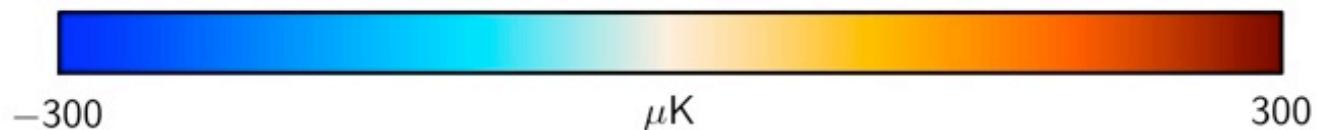
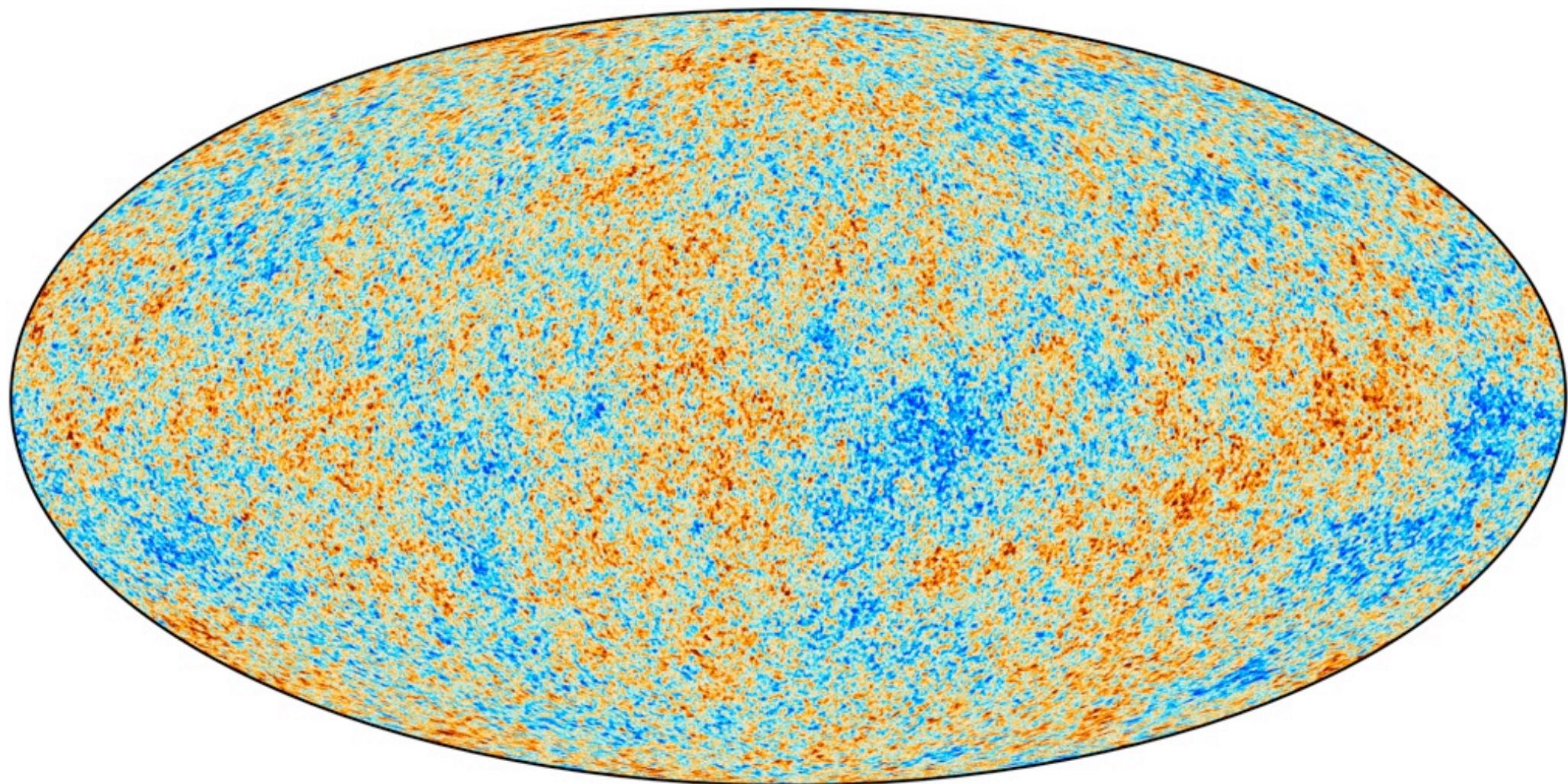
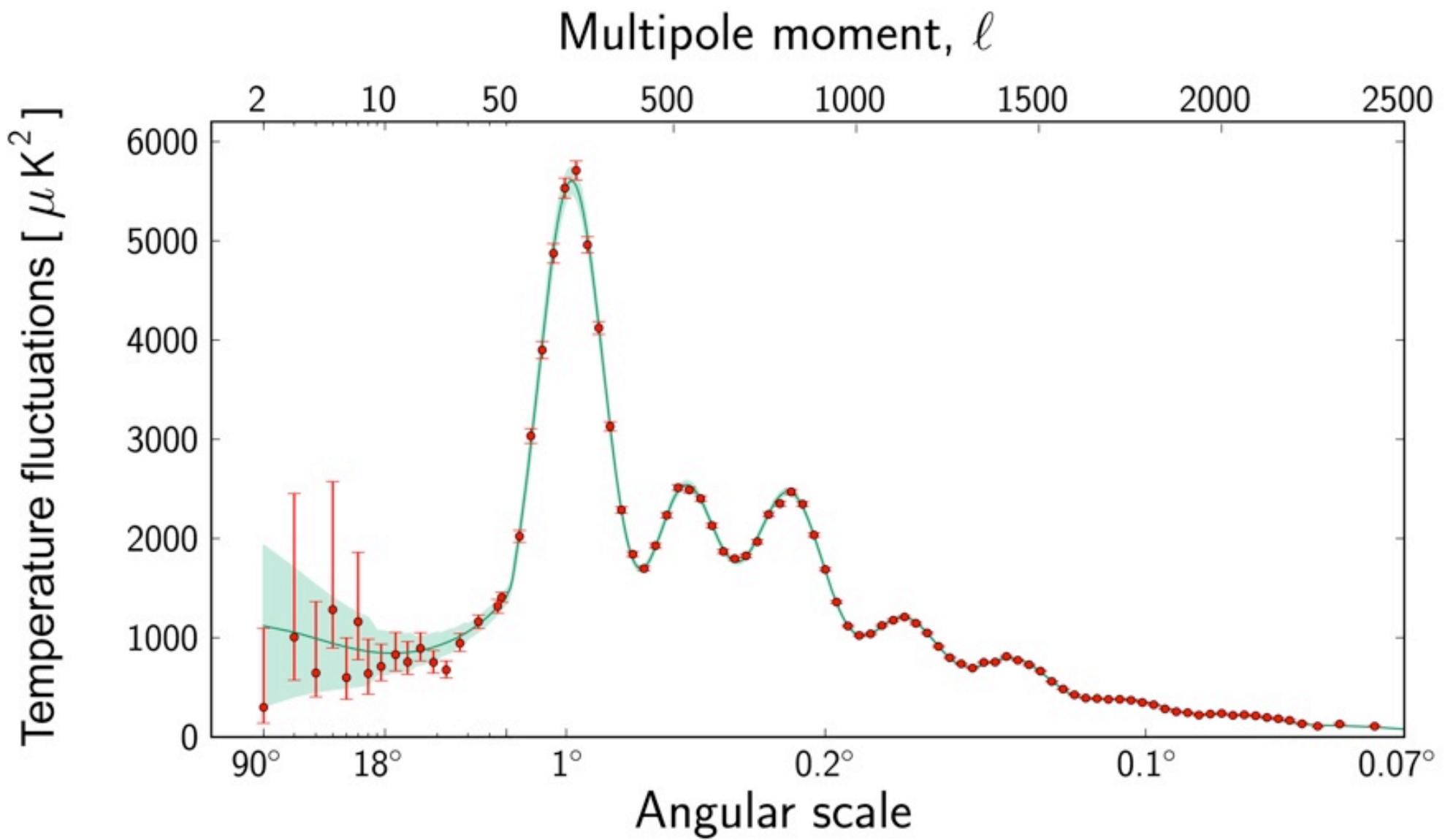


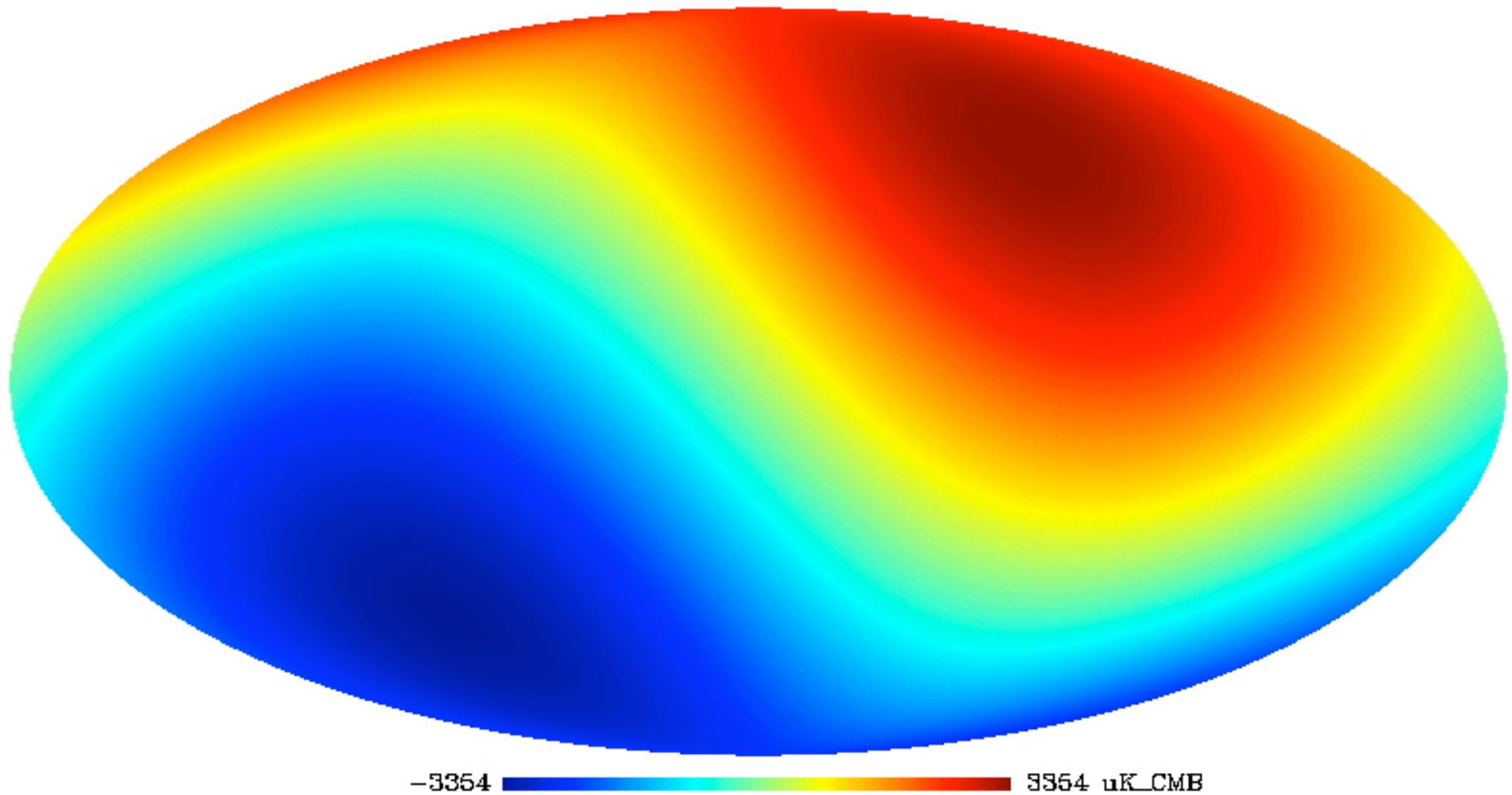
# Introduction to Cosmology

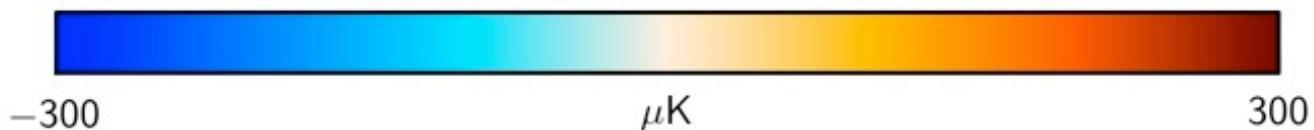
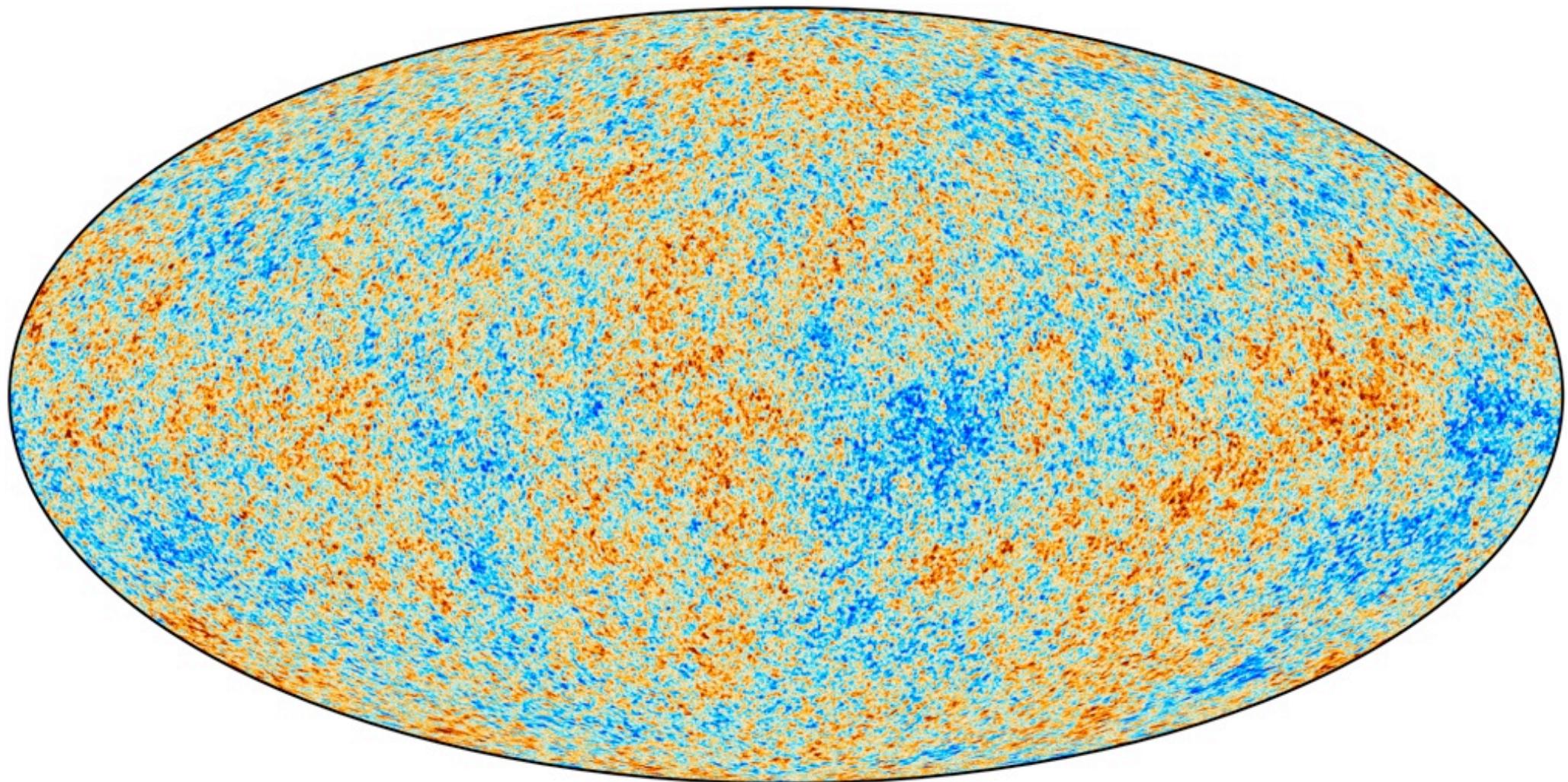
## Lecture 10



