Topics in Astrophysics

Michaelmas Term, 2010: Prof Craig Mackay

Module 5:

- Binary Stars.
- Basics, visual binaries, spectroscopic binaries, eclipsing binaries.
- Binary masses and radii.
- Supernovae, gravitational potentials, mass transfer, the Eddington limit, accretion.
- Accretion discs, cataclysmic variables.
- Accretion on to a magnetic star, types of high-energy binaries.

Binary Systems: Basic Properties

- Most stars are in binary systems. Approximately 70% of all stars are in binaries, triples etc. Single stars are not the norm. Binary stars help us to explain quite a number of astrophysical phenomena which are otherwise difficult to understand.
- Binaries provide an explanation for Type Ia supernovae. These are thought to be a consequence of accretion onto a white dwarf.
- Binaries explain why the progenitor of SN1987A was a blue supergiant: its outer envelope was lost to a companion.
- Binaries may have members which are black holes and/or neutron stars, provided they stay bound after a supernova explosion.
- Binaries are involved in some Gamma Ray Bursts (GRBs).
- Binaries are associated with millisecond pulsars.
- The evolutionary stages of the two stars in a binary are often at odds with standard stellar evolution theory. This is a consequence of mass transfer between the two components.
- Binaries are extremely important in providing information on the masses and radii of stars. They provide a critical test of stellar structure and evolution theories.

Binary Star Basics

- A ***visual binary*** (such as Sirius A + B) is one in which both stars can be seen to be orbiting about the common centre of mass. If their masses are M_1 and M_2, and they are orbiting at semi-major axes a_1 and a_2 from the common centre of mass then we know that $M_1.a_1 = M_2.a_2$ and so $M_1/M_2=a_2/a_1$.
- This means that we can work out the mass ratio of a binary even if we do not know its distance (the projection of the orbits on to the sky due to the inclination has to be taken in to account).

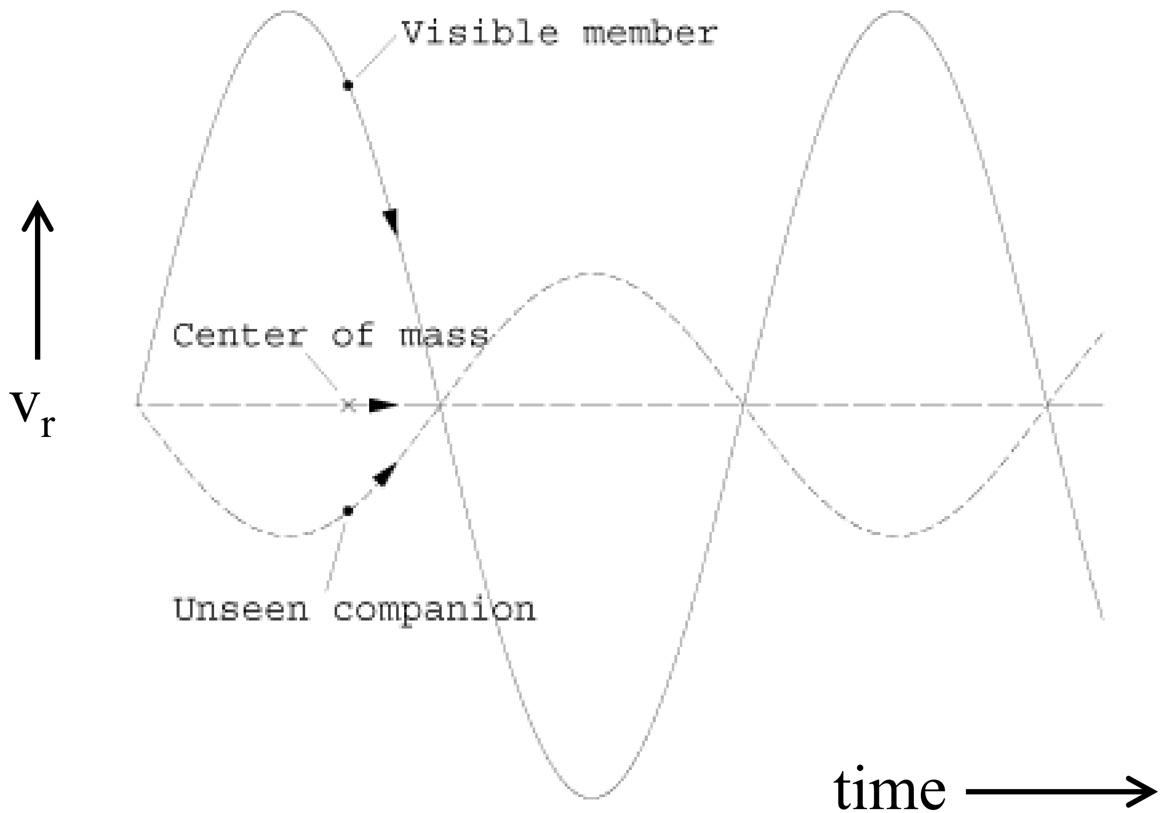
- Kepler's third law tells us that:

$$P^2 = \frac{4\pi^2 (a_1 + a_2)^3}{G(M_1 + M_2)}$$

- This means that if we know the distance to the binary we can work out the orbital radii and therefore deduce the masses of both stars. (case 1 in the table on slide 11).

Spectroscopic Binary Stars

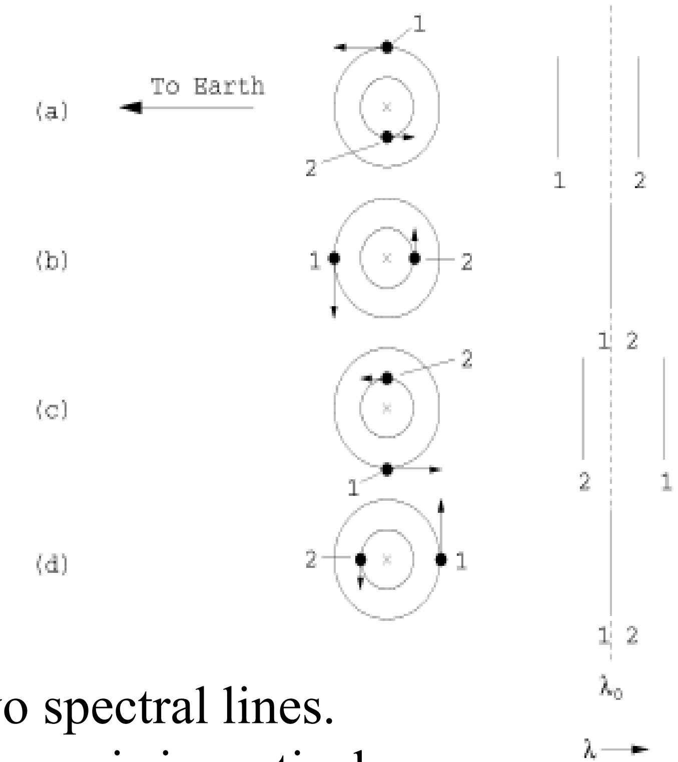
The radial velocity v_r is obtained from the observed shift of spectral lines.



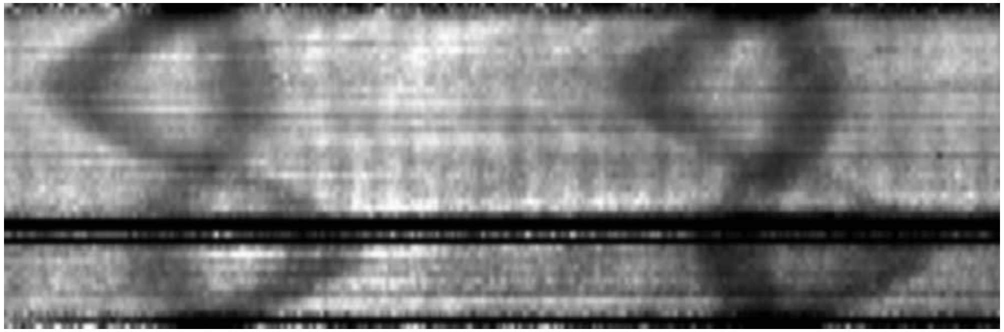
As shown, the unseen companion is the more massive component of the binary.

Spectroscopic Binary Stars

- With spectroscopic binaries (binaries that we cannot resolve into the individual stars because they are too close together or too distant) we deduce that they are binary because of a variable Doppler shift in the wavelength of the light emitted by the stars.
- The Doppler shift gives the velocity of the stars along the line of sight.
- We generally do not know the inclination of the orbit which could be anything from i = 0 (indicating face on) up to i=90 degrees (indicating edge on).
- What we actually measure is not the velocity, v, but the radial component of the velocity $v_r = v.\sin(i)$.
- If we measure the ratio of radial velocities of the two components we can deduce the ratio of the masses just as we could for visual binaries when we do not know the distance.
- Once we know the distance we can apply Kepler's third law as before and deduce the masses of the components. (see case 5 in the table).



Two spectral lines.
Time axis is vertical



Spectroscopic Binary Stars

In many systems one of the components is too faint to contribute to the spectrum (e.g. it is a neutron star, a black hole or a brown dwarfs, etc.) so we see only one component of the binary. This means that v_{2r} is unknown. However $v_{2r} = (M_1/M_2).v_{1r}$. Using Kepler's third law and

$$v_1 = \frac{2\pi a_1}{P}, \qquad v_2 = \frac{2\pi a_2}{P}$$

$$v_{1r} = v_1 \sin i, \quad v_{2r} = v_2 \sin i$$

We get

$$(M_1 + M_2)\sin^3 i = \frac{P v_{1r}^{\ 3}\left(1 + M_1/M_2\right)^3}{2\pi G}$$

This equation can be rearranged to give

$$\frac{M_2^{\ 3}}{(M_1 + M_2)^2}\sin^3 i = \frac{P v_{1r}^{\ 3}}{2\pi G}$$

The left-hand side is called ***the mass function*** and the right-hand side is what is actually observed. Since sin i < 1 and $M_1 > 0$ we get:

$$\frac{M_2^{\ 3}}{(M_1 + M_2)^2}\sin^3 i < M_2$$

 The mass function therefore gives us a lower limit to the mass of the companion

Spectroscopic Binary Stars

- We can get yet more information if we see an eclipse or a transit. This tells us immediately that i ~ 90 (except for very close binaries where the stellar radii are a significant fraction of the separation of the binary):

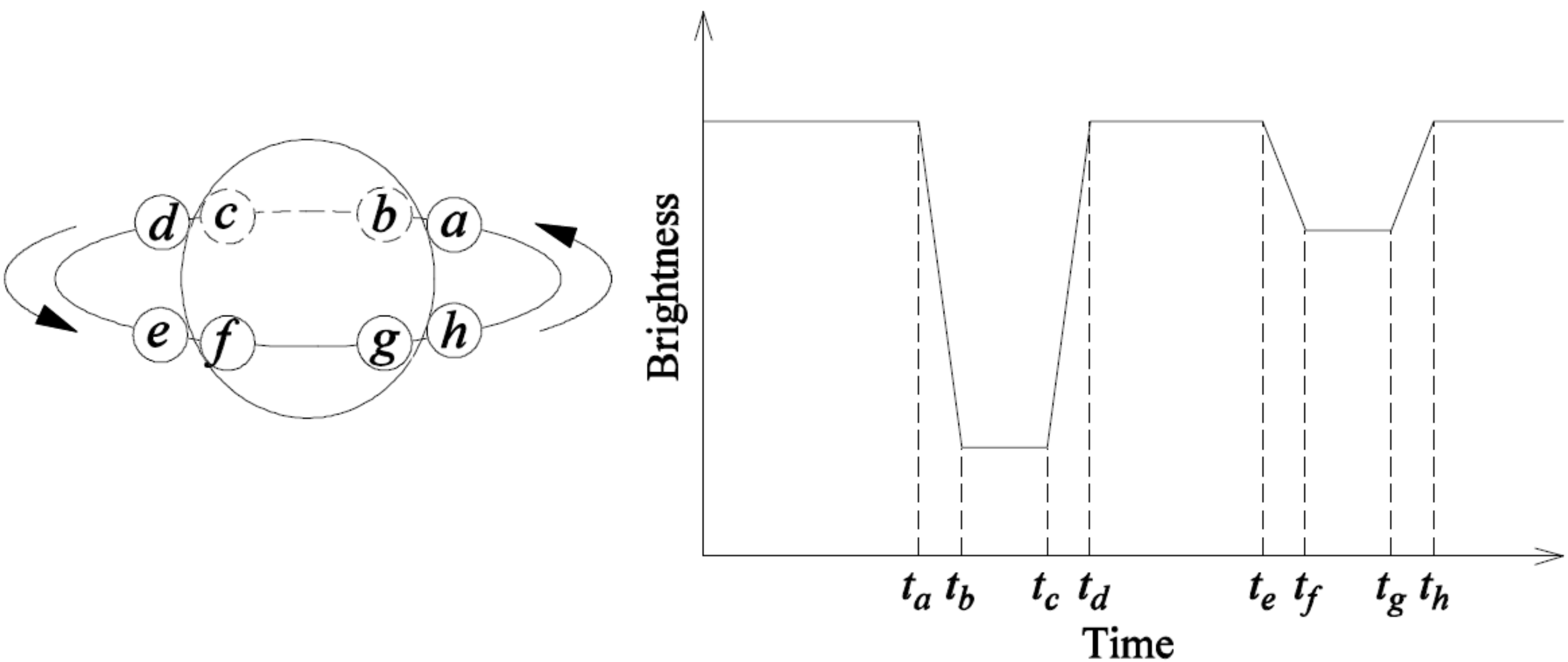
$$(M_1 + M_2) = \frac{4\pi^2 (a_1 + a_2)^3}{GP^2} \qquad \frac{v_1}{v_2} = \frac{a_1}{a_2} = \frac{M_2}{M_1}$$

- And so for an eclipsing binary when one velocity and the distance to the system are known, both masses can be determined.

