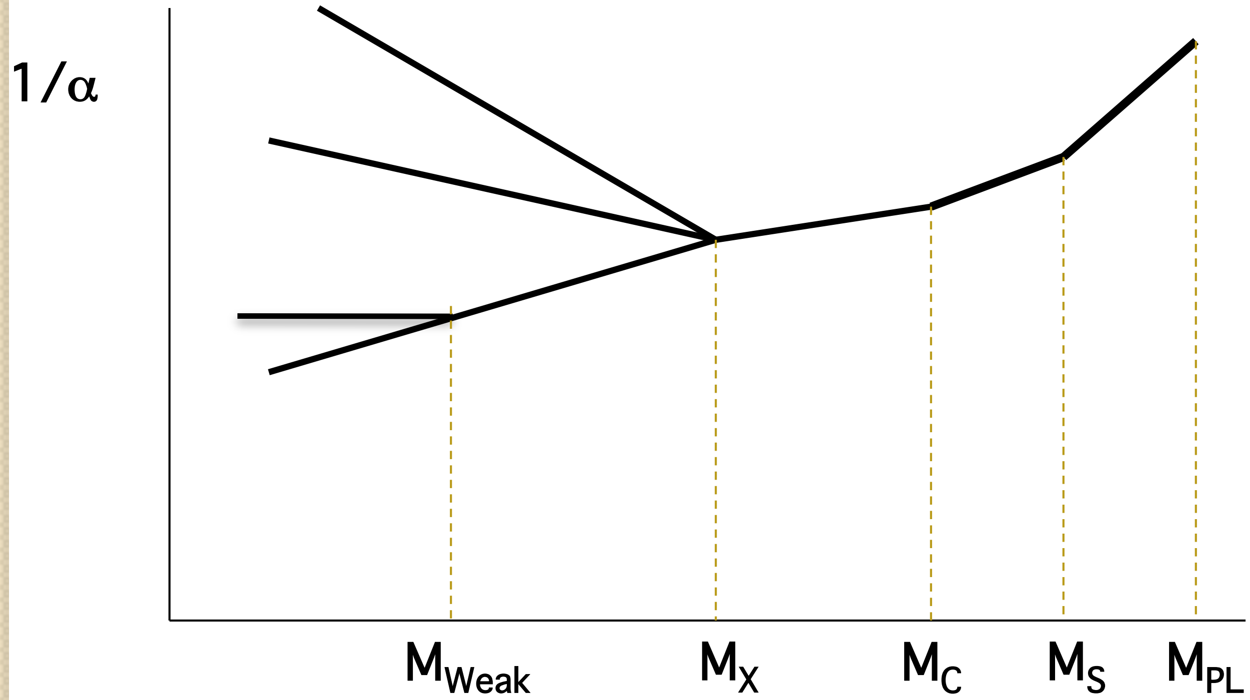
Relics from the Early Universe

Domain walls, cosmic strings and monopoles, dark matter

Domain walls

- Symmetry breaking causes phase transitions in any system.
- With Higgs and other vacuum fields, this means that the vacuum itself drops into a new phase.
- Many such phase transitions possible as Universe evolves.

Scales at high mass



M_{weak} = Electroweak scale, where Weak/EM diverge. 100 GeV

M_X = grand unification scale - where couplings meet. 2×10^{16} GeV?

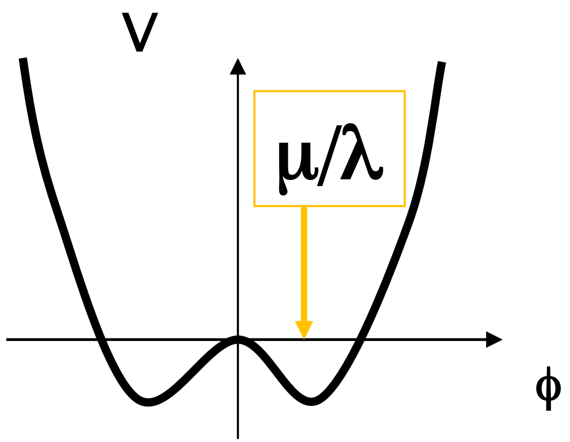
M_C = compactification scale - where extra dimensions of string theory curl up

M_S = string scale - given by natural lengths of string. 10^{17}-10^{18} GeV?

M_{PL} = Planck Mass - where quantum gravity effects become large. 10^{19} GeV

Scalar fields in cosmology

Recall higgs field from Lecture 1:



Field strength given by vacuum
Expectation value – VEV

Low T – Higgs VEV=μ/λ = 174 GeV

High T – Higgs VEV = 0

Phase change when energy drops below peak of potential.

VEV means that energy is stored in the vacuum, and must be taken into account in the energy budget of the Universe.

What happens if parts of the Universe drop into different minima?

With this potential, present day vacuum energy is negative:

$$V = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4 \qquad \phi_{\min} = \pm\frac{\mu}{\lambda}$$

$$V_{\min} = -\frac{\mu^4}{4\lambda^2}$$

Not good for cosmology! Add term to make V_{min}=0

$$V = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4 + \frac{\mu^4}{4\lambda^2}$$

$$= \frac{\lambda^2}{4}\left(\phi^2 - \phi_{\min}^2\right)^2$$

$$V_{\min} = 0$$

Equation of motion of field given by

$$-\frac{d^2\phi}{dx^2} = -\frac{dV}{d\phi} = -\frac{d}{d\phi}\left(\frac{\lambda^2}{4}\left(\phi(x)^2 - \phi_{\min}^2\right)^2\right)$$

For those doing QFT, here is proof of statement of eq. of motion:

Euler - Lagrange equations:

$$\partial_\mu\left(\frac{\partial L}{\partial\left(\partial_\mu \phi\right)}\right) = -\frac{\partial L}{\partial \phi} \quad \text{with } L = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - V(\phi)$$

$$\frac{dL}{\partial\left(\partial_\mu \phi\right)} = \partial^\mu \phi$$

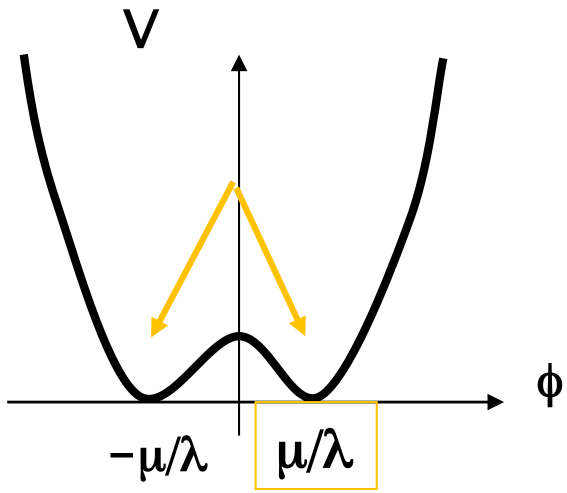
$$\text{Hence} \quad \partial_\mu\left(\partial^\mu \phi\right) = -\frac{dL}{d\phi} = -\frac{dV}{d\phi}$$