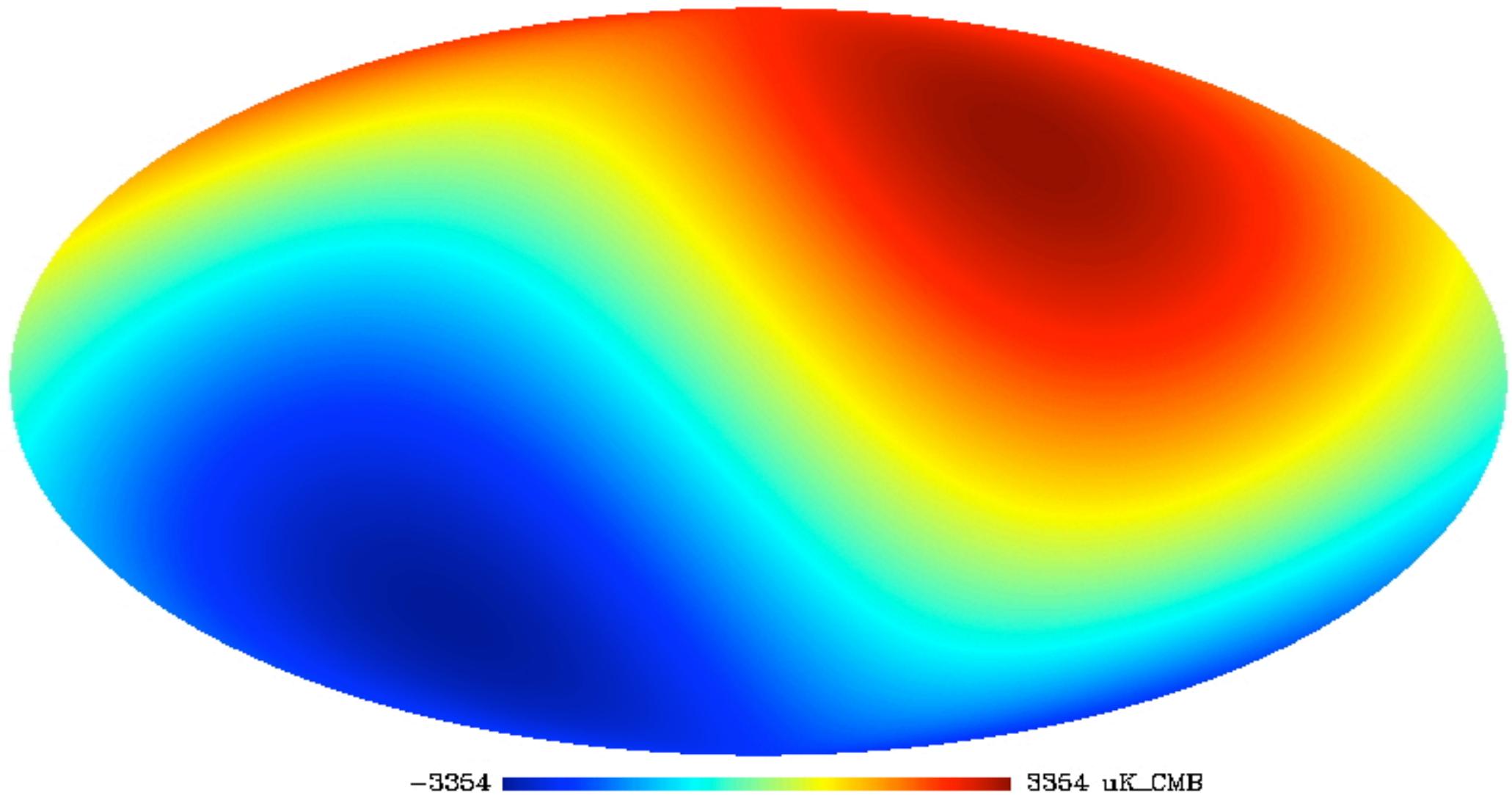


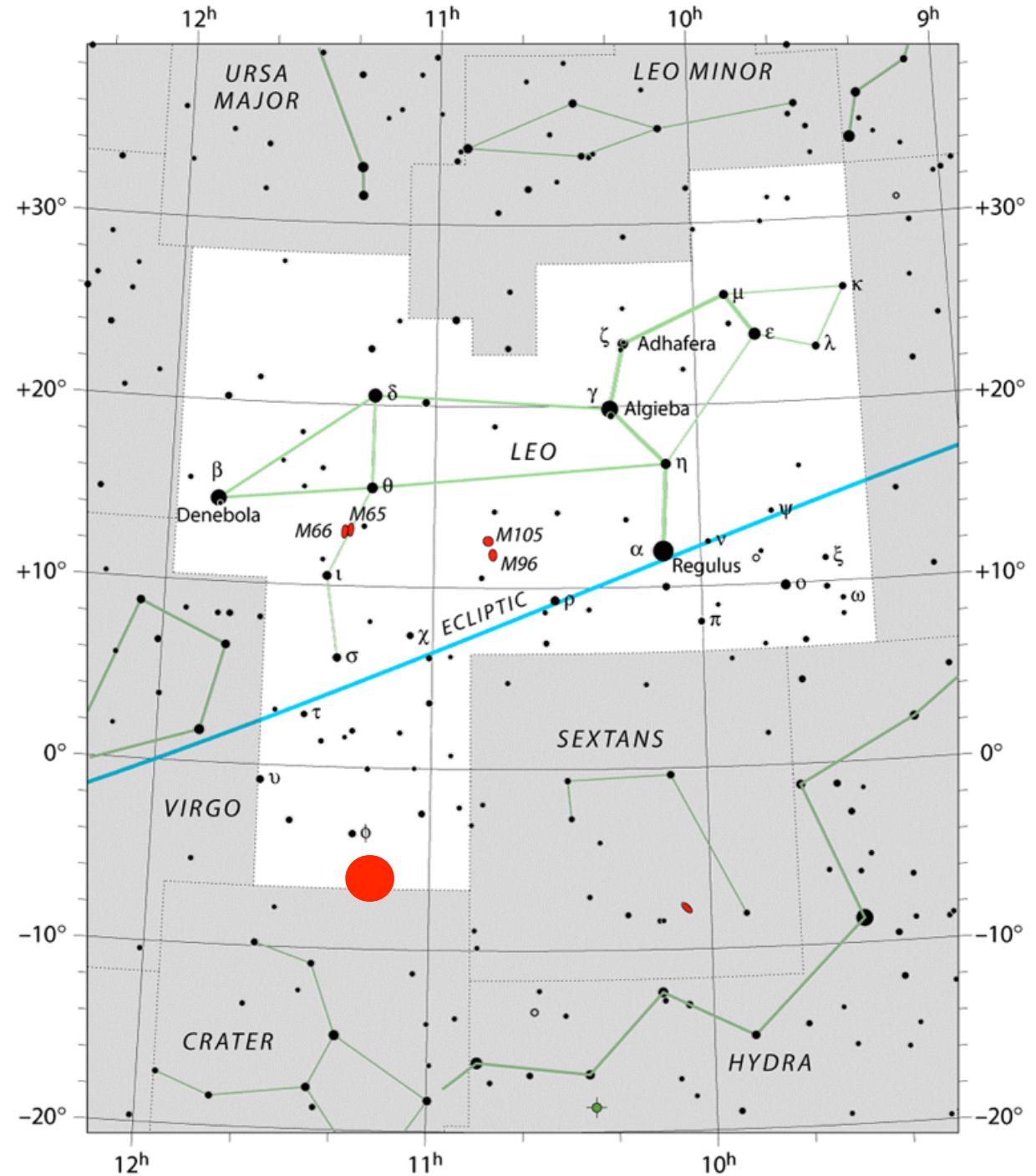
CMB dipole



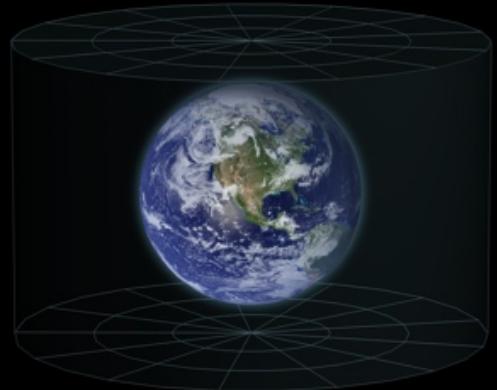


CMB dipole

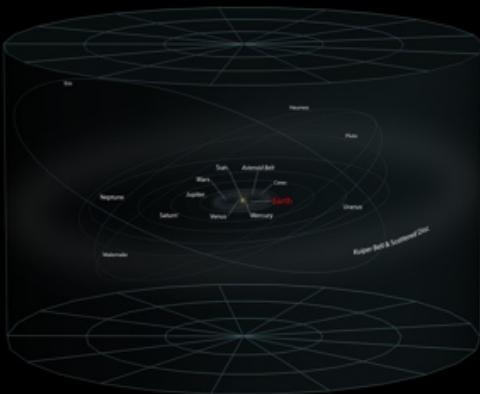




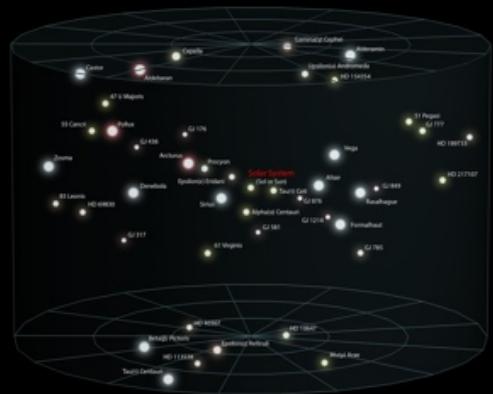
Earth



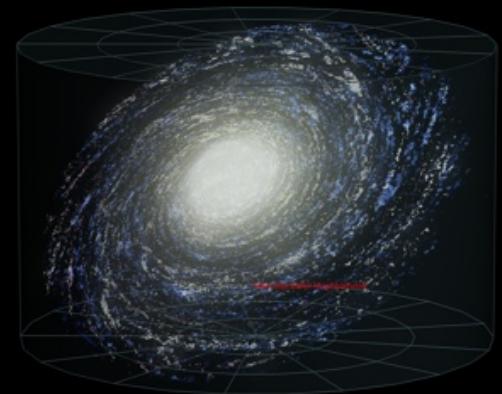
Solar System



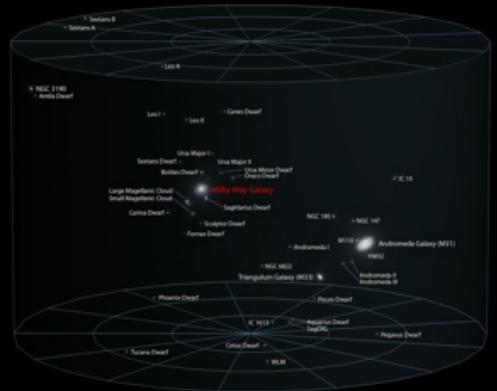
Solar Interstellar Neighborhood



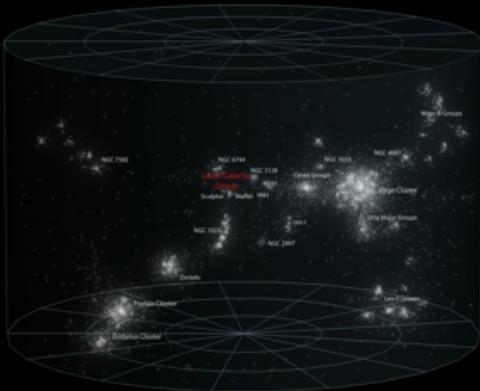
Milky Way Galaxy



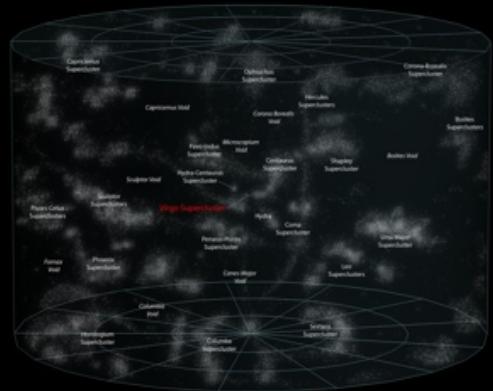
Local Galactic Group



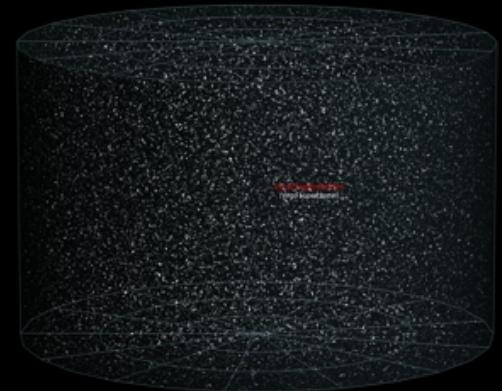
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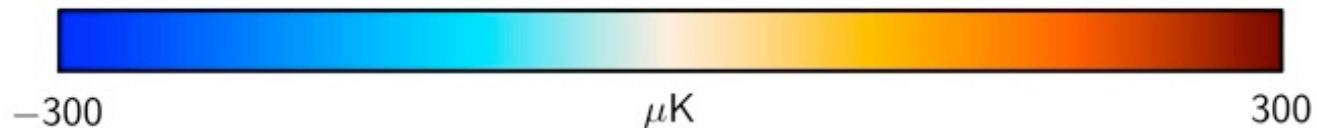
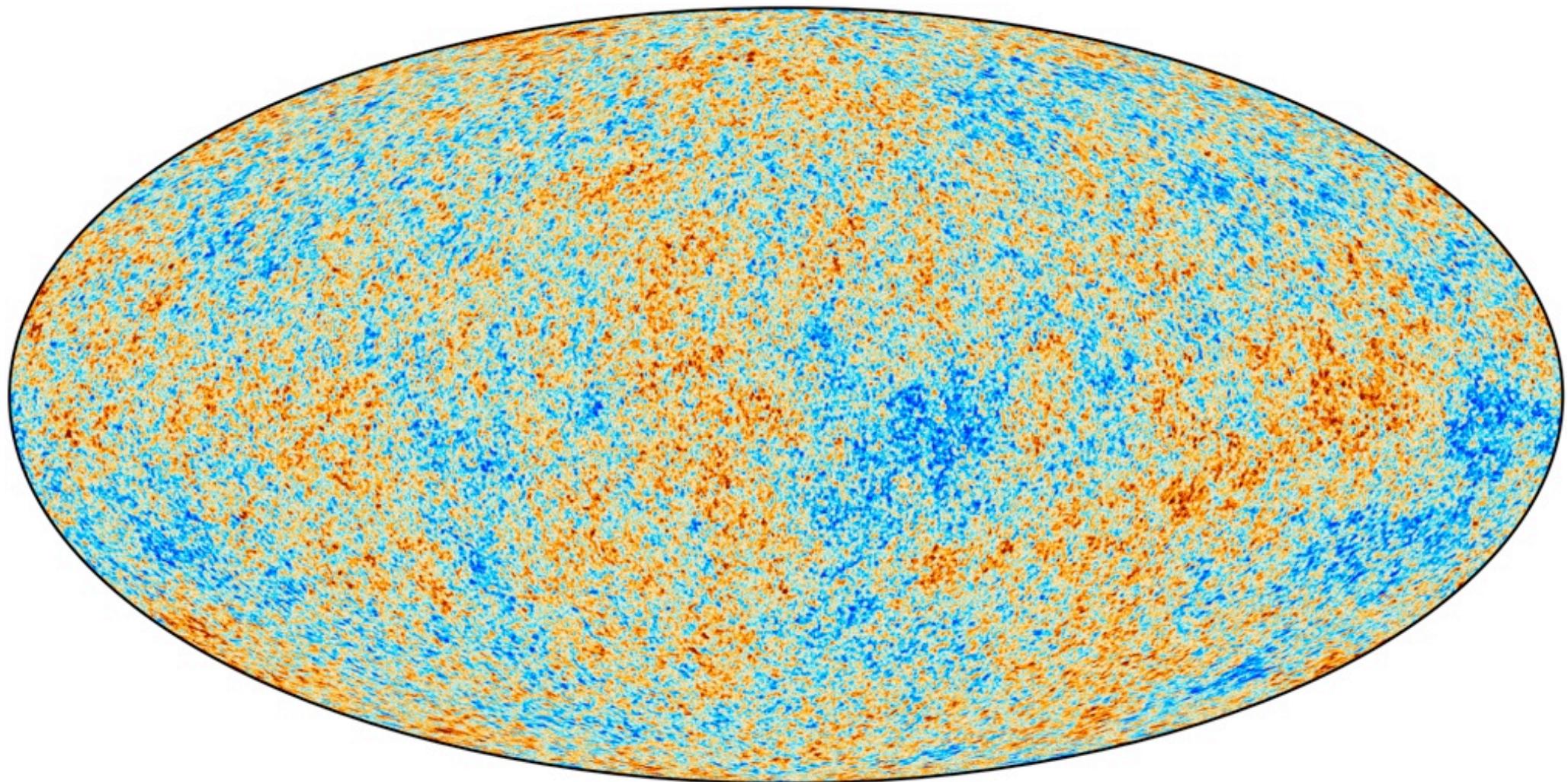