- In addition, when we have the light curves of the binary we have information on the radii of the stars. (see case 7 in table on slide 11).

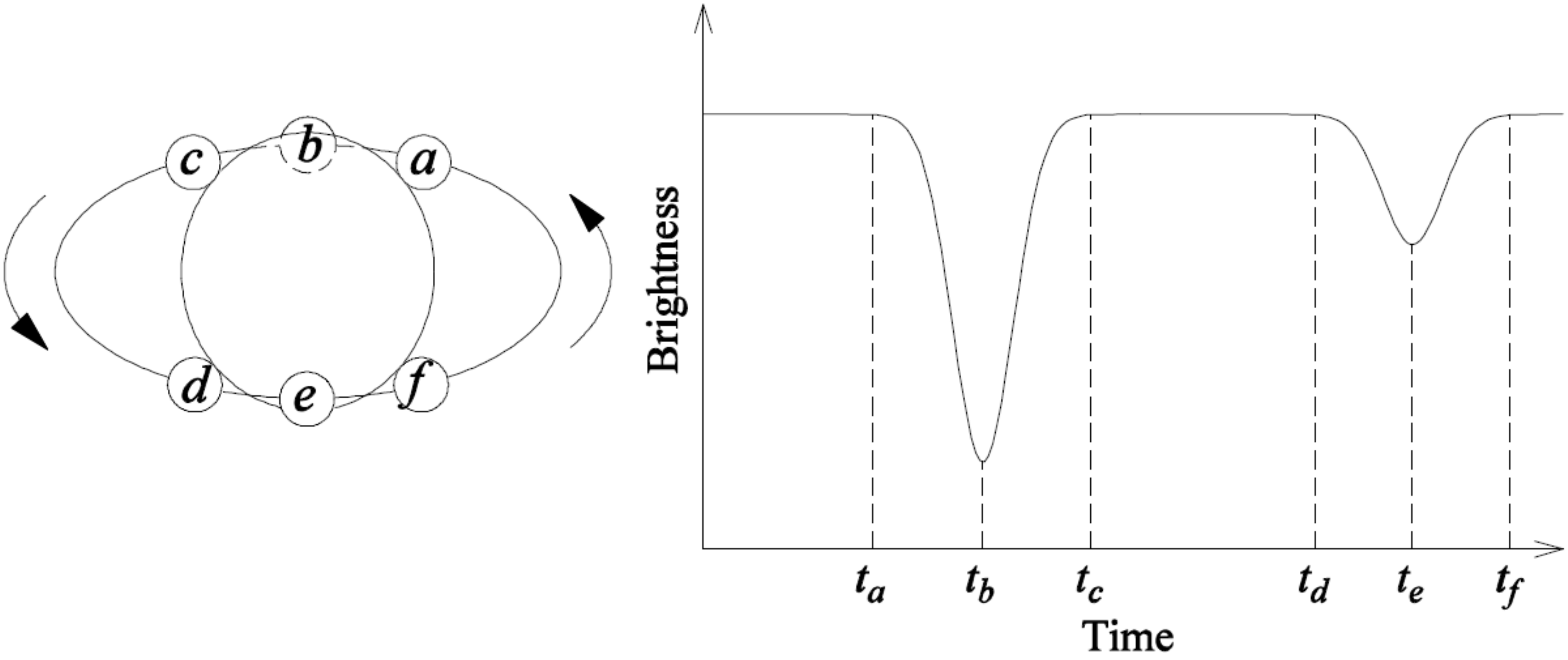
Eclipsing Binaries

This figure shows the light curve of an eclipsing binary for which i ~ 90 degrees. The times indicated on the light curve correspond to the positions of the smallest star relative to its larger companion. It is assumed in this example that the smaller star is hotter than the larger one.



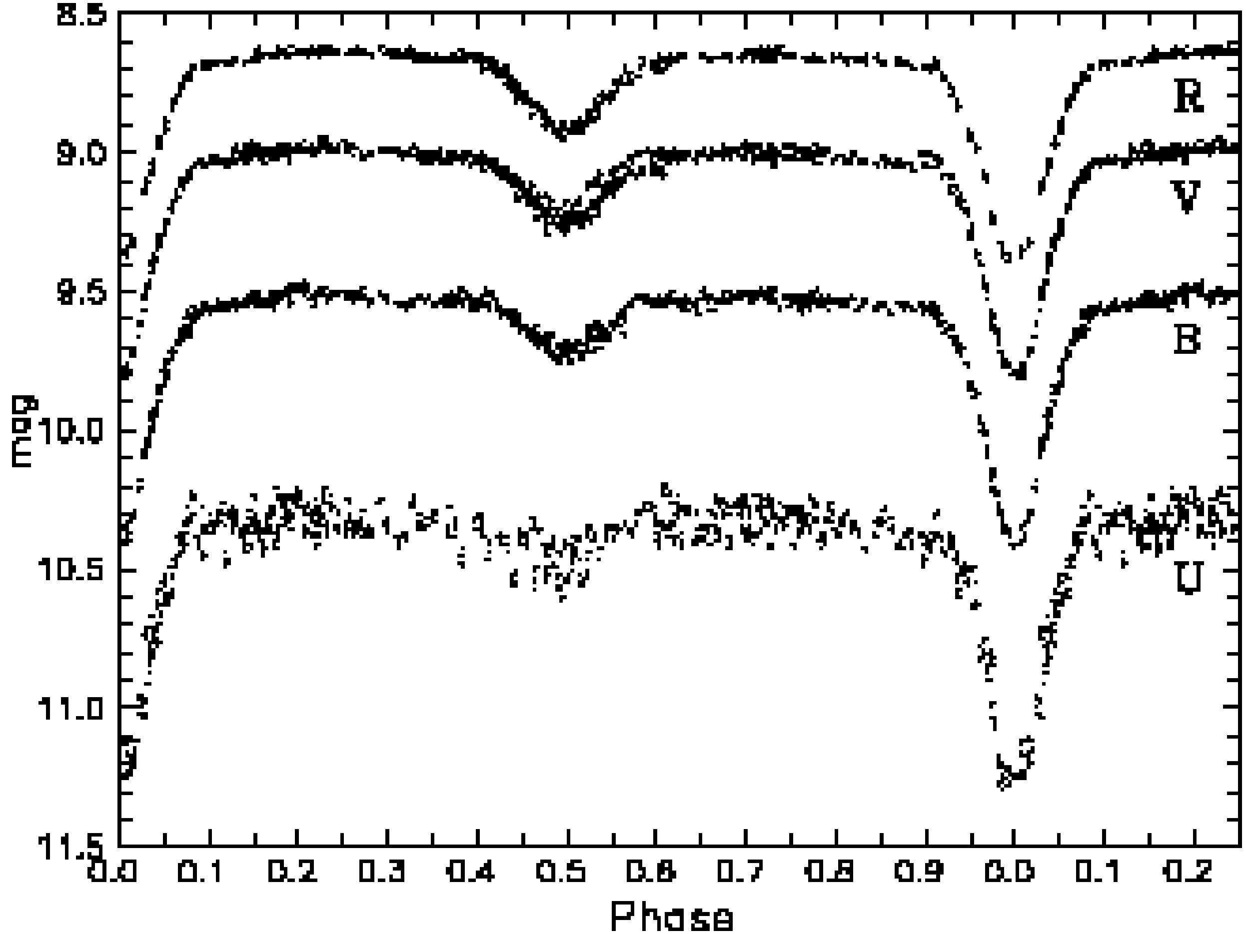
Eclipsing Binaries

This figure shows the light curve of a partially eclipsing binary. Again it is assumed here that the smaller star is hotter than its companion.



Eclipsing Binaries

This figure shows the phase diagram of the partially eclipsing binary RT Andromedae (Pribulla et al, A&A,362, 169-188, 2000).



Masses and Radii from Binary star observations

CASE	Observations					Stellar masses and sizes determined				
	astrometry		spectroscopy		photometry					
	See orbit 1	See orbit 2	V_1	V_2	Transits	Mass function	M_1	M_2	R_1	R_2
1	yes	yes	no	no	no	-	yes	yes	no	no
2	yes	no	no	no	no	-	no	no	no	no
3	yes	yes	yes	yes	no	yes	yes	yes	no	no
4	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
5	no	no	yes	yes	no	yes	yes	yes	no	no
6	no	no	yes	yes	yes	yes	yes	yes	yes	yes
7	no	no	yes	no	yes	yes	yes	yes	yes	yes
8	no	no	yes	no	no	yes	no	no	no	no

Table assumes distance to binary is known

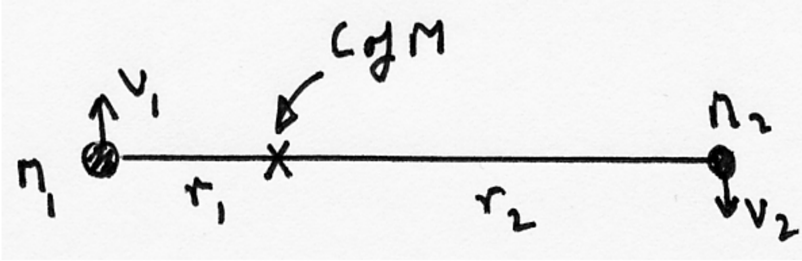
Supernovae in Binary Systems

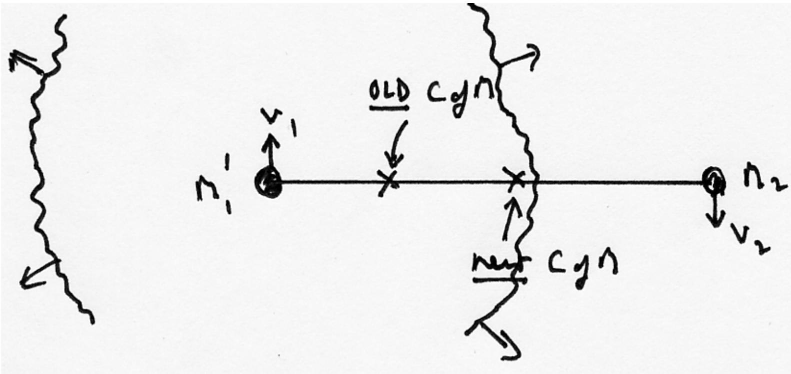
- We must expect that supernovae will be quite common in binary systems as 70% of the stars in our galaxy are in multiple systems.
- What is going to be the fate of a binary system after a supernova explosion?
- We have to calculate the velocity of the system after the supernova event:
- In the frame with the centre of mass at rest before the supernova event we have:

- $M_1\underline{v}_1 + M_2\underline{v}_2 = 0$ (total momentum of system is zero)
- $M_1\underline{r}_1 + M_2\underline{r}_2 = 0$

- M_1 then explodes. Let's suppose that $M_1 > M_2$ (for normal stars with no interchange of mass this is always true but this becomes a lot more complicated in the presence of substantial quantities of mass exchange).
- The new mass $M_1' = M_1 - \Delta M$.

$M_1 > M_2$ as drawn





Supernovae in Binary Systems.

- The binary system is no longer at rest and has a velocity v_c = the velocity of the centre of mass of the binary relative to its old one.
- The sum of the momenta of the two stars equals the total system momentum [1]

$$M_1' v_1 + M_2 v_2 = (M_1' + M_2) v_c$$

$$v_1 = -\frac{M_2 v_2}{M_1} \quad \text{and} \quad M_1' = M_1 - \Delta M \qquad \text{- [from previous page]}$$

$$>> (M_1 - \Delta M)\left(-\frac{M_2}{M_1}.v_2\right) + M_2 v_2 = (M_1 - \Delta M + M_2) v_c$$

$$>> v_c = \frac{\Delta M . M_2}{M_1(M_1 - \Delta M + M_2)}.v_2$$

- So we get the new system velocity in terms of M_1, M_2, ΔM and v_2.
- For Type Ia supernovae, $M_1' = 0$ since $\Delta M = M_1$ and $v_c = v_2$.
- If we take a representative set of parameters for a Type II supernova, for example, $M_1 = 15M_\odot$, $M_2 = 5\ M_\odot$, $\Delta M = 12\ M_\odot$ and the remnant neutron star has a mass $M_1' = 3\ M_\odot$ then we find that $v_c = 0.5\ v_2$.
- Binary star orbits can have v_2 large - many hundreds of kilometres per second.

Binaries and SN explosions

- Does the binary remain bound after the explosion?
- When one of the stars in a binary system goes supernova the mass lost in to the ISM goes past the non-exploding star in a very short time so we can assume the mass lost from the binary is instantaneous. (Gravity from a shell of matter is zero inside).
- Let M_1 be the initial mass of the exploding star and M_C its mass after the explosion.
- Let M_2 be the mass of the other star.
- Let us assume that the stars are initially in a circular orbit.
- When mass is lost in the explosion the stars will assume a different non-circular orbit around one another. If this is parabolic or hyperbolic then the binary is no longer bound.
- Let a_i and a_f be the semi-major axes of the orbit before and after the explosion respectively.