In 1D, such that ϕ(t,x,y,z) is ϕ(x), $\partial_\mu \rightarrow -\frac{\partial}{\partial x}$

$$-\frac{\partial^2 \phi}{\partial \mathrm{x}^2} = -\frac{dV}{d\phi}$$

From eq. of motion we get

$$-\frac{d^2\phi}{dx^2} + \lambda^2\phi\left(\phi^2 - \phi_{\min}^2\right) = 0$$



Imagine two space-time bubbles in different vacua, separated by a domain wall at x=0

At x=+∞ Φ=+μ/λ

x=-∞ Φ=−μ/λ

With these boundary conditions, the solution is

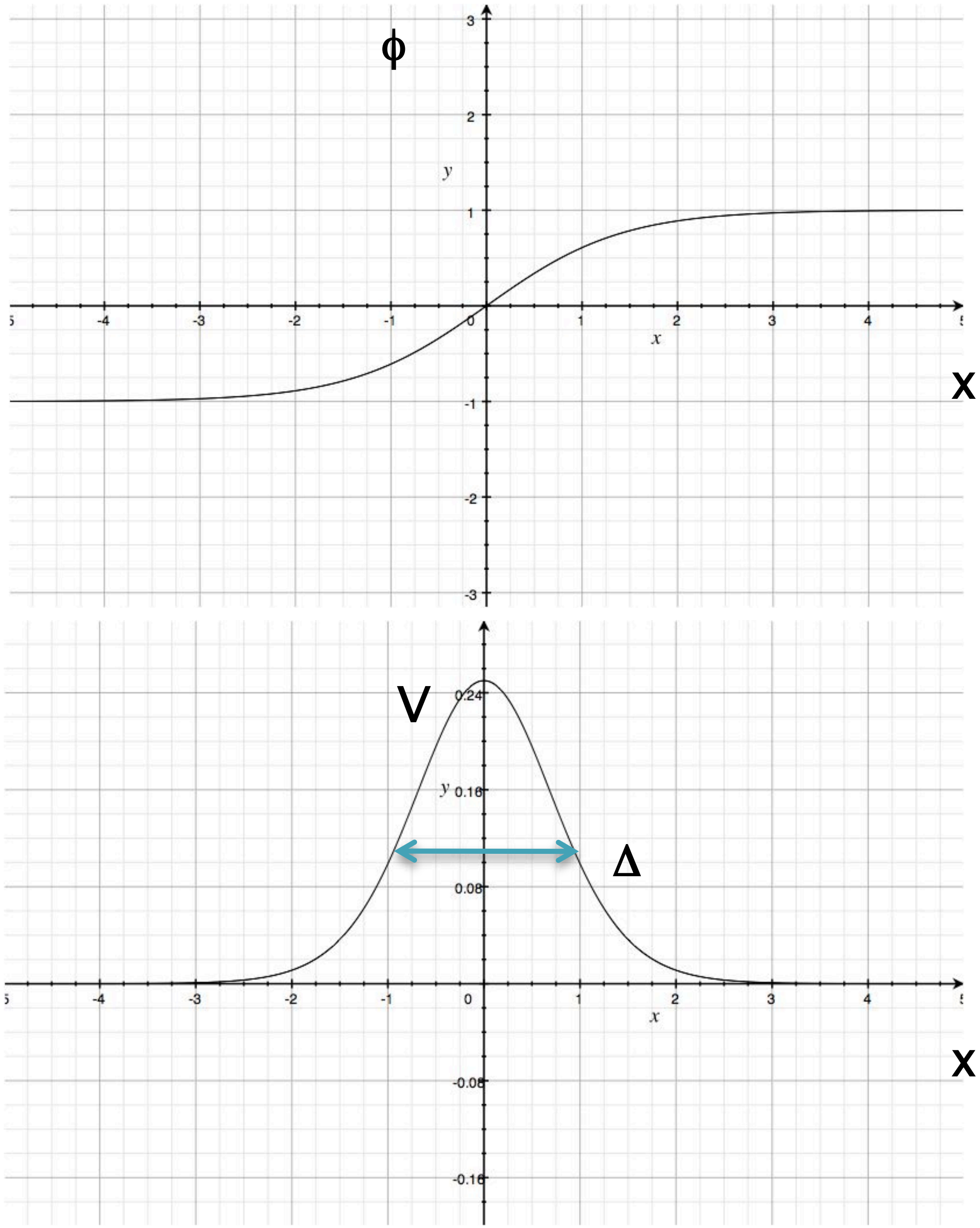
$$\phi(x) = \frac{\mu}{\lambda}\tanh(x/\Delta)$$

$$\text{where} \qquad \Delta = \frac{\sqrt{2}}{\lambda\phi_{\min}}$$

Since ϕ must change from negative to positive, ϕ=0 at the wall.

At this point, V must rise to the false vacuum value between the two minima

Wall thickness given by Δ



Domain walls separate regions of space with different vacuum topology.

Configuration of the field is stable: wall can only be removed by annihilation with wall of opposite topology.

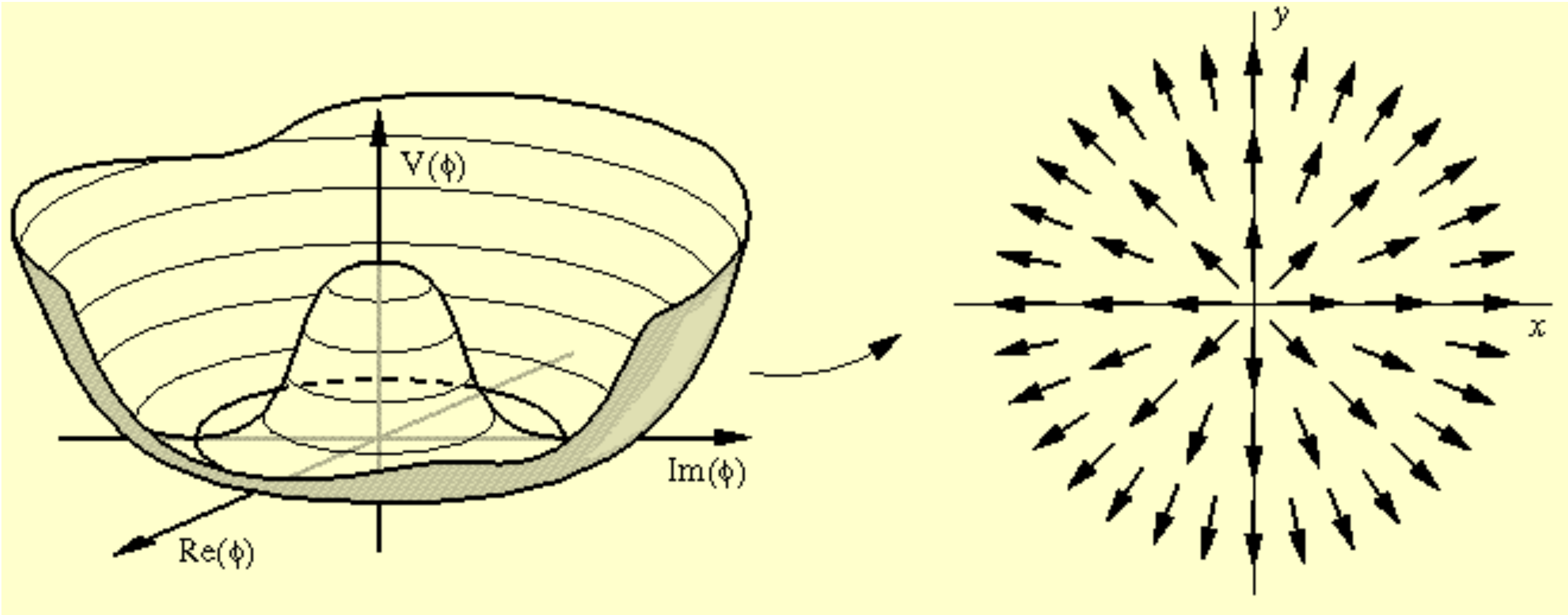
Chance of two causally unconnected regions of space landing in different vacua is 50% in this simple model: so domain walls should be common, and evolve to form the lowest energy network of domains, just like a ferromagnet.