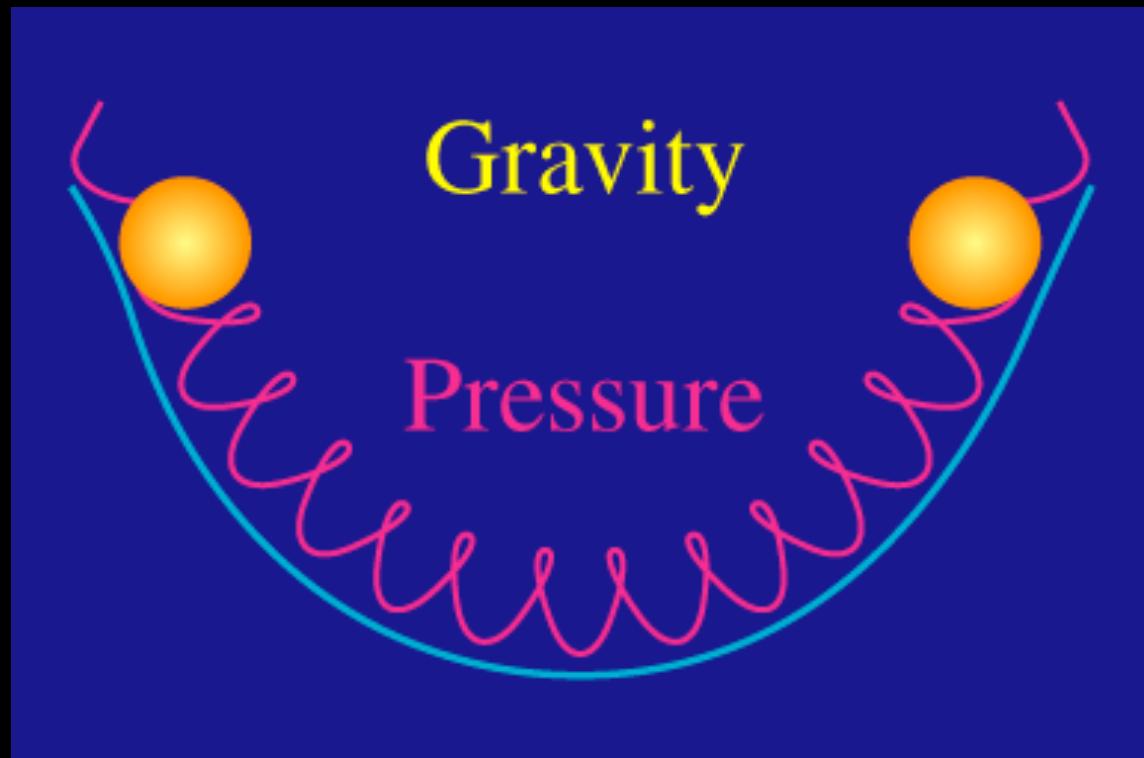
Local Superclusters

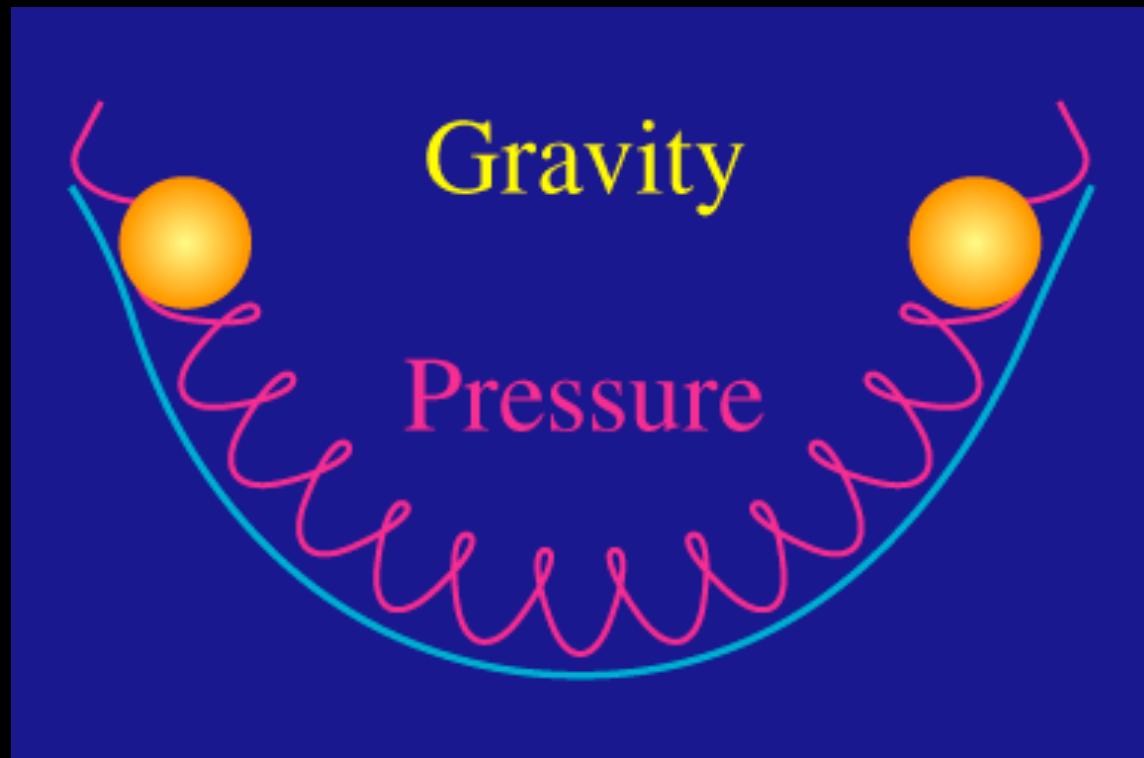


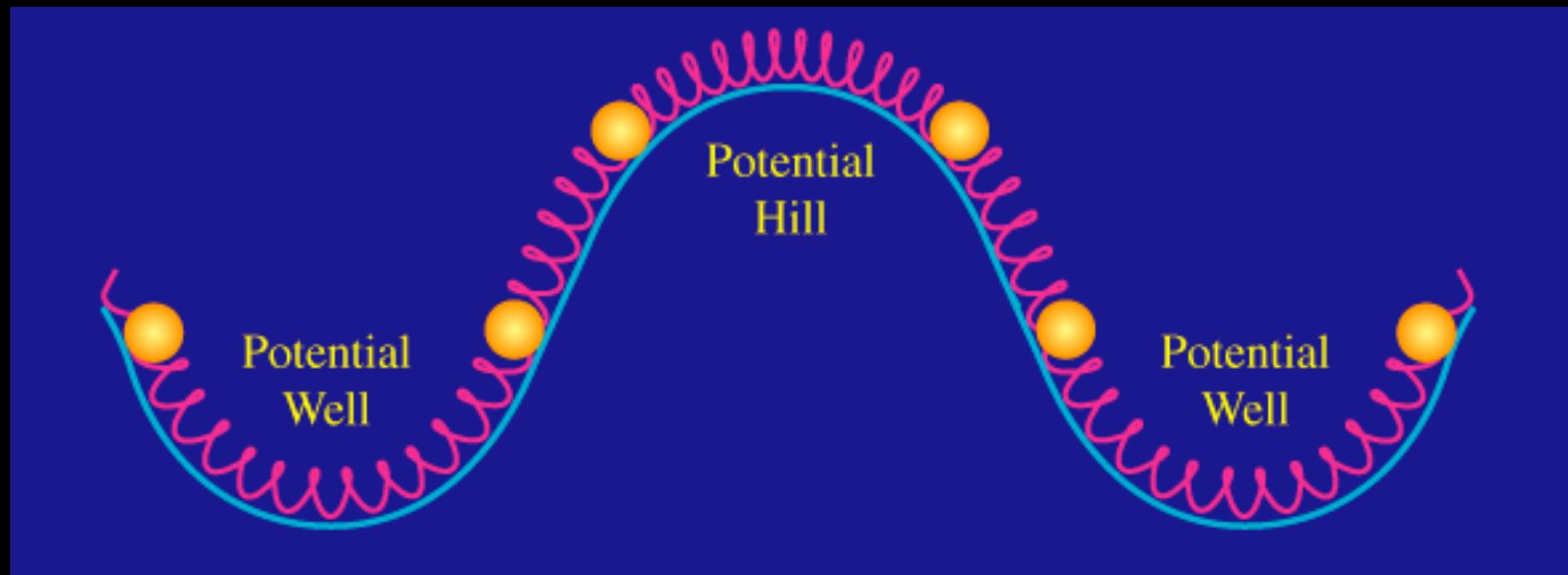
Observable Universe

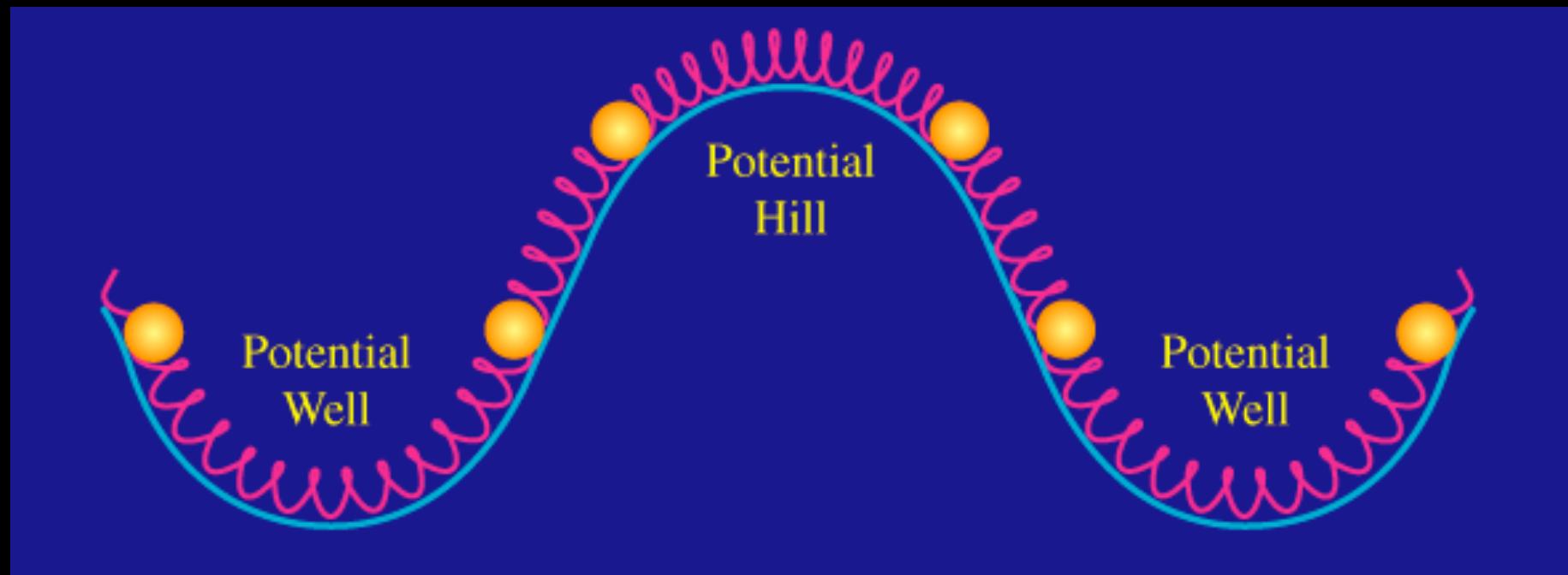


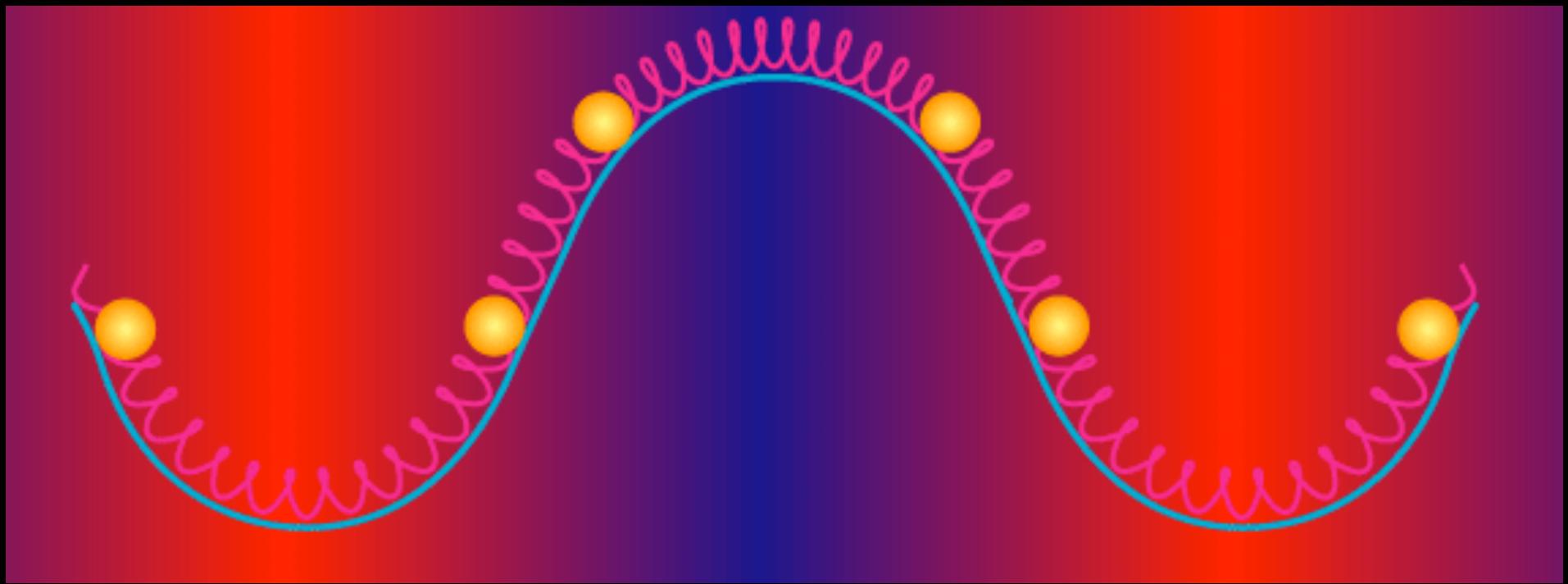


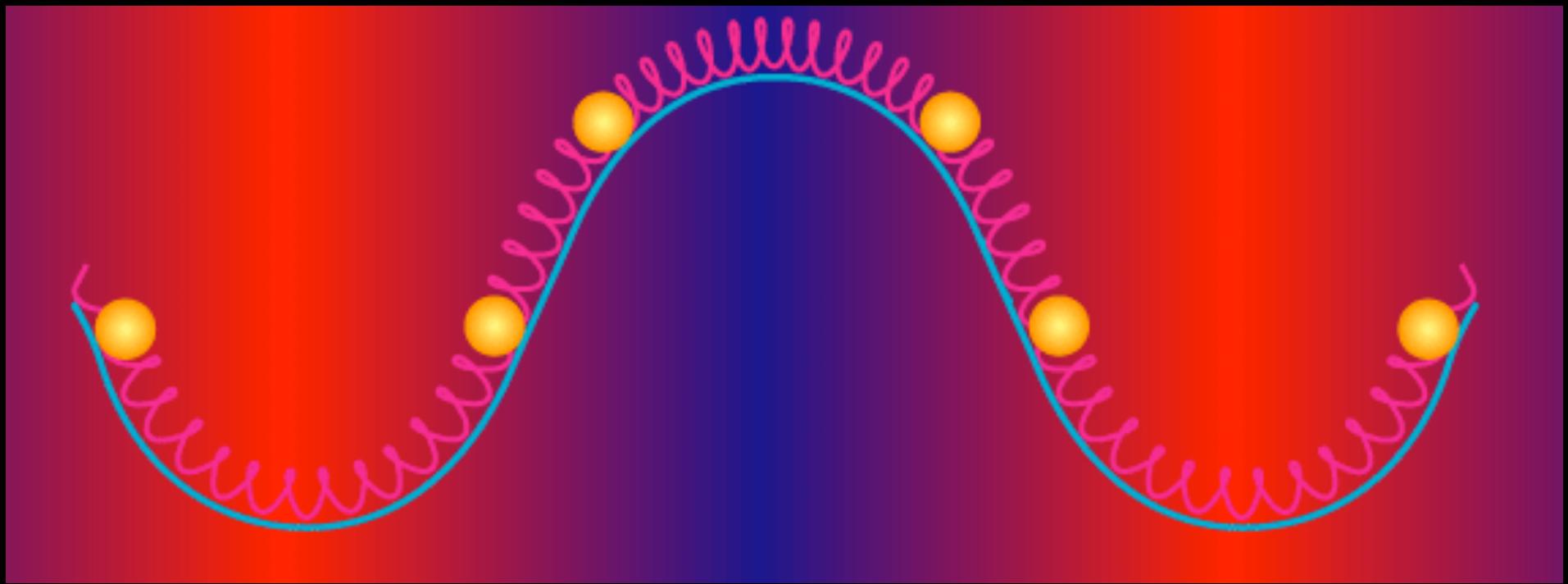


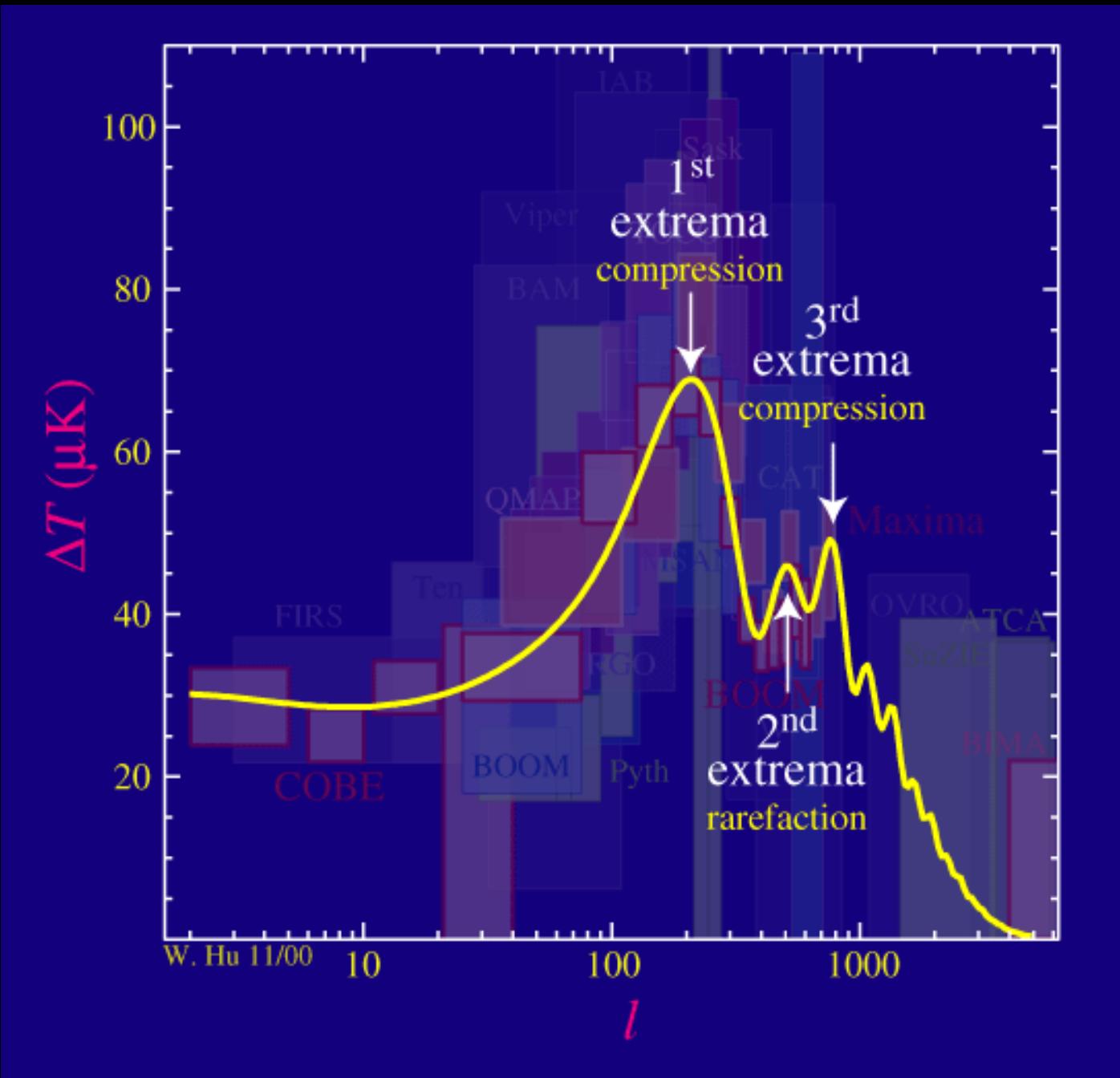