Binaries and SN explosions

The ***reduced mass*** of the binary after the explosion is given by

$$\mu_f = \frac{M_C M_2}{M_C + M_2}$$

The ratio of the binary system's before and after masses is

$$\chi = \frac{M_1 + M_2}{M_C + M_2}$$

The relative velocity of star 1 with respect to star 2 before the explosion is

$$V_i^2 = \frac{G(M_1 + M_2)}{a_i} \qquad *$$

Binaries and SN explosions

The total energy of the binary just after the explosion is simply the KE + the PE, i.e.

$$\text{Total Energy} = \mu_f \frac{V_f^{\,2}}{2} - \frac{GM_C M_2}{a_i}$$

Where V_f is the relative velocity of the system. By setting the total energy = 0 and eliminating a_i by using * we find

$$\frac{V_f}{V_i} = \sqrt{\frac{2}{\chi}}$$

In the instant just after the explosion we have $V_f = V_i$ so when the total energy is zero (i.e. the system is just unbound) we have $\chi=2$. When the total energy is > 0 the system is unbound and so the binary is disrupted if $\Delta M > M/2$.
In the example we had earlier $\Delta M = 12M_\odot$ so more than half the total mass was ejected and therefore the binary was disrupted.

Supernovae in Binary Systems.

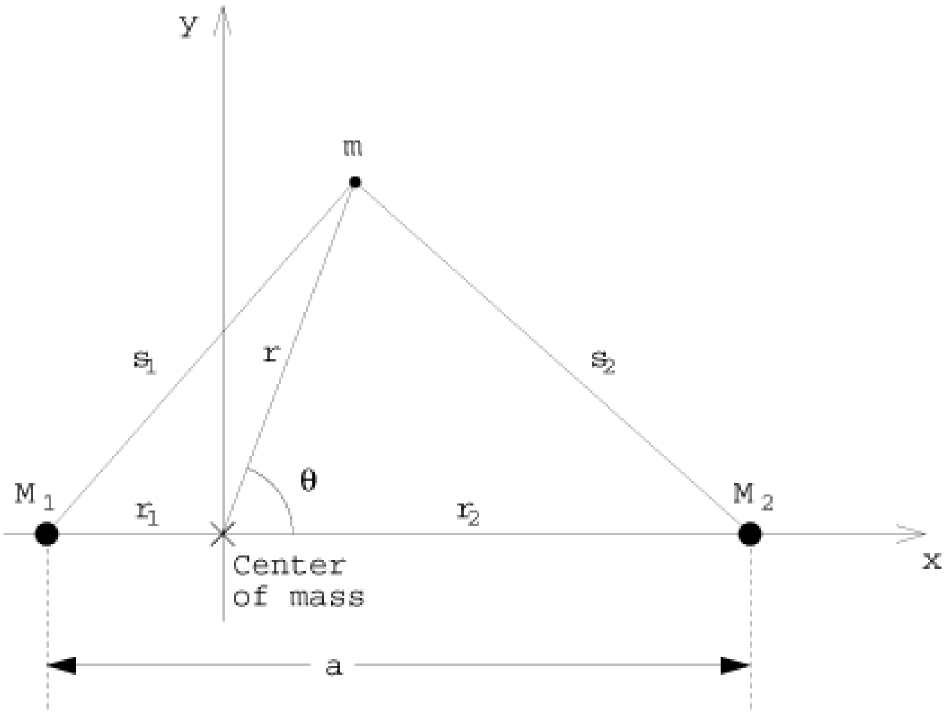
- It is also likely that the neutron star or black hole that is created by the supernova explosion is given a kick by the explosion (not taken in to account in the previous pages). The kick will be due to the explosion not having perfect spherical symmetry.
- The disruption of the binary gives us a natural explanation for high velocity neutron stars. We have evidence of relatively high pulsar space velocities and we also find that pulsars have an extended distribution with galactic latitude.
- It is also a natural explanation for higher velocity normal stars at substantial distances from the galactic plane. We find Type A stars (perfectly normal main sequence type stars) moving at velocities even exceeding 400 kilometres per second. If they were gravitationally bound this observation would be seriously discrepant with estimates of the mass of the Milky Way Galaxy.
- Many binary systems remain bound after a supernova. Therefore we expect a population of binary systems with neutron stars or black holes being one of the components.

Gravitational Potentials in Binary Star Systems

- We start by setting up a co-rotating coordinate system within which we will work out the total potential associated with the binary star system.
- The binary system consists of two masses, M_1 and M_2 separated by a distance *a*, rotating in a circular orbit in the xy plane.
- The potential at any point (x,y,z) is φ(x,y,z).
- φ is the sum of the gravitational potential due to both stars plus the centrifugal potential. We compute the centrifugal potential from the rotation of the system and therefore we get that:

$$\varphi = -\frac{GM_1}{S_1} - \frac{GM_2}{S_2} - \frac{\Omega^2 r^2}{2}$$

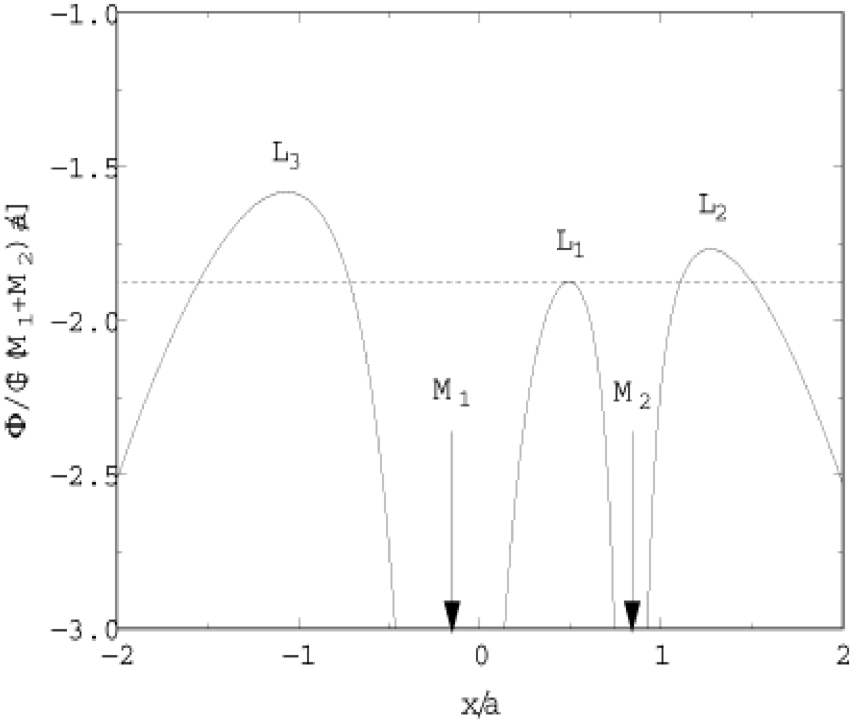
- φ is the ***effective*** gravitational potential.



Note that the point m is generally not in the xy plane so S_1, S_2 and r are functions of x, y and z.

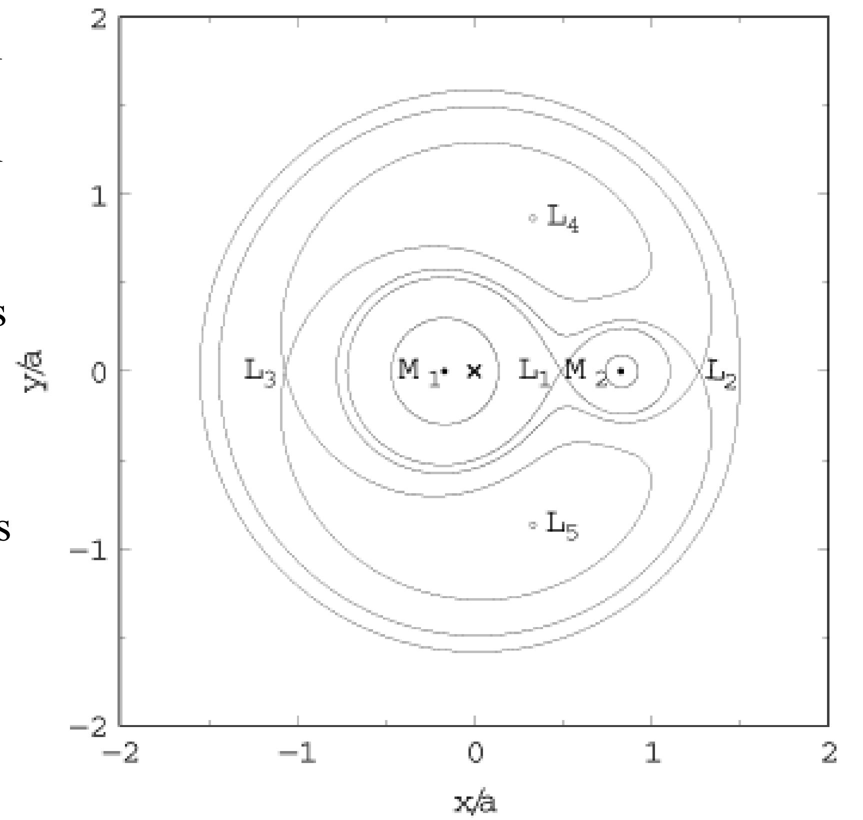
Gravitational Potentials in Binary Star Systems

- We can simply plot out the effective gravitational potential along the line connecting the two stars of the binary system.
- This is shown in the figure, with the origin as the location of the centre of mass. We see that there are points where there is no net force on the test particle ($d\varphi/dx = 0$).
- These are called the Lagrangian points and are marked L_1, L_2 ,L_3.
- These Lagrangian points occur in any binary system including, for example, the system comprising the Earth and the Moon.
- The L2 point in the Earth-Sun system is a good location for space missions which need to avoid the light from the Earth.
- This L2 position is about one million miles away from Earth. JWST, Planck, Herschel, GAIA, Darwin and TPF will all orbit the L2 position. WMAP is already there.
- SOHO and other solar observatories use the L1 point.



Gravitational Potentials in Binary Star Systems

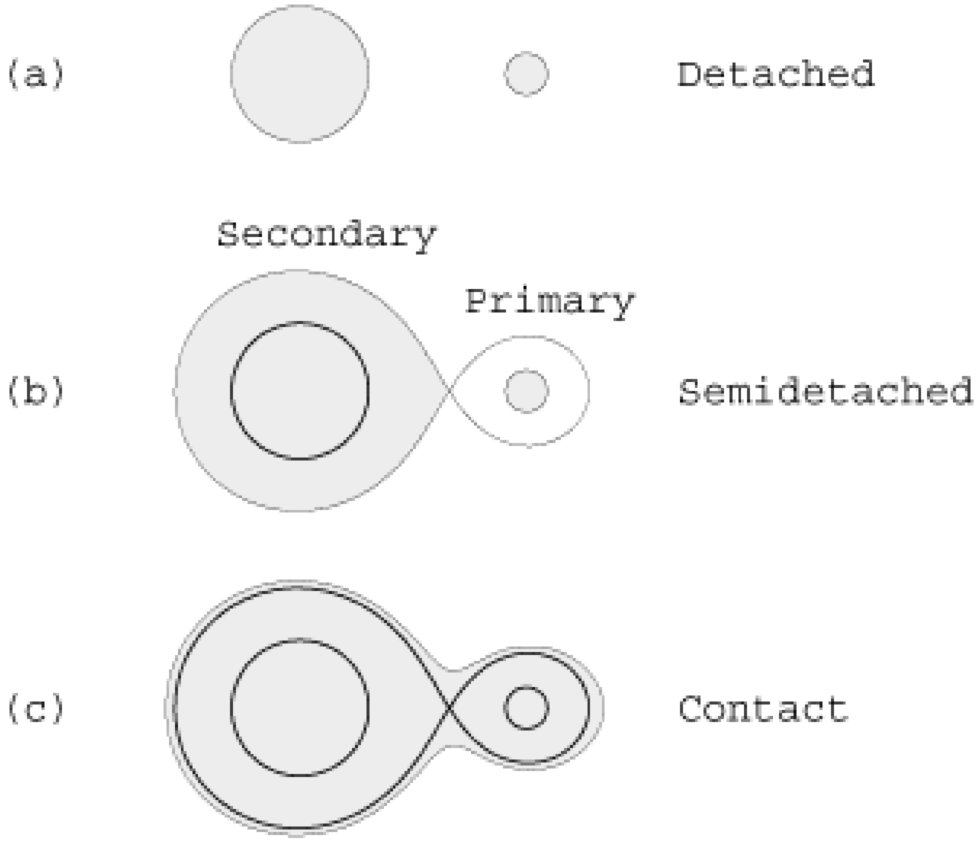
- We can plot out lines of equal effective gravitational potential on the XY plane. We can also have equipotential surfaces in 3D space. These are known as Roche equipotentials.
- Close to the individual stars the gravitational potential of the star dominates and the equipotentials are essentially spheres, just as for single stars. With two stars we have two spheres.
- At the largest radii the individual stars are essentially seen as being at a common centre of mass and again the equipotential is a single sphere.
- At intermediate distances the equipotentials are significantly more complicated.
- These equipotentials are essentially level surfaces for binary stars. If the atmosphere of the star extends to a specific equipotential then the envelope of the star will assume the shape of that equipotential.
- The appearance of the binary star system will depend on which equipotential surfaces are filled by the star's atmosphere.