Their energy density would be huge (10^{10} x current density), and they would imprint very large fluctuations on the CMB. Existence of domain walls would be cosmic catastrophe! These problems are solved by inflation.

Interestingly, infinite domain walls exhibit repulsive gravitational forces.

Cosmic strings

Higgs field is SU(2) and so is complex scalar: minimum in potential is a circle in the Re/Im plane. Field can point in different SU(2) directions in different regions.

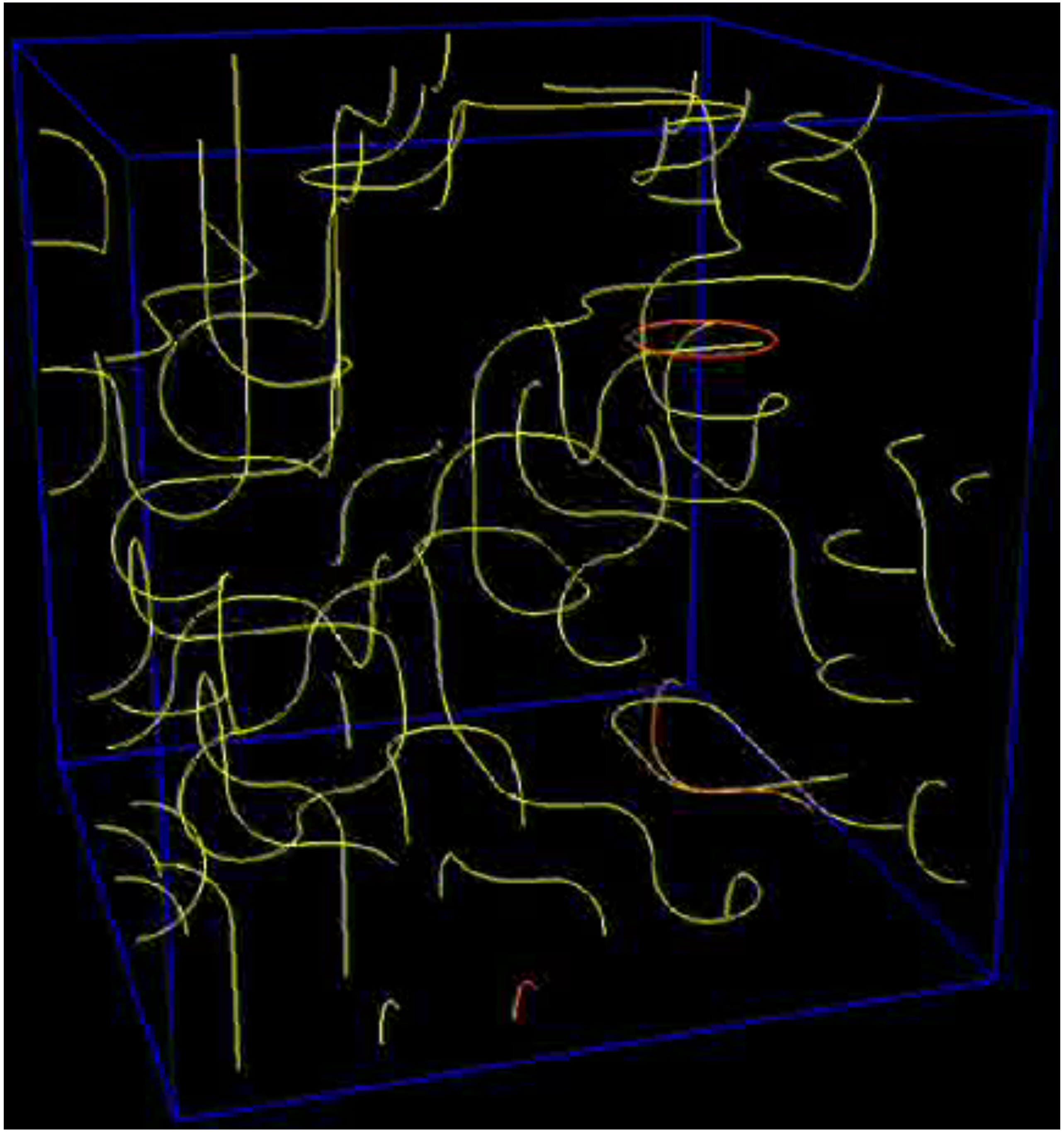


Consider region where phase changes smoothly from 0->2π .
At centre there must be a point where the field is zero, and hence a line of false vacuum – Cosmic string.

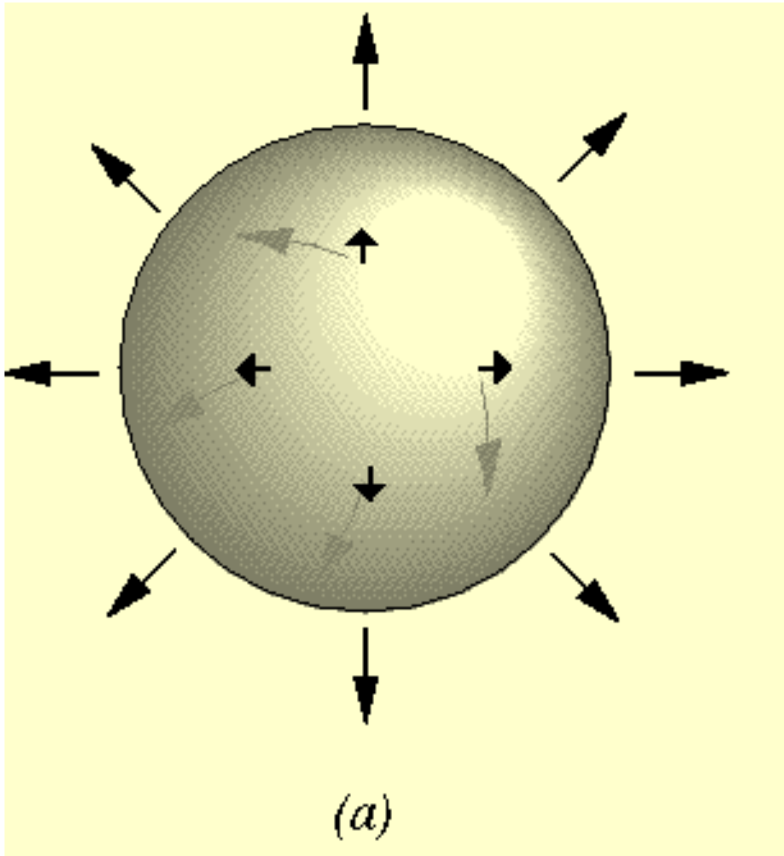
Pictures from http://www.damtp.cam.ac.uk/research/gr/public/

- Strings can be infinitely long or closed loops.
- Loops radiate gravitational waves and evaporate, but long strings interacting will create new loops, so get population of lines and loops which tend to stable "scaling solution".
- Energy density of strings is tolerable because creation and decay of loops moves energy into gravitational waves which are diluted by the expansion.
- Search for cosmic strings by seeking:
    - Relic graviational waves
    - Effect on dT in CMB
    - Gravitational lens effects
    - Effect on structure formation
- So far no evidence for cosmic strings in data.

