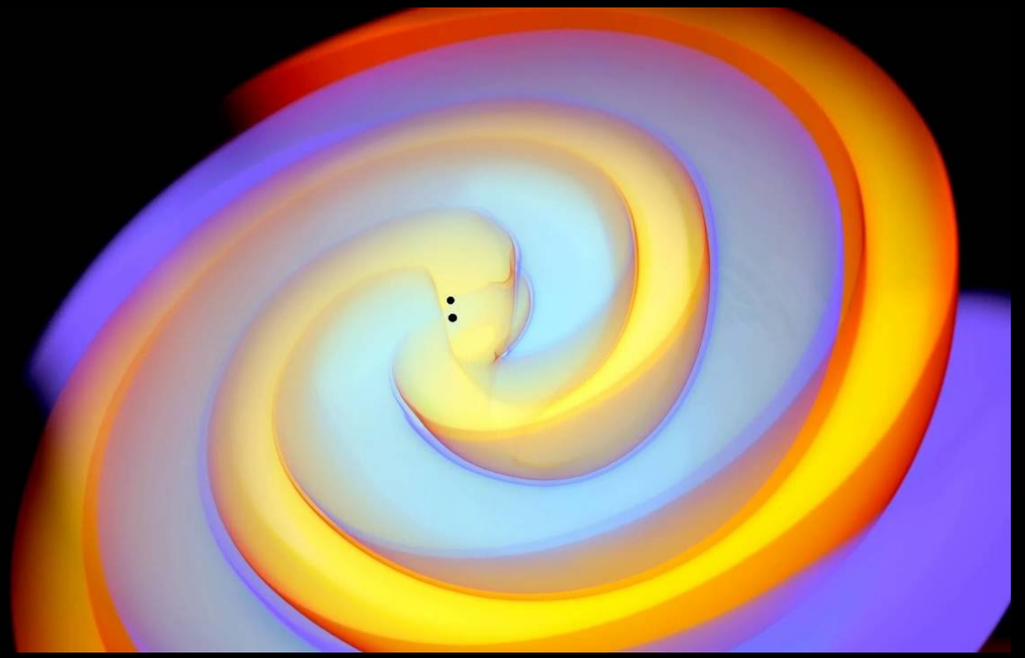


Part-II Physics/Astrophysics

Relativity – Lecture 1

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ADMINISTRATIVE DETAILS

- 24 lectures/videos
- Full notes provided (e.g., on Physics TiS)
- Four examples sheets
- Recommended books:
  - *General Relativity: An Introduction for Physicists*, Hobson, Efstathiou and Lasenby
  - *Spacetime and Geometry: An Introduction to General Relativity*, Carroll
  - *Introducing Einstein's Relativity*, d'Inverno
  - *General Relativity*, Kenyon
  - *Gravitation and Cosmology*, Weinberg
  - *Gravitation*, Misner, Thorne and Wheeler
  - *General Theory of Relativity*, Dirac

1. NEWTONIAN GRAVITY

- Described by gravitational potential $\Phi(\vec{x}, t)$
- Force on test particle

$$\vec{f} = -m_G \vec{\nabla} \Phi$$

- Potential determined by Poisson's equation:

$$\vec{\nabla}^2 \Phi = 4\pi G \rho$$

Inconsistent with Special Relativity since instantaneous propagation

- Mass density of point particle at $\vec{y}(t)$:

$$\rho(\vec{x}, t) = m_A \delta^{(3)} \left(\vec{x} - \vec{y}(t)\right)$$

and potential is

$$\Phi(\vec{x}, t) = -\frac{G m_A}{|\vec{x} - \vec{y}(t)|}$$

- Momentum conservation

$$\Rightarrow \vec{f}_{1 \text{ on } 2} = -\vec{f}_{2 \text{ on } 1} \Rightarrow m_{G,1} m_{A,2} = m_{G,2} m_{A,1}$$

  - Holds for arbitrary masses so

$$\boxed{\frac{m_{G,1}}{m_{A,1}} = \frac{m_{G,2}}{m_{A,2}}} \quad \text{Universal ratio} \quad \Rightarrow m_G = m_A$$