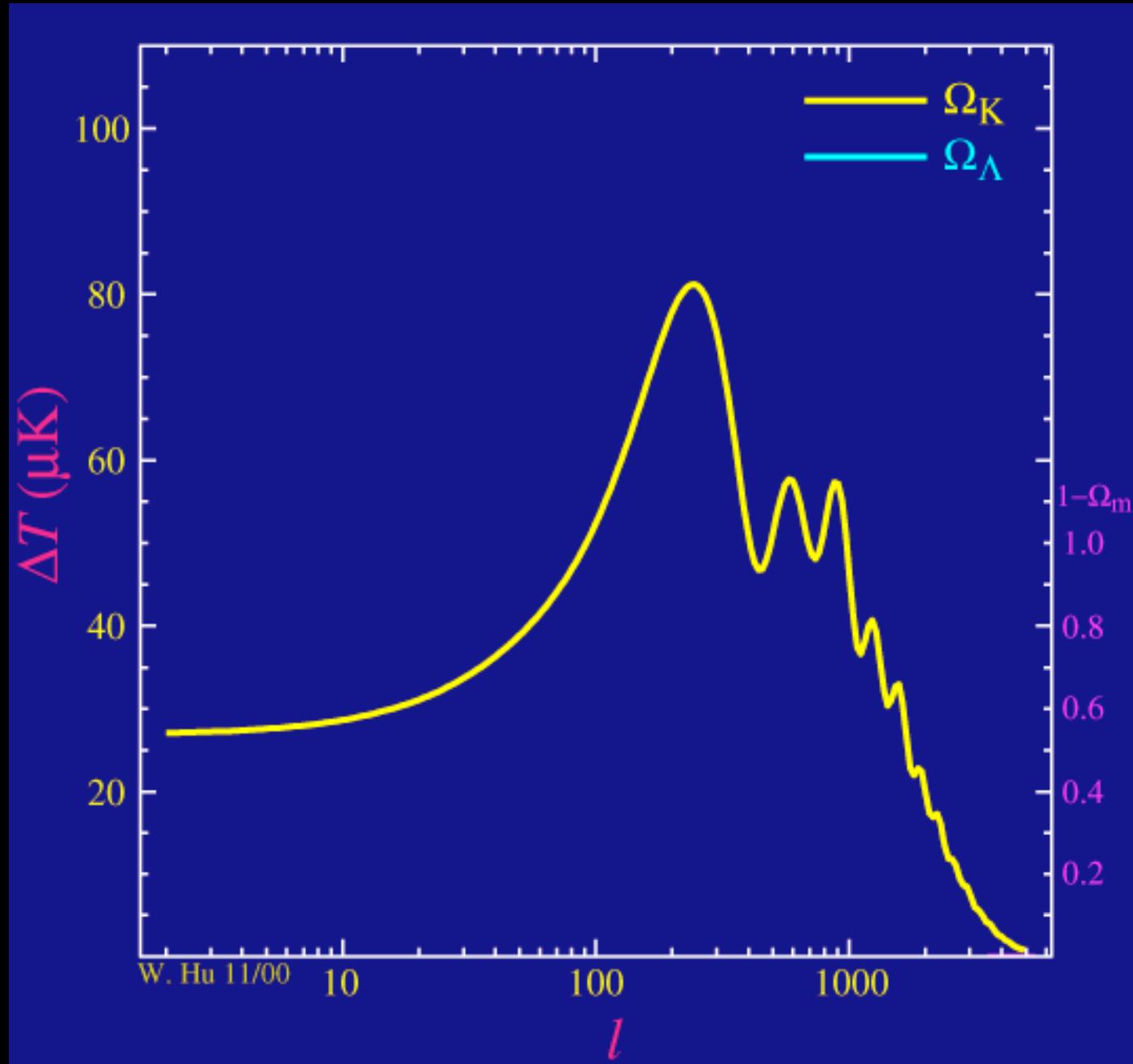


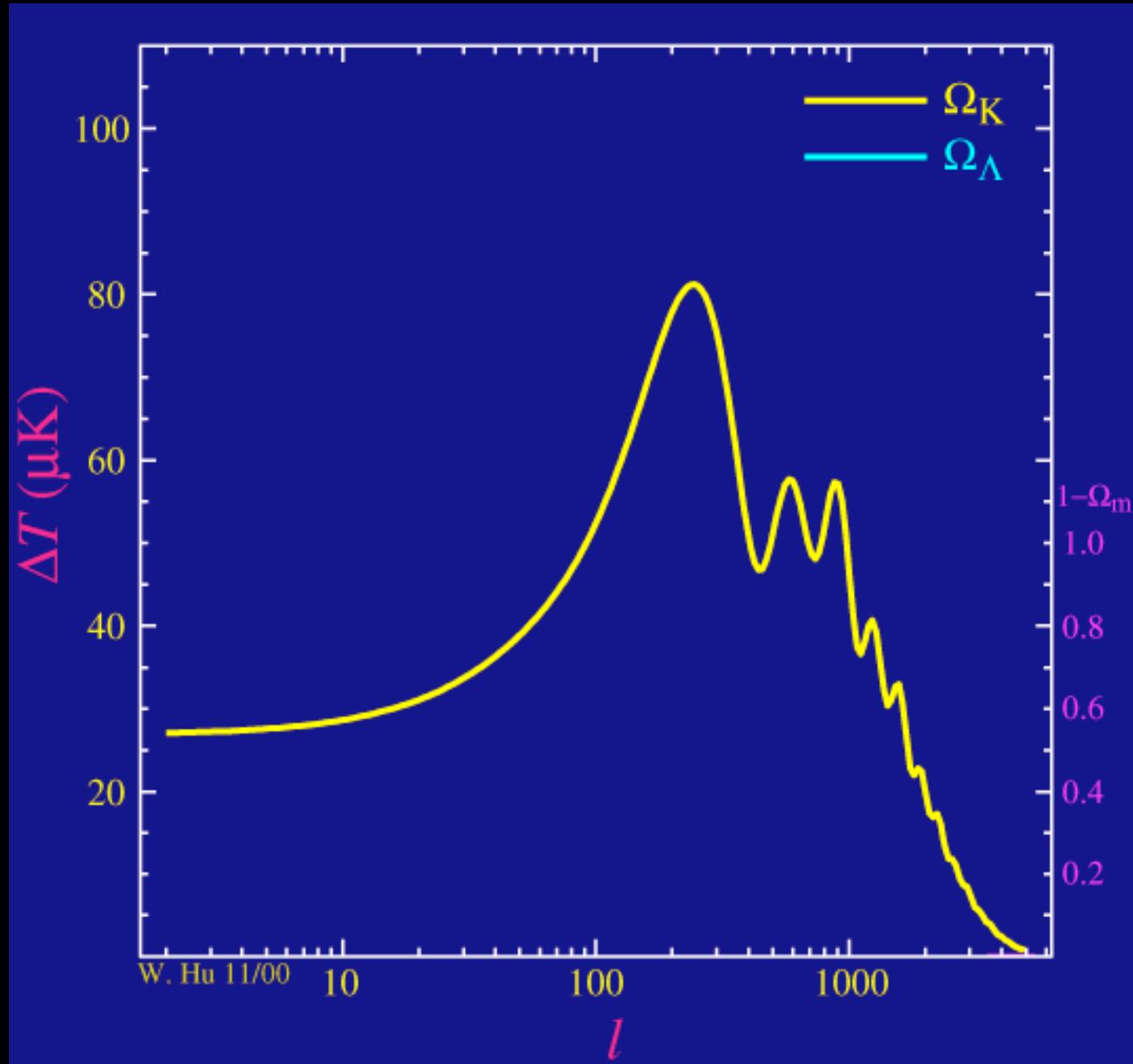




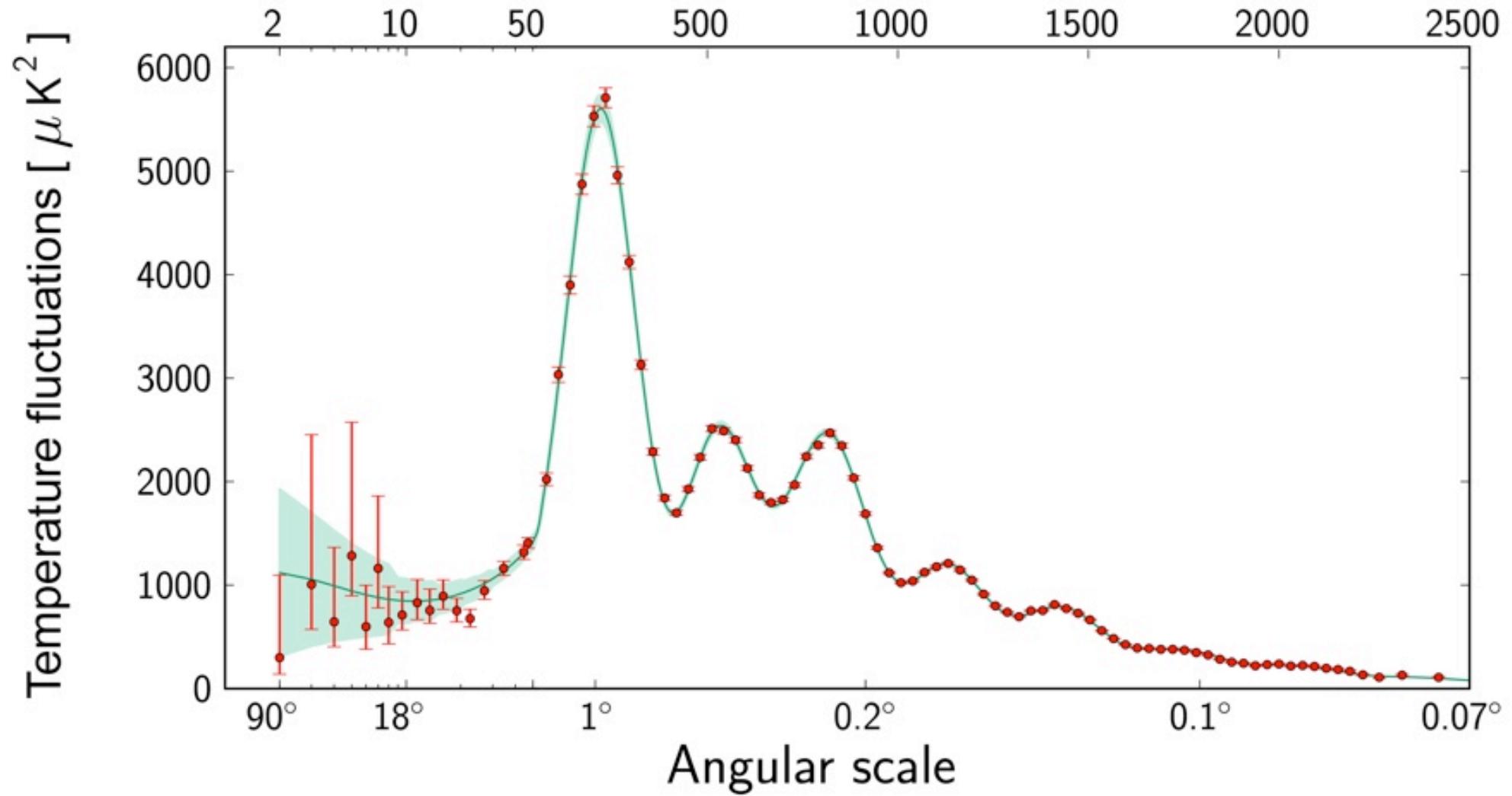




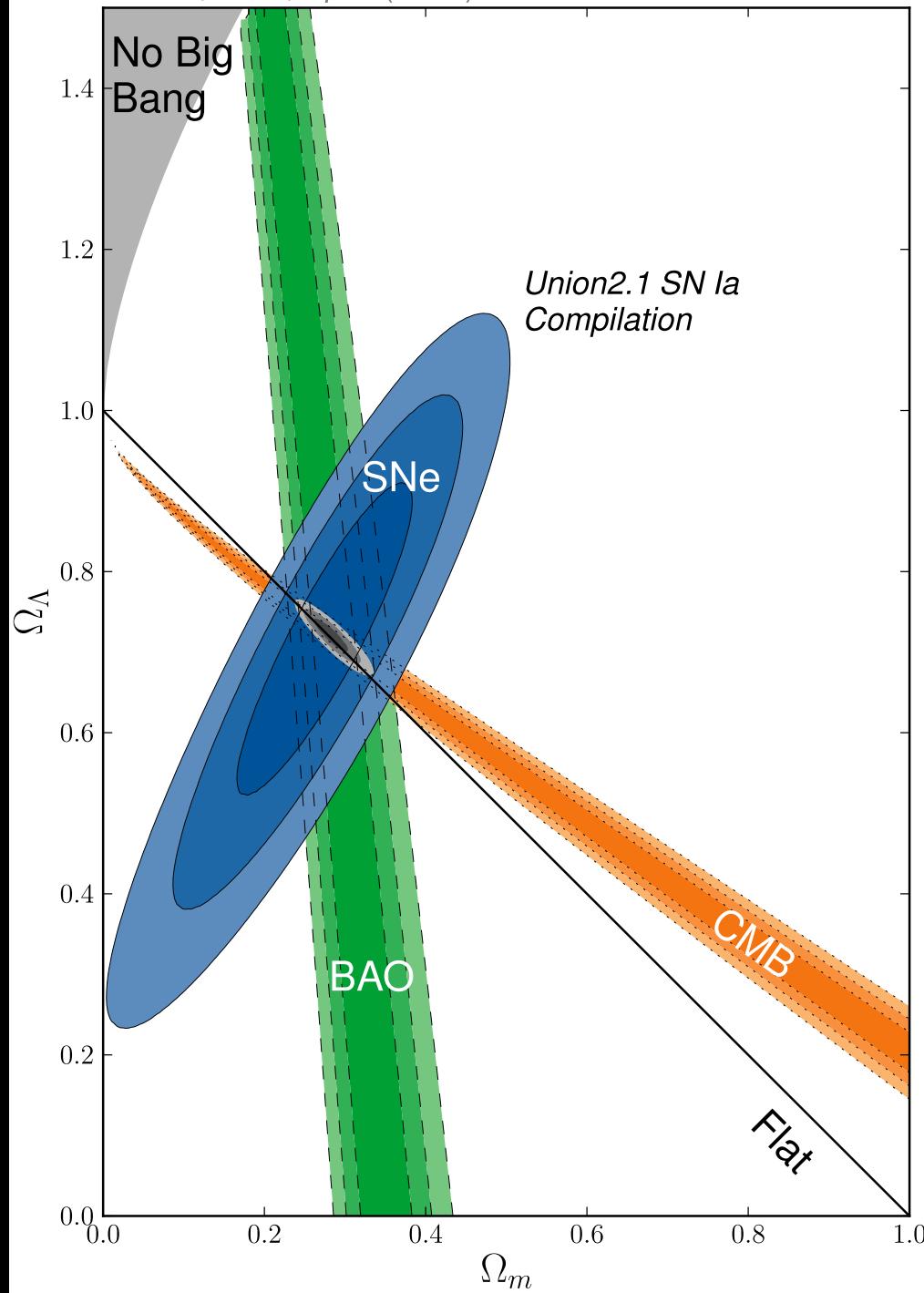


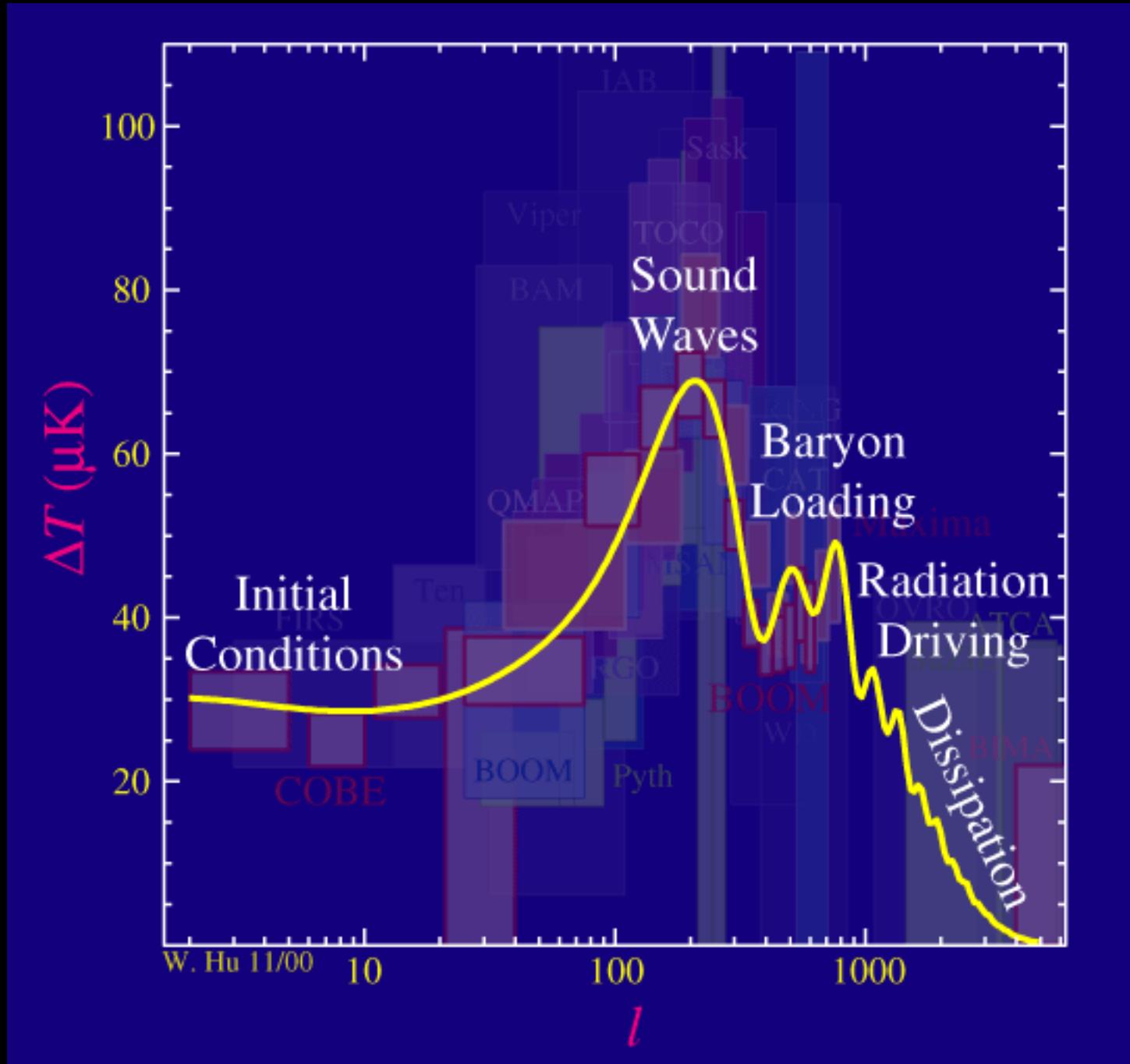


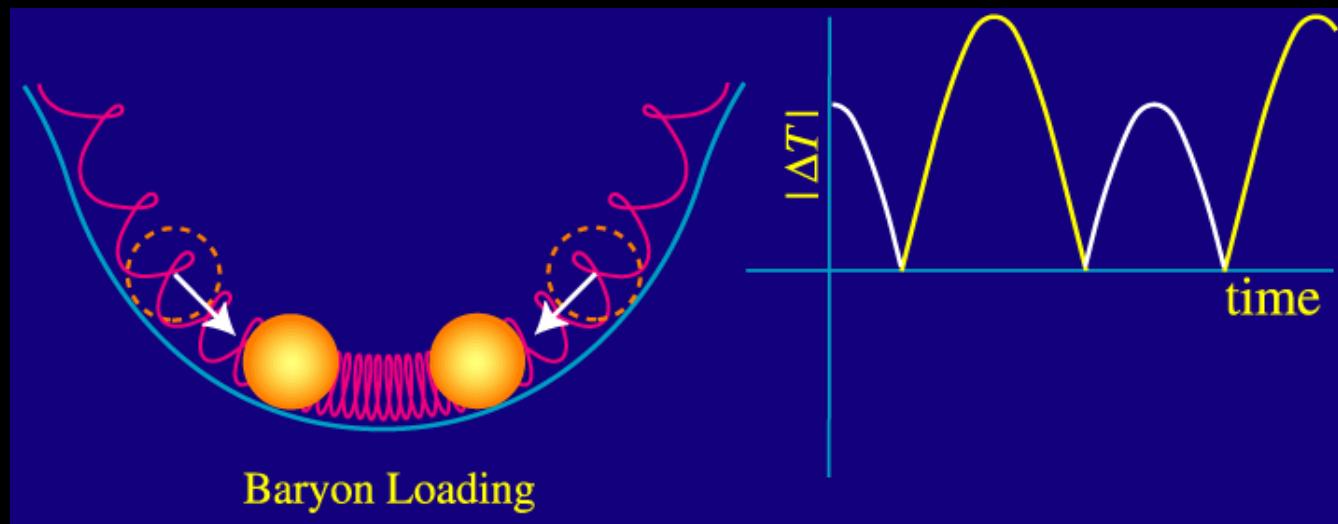
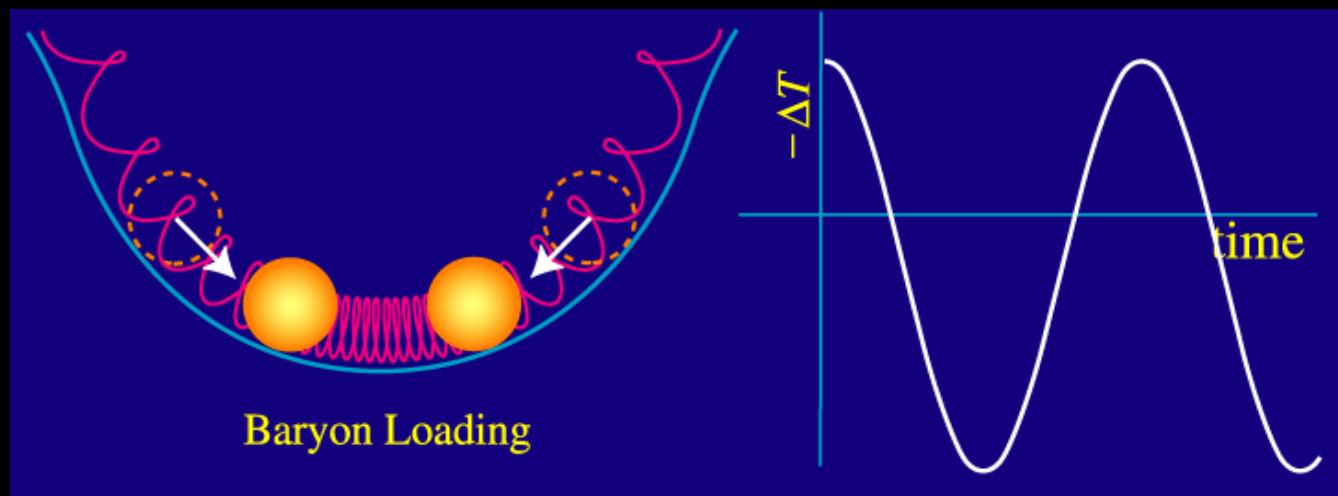
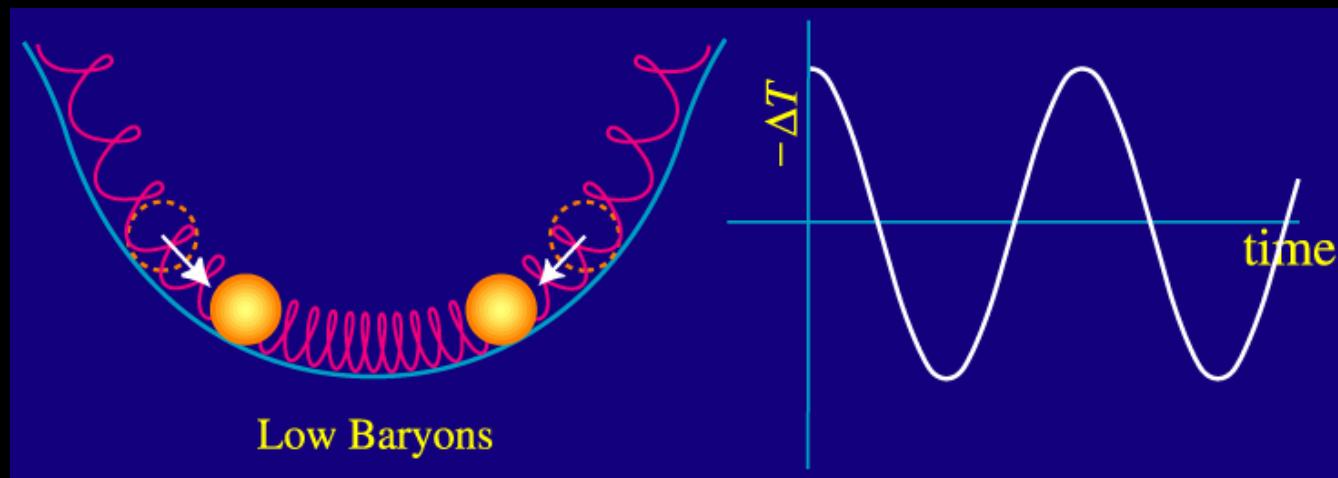
Multipole moment,  $\ell$