Scaling solution for cosmic strings. Co-moving volume: strings "fall" into volume as it grows. Closed loops (red) created and evaporate.



Monopoles



Dirac, P. A. M. *Proc. Roy. Soc. London A 133, 60, 1931.*

Monopoles are pointlike topological defects arising from broken spherical symmetries.

First predicted by Dirac to explain charge quantization in QED. A single monopole in the Universe quantizes charge everywhere!

No evidence seen so far - still agree with Maxwell

$$\nabla B = 0$$

But monopoles arise naturally in GUTs and other theories based on large gauge groups.

Monopoles

Available to buy now!

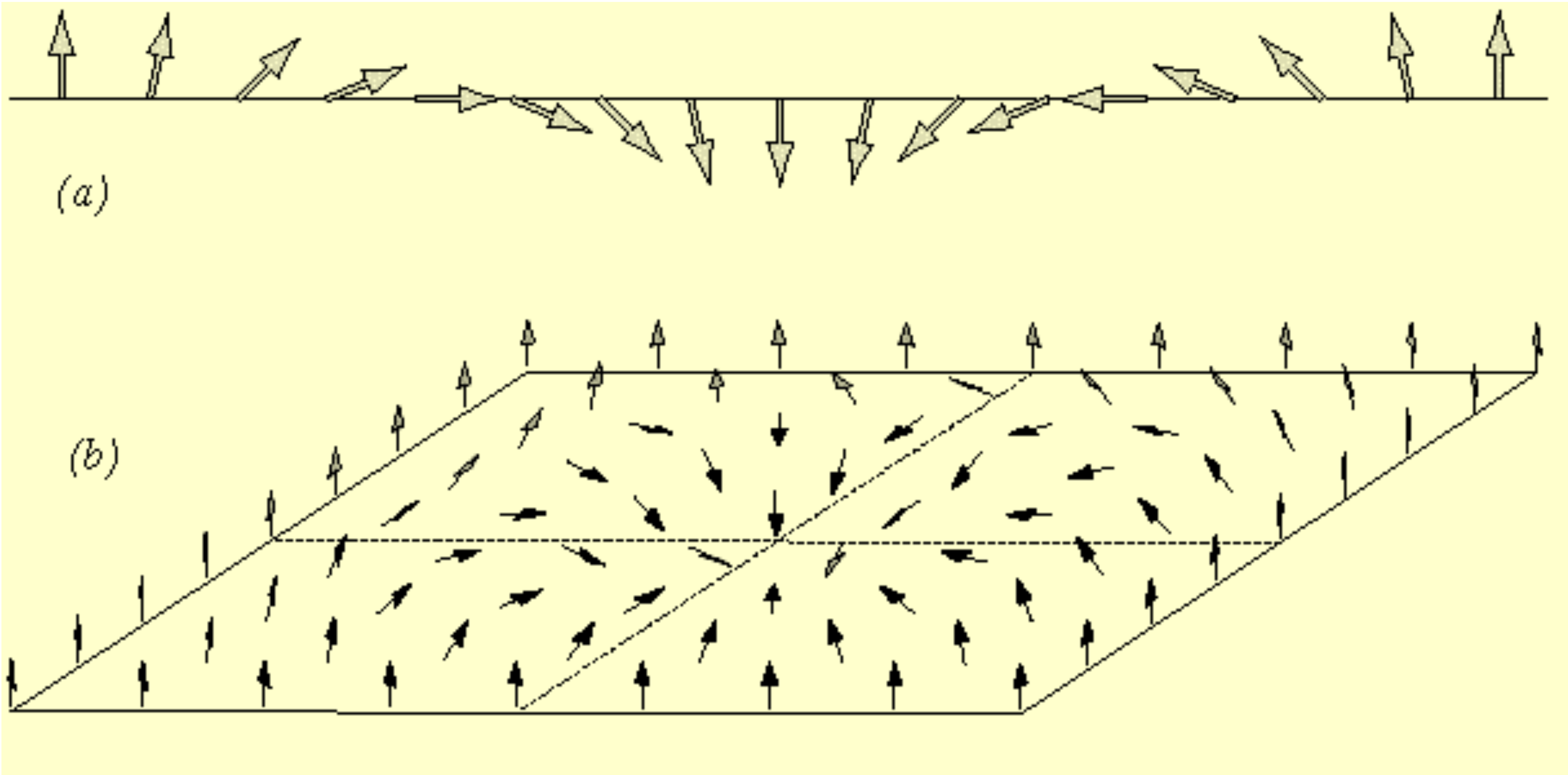
Monopole density

- Monopoles arise inevitably (as do other topological defects) in theories with symmetry breaking.
- Leads in general to large population of primordial stable monopoles, in conflict with data.
- Inflation cures this by inflating a small region with O(1) monopole to size of visible Universe.
- Best cosmic-ray supermassive monopole flux limit:

  $< 1.0\times10^{-15}\ \mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\mathrm{s}^{-1}$ for $1.1\times10^{-4} < \beta < 0.1$

