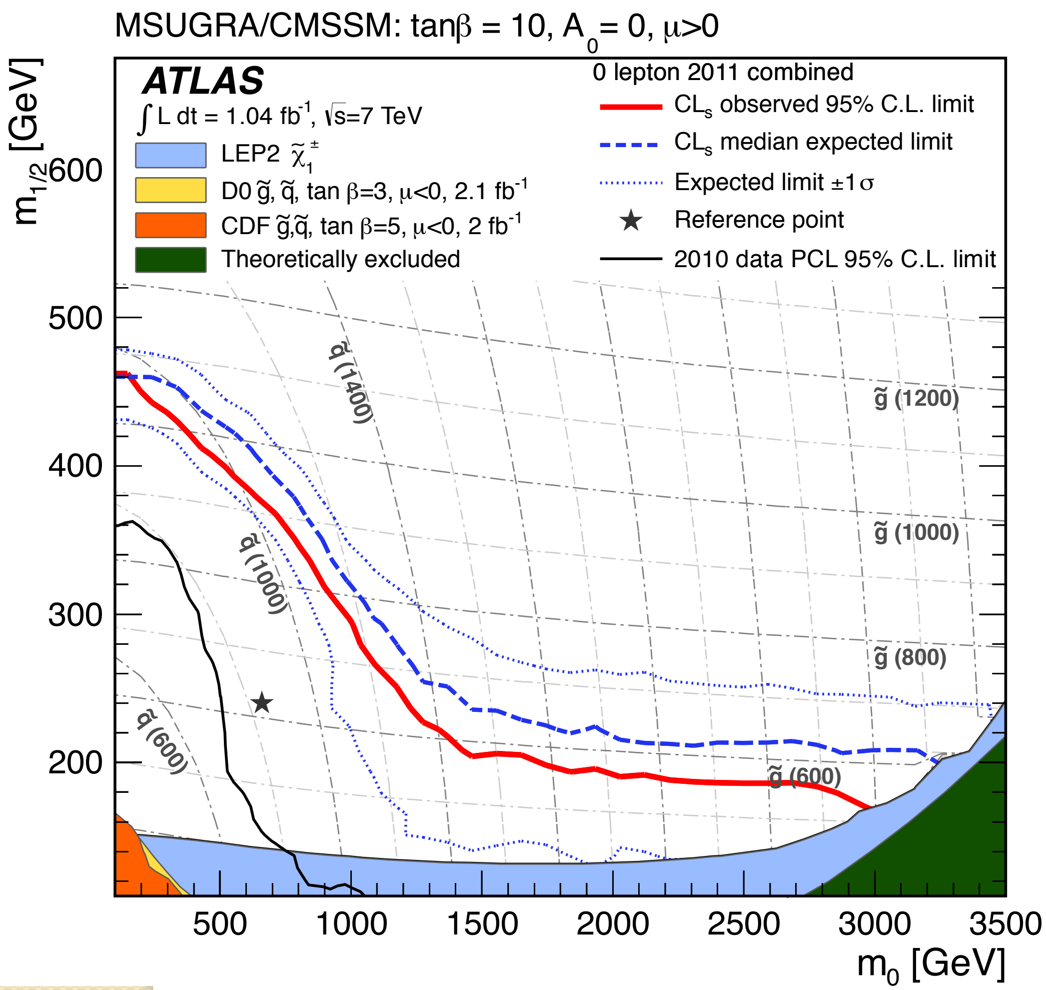
- Look at invariant mass of events with >=4 jets (ie quarks) and large missing energy.
- SUSY events would show excess (dotted line).

Exclusion of low squark and gluino masses



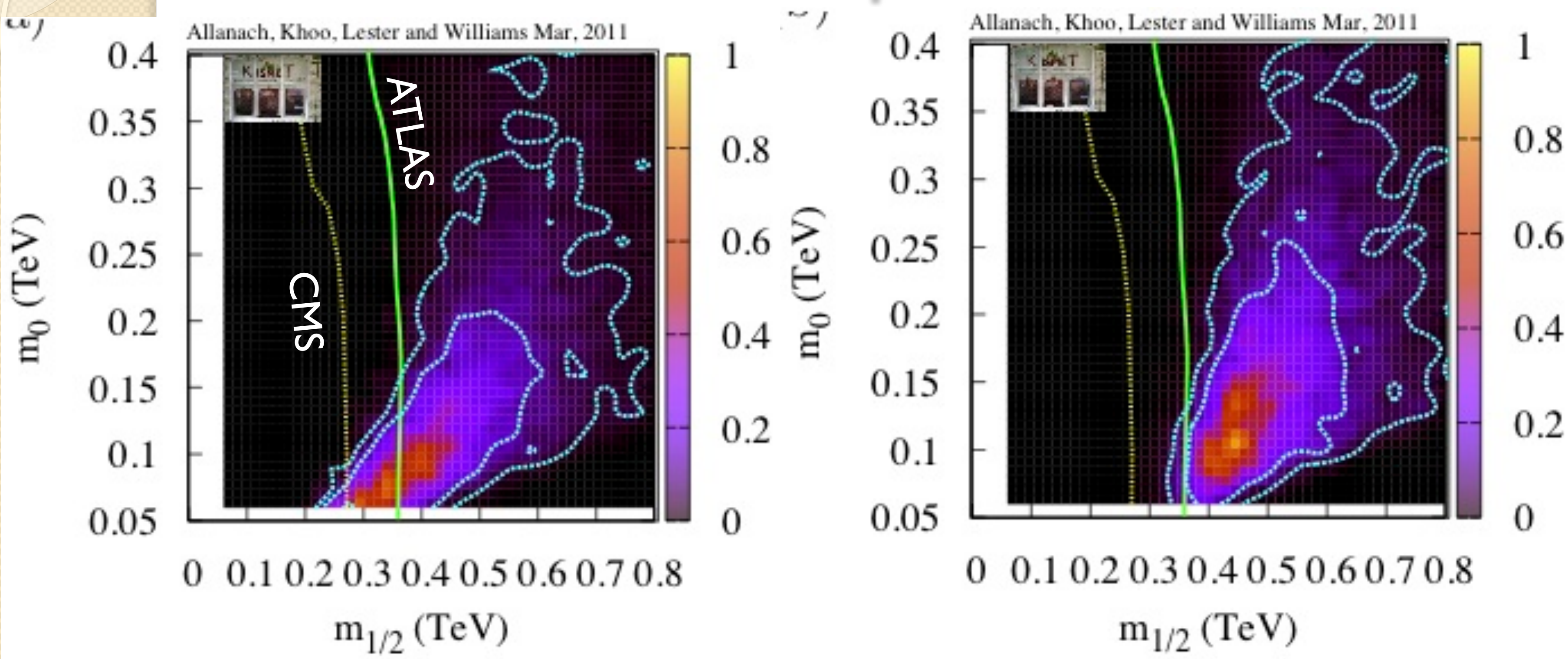
- Direct search for events with jets and missing energy (ie an LSP).Gluino and squark masses below the red line are excluded.
- These are the particles most likely to be produced at the LHC