There are 5 Lagrangian points. L1, L2 and L3 are saddle points. L4 and L5 are hills.

Gravitational Potentials in Binary Star Systems

- Binary stars with radii much less than their separation are nearly spherical. These are called ***detached binaries*** in which the stars evolve nearly independently. Most binaries are like this even when the stars have evolved to be red giants.
- In a ***close binary*** system if one star has expanded enough to fill the "figure-of-eight" contour then some of the gas from its atmosphere can escape through the inner Lagrangian point L_1 and be drawn on to the second star. These teardrop shaped regions bounded by this particular equipotential are called ***Roche lobes***.
- These are called ***semi-detached binaries***, and a star that fills its Roche lobe and loses mass is usually called the secondary star and its companion the primary star. The primary star may be either more or less massive than the secondary star.
- If both stars' atmospheres fill (or indeed overfill) their Roche lobes then the two stars will share a common atmosphere bounded by a dumbbell shaped equipotential surface. These systems are called ***contact binaries***.



Binary Stars: Mass Transfer

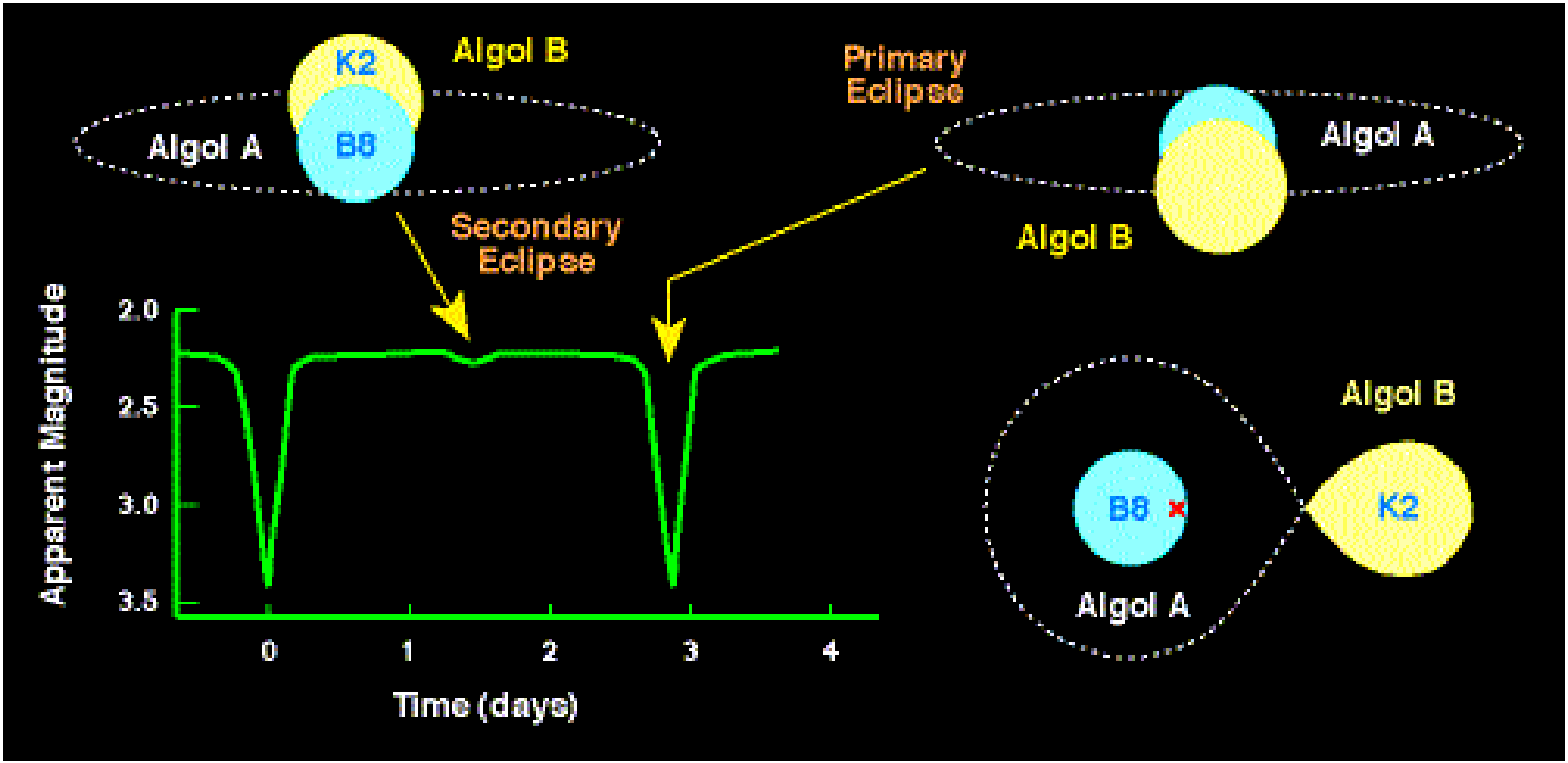
- When a star evolves away from the main sequence the radius increases substantially. For example, the sun will evolve into a red giant eventually.
- If the star is in a binary system it can expand to fill its Roche lobe. This allows mass transfer towards the companion to take place through the inner Lagrangian point L_1.
- This lets us understand the "Algol Paradox". Algol is a naked eye, double lined eclipsing binary. The masses and orbits of the components stars are well-known.
- They are:

  B8 main sequence star with $M = 3.3\ M_\odot$, $T_{eff} = 12000K$

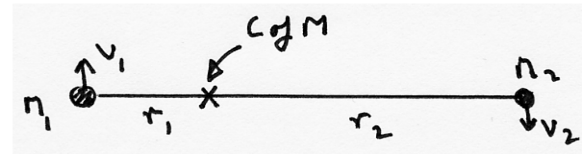
  K2 sub-giant star with $M = 0.8\ M_\odot$, $T_{eff} = 4900K$

- The rotation period is approximately 29 days.
- We know that the lifetime on the main sequence of the star goes as $\sim 1/M^{2.5}$ (luminosity $\sim M^{3.5}$, $E \sim M$).
- How can a more massive star still be on the main sequence while the low mass star has begun to evolve off the main sequence?
- It is because Algol is part of a binary system in which mass transfer is occurring. The stars are very close and the K-giant star is dumping mass onto the B-star changing its properties. Before this mass exchange started the K-giant star was the more massive star which is why it became a giant first.
- In fact Algol is part of a triple system with the outer binary period being approximately 680 days. The third star is type A5V, $M = 1.7\ M_\odot$ and $T_{eff} = 8500K$. It is far enough away that it is outside the complex gravitational potential of the compact pair.

The Algol System



Mass Transfer: Angular Momentum Conservation



- We need to look carefully at the consequences of mass transfer on the behaviour of the binary system.
- We work out the total angular momentum, J, neglecting the rotational angular momentum of the individual stars. This gives us:

(Assume that angular momentum is conserved

and that there is no mass loss from the system)

- We can work out the change in total angular momentum by differentiating [1].
- $P \propto \Omega^{-1}$
- Kepler's third law gives us $P \propto r^{3/2}$
- substitute this into the expression for the change in angular momentum (=0) and we get:
- This allows us to relate the individual stellar masses and the rate of change of the secondary star mass to the period and change in period.

$$J \approx \left(M_1 r_1^{\,2} + M_2 r_2^{\,2}\right)\Omega$$

$$M_1 r_1 = M_2 r_2$$

$$r = r_1 + r_2$$

$$M_{tot} = M_1 + M_2$$

$$>> J = \frac{M_1 M_2}{\left(M_1 + M_2\right)} . r^2 \Omega \qquad -[1]$$

$$\dot{J} = 0$$

$$\dot{M}_{tot} = 0$$

$$\dot{J} = \frac{\dot{M}_1 M_2 r^2 \Omega + M_1 \dot{M}_2 r^2 \Omega + 2 M_1 M_2 r \dot{r} \Omega + M_1 M_2 r^2 \dot{\Omega}}{\left(M_1 + M_2\right)}$$

$$P^2 = \frac{4\pi^2 r^3}{G\left(M_1 + M_2\right)}, \frac{P}{\dot{P}} = \frac{-\Omega}{\dot{\Omega}} = \frac{3}{2} . \frac{\dot{r}}{r}$$

$$>> \frac{3\dot{M}_1\left(M_1 - M_2\right)}{M_1 M_2} = \frac{-\Omega}{\dot{\Omega}} = \frac{P}{\dot{P}}, \dot{P} \propto \left|\dot{M}\right|$$

Mass Transfer: Angular Momentum Conservation

- If the star M_1 is losing mass and $M_1 < M_2$ then the orbital period and the orbital radius will increase. (i.e. when the less massive star loses mass the period and radius increase).
- Alternatively if $M_1 > M_2$, then the orbital period and the orbital radius will decrease. (i.e. when the more massive star loses mass the period and radius decrease).
- In the first case where the orbital period is increasing, the mass transfer is stable. However if the orbital period and radius are decreasing then the mass transfer becomes unstable.
- There is a form of positive feedback: as the orbit shrinks, the mass transfer increases making the orbital shrinkage more acute and so the situation is unstable.
- These effects will produce quite significant changes to the period of the orbit and to the masses of the stars involved.
- For pairs of normal stars with $M_1 > M_2$, and remembering that the more massive star will evolve fastest, we get rapid transfer of mass until $M_1 \sim M_{2,}$, where a more stable situation will be reached.

Mass Transfer: Accretion

- What happens when the material from star 1 flows, through this mechanism of accretion, onto the surface of star 2 (with radius = R_2) and the energy of the incoming material is thermalised?
- The energy supply ~ PE ~ GM_2/R_2 per unit mass, and
- the luminosity ~ $GM_2(dM/dt)/R_2 \sim 4\pi\sigma R_2^2 T^4$
- which is essentially black body radiation from the surface of the second star.
- If the situation leads to accretion onto a relatively normal star with $R \sim R_\odot$, or even onto a white dwarf, the energy liberated is not particularly high.
- However for small radii, such as a neutron star where $R \sim 10^4$ m, and $M \sim 1.4\ M_\odot$ then we find that > 20% of the rest mass energy is liberated as the material falls onto the surface of the neutron star.
- This is extremely efficient. With nuclear fusion processes we only get about 1% of the rest mass converted into energy.
- In addition, the luminosity ~ $GM\,(dM/dt)/R \sim 4\pi\sigma R^2 T^4$ will be very large for a significant (dM/dt). As the radius is tiny we get very high effective temperatures and therefore a great deal of this energy is emitted in the x-ray region of the spectrum.

Accretion: The Eddington Limit

- Material is accreted onto the surface of the star and, on impact, will generate a luminosity L.
- The energy flux at a radius r is given by $S = L/4\pi r^2$ (the units are J $s^{-1}m^{-2}$).
- The protons and the electrons in the infalling material present a cross-section to the photons. The classical radius of the electron is the Thompson cross-section = 6.7×10^{-29} m^2.
- The cross-section provided by the electrons completely dominates as the cross-section is inversely proportional to the square of the particle mass.
- However the Coulomb attraction between the electrons and the protons means the protons get dragged along by the electrons.
- There is essentially an outward radial force on the electrons which is equal to the rate at which the electrons absorb momentum from the photons.

Accretion: The Eddington Limit

- Force on an electron = (photon energy per sec)/c = σ_T.S/c = $L\sigma_T/(4\pi r^2 c)$
- so the net inward force on an electron is given by:

$$\left(GMm_p - \frac{L\sigma_T}{4\pi c} \right) \frac{1}{r^2}$$

- The Eddington luminosity is the luminosity where the inflow and the outflow are in balance i.e. where this expression (force) is zero.
- This gives us the Eddington luminosity, L_{edd},: the maximum luminosity for accretion.

$$L_{edd} = \frac{4\pi GMm_p c}{\sigma_T} \approx 1.3.10^{31} \left(\frac{M}{Msun} \right) \text{Watts}$$

- For a given observed luminosity then the assumption that L = L_{edd} gives a lower limit to the mass of the accreting object.
- If we imagine a neutron star with M = $3M_\odot$, and R = 1.4×10^4m then we find that GM/R ~ 2.9×10^{16} Jkg^{-1}, $L_{edd} = 3.9 \times 10^{31}$ W. The maximum accretion rate is then ~ 2.2×10^{-8} $M_\odot$ /year. (= L_{edd}/[GM/R])
- In this case the luminosity corresponding to the Eddington limit is L_{edd} ~ 10^5 $L_\odot$.