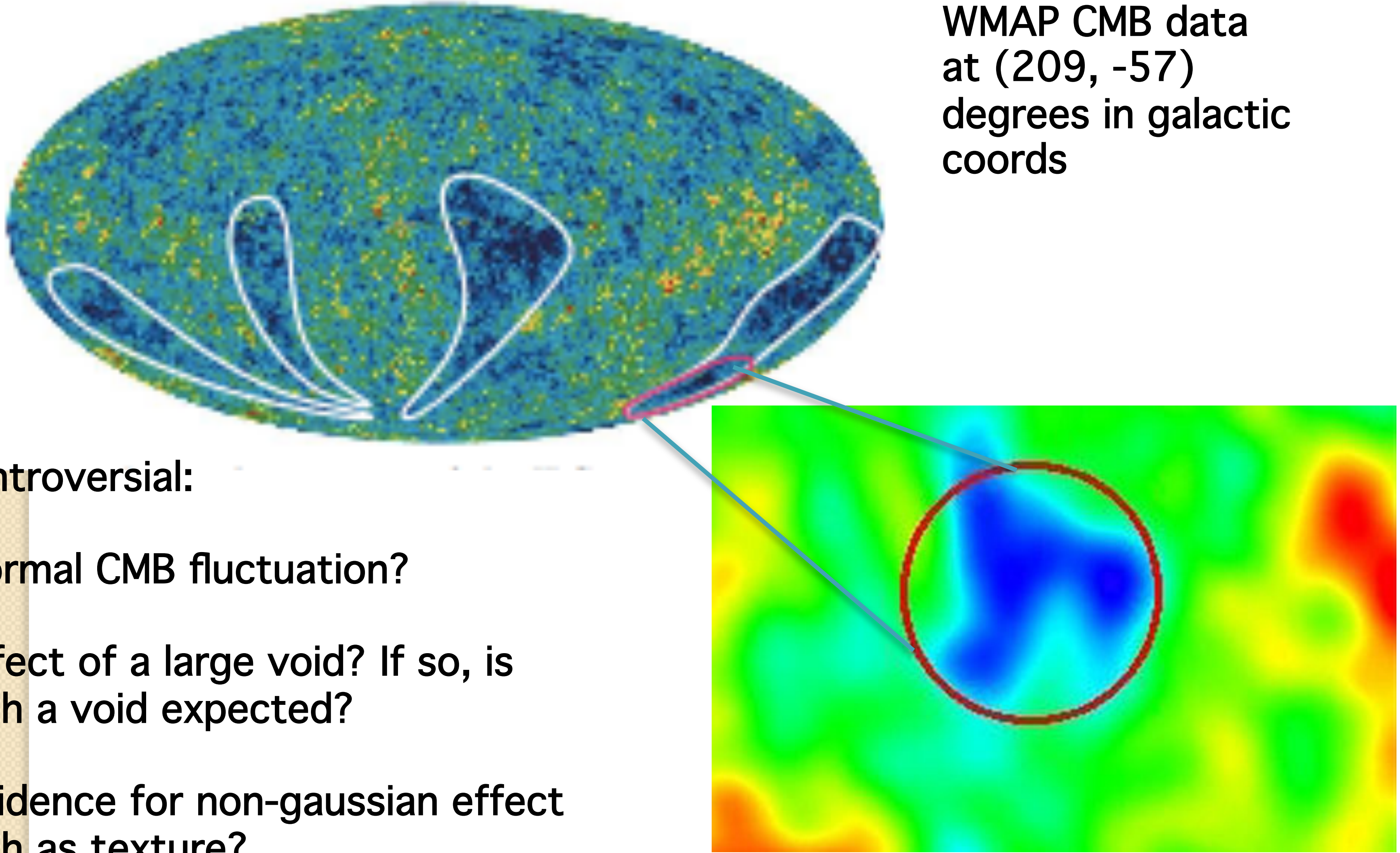
Textures

Symmetry breaking can occur in large gauge groups giving rise to more complex unstable topological defects called textures. These can also affect the CMB and structure formation.



A cosmic texture?

Cold spot II in the WMAP CMB data at (209, -57) degrees in galactic coords



Controversial:

-Normal CMB fluctuation?

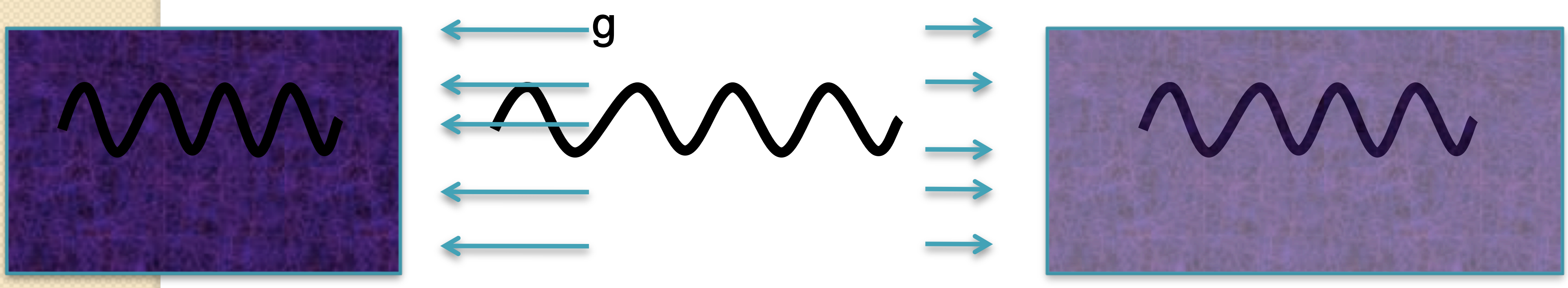
-Effect of a large void? If so, is such a void expected?

-Evidence for non-gaussian effect such as texture?

Integrated Sachs-Wolfe effect

- Photon entering large void loses energy as it climbs potential hill (gravitational red-shift).
- If void is large, it can expand significantly during the passage of the photon.
- Potential is less steep on exit, and so photon does not recover full energy, causing net red-shift and apparent cold spot.

Astrophysical Journal ***147: 73***



Photon red-shifted on entry by field of matter behind.

Photon blue-shifted less on exit

Dark Matter Density

- Stable dark matter particles (mass m_x) kept at equilibrium density by production reactions at same rate as annihilation.
- Freeze out when rate of collisions = rate of expansion. After this time (at temperature T_f), density is frozen at equilibrium value

$$n_{eq}(T_f) \propto \left(\frac{m_x}{T_f}\right)^{\frac{3}{2}} \exp\left(-\frac{m_x}{kT_f}\right)$$

- Density then reduced to present value by expansion.

- Since m_x is large, freeze-out happens at high T in radiation dominated era, when $m_x c^2$ >kT. Let $m_x c^2 = P\, kT_f$, with P=25.
- To find freeze-out point, require

$$t_{\text{coll}} = t_{\text{exp}}$$

$$t_{\text{coll}} = \left(n_{eq}\sigma_W v\right)^{-1}$$

$$t_{\text{exp}} = H(t)^{-1}$$

σ_W is annihilation cross-section (weak)

v is relative velocity of colliding particles.

- In radiation dominated era:

$$\rho_r \propto R^{-4}$$

$$\frac{\dot{\rho}_r}{\rho_r} = -4\frac{\dot{R}}{R} = -4H(t) = -4\left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} \rho_r^{1/2}$$

Integrating :

$$\rho_r = \left(\frac{3}{32\pi G}\right)\frac{1}{t^2} = \frac{4\sigma T^4}{c}$$

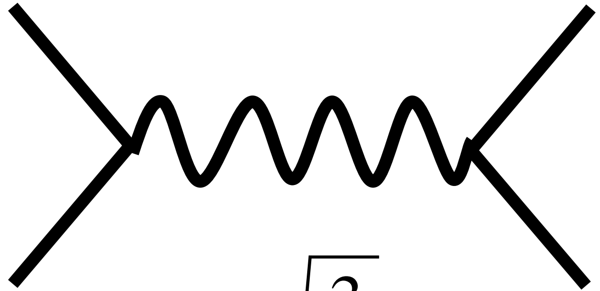
Energy density of black-body radiation

and hence :

$$H(t) = \frac{1}{2t} \qquad t = \left(\frac{1.3\,\mathrm{MeV\,s}^{1/2}}{kT}\right)^2$$

t in seconds
kT in MeV

Now consider annihilation reaction



Velocity of particles is given by

$$\frac{1}{2} m_X v^2 = \frac{3}{2} kT_f \qquad m_X c^2 = PkT_f \Rightarrow \frac{v}{c} = \sqrt{\frac{3}{P}}$$

Weak cross-section (for $m_x > m_Z/2$)

$$\sigma_W \approx \frac{\alpha_W^2}{m_x^2}$$

α_W from vertices, m_X from propagator

Note: this cross-section formula is appropriate for high mass. The behaviour is different for $m_x < m_Z/2$