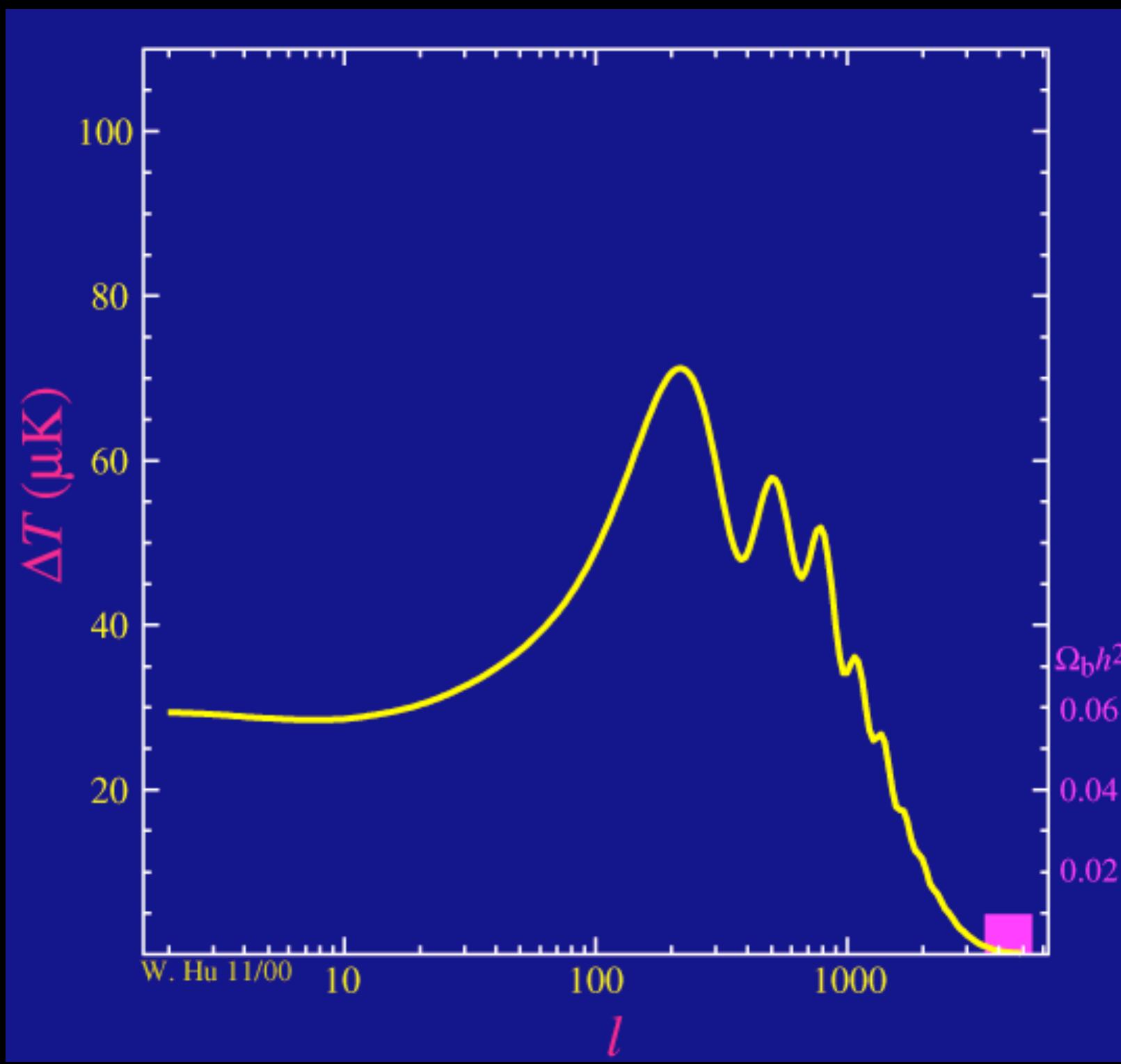


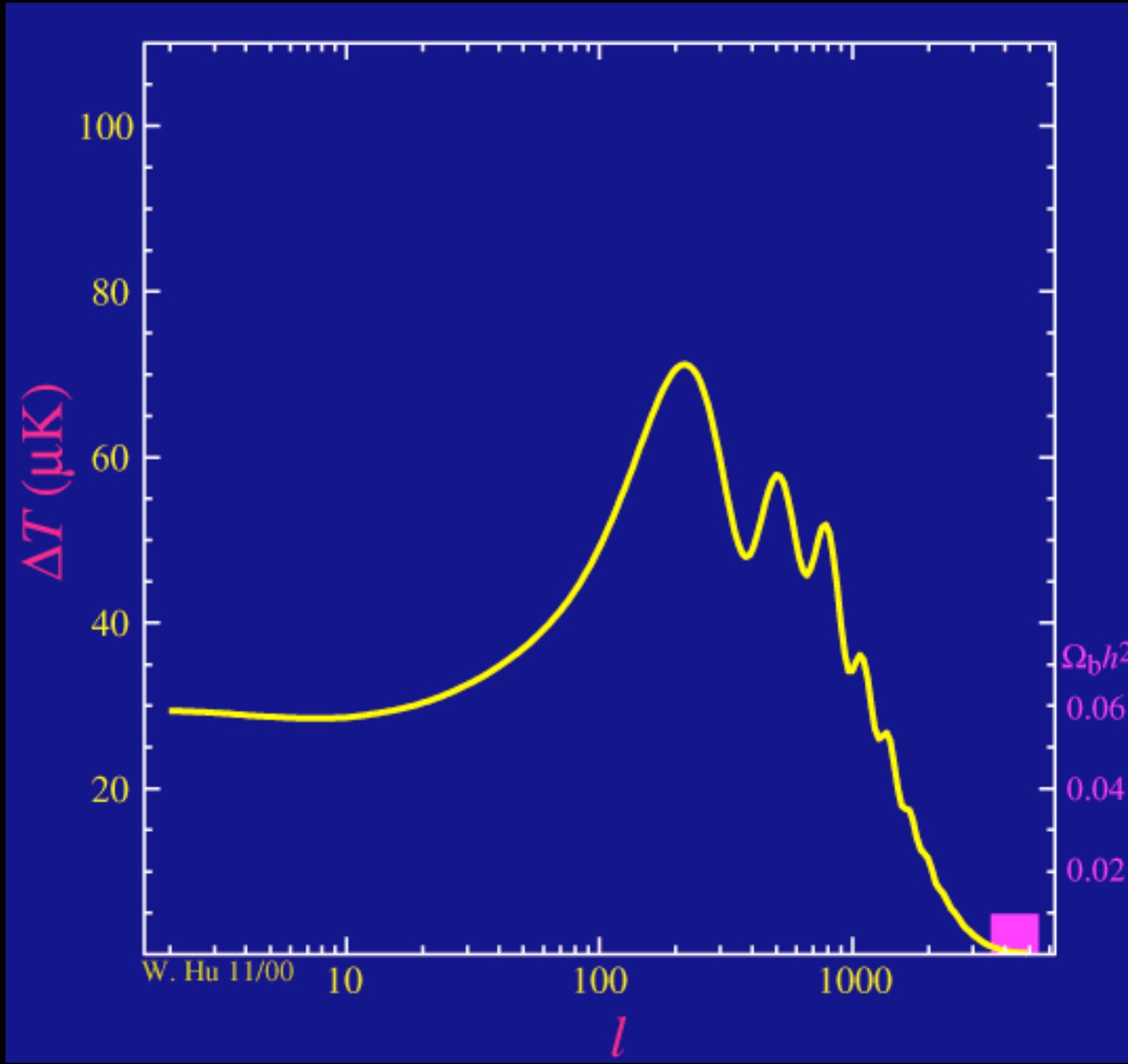
Supernova Cosmology Project  
Suzuki, et al., Ap.J. (2011)