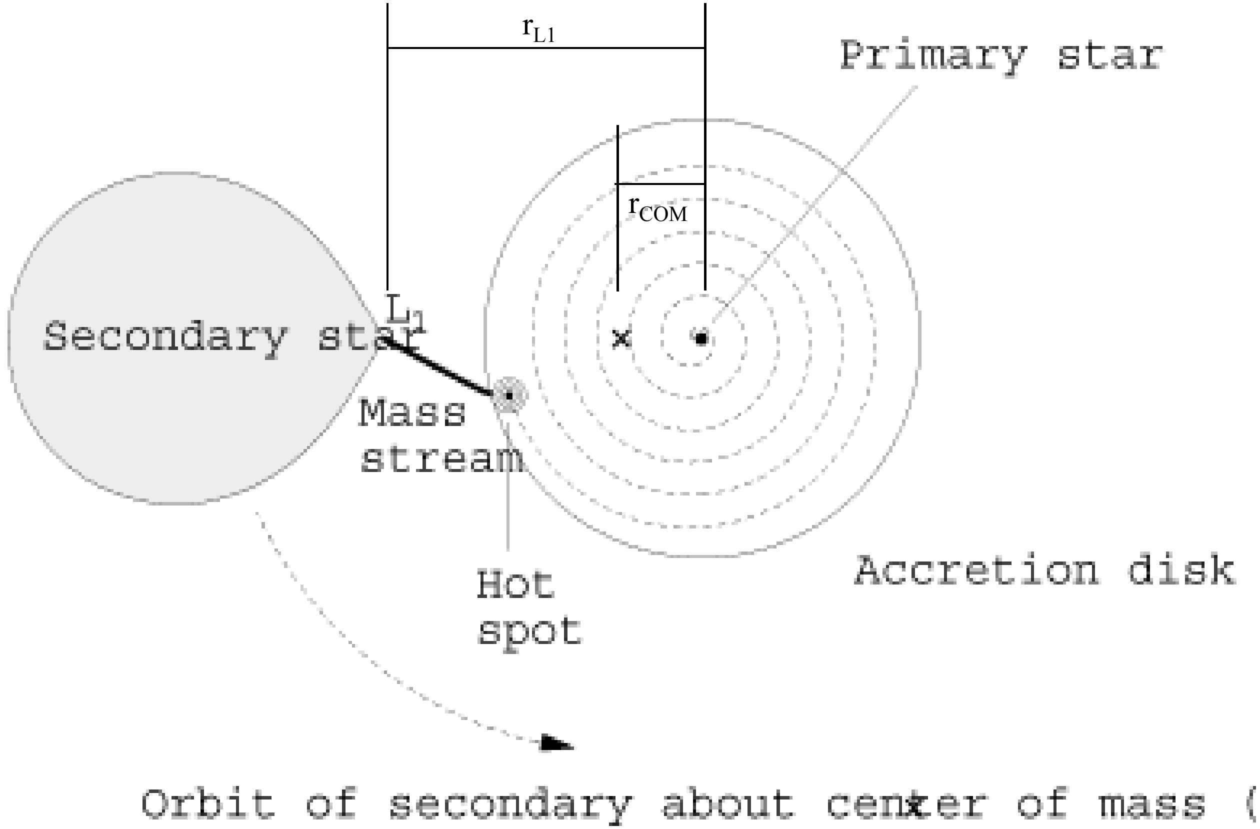
Accretion: With Low Angular Momentum

- If we have a situation where the accretion occurs with little angular momentum then the material which might start as a stellar wind from the companion star free-falls onto the surface of the accreting object.
- The velocity at impact will be $v^2 \sim 2GM/r$ where r is the distance from which the material falls.
- If the material falls on to a compact object the rapid deceleration at the surface of the star produces a shock and a very high temperature.
- At the surface of the star there will be a layer of material. It is this material which will be emitting the radiation and by cooling eventually will settle onto the surface.
- In the case of a black hole, however, the material can cross the event horizon without producing significant radiation.

Accretion: With Significant Angular Momentum

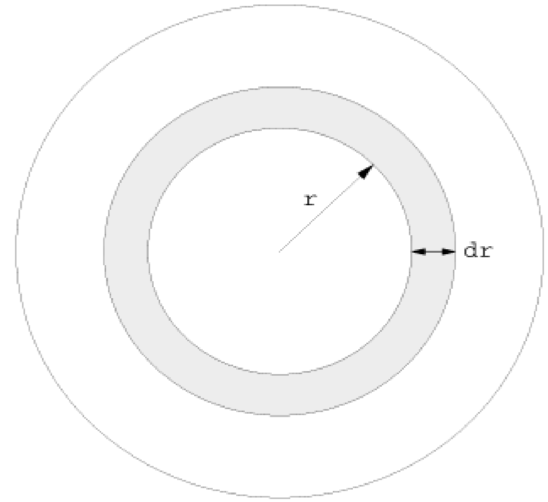
- In most cases the angular momentum of the material that is being accreted is very important.
- For binary stars the mass transfer is via the Lagrangian point ,L_1. The angular momentum is given by $m(r_{L1} - r_{com})^2 \Omega$.
- If the components masses are equal then the centre of mass and the Lagrangian point L_1 coincide and therefore there is zero angular momentum transferred.
- However, in most cases the two masses are not identical and the angular momentum can be large and the angular momentum transfer very significant.
- When angular momentum is involved, the accreted material spins into the preferred orbital plane and forms an **accretion disk**.

Accretion: With Significant Angular Momentum



The figure shows a semi-detached binary with an accretion disk around the primary star and a hot spot with mass streaming through the inner Lagrangian point to impact the disk. Inside the disk, viscosity allows matter to spiral slowly into the central compact object. The release of orbital energy in this way heats up the disk and causes it to radiate. In some stars (cataclysmic variables) the shock impact heating at the hot spot is observable.

Accretion Discs: Simple Model



- We will use a very simple model for the accretion disk: we will assume it is a steady state disk with constant mass transfer rates, with mass entering at the outer boundary in time t. The material leaving the inner boundary at the stellar surface is then (dM/dt)t.
- The disk is made up of annuli, of radius r and thickness dr. The mass of the central star is M. The *orbital* energy of the mass *m* that enters the annulus at a distance r is then E= -GMm/2r.
- Matter spirals inwards so that as the radius decreases the gravitational energy also decreases and the energy will be liberated as radiation.
- Conserve energy within one annulus.

$$m = \dot{M}\,t$$

$$\mathrm{dE} = \frac{\mathrm{d}}{dr}\left(\frac{-GMm}{2r}\right)dr = \frac{GM\ddot{M}}{2r^2}.t.dr = tdL$$

- If the disk radiates as a black body then (and remembering it is a disc and not a sphere) $dL = 4\pi\, r dr\, \sigma\, T^4$ and we find that:

$$T = \left(\frac{GM\ddot{m}}{8\pi\sigma R^3}\right)^{\frac{1}{4}}\left(\frac{R}{r}\right)^{\frac{3}{4}}$$

- Where T(r) is normalised to the radius R of the accreting object.

Accretion Discs: Simple Model

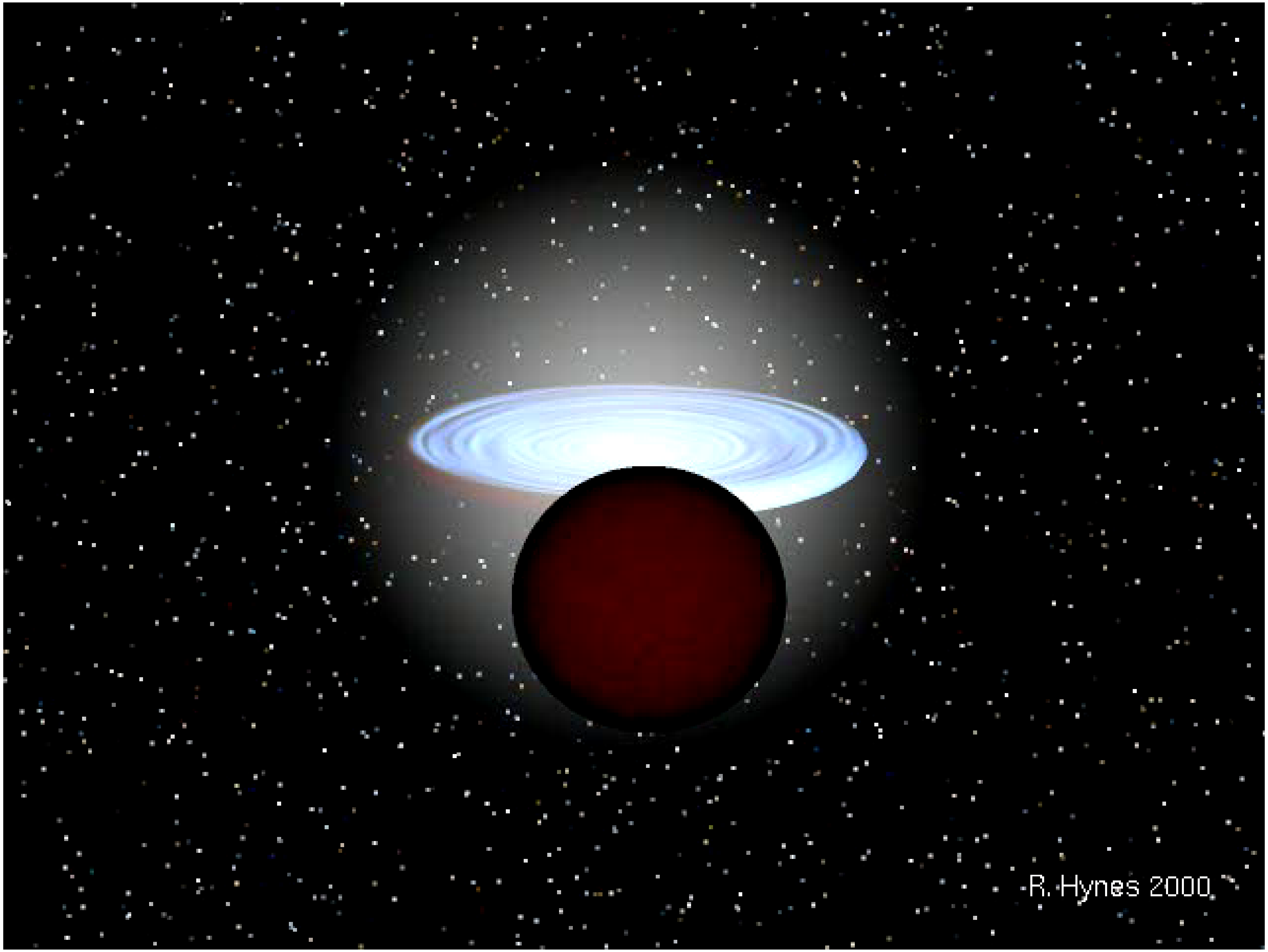
- For an individual ring we work things out as follows:
- We integrate to give the total luminosity of the ring between r = R and infinity and we find that:

$$dL = 4\pi r dr \sigma T^4 = \frac{GM\dot{m}}{2r^2}.dr$$

$$L = \frac{GM\dot{m}}{2R}$$

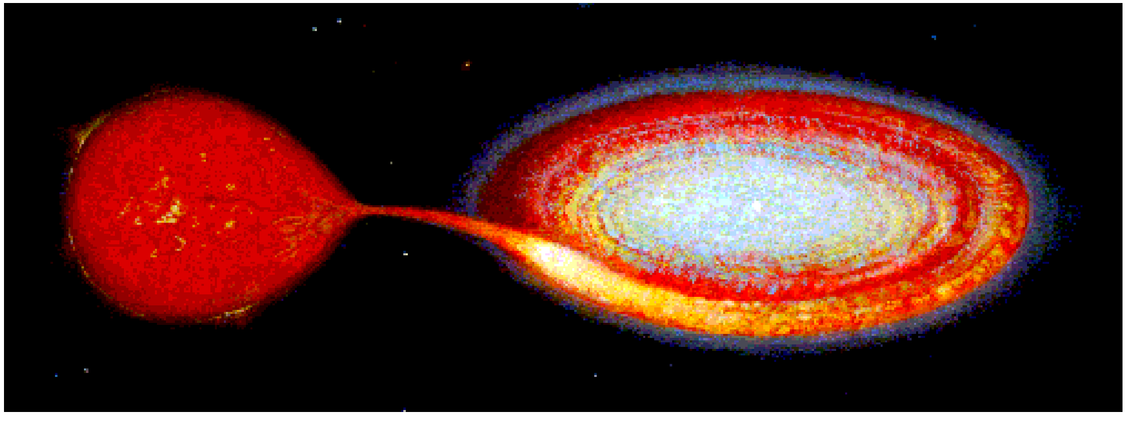
- This is half the energy that you get from dropping material directly onto the central object (material is orbiting – not freefalling). The remainder of the energy goes into the boundary layer between the disk and the star and is deposited onto the star.
- We now have estimates of the radial temperature dependence and the luminosity of the disk using a very simple model.

Accretion Discs: Simple Model



Z Chamaeleontis: Accreting White Dwarf

- Z Cha is one of approximately 400 dwarf novae that are known.
- It is a very close binary star consisting of a 0.85 $M_\odot$ white dwarf, radius $R = 9.5 \times 10^{-3} R_\odot$ and an M6 main sequence companion of 0.17 $M_\odot$. The less massive star is losing mass and the period and radius increase.
- The period is 6451 seconds and therefore the separation of the stars is $a = 5.2 \times 10^8$m or 0.75 $R_\odot$.
- The luminosity of the disk = $GM(dM/dt)/2R \sim 7 \times 10^{26}$ W and the calculated mass loss rate is therefore $(dM/dt) = 1.3 \times 10^{-9} M_\odot \, yr^{-1}$.
- It is a "cataclysmic binary".
- It experiences a sudden increase in the rate of mass loss through the disk. This causes an increasing luminosity by a factor of 10-200 for between five and 20 days. The periodic outbursts are separated by an interval of 30-300 days.