Combine everything, and get density of X at freeze-out:

$$n_{eq} = \frac{1}{2\sigma_W v t} \approx 3 \times 10^{32} m_x^4 P^{-3/2} \quad m^{-3}$$

$$\rho_X = n_{eq} m_x c^2$$

$$\text{Dilution due to expansion} = \left(\frac{T_f}{T_0}\right)^3 = \left(\frac{m_x c^2}{PkT_0}\right)^3$$

$$\rho_X(0) = 4 \times 10^{-4} m_x^2 \quad \text{GeVm}^{-3} \text{ for } P = 25$$

To explain DM, want 20% of critical density:

$$\rho_X(0) = 0.2\rho_C \qquad \rho_C \approx 5 \text{ GeVm}^{-3}$$

$$\therefore\ m_X = 50 \text{ GeV}$$

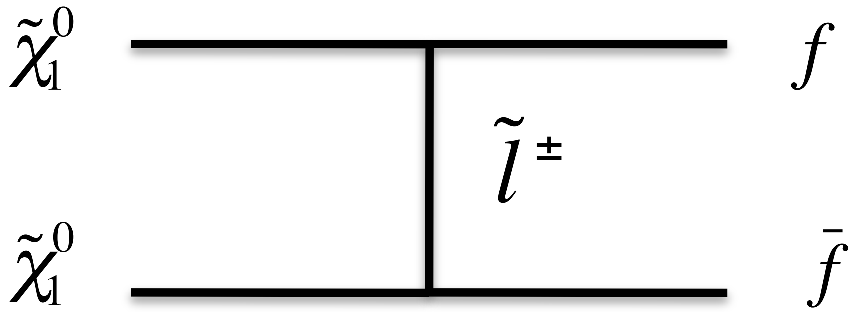
The "WIMP Miracle": weak interactions of sensible mass particles produce correct DM density now. Can make same argument for wide range of masses up to a few TeV but not larger.

WMAP + LHC

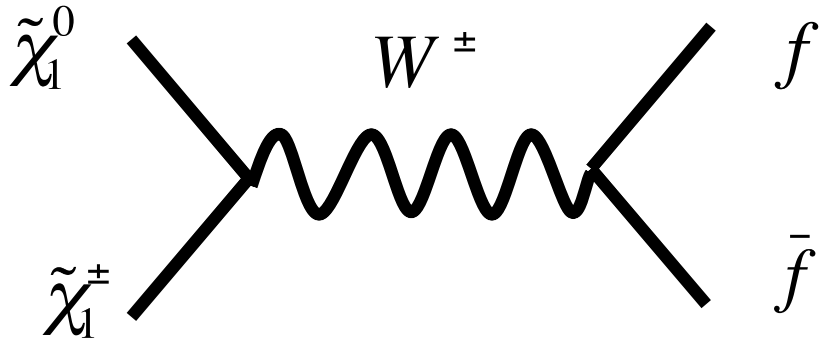
- Can run specific SUSY models, such as MSSM, and make detailed predictions of particle masses and annihilation processes.
- 105 parameters in MSSM, reduced to 4+1 in CMSSM. Most important are the base masses of the scalar and fermion states m_0 and $m_{1/2}$
- Plot allowed regions on m_0 , $m_{1/2}$ plane – thin linear regions where two particles are close enough in mass to "coannihilate".
- Compare to region of plane which the LHC experiments will be able to exclude.