Plot in SUSY parameter space



- Values of $m_0/m_{1/2}$ below the red line are excluded.
- SUSY will be in real trouble if the range above 1 TeV is reached without evidence of new particles.

Theoretical interpretation

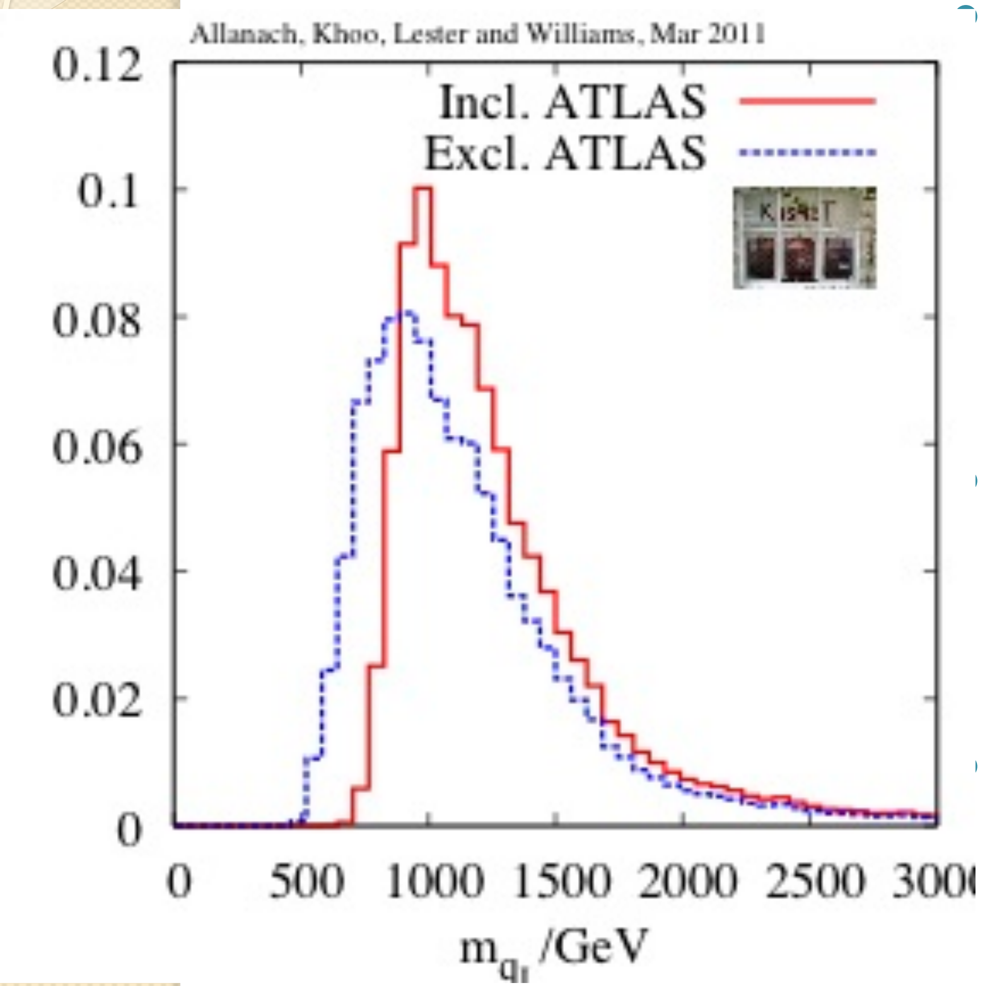


Fit using CMS data alone

Fit including CMS and ATLAS

- Ben Allanach, Chris Lester and students (DAMTP/Cavendish) have analyzed the data from CMS and ATLAS. Plot shows likelihood of solutions in mSUGRA parameter space, using SM electroweak data and new LHC results.

Beware – axes are reversed wrt experimental plots



- Blue/red curves shows probability of given squark masses being correct, before/after inclusion of CMS and ATLAS data.
- Result peaks close to lower limit of ~600 GeV from experiment.
- V low probability of viable solution above 2 TeV – LHC should exclude much of the parameter space for this class of model in next run.

Conclusions

- Supersymmetry is the most popular extension of the SM, and predicts dark matter particles with correct properties
- Data from LHC is now excluding light SUSY particles. In next year, will explore most probable regions of parameter space.
- Still some room for theory to move into unconventional models.