THERMAL HISTORY OF THE UNIVERSE

The Universe today is bathed in an all-pervasive radiation field, the Cosmic Microwave Background (CMB) which we introduced in Lecture 5. The spectrum of this radiation is that of a blackbody:

$$B_\lambda(T)\ [\text{erg s}^{-1}\ \text{cm}^{-2}\ \text{Å}^{-1}\ \text{sr}^{-1}] \;=\; \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT}-1} \tag{7.1}$$

$$B_\nu(T)\ [\text{erg s}^{-1}\ \text{cm}^{-2}\ \text{Hz}^{-1}\ \text{sr}^{-1}] \;=\; \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1} \tag{7.2}$$

with $T = 2.7255$ to a very high precision (see Figure 7.1).

Let us calculate the ratio of number of photons to baryons today. The total energy density of a blackbody radiation field is:

$$u = aT^4 \tag{7.3}$$

where $a = 4\sigma/c = 7.57 \times 10^{-15}\,\text{erg cm}^{-3}\ \text{K}^{-4}$ is the radiation constant[1] and σ is the Stefan-Boltzmann constant.[2] The average energy per photon of blackbody radiation is $\langle u \rangle = 2.70\,k\,T$, where k is Boltzmann constant

[1]Not to be confused with the scale factor of the universe, $a = (1+z)^{-1}$!

[2]You should have already encountered these relations in the *Stellar Structure and Evolution* course.

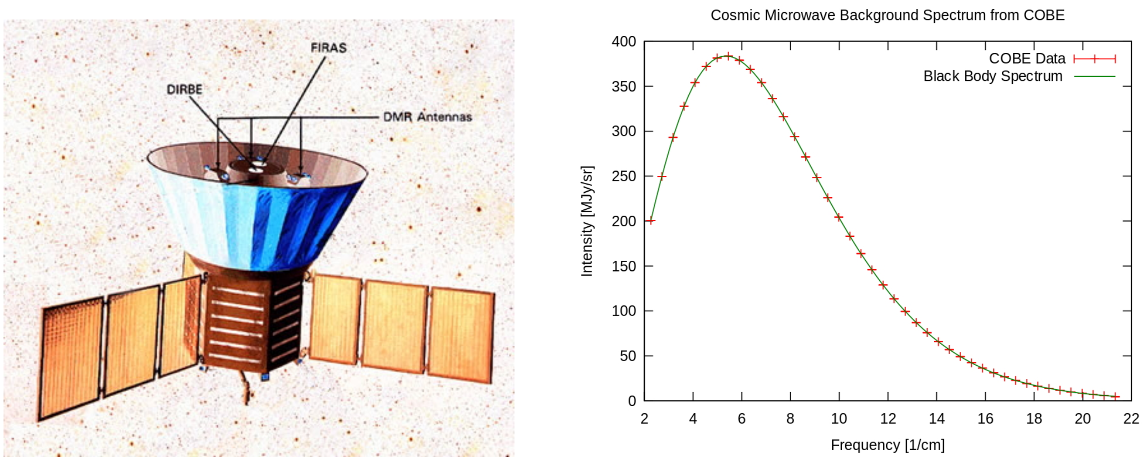

Figure 7.1: The spectral shape of the Cosmic Microwave Background measured by the COBE satellite is that of a blackbody with temperature $T = 2.7255\,\text{K}$.

($k = 1.38 \times 10^{-16}\,$erg K^{-1}). Thus, today the number density of CMB photons is:

$$n_{\gamma,0} = \frac{u}{\langle u \rangle} = \frac{a}{2.70\,k} T^3 = \frac{7.57 \times 10^{-15}}{2.70 \cdot 1.38 \times 10^{-16}} \cdot 2.73^3 = 4.1 \times 10^2\,\mathrm{cm}^{-3} \quad (7.4)$$

On the other hand, the average density of baryons is given by:

$$n_{\mathrm{b},0} = \frac{\Omega_{\mathrm{b},0} \cdot \rho_{\mathrm{crit}}}{\langle m_{\mathrm{b}} \rangle} = \frac{0.0488 \cdot 8.5 \times 10^{-30}\,\mathrm{g\ cm}^{-3}}{1.67 \times 10^{-24}\,\mathrm{g}} = 2.5 \times 10^{-7}\,\mathrm{cm}^{-3} \quad (7.5)$$

where $\rho_{\mathrm{crit}} = 3H_0^2/8\pi G$ is the critical density (see Lecture 1) and $\langle m_{\mathrm{b}} \rangle$ is the mean mass per baryon. Thus:

$$\frac{n_{\mathrm{b},0}}{n_{\gamma,0}} = 6.1 \times 10^{-10} \quad (7.6)$$

i.e. there are more than a billion photons per baryon. This ratio has remained approximately constant since $\sim 1\,$s after the Big Bang.