(Graphic by Dale Bryner)

Accretion onto Magnetic Stars

- Most stars have weak magnetic fields. When these stars collapse to form a white dwarf or a neutron star then the magnetic fields become very large.
- The material that is accreted onto a star is very hot and therefore fully ionised. As such it is a conducting plasma and the magnetic field has a considerable influence on the way that the material is accreted onto the surface of the compact star.
- Because of its internal magnetic field, white dwarfs and neutron stars are effectively giant magnetic dipoles with $B \propto R^{-3}$ (i.e. the field strength varies inversely with the cube of the distance from the centre of the star).
- This dipole field dependence means that the magnetic field is very strong at small radii.
- As with all charged material it has a strong preference to travel along magnetic field lines rather than to cross them. This means that the magnetic field channels accreting material along the field lines and that material is drawn down onto the magnetic poles of the star.
- We can get some estimate as to when the magnetic field becomes important.

Accretion onto Magnetic Stars

- The first term here is the energy density (the magnetic pressure). The second term is the kinetic energy density or ram pressure which we can write (third term) in terms of the accretion rate, the mass and the radius.

$$\frac{B^2}{2\mu_o} \equiv \rho \mathrm{v}_{ff}^{\ 2} = \frac{\dot{M}}{4\pi R^2}\sqrt{\frac{2GM}{R}}$$

- The magnetic dipole moment M_D is given by: (just as for pulsars).

$$\frac{BR^3}{2}\left(\frac{4\pi}{\mu_o}\right) = M_D$$

- And we can use this to rewrite the magnetic energy density as

- Equate the two expressions and solve for R to derive the radius, R_m, at which the magnetic pressure equals the ram pressure.

$$>> \frac{B^2}{2\mu_o} = \frac{M_D^{\ 2}\mu_o}{8\pi^2 R^6} = \frac{\dot{M}}{4\pi R^2}\sqrt{\frac{2GM}{R}}$$

- We ignore constants and note that (for accretion) L is proportional to (dM/dt):

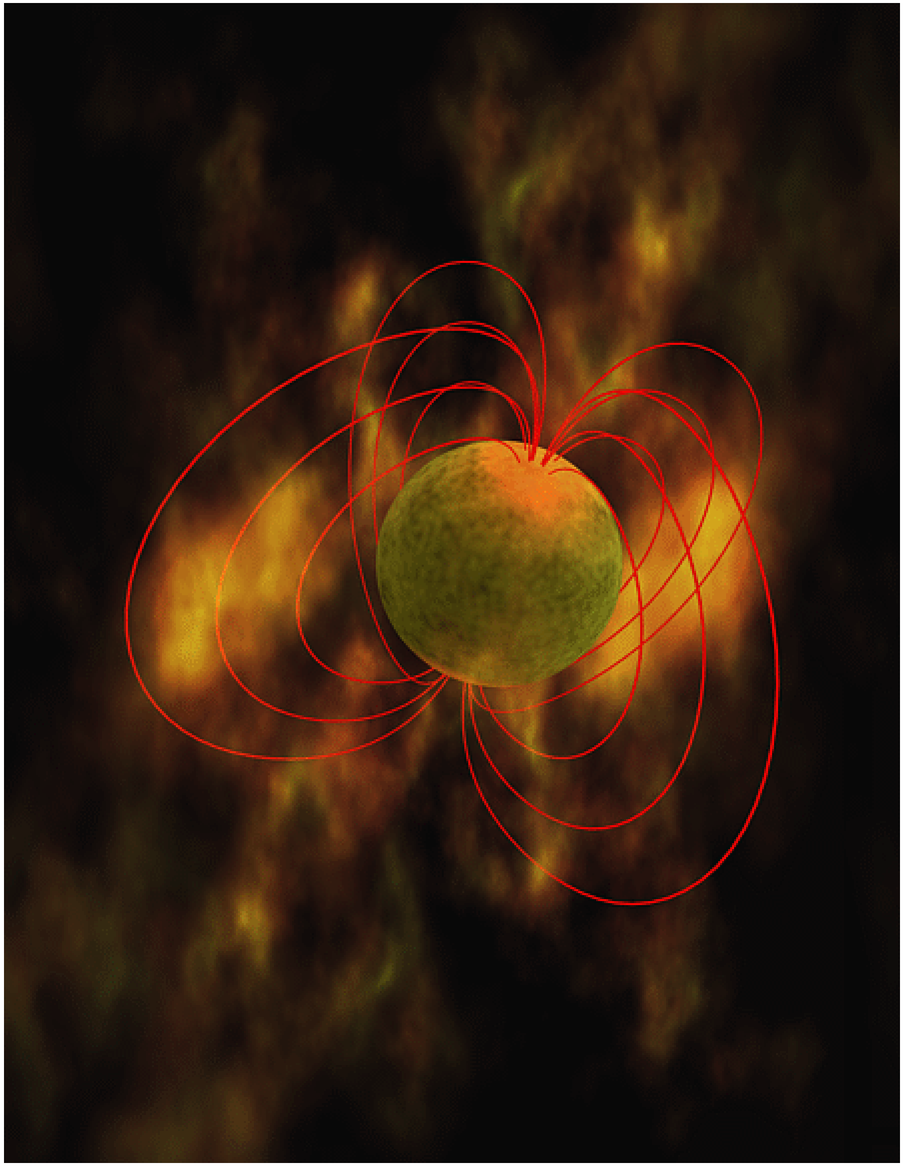
$$>> R_M = M_D^{\frac{4}{7}} . \dot{M}^{\frac{-2}{7}} (2G)^{\frac{-1}{7}} M^{\frac{-1}{7}} \left(\frac{2\pi}{\mu_o}\right)^{\frac{-2}{7}}$$

- R_m is the radius within which the magnetic field dominates the motion of the accreting material.

$$>> R_M \propto L^{\frac{-2}{7}} M_D^{\frac{4}{7}}$$

Accretion onto Magnetic Stars

White dwarfs where the B field is very strong and the accretion is channelled via the stellar poles are known as Polars.



Schematic of a POLAR

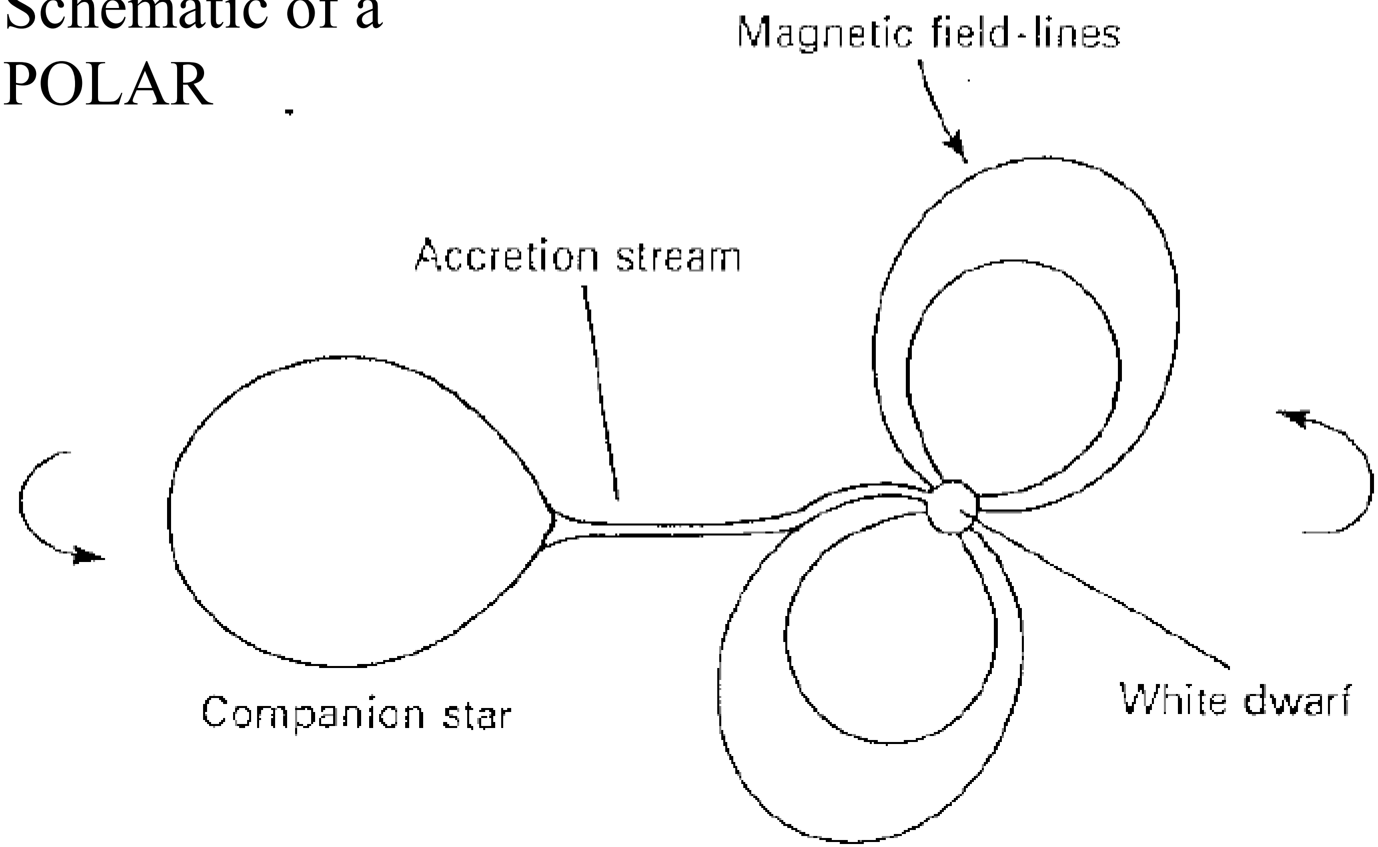


Figure 43. Schematic view of an AM Herculis system. The rotation of the strongly magnetic ($\gtrsim 10^7$ G) white dwarf is locked to that of the binary ($P \lesssim$ 4 h). No accretion disc forms, matter impinging directly on the magnetosphere and following field-lines down to the white dwarf surface.

Accretion onto Magnetic Stars

- If the accreting material forms a disc around the star then angular momentum can be transferred from the disc to the accreting object.
- If the disc has a significant amount of angular momentum then it is possible to "spin-up" the accreting object (i.e. the central star).

- LHS of [1] is the gain in angular momentum of the star. RHS of [1] is the gain in angular momentum of the disc due to material being injected into it at a particular radius R_A with a Keplerian velocity. (We assume the angular momentum of the disc is constant as is the total angular momentum).
- We can assume that the star's moment of inertia, I, is approximately constant because the mass accreted is small compared to the mass of the star.
- Getting dΩ/dt in terms of the period P and substitute into [1] we get [2].
- [2] tells us that if the accretion rate is positive then the star spins faster.
- Letting R_A equal to the accretion radius for a magnetic star R_M and $L \propto dM/dt$ we get [3].

$$I\dot{\Omega} = \dot{M}\sqrt{GMR_A} \qquad [1]$$

$$\frac{\mathrm{d}\Omega}{dt} = \frac{d}{dt}\left(\frac{2\pi}{P}\right) = -2\pi\frac{\dot{P}}{P^2}$$

$$\dot{P} = \frac{-P^2\dot{M}\sqrt{GMR_A}}{2\pi . I} \qquad [2]$$

$$R_A \approx R_M \propto \mathrm{M_D}^{\frac{4}{7}} . \mathrm{L}^{\frac{-2}{7}}$$

$$\frac{\dot{P}}{P} \propto -\frac{P . \mathrm{M_D}^{\frac{2}{7}} . \mathrm{L}^{\frac{6}{7}}}{I} \qquad [3]$$

Accretion onto Magnetic Stars

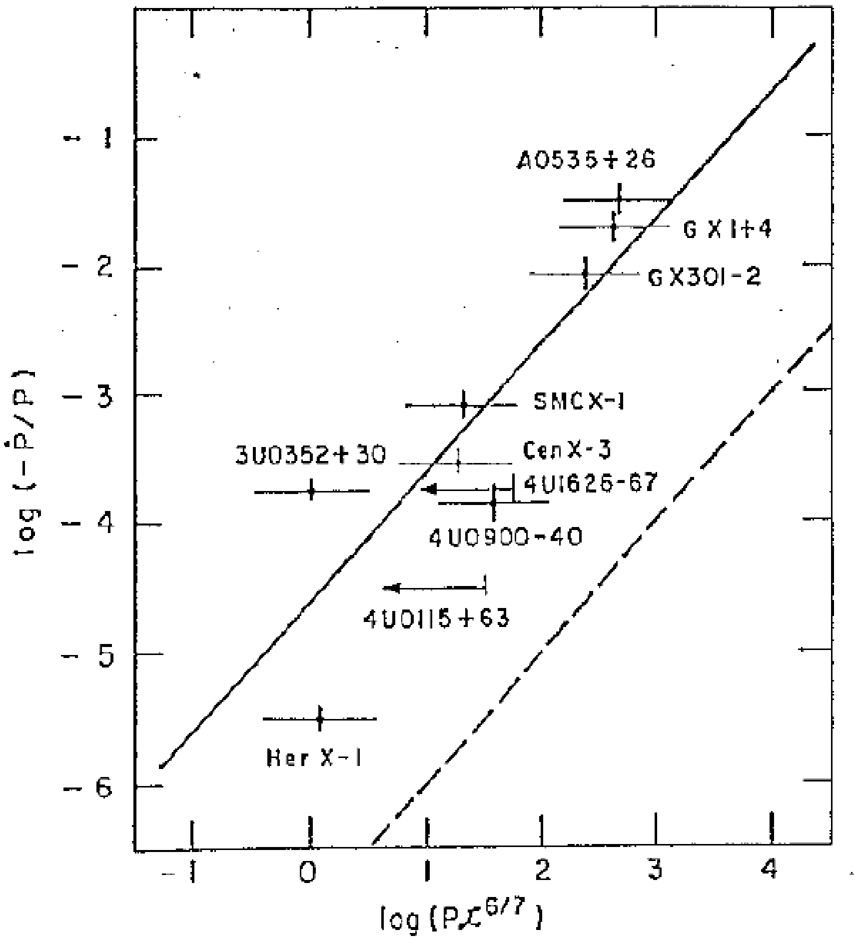


Figure 42. Observed values of the spinup rates $\dot{P}_{pulse}/P_{pulse}$ plotted versus P_{pulse} $L^{6/7}$ for ten pulsating binary X-ray sources. The solid line is the best fit straight line with logarithmic slope 1, and is in good agreement with equation (6.16), the theoretical prediction for neutron stars. The dashed line is the expected relation for a 1 $M_{\odot}$ white dwarf. (Reproduced from Rappaport & Joss in *Accretion-Driven Stellar X-ray Sources*, eds. W. H. G. Lewin & E. P. J. van den Heuvel (1983), Cambridge University Press.)