Annihilation processes

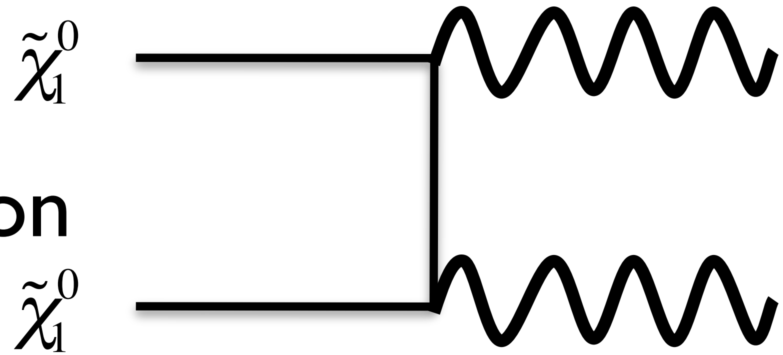
- Bulk region



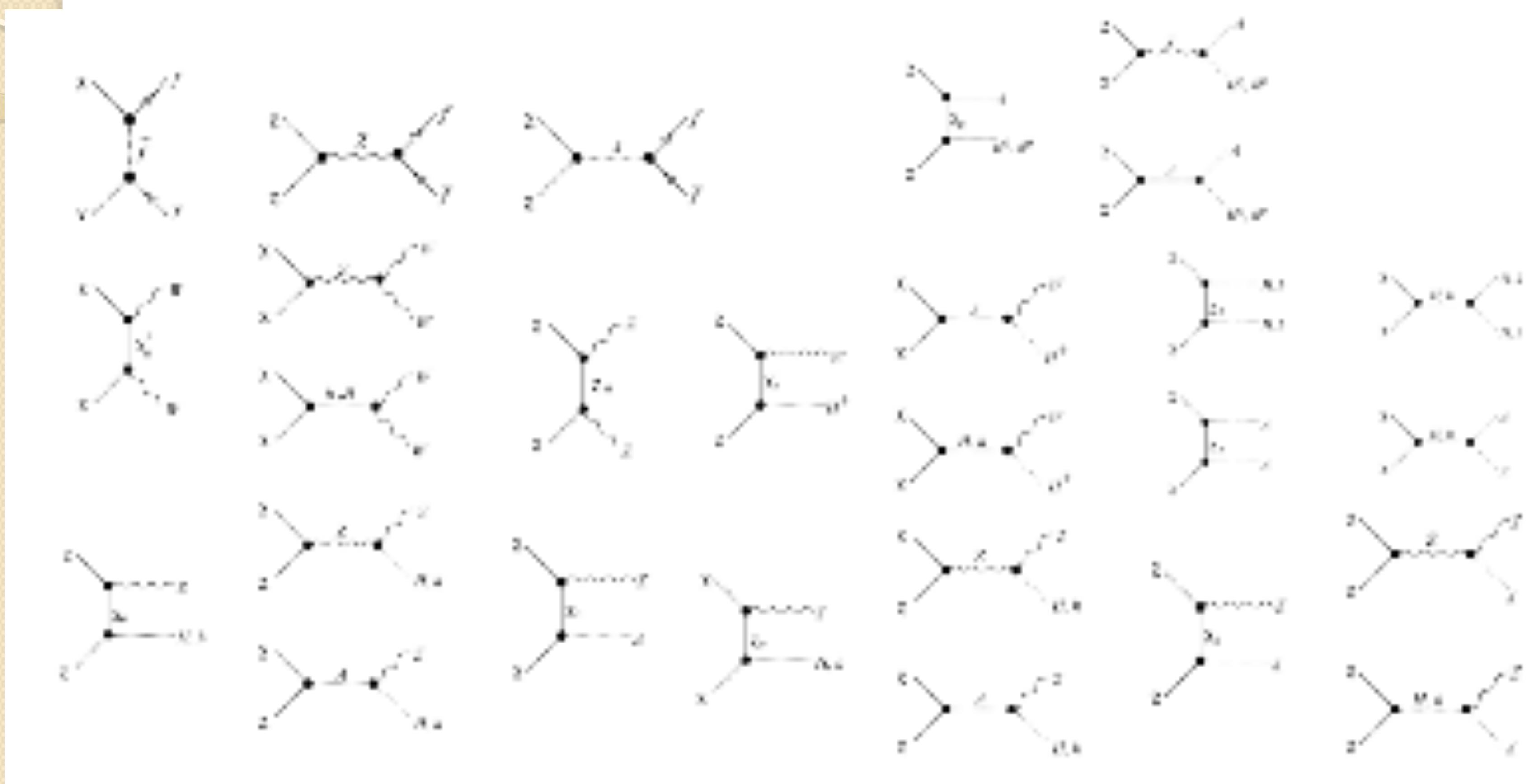
- Co-annihilation region



- Focus point region

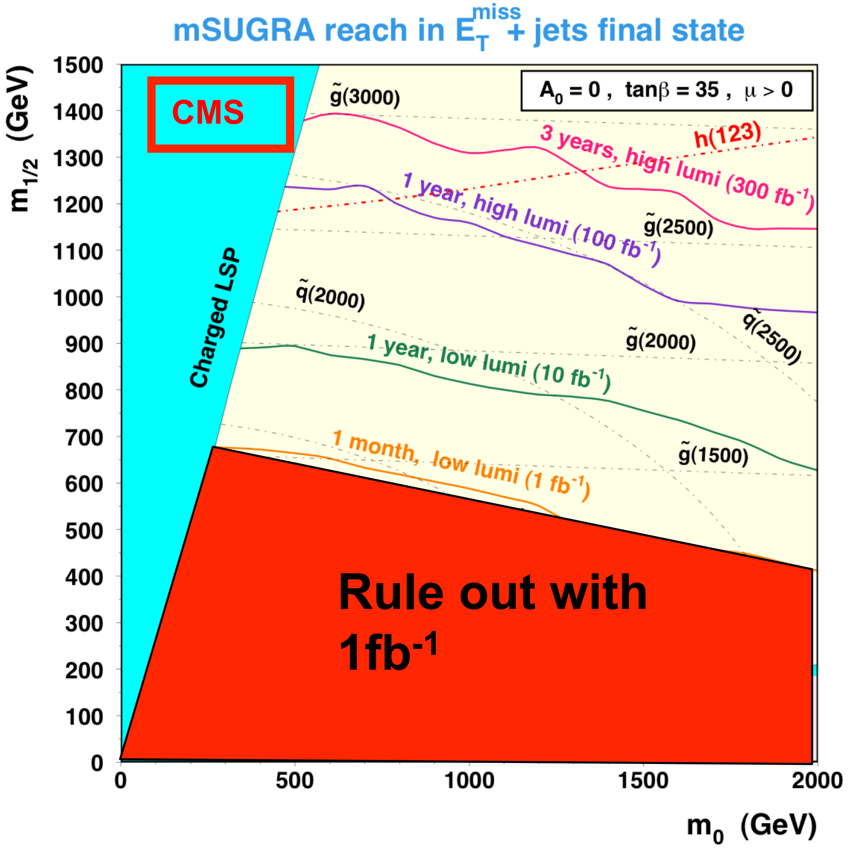


A more complete set…



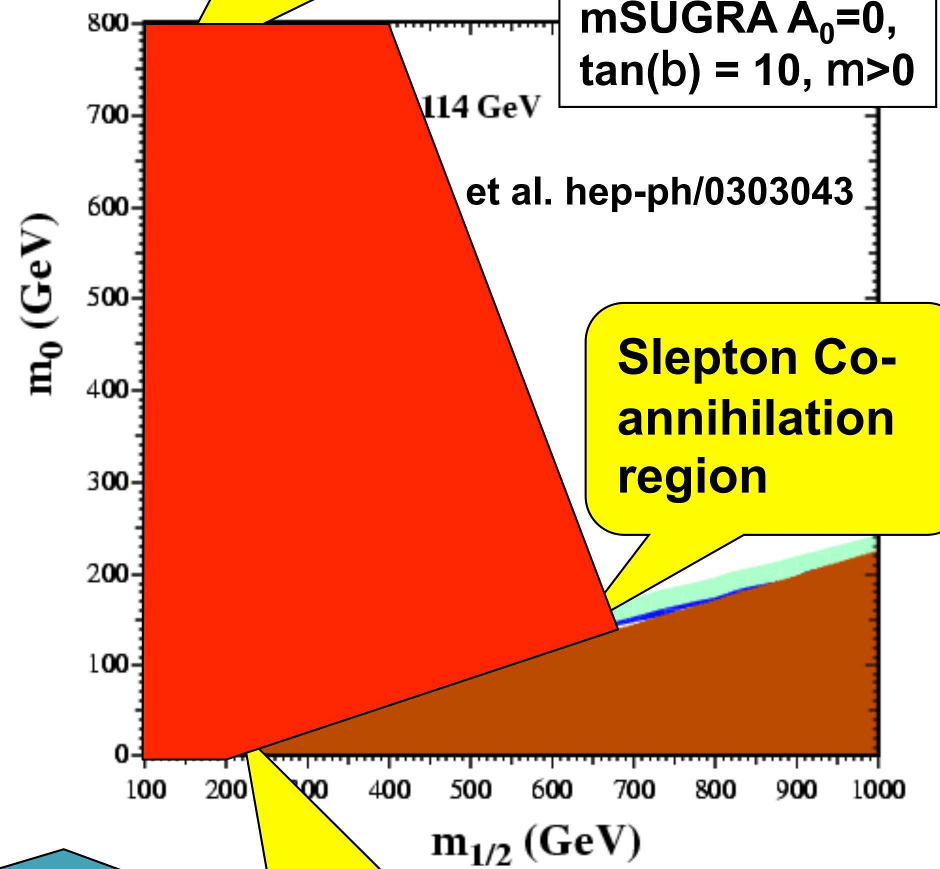
SUSY at LHC

Discovery reach of CMS experiment



WMAP constraints

'Focus point' region: annihilation to gauge bosons

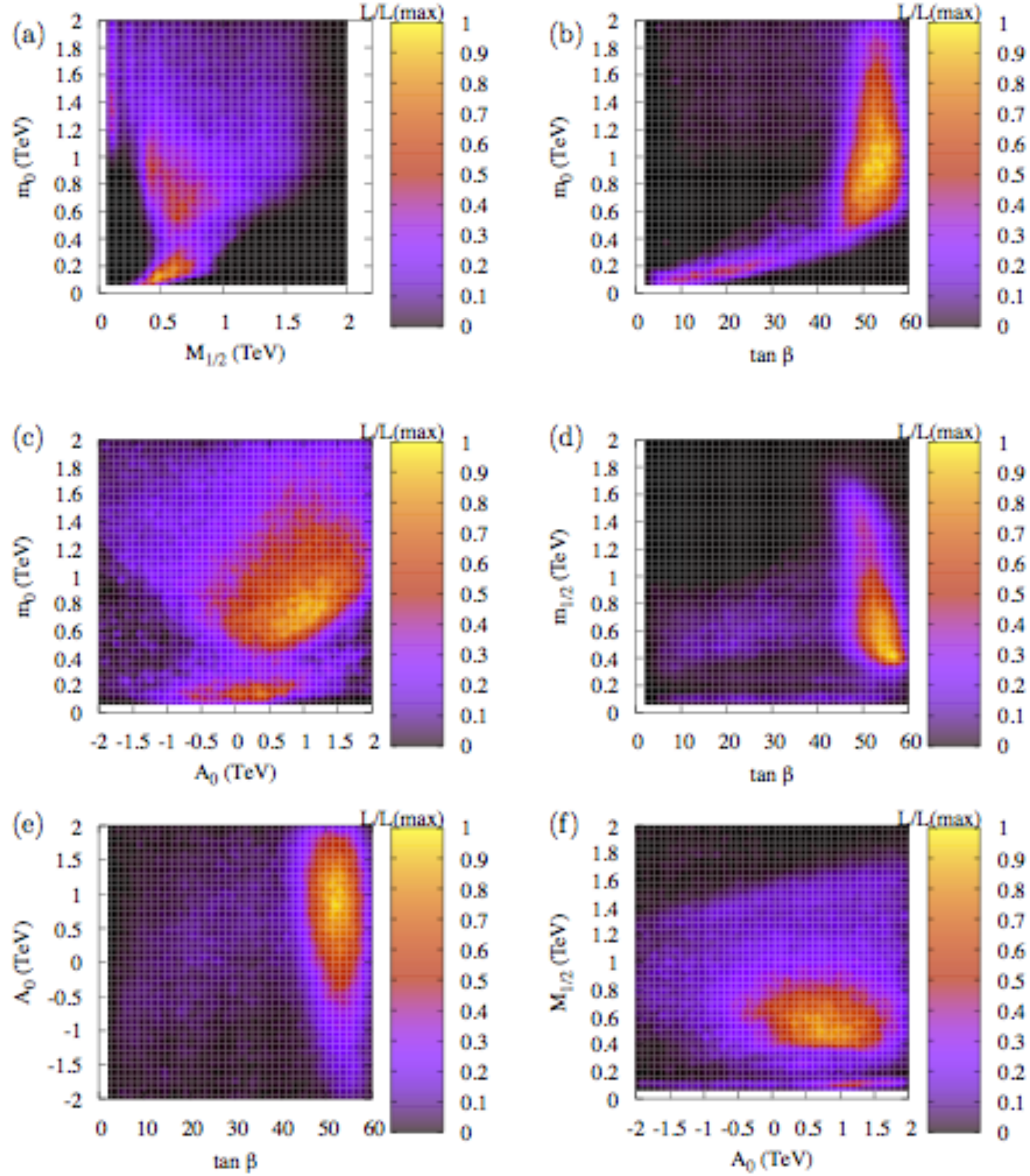


Slepton Co-annihilation region

'Bulk' region: t-channel slepton exchange

SUSY parameter space

- **Normally shown plots with just 2 parameters (m_0 and $m_{1/2}$), based on mSUGRA -> small part of real parameter space (even within MSSM)**
- **Cannot be sure that even if SUSY provides a WIMP, it is the only component of dark matter.**