- Not unusual: e.g., passive and active charges equal in electromagnetism

- Newton's second law:

$$\vec{f} = m_I \frac{d^2\vec{x}}{dt^2}$$

- Motion under gravity:

$$\frac{d^2\vec{x}}{dt^2} = -\frac{m_G}{m_I}\vec{\nabla}\Phi$$

- Ratio m_G/m_I measured to be the same for all particles, irrespective of their composition, to 1 part in 10^{13}

Can take $m_G = m_I$

- This universal acceleration is odd – definitely not the case that q/m_I universal in EM!
- Often rephrased as the Weak Equivalence Principle:

All free-falling particles follow the same path through space and time if they have the same initial position and velocity

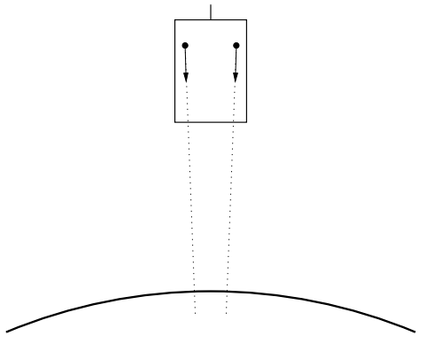
2. Implications of the equivalence principle

- Consider observer in free-falling, non-rotating elevator in uniform gravitational field
  - Sees any free-falling particle follow straight-line path at constant velocity
  - Exactly as motion relative to an inertial reference frame in the absence of gravity!

- Strong Equivalence Principle extends this to all of physics and non-uniform gravitational field:

> In arbitrary gravitational field, all laws of physics in free-falling, non-rotating laboratory occupying a sufficiently small region of spacetime look locally like SR (with no gravity)

- Why local?
  - Tidal effects are observable manifestation of gravitational field, but undetectable for sufficiently local measurements



- SEP $\Rightarrow$ local equivalence of gravity and acceleration
  - Cannot distinguish uniform gravitation field from constant acceleration
- Given in SR without gravity, physics looks simple in inertial frames, but with gravity looks equally simple locally in free-falling reference frames

Local inertial reference frame = free-falling, non-rotating

- Acceleration should be defined relative to such frames and will arise from non-gravitational forces

2.1 Gravity as spacetime curvature

- Universal free-fall suggests trajectories of free-falling particles determined by local structure of spacetime itself
  - Replaces idea of gravitational force with mysterious universal coupling to matter

- Local inertial frames become system of local coordinates over small region of spacetime such that geometry of spacetime locally like that in SR in inertial coordinates

Gravity $\Rightarrow$ cannot extend such a frame throughout all of spacetime $\Rightarrow$ Spacetime curvature

3 Further motivation: extreme gravity

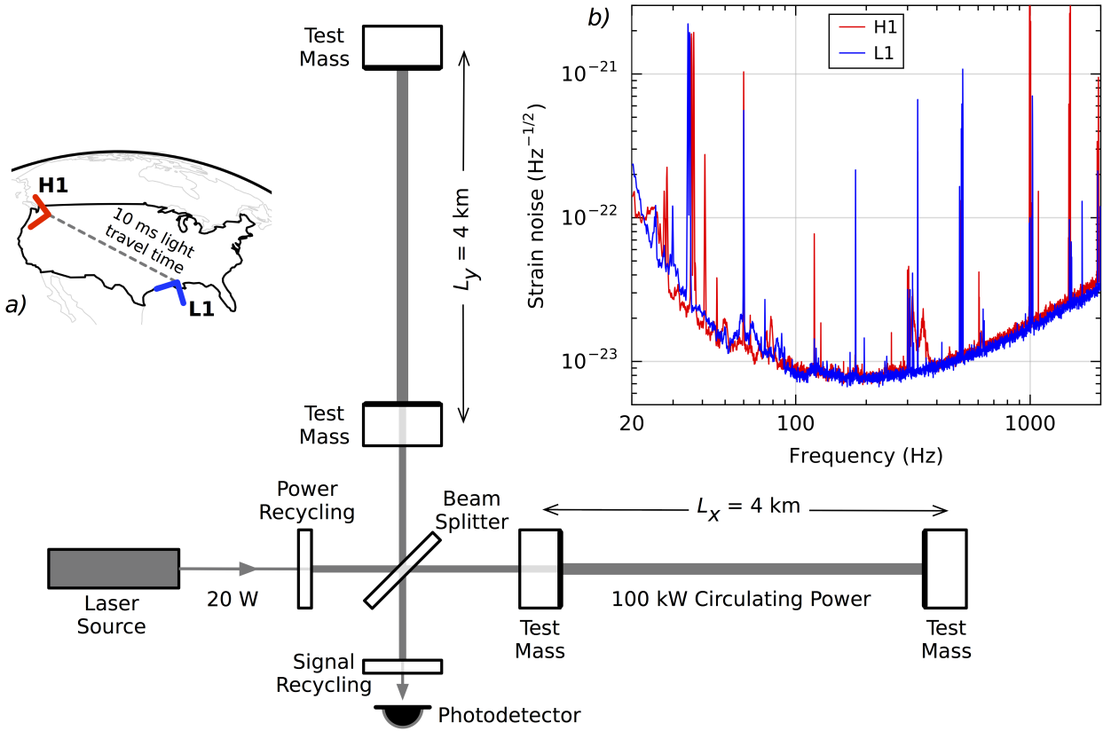
- Expect Newtonian gravity valid when $v \ll c$ and $|\Phi| \ll c^2$
  - Generally equivalent when gravity determines dynamics