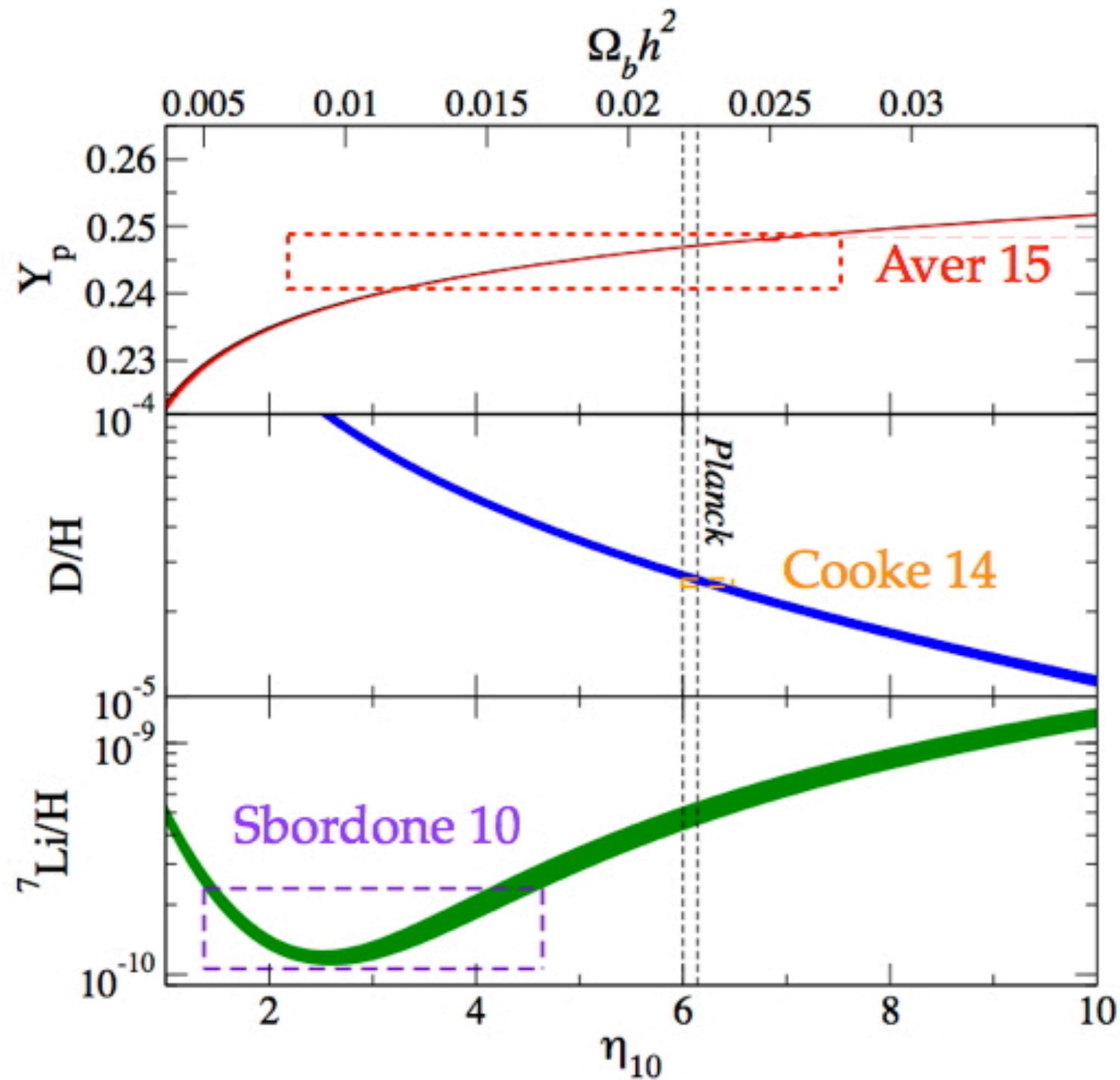


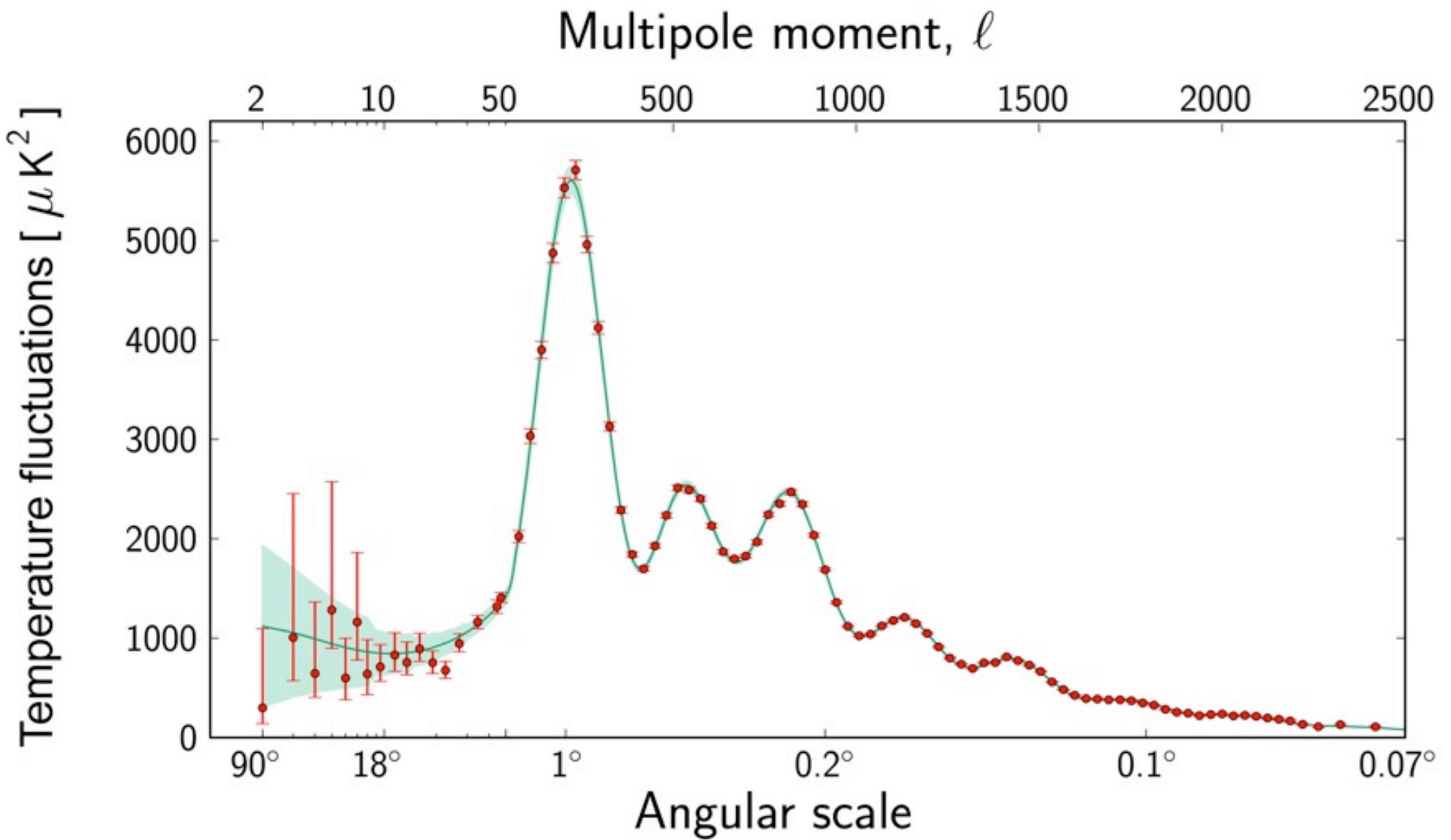




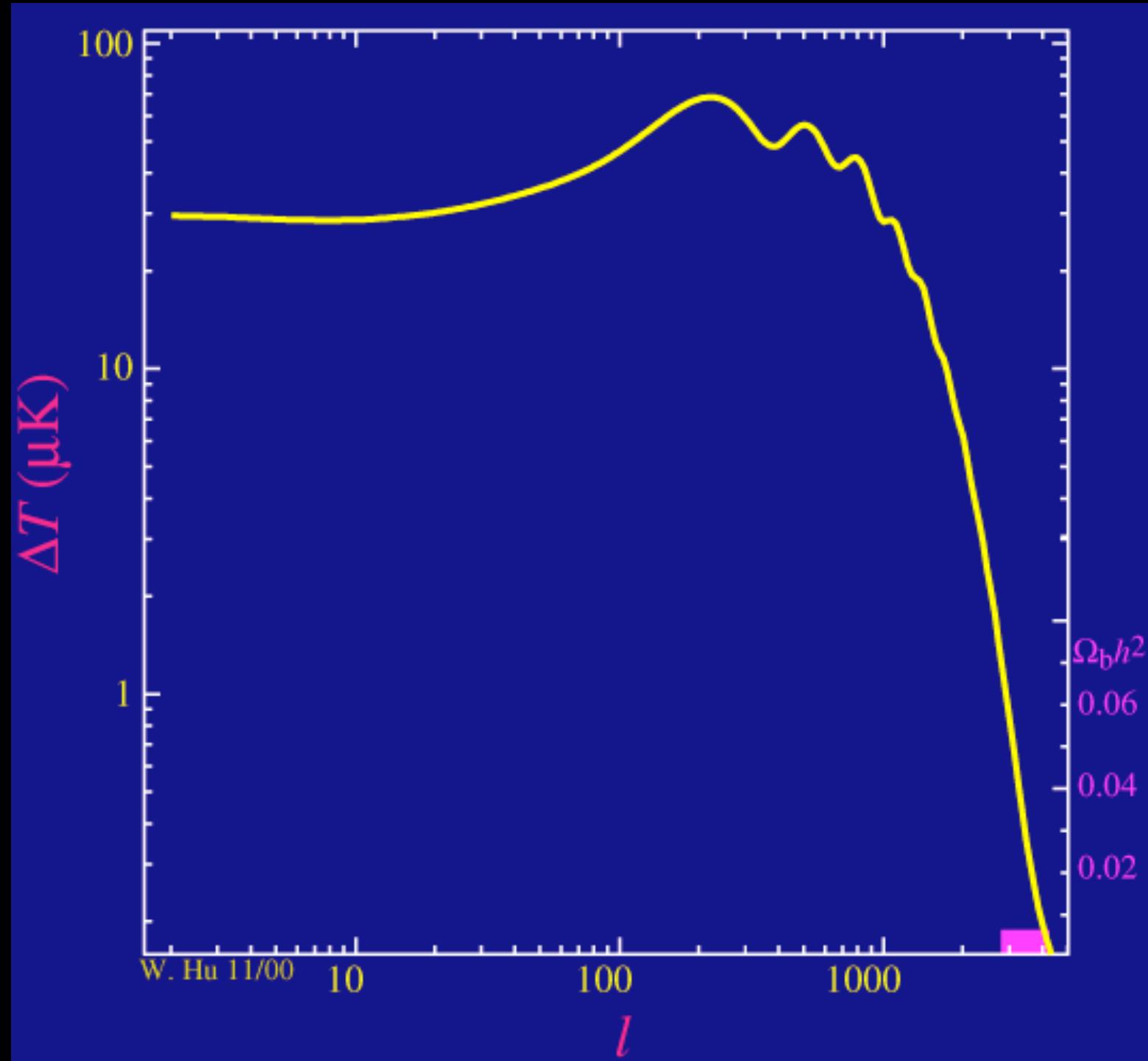


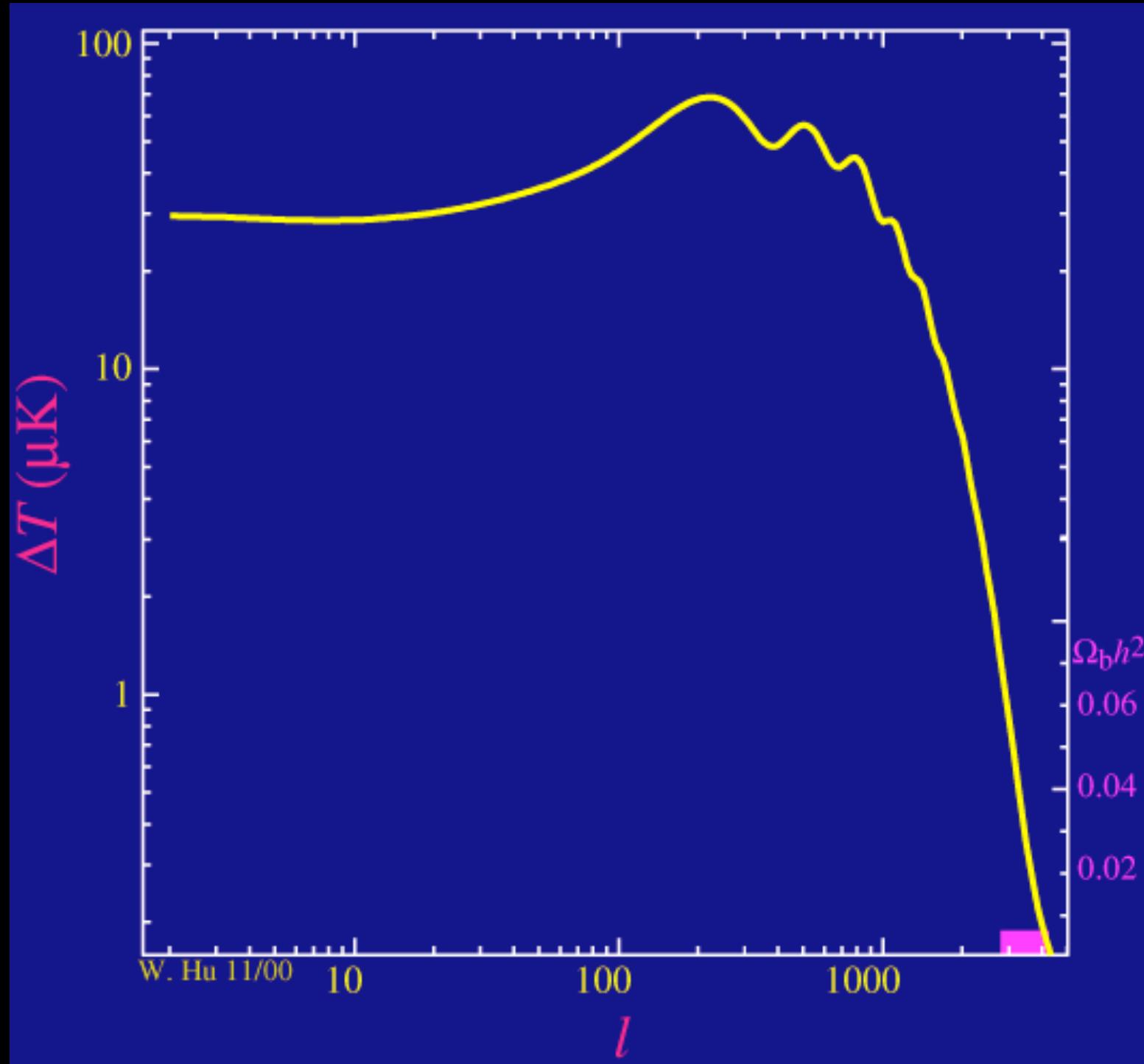


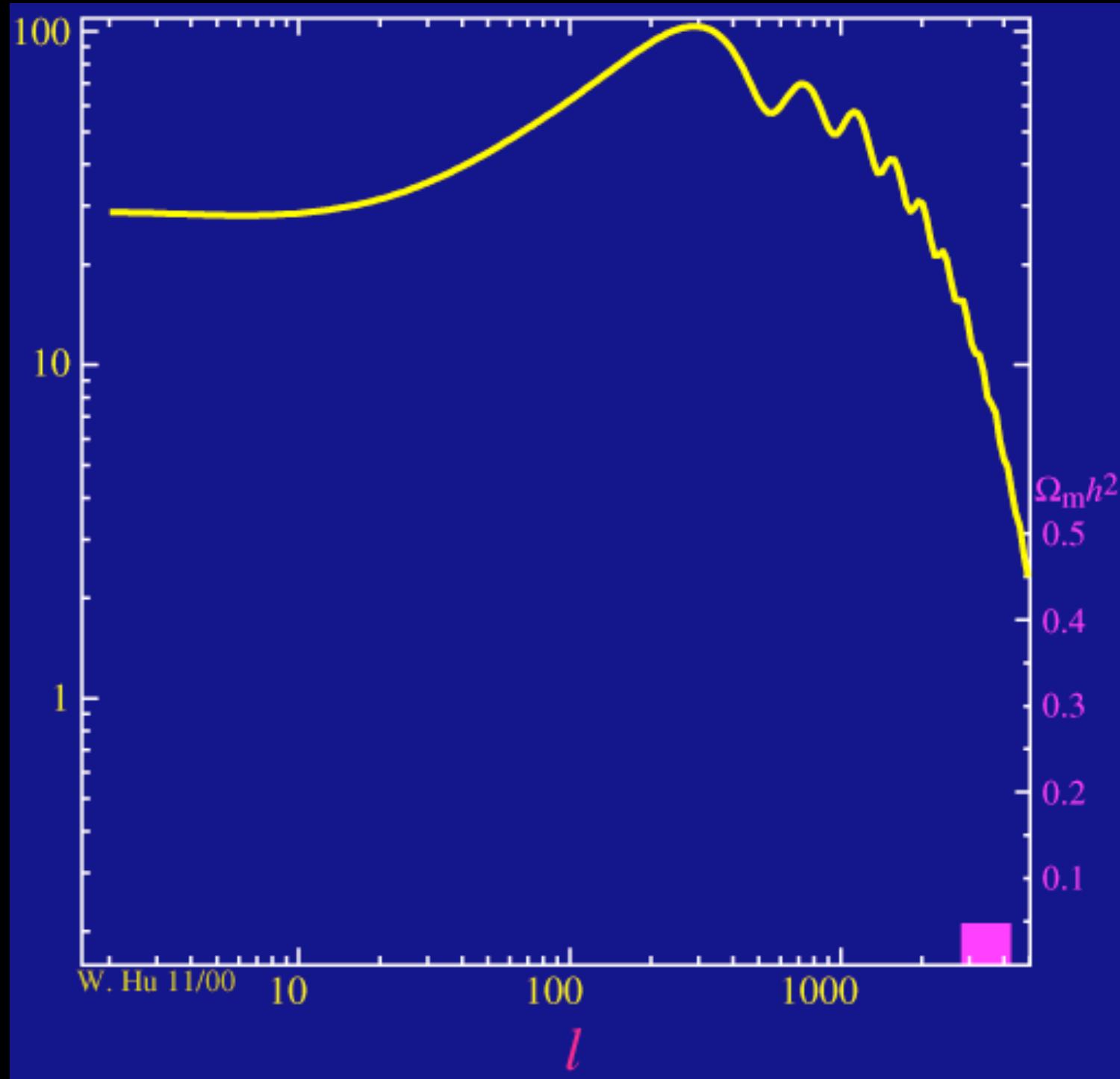












W. Hu 11/00

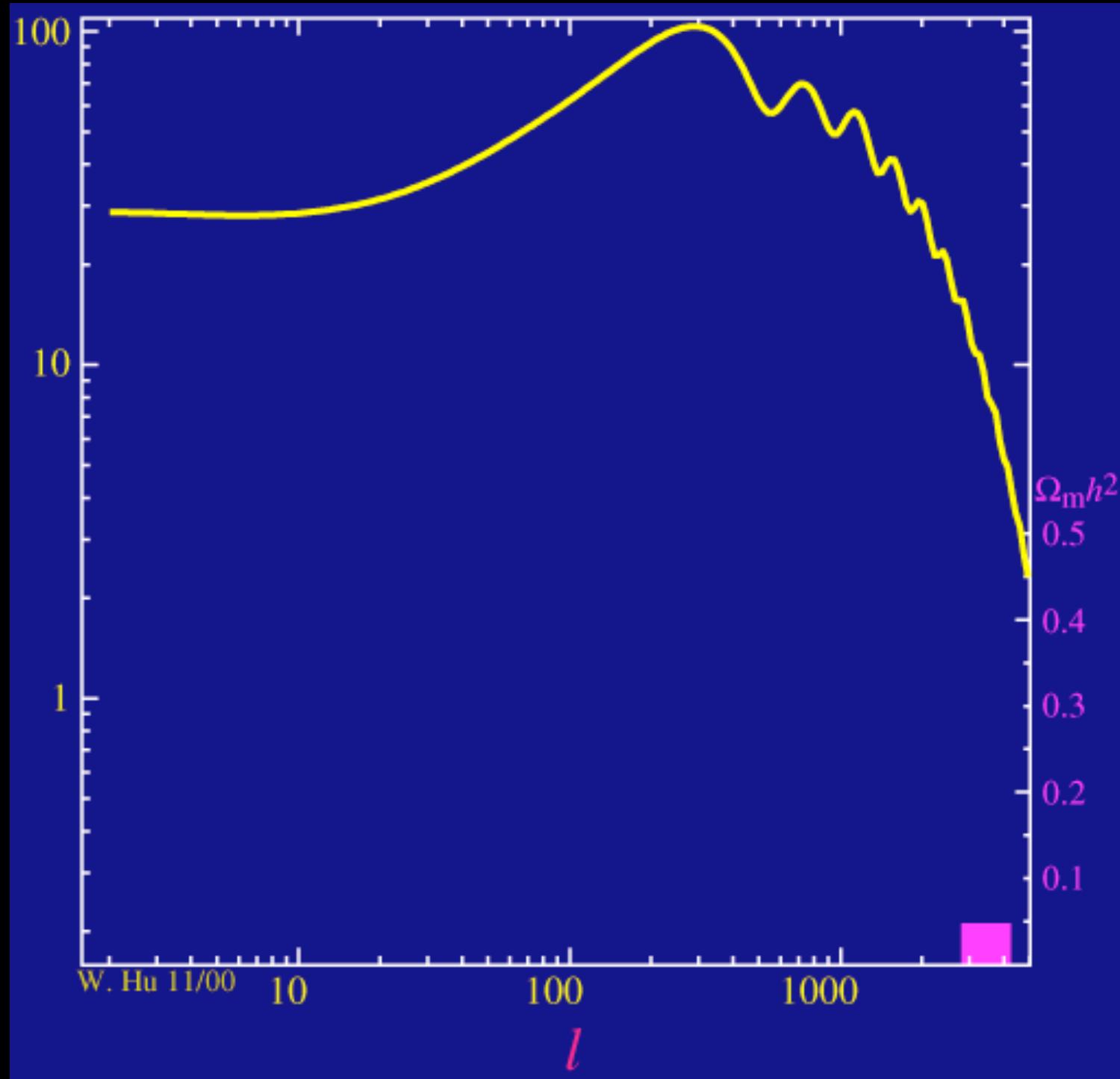
10

100

1000

$l$

$\Omega_m h^2$   
0.5  
0.4  
0.3  
0.2  
0.1



W. Hu 11/00

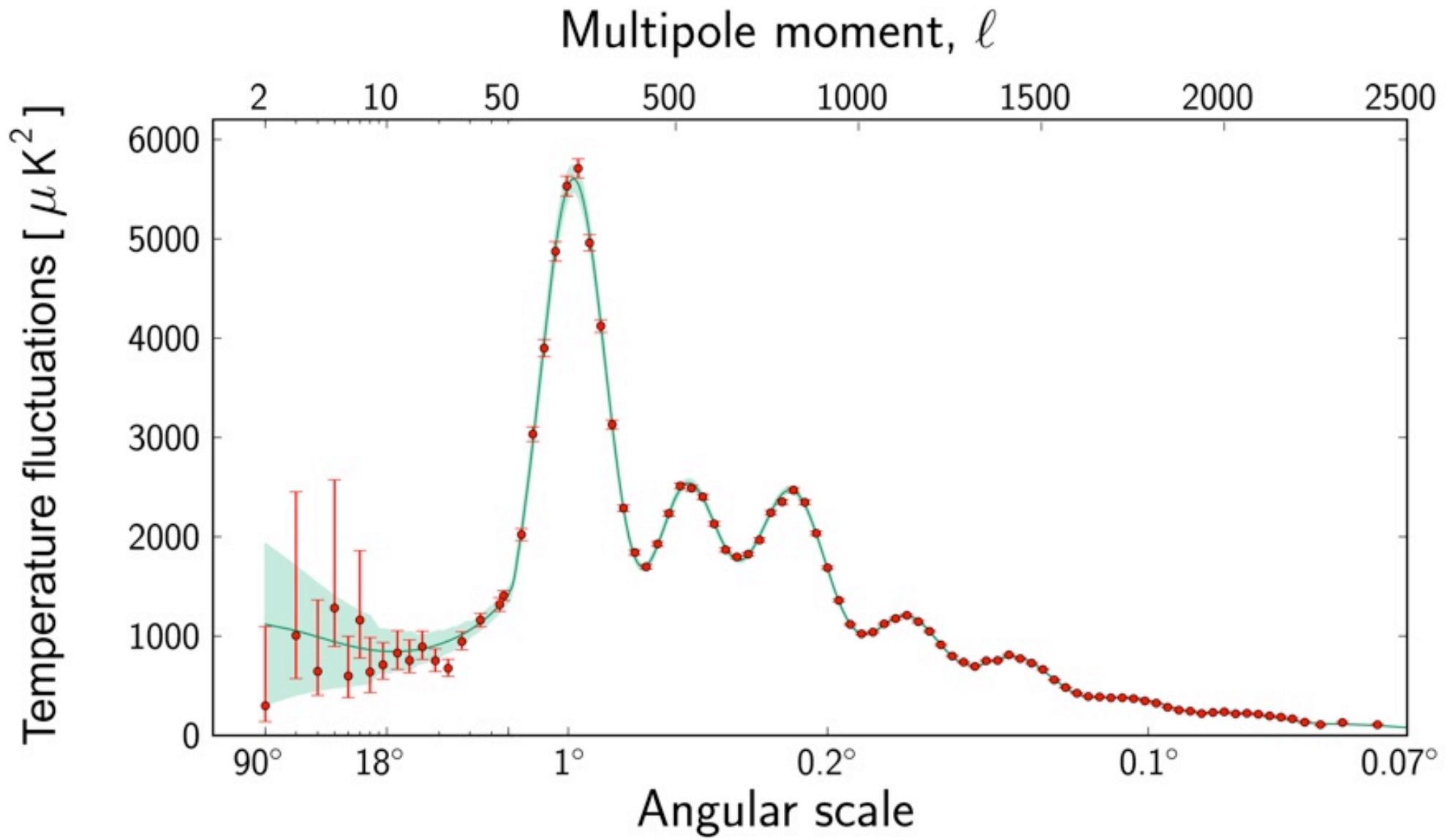
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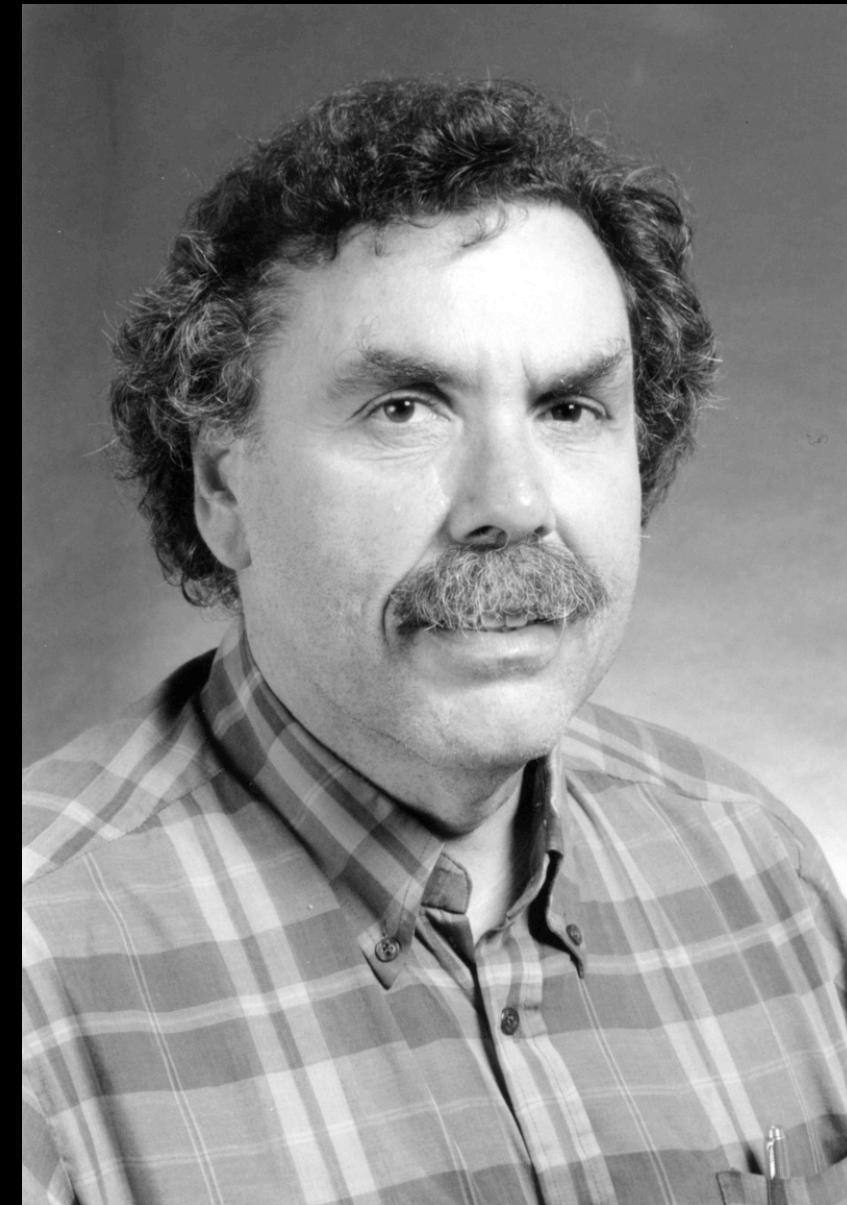
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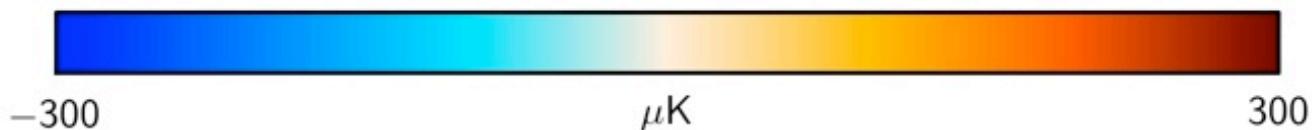
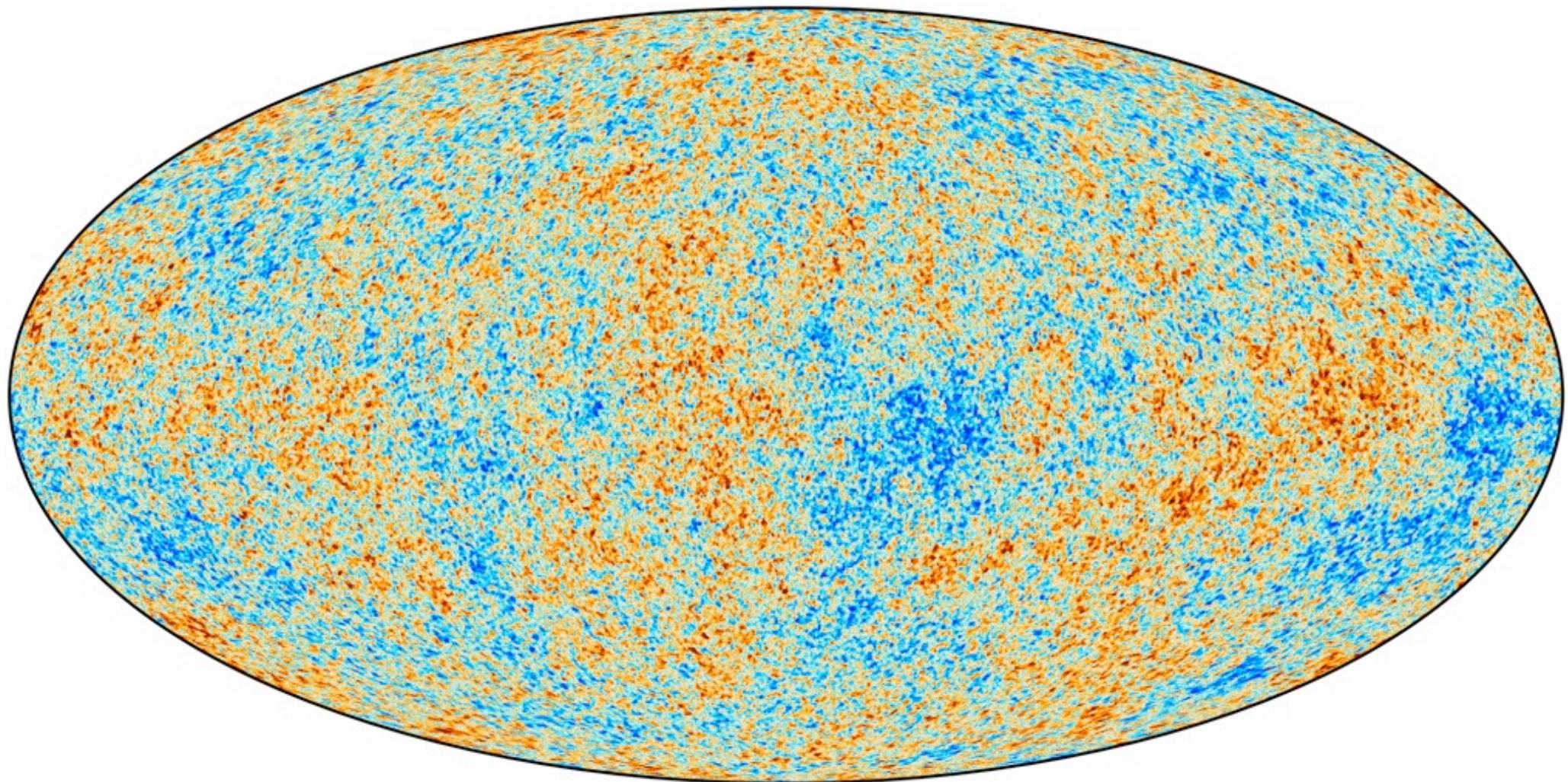
$\Omega_m h^2$   
0.5  
0.4  
0.3  
0.2  
0.1

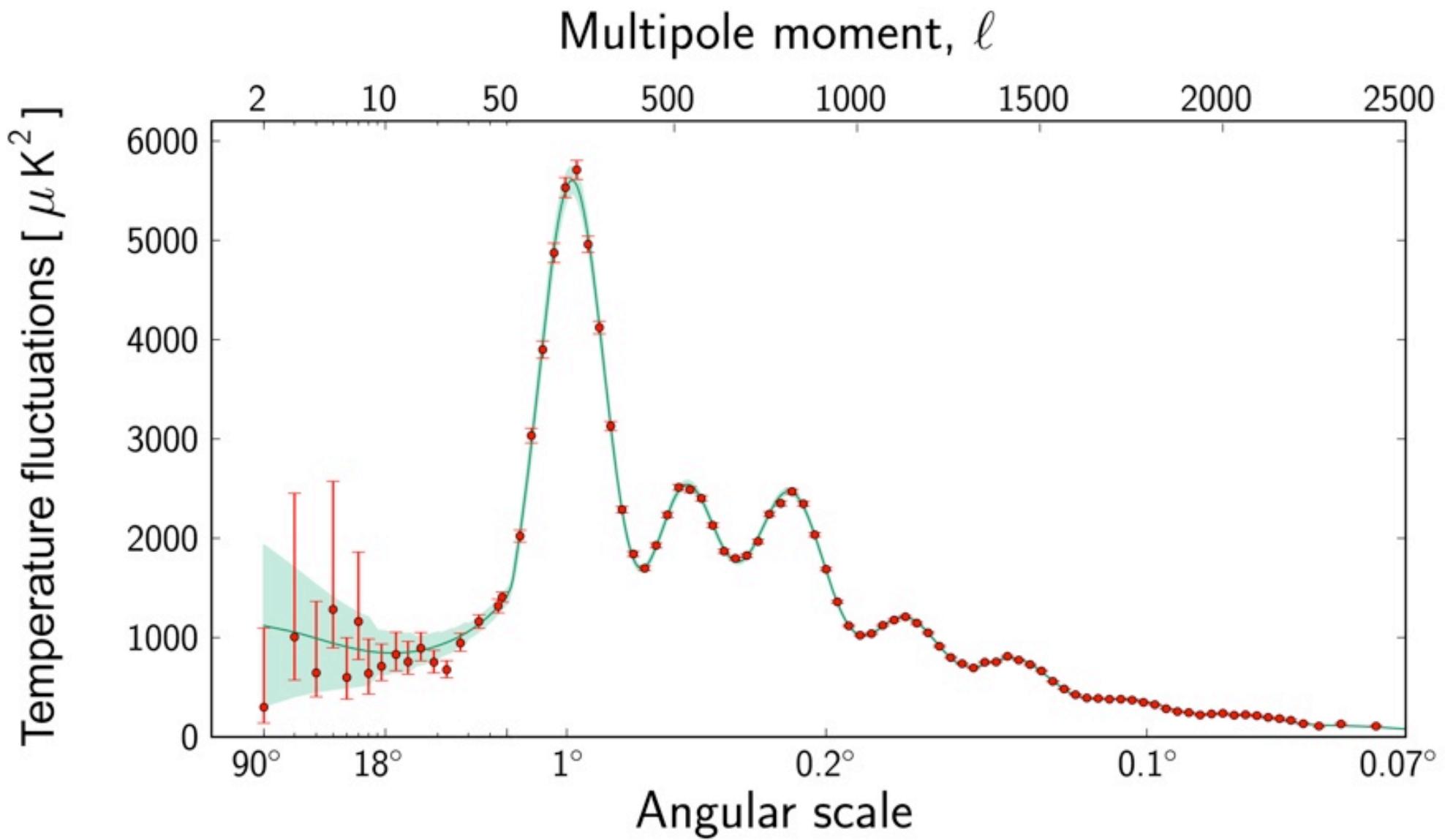


R K Sachs



A M Wolfe





## Planck 2015 results. XIII. Cosmological parameters

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Pasian<sup>56</sup>, G. Patanchon<sup>1</sup>, T. J. Pearson<sup>13,67</sup>, O. Perdereau<sup>81</sup>, L. Perotto<sup>86</sup>, F. Perrotta<sup>99</sup>, V. Pettorino<sup>50</sup>, F. Piacentini<sup>38</sup>, M. Piat<sup>1</sup>, E. Pierpaoli<sup>25</sup>, D. Pietrobon<sup>78</sup>, S. Plaszczynski<sup>81</sup>, E. Pointecouteau<sup>111,11</sup>, G. Polenta<sup>4,5</sup>, L. Popa<sup>72</sup>, G. W. Pratt<sup>84</sup>, G. Prézeau<sup>13,78</sup>, S. Prunet<sup>71,110</sup>, J.-L. Puget<sup>70</sup>, J. P. Rachén<sup>23,91</sup>, W. T. Reach<sup>112</sup>, R. Rebolo<sup>75,17,44</sup>, M. Reinecke<sup>91</sup>, M. Remazeilles<sup>79,70,1</sup>, C. Renaul<sup>86</sup>, A. Renzi<sup>42,61</sup>, L. Ristorcelli<sup>111,11</sup>, G. Rocha<sup>78,13</sup>, C. Rossetti<sup>1</sup>, M. Rossetti<sup>39,53</sup>, G. Roudier<sup>1,83,78</sup>, B. Rouillé d'Orfeuil<sup>81</sup>, M. Rowan-Robinson<sup>66</sup>, J. A. Rubiño-Martín<sup>75,44</sup>, B. Rusholme<sup>67</sup>, N. 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Wehus<sup>78</sup>, M. White<sup>32</sup>, S. D. M. White<sup>91</sup>, A. Wilkinson<sup>79</sup>, D. Yvon<sup>18</sup>, A. Zacchei<sup>56</sup>, and A. Zonca<sup>34</sup>