- **Plots show likelihood density for different parts of SUSY parameter space (still using only 4+1 parameters).**



Allanach and Lester hep-ph/0507283

SUSY dark matter

- In order to solve the dark matter problem, need to observe candidate particle in the lab, measure cross sections and mass and show that they are consistent with WMAP/Planck data.
- Ideally, also observe particle in Milky Way directly in underground experiments, and indirectly from annihilation products.

IDENTIFYING DARK MATTER

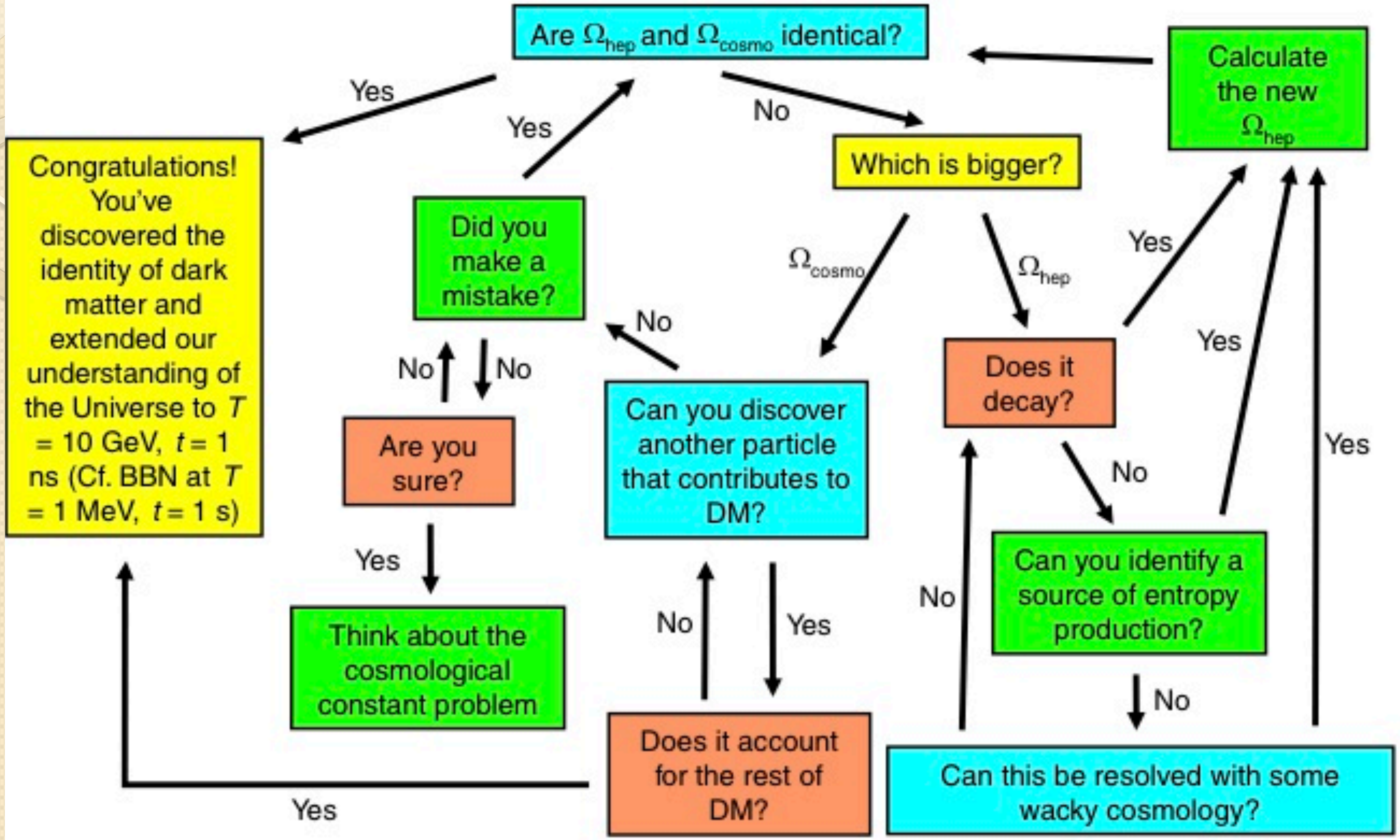


Figure 5. Flowchart illustrating the possible implications of comparing Ω_{hep}, the predicted dark matter thermal relic density determined from high energy physics, and Ω_{cosmo}, the actual dark matter relic density determined by cosmological observations.