FRANK KING p12

- This plot shows good agreement between equation [3] and observations of spin-up rates for neutron stars (pulsars).
- With white dwarfs the fields are too weak, and the moments of inertia, I, too large for useful observations to be possible

$$\frac{\dot{P}}{P} \propto -\frac{P.M_D^{\frac{2}{7}}.L^{\frac{6}{7}}}{I}$$

The Binary Star Zoo

- There are many binary star types.
- The precursor of a semi-detached binary is a detached binary with two main sequence stars.
- The components are not compact and therefore the temperature of the accreted material is low so the luminosity that can be created by accretion is not particularly high. However very large masses can be transferred. This explains the "Algol paradox".
- This configuration is the precursor of later evolution where we have:

1. **Cataclysmic variable binaries** where one of the stars is a white dwarf.
2. **Low mass x-ray binaries**: a neutron star or a black hole with a low mass companion
3. **High mass x-ray binaries**: a neutron star or black hole with a high mass companion.

Cataclysmic variables: Classical Novae

- Here a white dwarf accretes material with high mass accretion rates, typically in the range from 10^{-9} to 10^{-8} $M_{\odot}$ yr^{-1}.
- These objects exhibit massive outbursts in the range of 5-20 magnitudes (10^2 to 10^8 change in luminosity) though 10-12 magnitudes (factors of 10^4-10^5) are more typical. This gives luminosities up to $10^5 L_{\odot}$.
- There is a rapid rise in luminosity over a few days and a much lower decline, typically over a few months. (Fast or slow subclasses depending on decline rate)
- We also see in the spectrum of these objects the ejection of 10^{-4} to 10^{-5} $M_{\odot}$ of material at velocities in the range of 200-2000 kilometres per second.

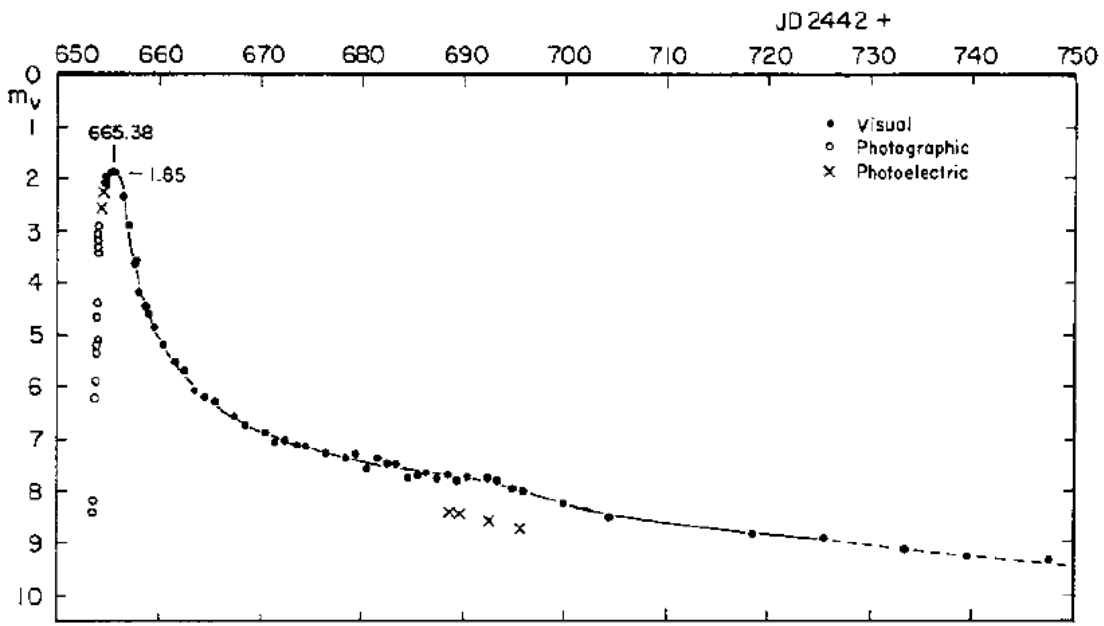


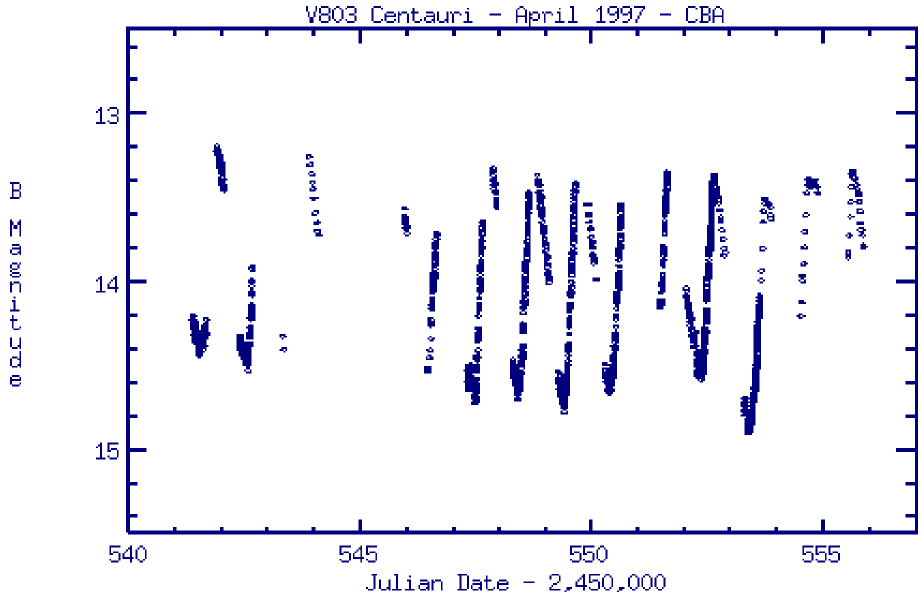
Figure 17.17 The light curve of V1500 Cyg, a fast nova. (Figure adapted from Young, Corwin, Bryan, and De Vaucouleurs, *Ap. J.*, *209*, 882, 1976.)

Cataclysmic Variables: Classical Novae

- Most of the material accreted onto the surface of the white dwarf is hydrogen. Eventually it accumulates on the surface of the white dwarf and mixes with C, N, O from the white dwarf.
- In the white dwarf interior, degeneracy pressure dominates and the temperature is very high, perhaps 2×10^6 K.
- As the material builds up on the surface eventually the bottom of the layer is hot enough for ***fusion*** from H >> He to occur.
- The fusion occurs in degenerate material ($P=P_{deg}$) so the balancing mechanism which occurs when $P=P_{gas}$ (T increases, density decreases, energy generation decreases) does not apply and the reaction runs away causing the temperature to increase to $T \sim 10^8$K.
- This luminosity then exceeds the Eddington luminosity limit and a large part of the shell of accreted material is blown off.
- In between outbursts the white dwarf radiates at a luminosity just below the Eddington limit so that the mass of the degenerate material grows.
- The recurrence time is long, $\sim 10^4 - 10^5$ years with an accretion over that period in the range of 10^{-4} to 10^{-5} $M_{\odot}$.
- Eventually perhaps, the mass of the white dwarf will reach the Chandrasekhar mass of ~ 1.4 $M_{\odot}$ and a supernova of Type 1a occurs.

Cataclysmic Variables: Dwarf Novae

- In the later stages of evolution of low mass systems one component evolves to the white dwarf stage with a mass between 0.6 and 0.8 solar masses.
- Material accreted onto the more compact object heats up greatly and therefore greatly increases its luminosity.
- Within the accretion disk, changes in the mass transfer rates produce flares and outbursts. This is what give ***dwarf novae*** their peculiar characteristics.
- Although mass transfer rates can get up to $10^{-9}\ M_{\odot}\ yr^{-1}$, levels of 10-100 times lower are much more usual.
- If the accreting object also has very large magnetic fields then the accretion column is directed onto the magnetic poles. These are the "Polars".
- The energy for Dwarf Novae comes from ***gravitational energy***. Time variation is due to instabilities in the accretion disk.



V803 Cen: A Dwarf Nova

The Binary Stars Zoo: Neutron Star Binaries

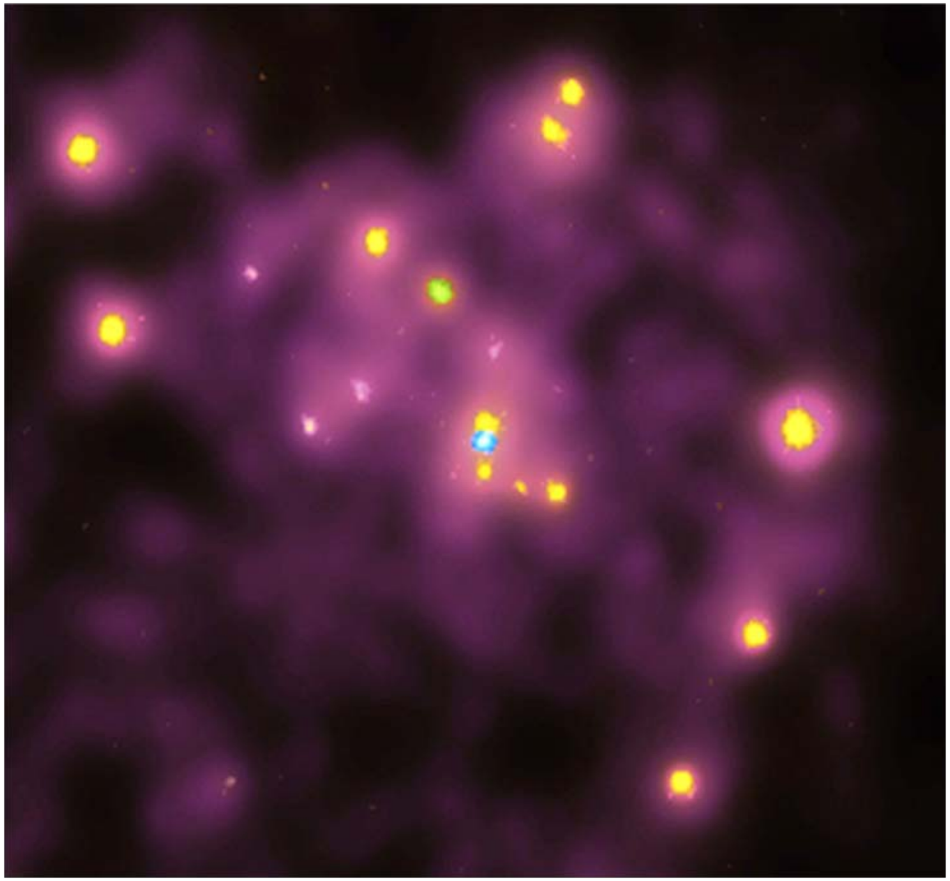
- In the case of neutron stars we have a star with a small radius and a large mass.
- The gravitational potential energy that is released by accretion is large.
- This gives us a high temperature and therefore a very large luminosity and the strong emission of x-rays and gamma rays.
- The very strong magnetic fields tend to funnel the material onto the magnetic poles of the neutron star giving us an x-ray pulsar.
- The accretion disk and the transfer of angular momentum to the neutron star spins it up explaining the millisecond pulsars that we observe.
- Neutron star binaries are grouped into two classes, low mass x-ray binaries (LMXB), and high mass x-ray binaries: HMXB

The Binary Stars Zoo: Low Mass X-ray Binaries

- The "low mass" refers to the mass of the companion star, which is less than about two solar masses.
- K-type main sequence stars are not uncommon. These have low luminosity and may be difficult to detect for distant binaries.
- Periods are generally < 1 day and so they must be close together for mass transfer to occur.
- Many of these objects are known via their x-ray emission but remain optically unidentified.
- In practice the accretion disk radiation often dominates over the companion star but it is the optical companion that is essential to give us the radial velocity information we need to be able to understand these objects.
- They are generally distributed in the bulge of our galaxy and in globular clusters. This implies that they are all old systems and are associated with population II stars.
- Their origins are uncertain: they could be formed by capture or by the collapse of a white dwarf.