However, the ratio of the *energy densities* in baryons and photons is:

$$\frac{u_{\mathrm{b},0}}{u_{\gamma,0}} = \frac{n_{\mathrm{b}}}{n_\gamma} \frac{\langle m_{\mathrm{b}} \rangle c^2}{\langle u \rangle} = 9.0 \times 10^2 \quad (7.7)$$

and, if we include non-relativistic (i.e. 'cold') dark matter and relativistic neutrinos:

$$\frac{u_{\mathrm{m},0}}{u_{\mathrm{rad},0}} \simeq 3.4 \times 10^3\,. \quad (7.8)$$

Thus, today the Universe is *matter dominated.*

However, this was not always the case. We have already seen in Lecture 2 (eqs. 2.25 and 2.26), the energy density of matter increases as $\rho_{\mathrm{m}} = \rho_0\,(1+z)^3$, while for radiation $\rho_{\mathrm{rad}} = \rho_0\,(1+z)^4$. As we look back in time then, there will come an epoch (at $z_{\mathrm{eq}} \simeq 3370$) when $u_{\mathrm{m},0}/u_{\gamma,0} = 1$; before that time the Universe was *radiation dominated.*

Another properties of blackbody spectra is that $\lambda_{\mathrm{max}} T =$ constant, where λ_{max} is the wavelength at which $B_\lambda(T)$ peaks. This relation, known as Wien's law, applies equally to stars (whose emergent spectrum is approximately a blackbody) as to the CMB photons. Thus, as the wavelengths of the CMB photons decrease as $(1+z)^{-1}$, the temperature of the background increases as $T_{\mathrm{CMB}} = 2.73\,(1+z)\,$K. The early Universe was a very hot place![3]

[3]The expectation that $T_{\mathrm{CMB}} = 2.73\,(1+z)$ has been verified experimentally up to $z \simeq 3$ using the rotational levels of the CO molecule as a 'thermometer'.

At early times, the typical photons are sufficiently energetic that they interact strongly with matter: the whole Universe sits at a temperature dictated by the radiation. That is why we speak of the 'Thermal History' of the Universe. Note that temperature and energy can be converted to one another via Boltzmann constant: $1\,\mathrm{eV} = 1.1605 \times 10^4\, k\,\mathrm{K}$.

We also saw in Lecture 2 (eq. 2.24) that in the radiation-dominated era the scale factor of the Universe evolves as $a(t) = (t/t_0)^{1/2}$. Given that $T \propto a^{-1}$, we have a relation between time and temperature:

$$t(\mathrm{s}) = \left(\frac{T}{1.5 \times 10^{10}\,\mathrm{K}}\right)^{-2} = \left(\frac{T}{1.3\,\mathrm{MeV}}\right)^{-2} \tag{7.9}$$

7.1 The Universe at $t < 1\,\mathrm{s}$

7.1.1 Planck Time

How can we even begin to speculate what the Universe was like before it was 1 s old? There are two things to keep in mind here. (1) Although we cannot explore this era empirically, we can test the consequences of our models with observations of the Universe at later times. (2) The underlying assumption is that the laws of physics are time-invariant. Of course we do not know that this is the case, but if some parameters have changed during the course of time (such as the cross-sections of some nuclear reactions), cosmological measurements may be the only way to find out.

The logical consequence of $T = 2.73/a$ is $\lim_{a\to 0} T = \infty$, but this extrapolation of classical physics eventually breaks down when the wavelength associated with a particle approaches its Schwarzschild radius, that is when:

$$\lambda_{\mathrm{dB}} = \frac{2\pi\hbar}{mc} = \pi r_{\mathrm{s}} = \frac{2\pi G m}{c^2}\,. \tag{7.10}$$

The above equality defines the Planck mass:

$$m_{\mathrm{P}} = \left(\frac{\hbar c}{G}\right)^{1/2} \simeq 10^{19}\,\mathrm{GeV}\,; \tag{7.11}$$

the Planck length:

$$l_{\mathrm{P}} = \frac{\hbar}{m_{\mathrm{P}} c} = \left(\frac{\hbar G}{c^3}\right)^{1/2} \simeq 10^{-33}\,\mathrm{cm}\,; \tag{7.12}$$

and the Planck time:

$$t_{\rm P} = \frac{l_{\rm P}}{c} = \left(\frac{\hbar G}{c^5}\right)^{1/2} \simeq 10^{-43}\,{\rm s}\,. \qquad (7.13)$$

At $t \simeq t_{\rm P}$ classical spacetime dissolves into a foam of quantum black holes; current physical theory does not take us beyond this limit. This is also referred to as the 'quantum gravity limit' because it involves new physical laws that unify quantum physics (which describes the strong and weak nuclear forces and the electromagnetic force) and gravity (described by Einstein's general relativity).