$$\frac{v^2}{R} = \frac{GM}{R^2}$$
$$\Rightarrow \quad \frac{v^2}{c^2} = \frac{GM}{Rc^2} = \frac{|\Phi|}{c^2}$$

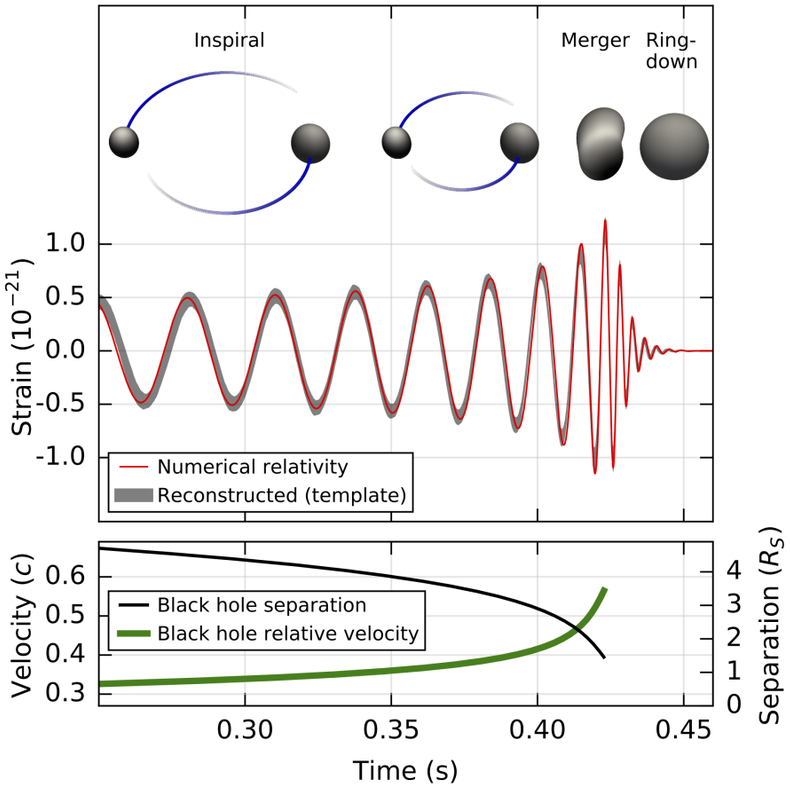
- Many astrophysical systems where Newtonian limit does not apply

LIGO AND GRAVITATIONAL WAVES

- Gravitational waves are wavelike disturbances in the geometry of spacetime that propagate at the speed of light
- Natural prediction of general relativity – don't exist in Newtonian theory
- Give rise to quadrupole oscillatory distortion (stretching in one direction, contraction in orthogonal direction) in two arms of a laser interferometer
  - First direct detection (14 September 2015) by US-led LIGO (2017 Nobel Prize in Physics)
  - LIGO now joined by European-led VIRGO (first joint detection of event on 14 August 2017)



First direct detection of GWs: GW150914



- Source: pair of merging black holes ($M \approx 30 M_\odot$) at distance of 2 Gly
- Probes strong-field regime of general relativity with relativistic speeds
- Peak luminosity in GWs: 3.6×10^{49} W or $200\, M_\odot c^2\, \mathrm{s}^{-1}$!