(Affiliations can be found after the references)

February 5 2015

### ABSTRACT

This paper presents cosmological results based on full-mission *Planck* observations of temperature and polarization anisotropies of the cosmic microwave background (CMB) radiation. Our results are in very good agreement with the 2013 analysis of the *Planck* nominal-mission temperature data, but with increased precision. The temperature and polarization power spectra are consistent with the standard spatially-flat six-parameter  $\Lambda$ CDM cosmology with a power-law spectrum of adiabatic scalar perturbations (denoted “base  $\Lambda$ CDM” in this paper). From the *Planck* temperature data combined with *Planck* lensing, for this cosmology we find a Hubble constant,  $H_0 = (67.8 \pm 0.9)$  km s $^{-1}$ Mpc $^{-1}$ , a matter density parameter  $\Omega_m = 0.308 \pm 0.012$ , and a tilted scalar spectral index with  $n_s = 0.968 \pm 0.006$ , consistent with the 2013 analysis. (In this abstract we quote 68 % confidence limits on measured parameters and 95 % upper limits on other parameters.) We present the first results of polarization measurements with the Low Frequency Instrument at large angular scales. Combined with the *Planck* temperature and lensing data, these measurements give a reionization optical depth of  $\tau = 0.066 \pm 0.016$ , corresponding to a reionization redshift of  $z_{\text{reion}} = 8.8^{+1.7}_{-1.4}$ . These results are consistent with those from WMAP polarization measurements cleaned for dust emission using 353 GHz polarization maps from the High Frequency Instrument. We find no evidence for any departure from base  $\Lambda$ CDM in the neutrino sector of the theory. For example, combining *Planck* observations with other astrophysical data we find  $N_{\text{eff}} = 3.15 \pm 0.23$  for the effective number of relativistic degrees of freedom, consistent with the value  $N_{\text{eff}} = 3.046$  of the Standard Model of particle physics. The sum of neutrino masses is constrained to  $\sum m_\nu < 0.23$  eV. The spatial curvature of our Universe is found to be very close to zero with  $|\Omega_K| < 0.005$ . Adding a tensor component as a single-parameter extension to base  $\Lambda$ CDM we find an upper limit on the tensor-to-scalar ratio of  $r_{0.002} < 0.11$ , consistent with the *Planck* 2013 results and consistent with the  $B$ -mode polarization constraints from a joint analysis of BICEP2, Keck Array, and *Planck* (BKP) data. Adding the BKP  $B$ -mode data to our analysis leads to a tighter constraint of  $r_{0.002} < 0.09$  and disfavors inflationary models with a  $V(\phi) \propto \phi^2$  potential. The addition of *Planck* polarization data leads to strong constraints on deviations from a purely adiabatic spectrum of fluctuations. We find no evidence for any contribution from isocurvature perturbations or from cosmic defects. Combining *Planck* data with other astrophysical data, including Type Ia supernovae, the equation of state of dark energy is constrained to  $w = -1.006 \pm 0.045$ , consistent with the expected value for a cosmological constant. The standard big bang nucleosynthesis predictions for the helium and deuterium abundances for the best-fit *Planck* base  $\Lambda$ CDM cosmology are in excellent agreement with observations. We also analyse constraints on annihilating dark matter and on possible deviations from the standard recombination history. In both cases, we find no evidence for new physics. The *Planck* results for base  $\Lambda$ CDM are in good agreement with baryon acoustic oscillation data and with the JLA sample of Type Ia supernovae. However, as in the 2013 analysis, the amplitude of the fluctuation spectrum is found to be higher than inferred from some analyses of rich cluster counts and weak gravitational lensing. We show that these tensions cannot easily be resolved with simple modifications of the base  $\Lambda$ CDM cosmology. Apart from these tensions, the base  $\Lambda$ CDM cosmology provides an excellent description of the *Planck* CMB observations and many other astrophysical data sets.

**Key words.** Cosmology: observations – Cosmology: theory – cosmic microwave background – cosmological parameters

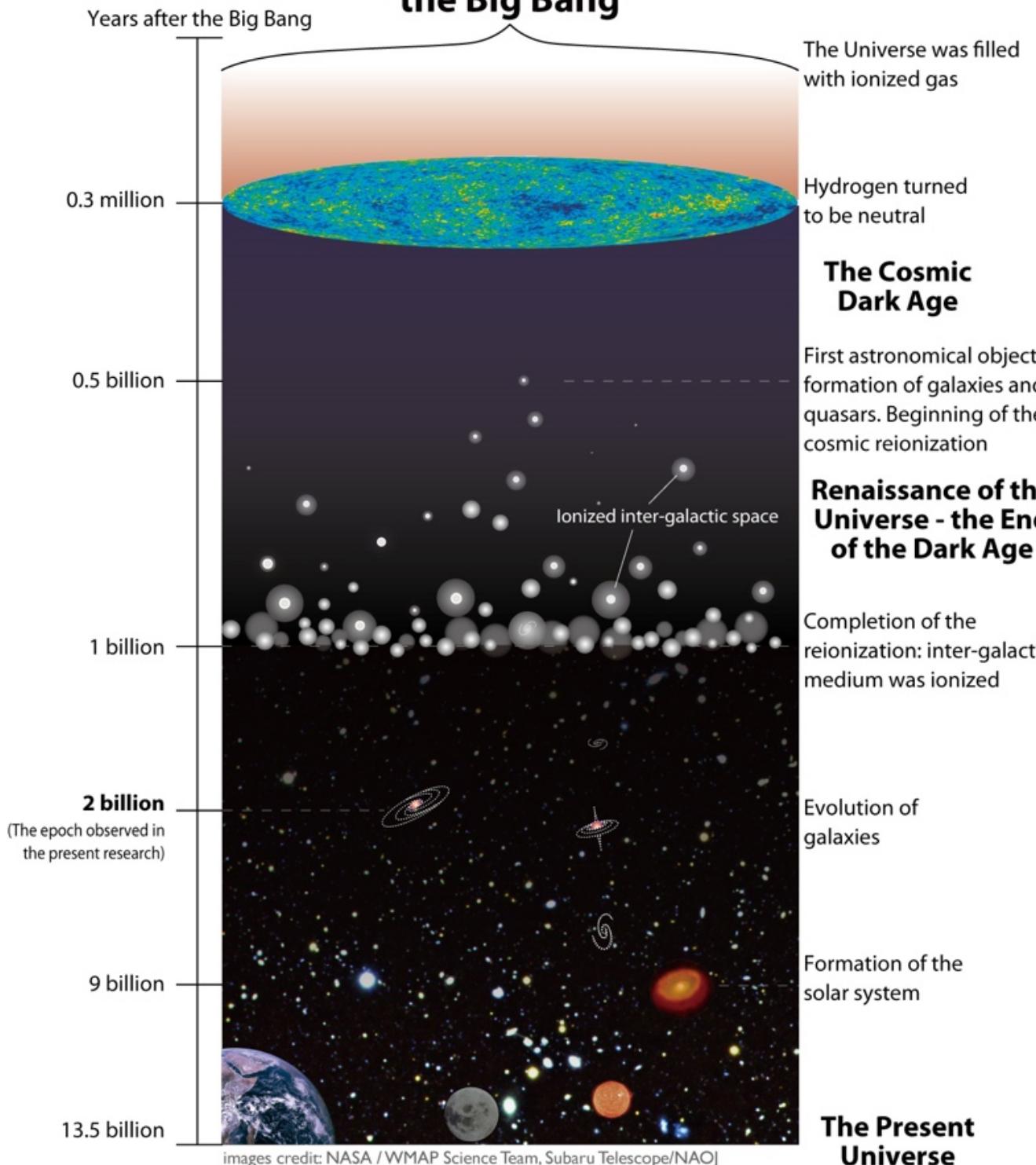
**Table 4.** Parameter 68 % confidence limits for the base  $\Lambda\text{CDM}$  model from *Planck* CMB power spectra, in combination with lensing reconstruction (“lensing”) and external data (“ext,” BAO+JLA+ $H_0$ ). Nuisance parameters are not listed for brevity (they can be found in the *Planck Legacy Archive* tables), but the last three parameters give a summary measure of the total foreground amplitude (in  $\mu\text{K}^2$ ) at  $\ell = 2000$  for the three high- $\ell$  temperature spectra used by the likelihood. In all cases the helium mass fraction used is predicted by BBN (posterior mean  $Y_{\text{P}} \approx 0.2453$ , with theoretical uncertainties in the BBN predictions dominating over the *Planck* error on  $\Omega_{\text{b}}h^2$ ).

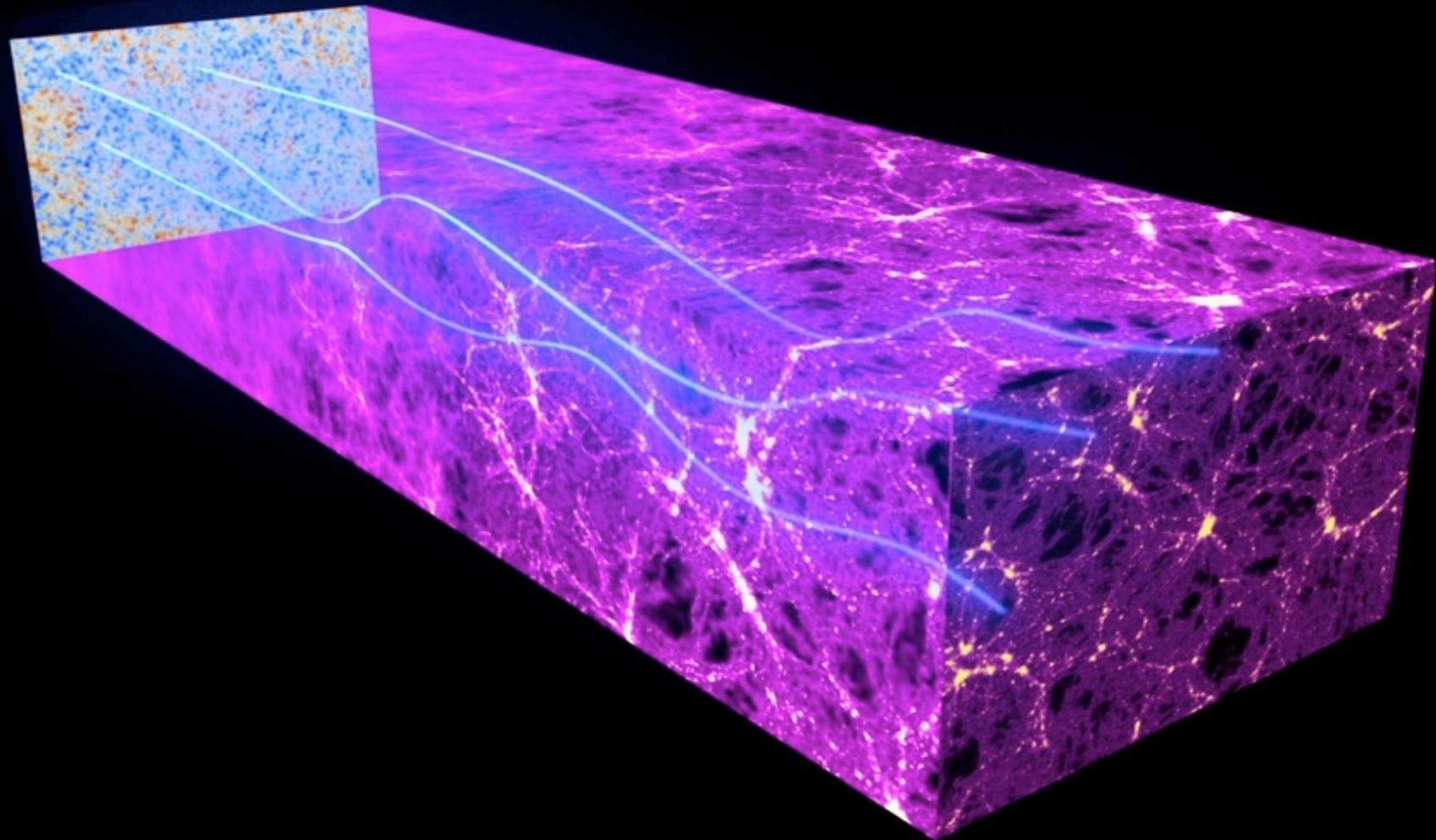
Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits	TT,TE,EE+lowP+lensing 68 % limits	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_{\text{b}}h^2$	$0.02222 \pm 0.00023$	$0.02226 \pm 0.00023$	$0.02227 \pm 0.00020$	$0.02225 \pm 0.00016$	$0.02226 \pm 0.00016$	$0.02230 \pm 0.00014$
$\Omega_{\text{c}}h^2$	$0.1197 \pm 0.0022$	$0.1186 \pm 0.0020$	$0.1184 \pm 0.0012$	$0.1198 \pm 0.0015$	$0.1193 \pm 0.0014$	$0.1188 \pm 0.0010$
$100\theta_{\text{MC}}$	$1.04085 \pm 0.00047$	$1.04103 \pm 0.00046$	$1.04106 \pm 0.00041$	$1.04077 \pm 0.00032$	$1.04087 \pm 0.00032$	$1.04093 \pm 0.00030$
$\tau$	$0.078 \pm 0.019$	$0.066 \pm 0.016$	$0.067 \pm 0.013$	$0.079 \pm 0.017$	$0.063 \pm 0.014$	$0.066 \pm 0.012$
$\ln(10^{10}A_s)$	$3.089 \pm 0.036$	$3.062 \pm 0.029$	$3.064 \pm 0.024$	$3.094 \pm 0.034$	$3.059 \pm 0.025$	$3.064 \pm 0.023$
$n_s$	$0.9655 \pm 0.0062$	$0.9677 \pm 0.0060$	$0.9681 \pm 0.0044$	$0.9645 \pm 0.0049$	$0.9653 \pm 0.0048$	$0.9667 \pm 0.0040$
$H_0$	$67.31 \pm 0.96$	$67.81 \pm 0.92$	$67.90 \pm 0.55$	$67.27 \pm 0.66$	$67.51 \pm 0.64$	$67.74 \pm 0.46$
$\Omega_{\Lambda}$	$0.685 \pm 0.013$	$0.692 \pm 0.012$	$0.6935 \pm 0.0072$	$0.6844 \pm 0.0091$	$0.6879 \pm 0.0087$	$0.6911 \pm 0.0062$
$\Omega_{\text{m}}$	$0.315 \pm 0.013$	$0.308 \pm 0.012$	$0.3065 \pm 0.0072$	$0.3156 \pm 0.0091$	$0.3121 \pm 0.0087$	$0.3089 \pm 0.0062$
$\Omega_{\text{m}}h^2$	$0.1426 \pm 0.0020$	$0.1415 \pm 0.0019$	$0.1413 \pm 0.0011$	$0.1427 \pm 0.0014$	$0.1422 \pm 0.0013$	$0.14170 \pm 0.00097$
$\Omega_{\text{m}}h^3$	$0.09597 \pm 0.00045$	$0.09591 \pm 0.00045$	$0.09593 \pm 0.00045$	$0.09601 \pm 0.00029$	$0.09596 \pm 0.00030$	$0.09598 \pm 0.00029$
$\sigma_8$	$0.829 \pm 0.014$	$0.8149 \pm 0.0093$	$0.8154 \pm 0.0090$	$0.831 \pm 0.013$	$0.8150 \pm 0.0087$	$0.8159 \pm 0.0086$
$\sigma_8\Omega_{\text{m}}^{0.5}$	$0.466 \pm 0.013$	$0.4521 \pm 0.0088$	$0.4514 \pm 0.0066$	$0.4668 \pm 0.0098$	$0.4553 \pm 0.0068$	$0.4535 \pm 0.0059$
$\sigma_8\Omega_{\text{m}}^{0.25}$	$0.621 \pm 0.013$	$0.6069 \pm 0.0076$	$0.6066 \pm 0.0070$	$0.623 \pm 0.011$	$0.6091 \pm 0.0067$	$0.6083 \pm 0.0066$
$z_{\text{re}}$	$9.9^{+1.8}_{-1.6}$	$8.8^{+1.7}_{-1.4}$	$8.9^{+1.3}_{-1.2}$	$10.0^{+1.7}_{-1.5}$	$8.5^{+1.4}_{-1.2}$	$8.8^{+1.2}_{-1.1}$
$10^3 A_s$	$2.198^{+0.076}_{-0.085}$	$2.139 \pm 0.063$	$2.143 \pm 0.051$	$2.207 \pm 0.074$	$2.130 \pm 0.053$	$2.142 \pm 0.049$
$10^3 A_s e^{-2r}$	$1.880 \pm 0.014$	$1.874 \pm 0.013$	$1.873 \pm 0.011$	$1.882 \pm 0.012$	$1.878 \pm 0.011$	$1.876 \pm 0.011$
Age/Gyr	$13.813 \pm 0.038$	$13.799 \pm 0.038$	$13.796 \pm 0.029$	$13.813 \pm 0.026$	$13.807 \pm 0.026$	$13.799 \pm 0.021$
$z_e$	$1090.09 \pm 0.42$	$1089.94 \pm 0.42$	$1089.90 \pm 0.30$	$1090.06 \pm 0.30$	$1090.00 \pm 0.29$	$1089.90 \pm 0.23$
$r_s$	$144.61 \pm 0.49$	$144.89 \pm 0.44$	$144.93 \pm 0.30$	$144.57 \pm 0.32$	$144.71 \pm 0.31$	$144.81 \pm 0.24$
$100\theta_s$	$1.04105 \pm 0.00046$	$1.04122 \pm 0.00045$	$1.04126 \pm 0.00041$	$1.04096 \pm 0.00032$	$1.04106 \pm 0.00031$	$1.04112 \pm 0.00029$
$z_{\text{drag}}$	$1059.57 \pm 0.46$	$1059.57 \pm 0.47$	$1059.60 \pm 0.44$	$1059.65 \pm 0.31$	$1059.62 \pm 0.31$	$1059.68 \pm 0.29$
$r_{\text{drag}}$	$147.33 \pm 0.49$	$147.60 \pm 0.43$	$147.63 \pm 0.32$	$147.27 \pm 0.31$	$147.41 \pm 0.30$	$147.50 \pm 0.24$
$k_D$	$0.14050 \pm 0.00052$	$0.14024 \pm 0.00047$	$0.14022 \pm 0.00042$	$0.14059 \pm 0.00032$	$0.14044 \pm 0.00032$	$0.14038 \pm 0.00029$
$z_{\text{eq}}$	$3393 \pm 49$	$3365 \pm 44$	$3361 \pm 27$	$3395 \pm 33$	$3382 \pm 32$	$3371 \pm 23$
$k_{\text{eq}}$	$0.01035 \pm 0.00015$	$0.01027 \pm 0.00014$	$0.010258 \pm 0.000083$	$0.01036 \pm 0.00010$	$0.010322 \pm 0.000096$	$0.010288 \pm 0.000071$
$100\theta_{\text{eq}}$	$0.4502 \pm 0.0047$	$0.4529 \pm 0.0044$	$0.4533 \pm 0.0026$	$0.4499 \pm 0.0032$	$0.4512 \pm 0.0031$	$0.4523 \pm 0.0023$
$f_{143}^{143}$	$29.9 \pm 2.9$	$30.4 \pm 2.9$	$30.3 \pm 2.8$	$29.5 \pm 2.7$	$30.2 \pm 2.7$	$30.0 \pm 2.7$
$f_{143 \times 217}^{143 \times 217}$	$32.4 \pm 2.1$	$32.8 \pm 2.1$	$32.7 \pm 2.0$	$32.2 \pm 1.9$	$32.8 \pm 1.9$	$32.6 \pm 1.9$
$f_{217}^{217}$	$106.0 \pm 2.0$	$106.3 \pm 2.0$	$106.2 \pm 2.0$	$105.8 \pm 1.9$	$106.2 \pm 1.9$	$106.1 \pm 1.8$