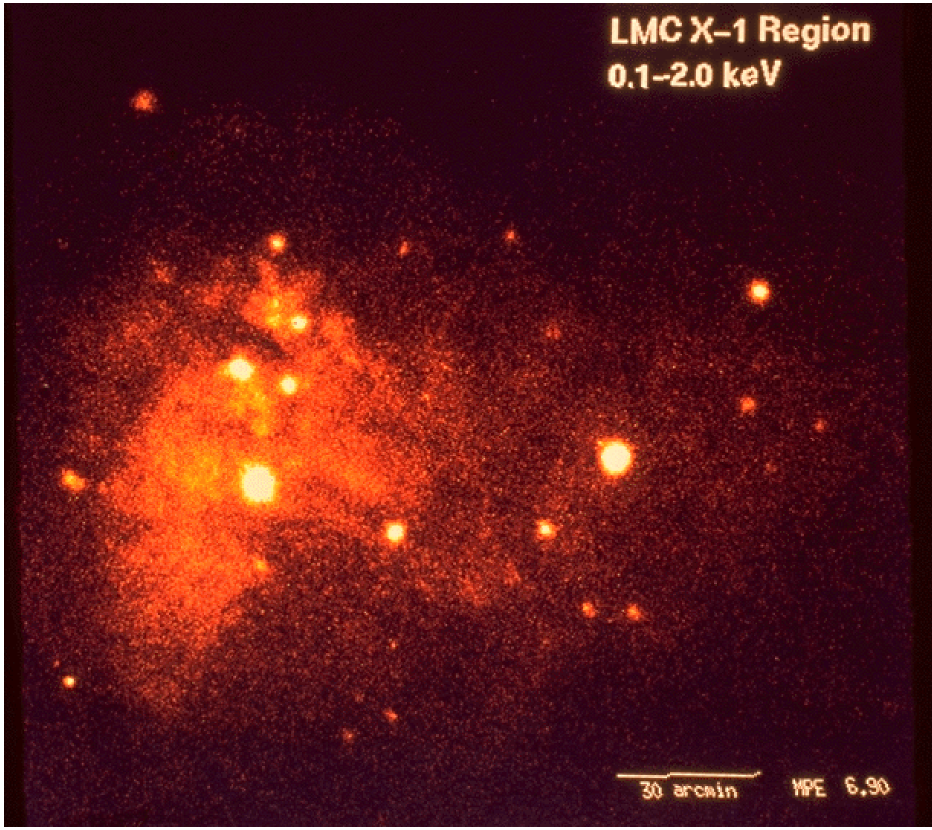
The Binary Stars Zoo: Neutron Star Binaries (LMXBs)

- This x-ray image was taken by the Chandra satellite in October, 1999.
- It is an image of the core of M 31, the Andromeda galaxy. This is a big nearby spiral galaxy, the nearest proper spiral galaxy to our own.
- The many unresolved x-ray sources are a population of (mainly) LMXBs.
- This image is taken in multiple energies. The blue source indicates a much cooler (1,000,000°K) source coincident with the centre of the galaxy. The other colours are only intended to show intensity
- This central cooler source is thought to be caused by x-rays from matter swirling towards the super massive black hole in the nucleus of the galaxy. This black hole is thought to be about 30 million solar masses in size.



The Binary Star Zoo: High Mass X-ray Binaries

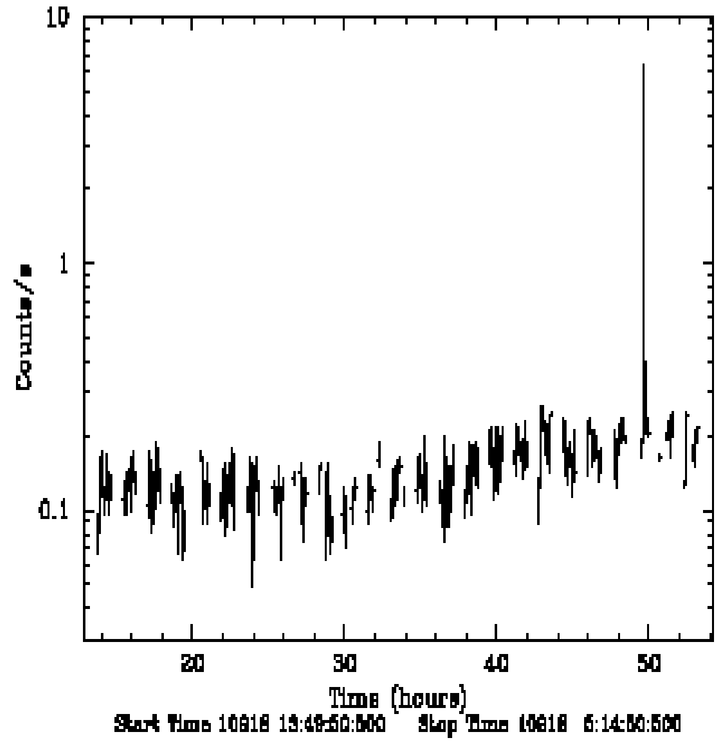
- These objects are neutron stars that have companion stars more massive than ~10 solar masses. These companions are therefore supergiants or Be stars.
- Their periods range from 0.2-200 days.
- As objects they are short lived with the lifetime in the range of ~ 10^4 - 10^5 years, due to the lifetime of the companion star.
- Most of them have optically identified companion stars because they are very luminous. Spectroscopically they are usually single line spectroscopic binaries which gives us valuable constraints on the mass of the unseen object.
- These objects also pulsate with periods in the range of 0.7-900 seconds. Usually the change in period implies that they are spinning up, and the rate of spin up appears to be related to their x-ray luminosities.



- These objects show x-ray variability on many timescales.
- This is a ROSAT x-ray image of LMC X-1, the brightest x-ray source in the Large Magellanic Cloud.
- The x-ray photons emitted (with a luminosity of about 10^5 $L_{\odot}$ due to accretion onto a neutron star or black hole) excite the gas clouds in the surrounding interstellar medium.

The Binary Zoo: X-ray Bursters

- In this case an accreting neutron star builds up a surface layer of hydrogen and helium which is several metres thick.
- The temperature is high, ~ $3\text{x}10^7$ K, and the helium layer ignites explosively under the degenerate conditions.
- At these temperatures bursts of x-rays of typically ~ 20 KeV energy are emitted for ~20 seconds.
- Each flash involves ~ 10^{18} kg, and the densities are as high as ~ 10^9 kgm^{-2}.
- The energy in a single flash is ~10^{32} J, and the recurrence time is from 10^5 to 10^6 seconds.
- The time averaged luminosity, L ~ 50 $L_{\odot}$, and the flux in the bursts only accounts for about 1% of the total luminosity, although when it goes off it is incredibly bright.
- Compare to classical nova: WD and H fusion.



•This plot shows observations of an LMXB in the galactic centre coincident with an x-ray transient source originally seen in 1976.

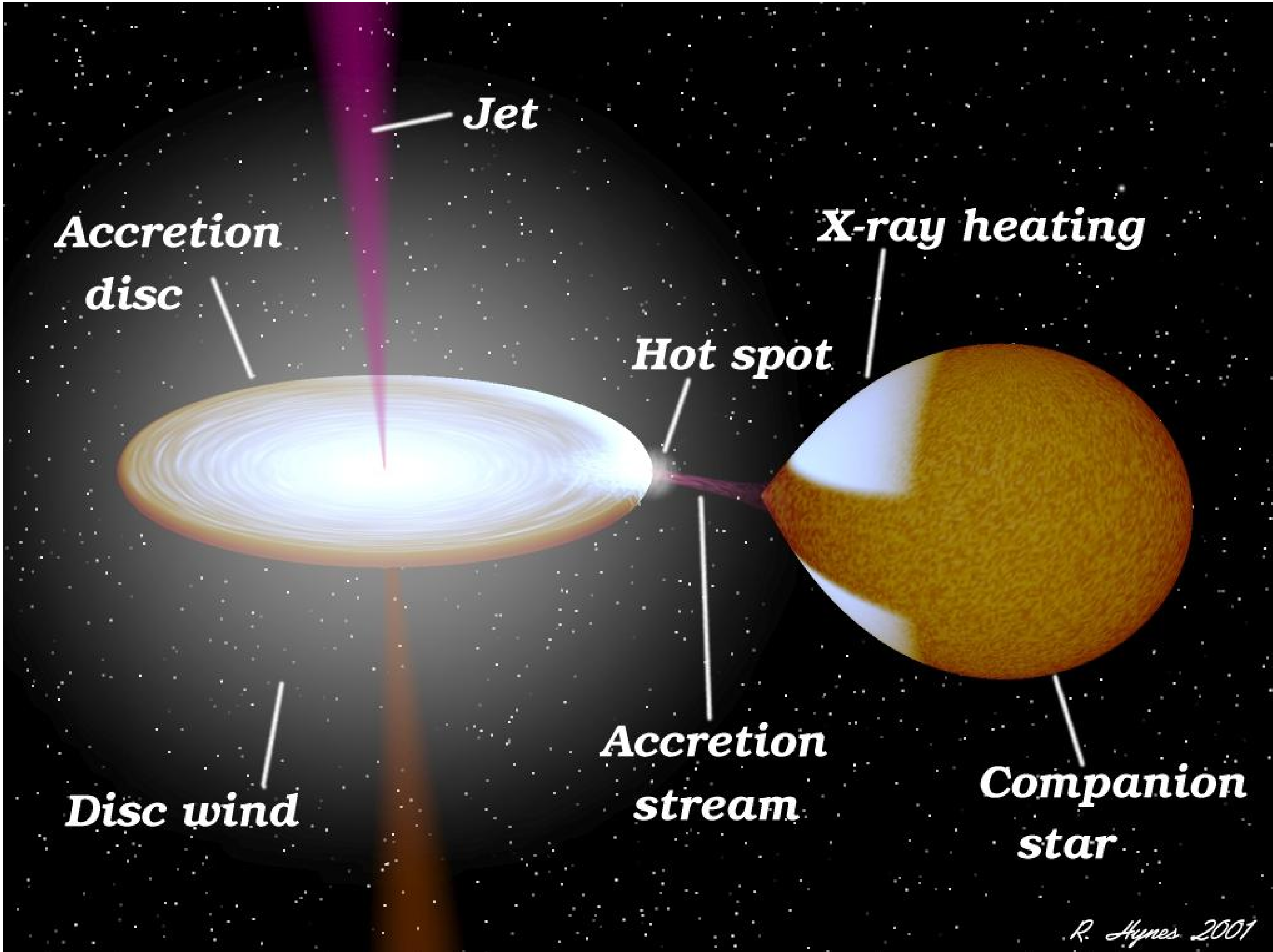
•There is no optical counterpart but there are 50 magnitudes of absorption towards the galactic centre at optical wavelengths (10^{-20} transmission!)

The Binary Pulsar

- We have already seen that there are many binary systems where one component is a pulsar. We can observe the radial velocities of these objects with great accuracy.
- A small fraction show eclipses and therefore we know the angle of inclination and therefore get a good mass for the pulsar.
- In 1974 the binary pulsar **PSR B1913+16** was discovered by Hulse & Taylor and this led to a Nobel prize in 1993 for both the supervisor and a graduate student (c.f. Bell & Hewish).
- The orbital velocity is 300 kms^{-1} ($\sim 10^{-3}$ c), and the period is about eight hours. The separation of the stars is $\sim 7 \times 10^5$ kilometres.
- The orbit is a very eccentric with e ~ 0.62. The masses of the two stars are very similar at 1.44 $M_\odot$ and 1.39 $M_\odot$.
- The pulsar provides a superb clock and the companion is most likely a neutron star.
- This configuration (two point masses moving at 0.001 c, one with an on-board precision clock) gives us a great opportunity to test General Relativity.
- The precession of the orbit is ~ 4.2 degrees per year (compare this with 0.0074 degrees per year for the planet Mercury).
- We find that the period is decreasing and the separation decreasing.
- The energy loss is $\sim 10^{25}$ W, and (dP/dt) = -2.4×10^{-12}.

Stellar Mass Black Holes?

- The more massive core-collapse supernova are thought to produce black holes.
- Are there any binaries where there might be clear evidence of a black hole?
- We expect x-ray binaries to involve accretion onto a compact object.
- We can observe the stellar companion, as a single line spectroscopic binary which gives us the orbital period P and $v_r = v \sin(i)$.
- One possible candidate is Cygnus X-1. It has P = 5.6 days, v.sin(i) = 70 kms^{-1}.
- The supergiant companion is between 8.5 and 15 $M_\odot$, and the orbit implies the unseen star has a mass > 3.3 $M_\odot$. (see mass function earlier).
- The companion is distorted and we can use the photometric variations around the orbit to deduce its inclination.
- This suggests that the mass of the unseen star is ~ 7 $M_\odot$. Too high to be a neutron star.
- However the companion does not fill its Roche lobe implying that there is uncertainty in this value.
- Another object, V404 Cygni, is now one of our best recent candidates.
- Its companion is believed to fill the Roche lobe therefore we know the shape of the star which give us the system parameters etc. which are well matched by the model.
- This implies here that the mass of the unseen object = 12 ± 2 $M_\odot$. Definitely too high to be a neutron star!



Jet
Accretion disc
X-ray heating
Hot spot
Accretion stream
Companion star
Disc wind
R. Hynes 2001