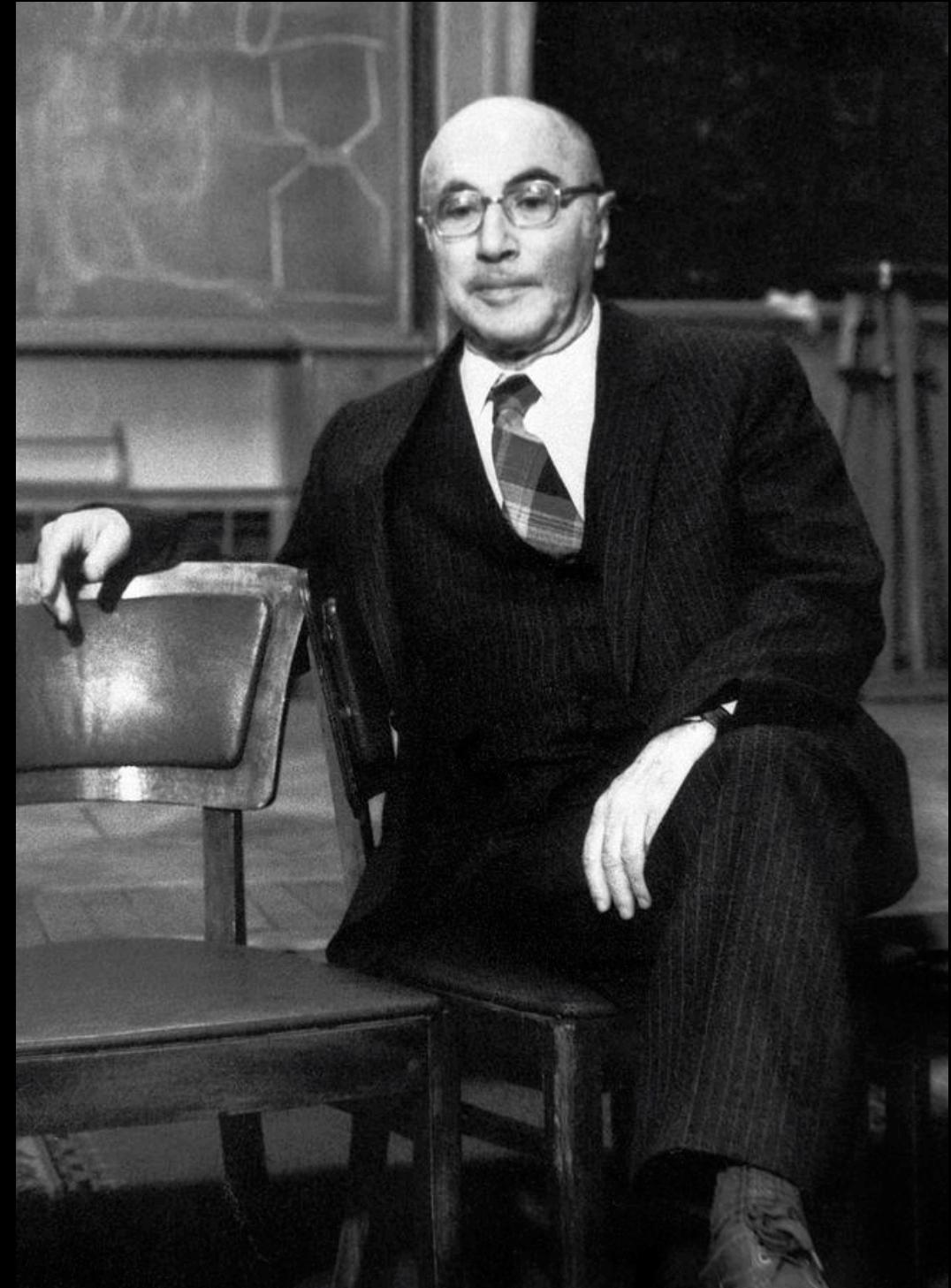
**Table 5.** Constraints on 1-parameter extensions to the base  $\Lambda\text{CDM}$  model for combinations of *Planck* power spectra, *Planck* lensing, and external data (BAO+JLA+ $H_0$ , denoted “ext”). Note that we quote 95 % limits here.

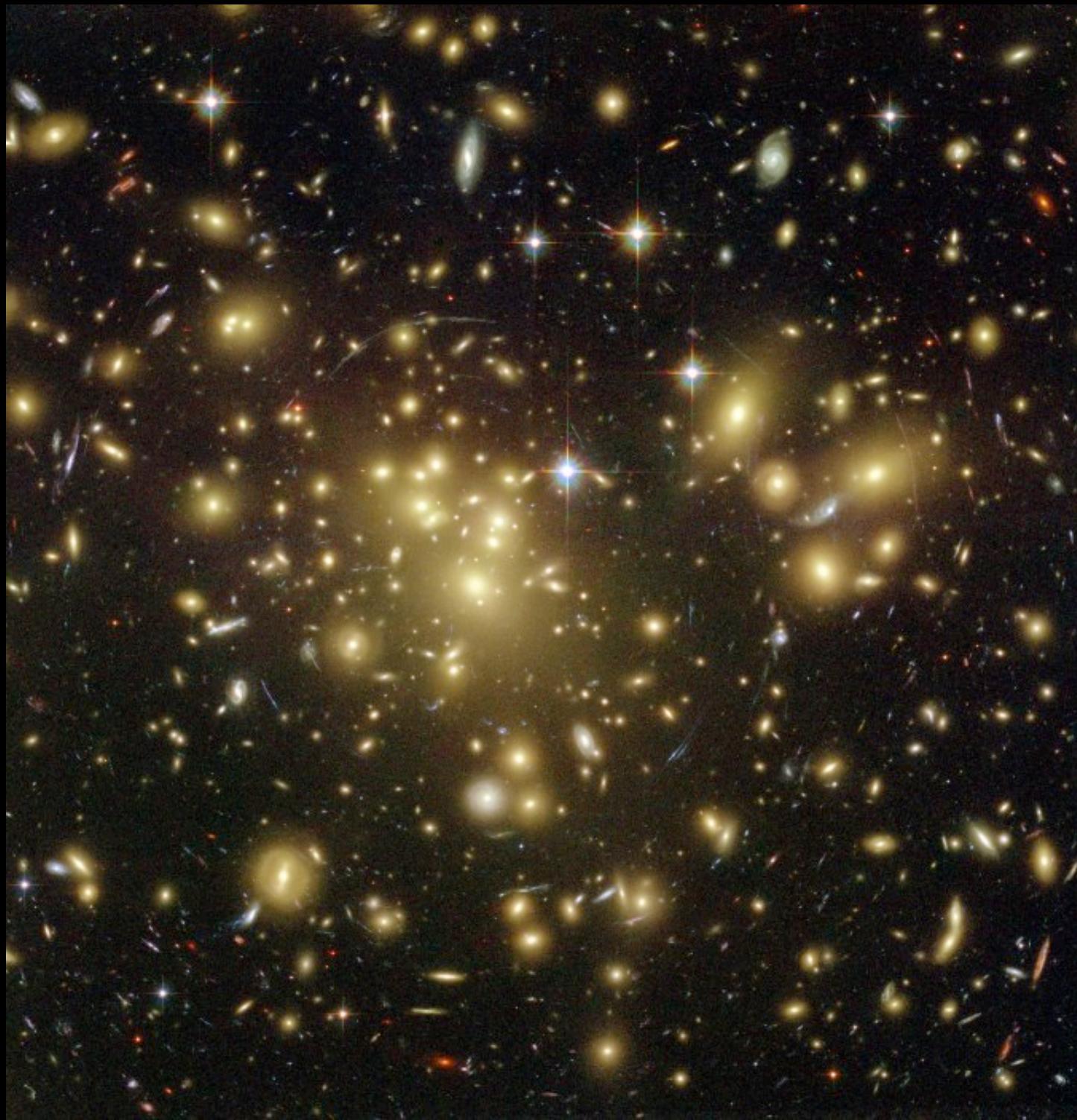
Parameter	TT	TT+lensing	TT+lensing+ext	TT, TE, EE	TT, TE, EE+lensing	TT, TE, EE+lensing+ext
$\Omega_K$	$-0.052^{+0.049}_{-0.055}$	$-0.005^{+0.016}_{-0.017}$	$-0.0001^{+0.0054}_{-0.0052}$	$-0.040^{+0.038}_{-0.041}$	$-0.004^{+0.015}_{-0.015}$	$0.0008^{+0.0040}_{-0.0039}$
$\Sigma m_\nu$ [eV]	$< 0.715$	$< 0.675$	$< 0.234$	$< 0.492$	$< 0.589$	$< 0.194$
$N_{\text{eff}}$	$3.13^{+0.64}_{-0.63}$	$3.13^{+0.62}_{-0.61}$	$3.15^{+0.41}_{-0.40}$	$2.99^{+0.41}_{-0.39}$	$2.94^{+0.38}_{-0.38}$	$3.04^{+0.33}_{-0.33}$
$Y_P$	$0.2552^{+0.041}_{-0.042}$	$0.251^{+0.040}_{-0.039}$	$0.251^{+0.035}_{-0.036}$	$0.250^{+0.026}_{-0.027}$	$0.247^{+0.026}_{-0.027}$	$0.249^{+0.025}_{-0.026}$
$dn_s/d\ln k$	$-0.008^{+0.016}_{-0.016}$	$-0.003^{+0.015}_{-0.015}$	$-0.003^{+0.015}_{-0.014}$	$-0.006^{+0.014}_{-0.014}$	$-0.002^{+0.013}_{-0.013}$	$-0.002^{+0.013}_{-0.013}$
$r_{0.02}$	$< 0.103$	$< 0.114$	$< 0.114$	$< 0.0987$	$< 0.112$	$< 0.113$
$w$	$-1.54^{+0.62}_{-0.50}$	$-1.41^{+0.64}_{-0.56}$	$-1.006^{+0.085}_{-0.091}$	$-1.55^{+0.58}_{-0.48}$	$-1.42^{+0.62}_{-0.56}$	$-1.019^{+0.075}_{-0.080}$

# the Big Bang

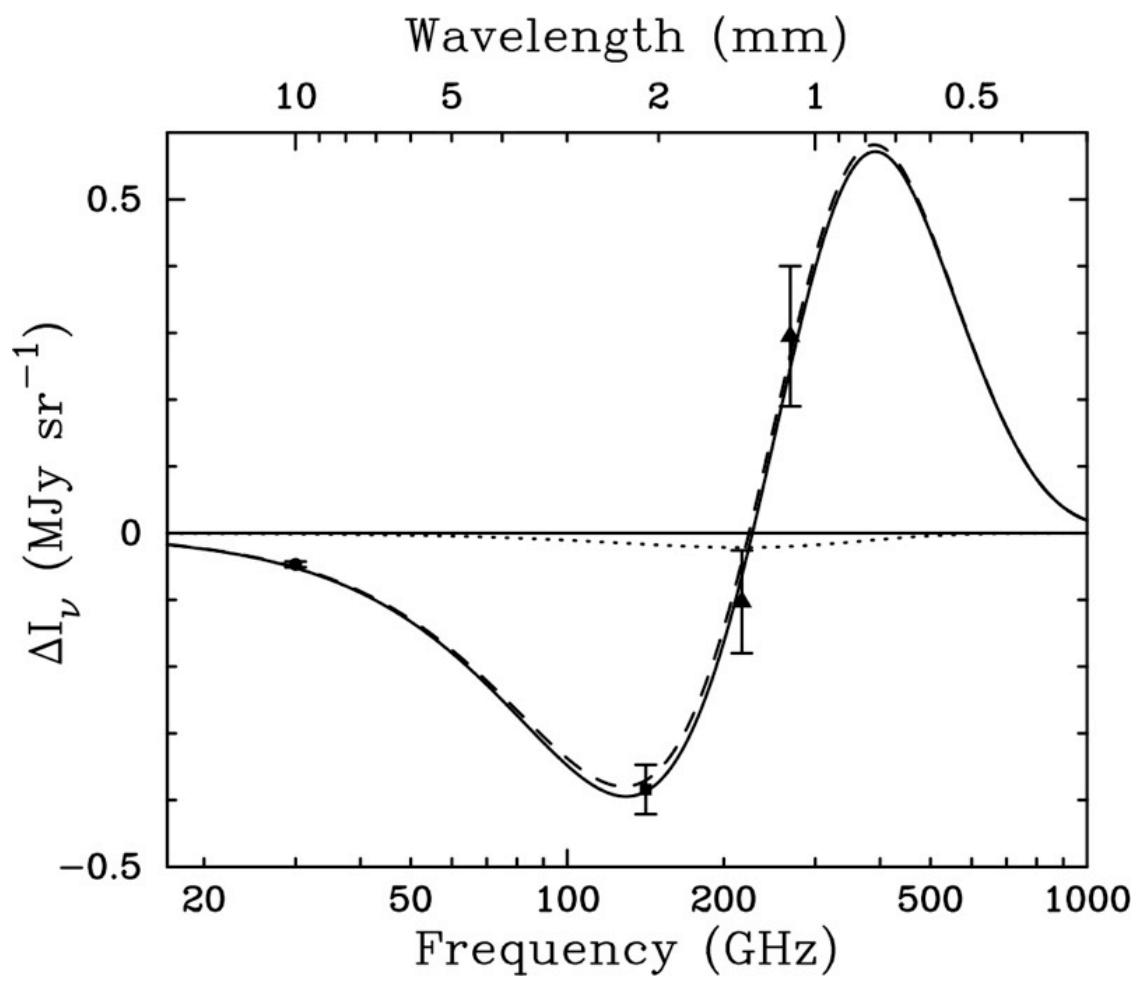
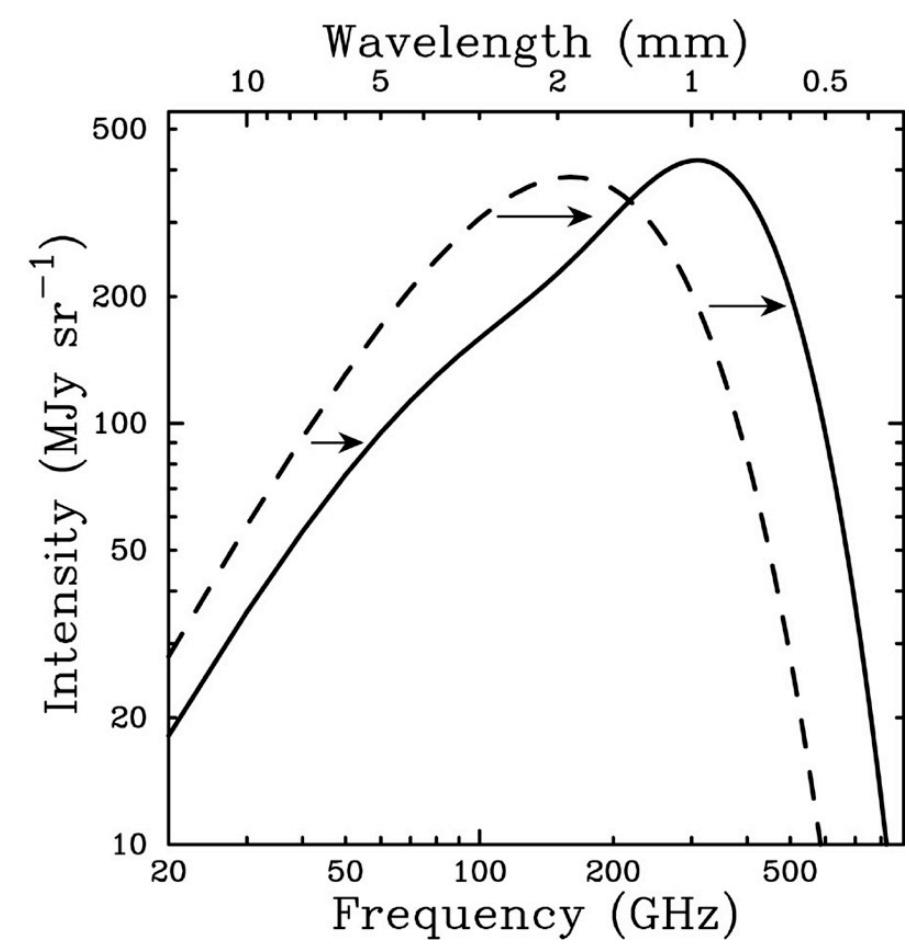




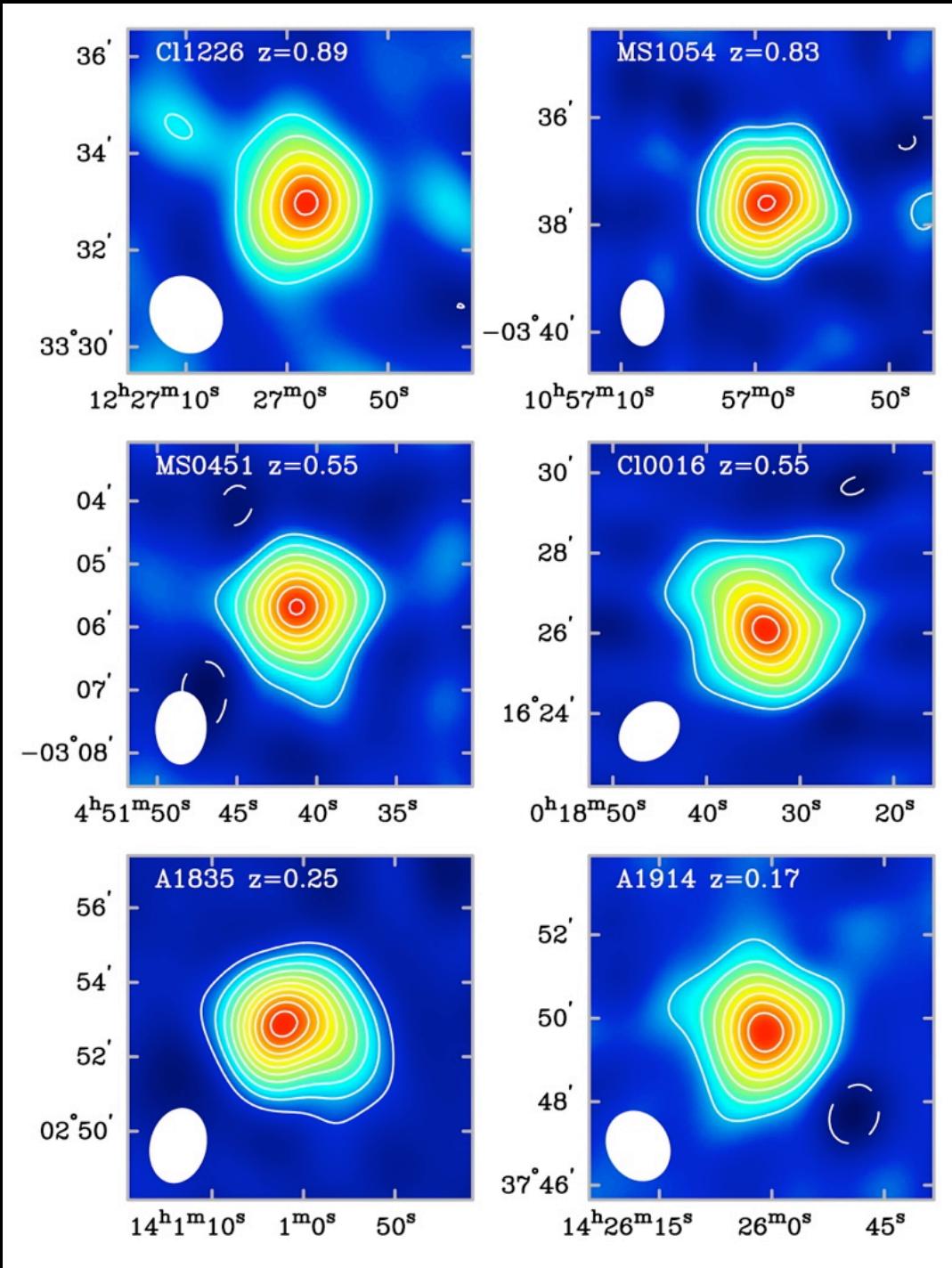








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