7.1.2 Freeze-out

The key to understanding the thermal history of the Universe is the comparison between the rate of interaction for a given process, Γ, and the expansion rate of the Universe, H. If the condition $\Gamma \gg H$ is satisfied, then the timescale of particle interactions is much smaller than the characteristic expansion timescale:

$$t_{\rm c} = \frac{1}{\Gamma} \ll t_{\rm H} = \frac{1}{H}\,, \qquad (7.14)$$

and local thermal equilibrium[4] is established before the effect of the expansion becomes relevant. As the Universe cools, Γ for some interactions may decrease faster than the expansion rate; when $t_{\rm c} \sim t_{\rm H}$, the particles in question *decouple* from the thermal plasma. Different particle species may have different interaction rates, and therefore may decouple at different times/temperatures.

7.1.3 Major Milestones

Physicists wonder whether the four fundamental interactions in nature: gravity, the strong and weak nuclear forces, and electromagnetism, may be manifestations of a single fundamental type of interaction. This approach may just reflect a human desire to reduce our description of the physical world to the smallest number of parameters. It is a logical extension of the

[4] A system of particles is said to be in thermodynamic equilibrium if the particles exchange energy and momentum efficiently.

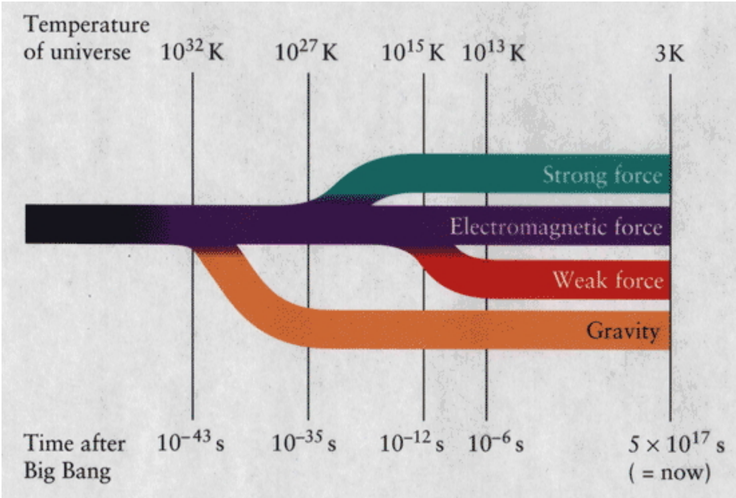

Figure 7.2: The four fundamental interactions became distinct with the falling temperature of the Universe.

scientific method that explains the wide variety of compounds on Earth as different combinations of the 98 chemical elements of the periodic table, themselves consisting of different numbers of protons and neutrons, and so on.

The unification of the four forces of nature implies that at sufficiently high energies their strengths become comparable, whereas today they are very different from each other. This idea appears to be supported by the results of experiments using particle accelerators which have shown that the strengths of the electromagnetic and weak interactions become closer to one another at high interaction energies.

If all four forces were unified, this must have happened before the Planck time; $t \simeq 10^{-43}$ s is taken as the time when gravity became separate from the other three forces and we enter the Grand Unified Theory (GUT) era (see Figure 7.2).

At $t \sim 10^{-35}$ s, $T \sim 10^{27}\,\mathrm{K} \simeq 10^{14}\,\mathrm{GeV}$ (**the GUT transition**), the Electroweak and Strong forces emerge, and quarks (that interact mostly through the strong force) and leptons (which interact mostly through the weak force) and their anti-particles acquire individual identities.

The end of the GUT era is thought to be associated with two major events in the history of the Universe: an exponential expansion between $t = 10^{-36}$ and 10^{-34} s called *inflation*, and *baryogenesis*. The first was postulated to explain, among other things, why we live in a flat Universe with $k = 0$ (recall the discussion at the end of Lecture 5.3.3), while the latter is required to explain the absence of anti-matter in today's Universe. We shall return to both topics in a later lecture.

At $t \sim 10^{-12}$ s, $T \sim 10^{15}$ K $\simeq 100$ GeV (**the Electro-Weak transition**), the electromagnetic and weak forces become separate. This is when leptons acquire mass. The corresponding bosons also appear: intermediate vector bosons of electroweak force decay into massive W^+, W^-, and Z^0 bosons that mediate the weak force, and the massless photon, that mediates electromagnetic force. The massive $W^\pm$, and Z^0 bosons (with masses $m \sim 80$–90 GeV) decay soon thereafter, at the temperature corresponding to their mass. Photons, being massless, do not. It is also possible that baryogenesis may have taken place at this time, rather than at the GUT transition.

At $t \sim 10^{-6}$ s, $T \sim 10^{12-13}$ K $\simeq 200$ MeV – 1 GeV (**the QCD**, or **Quark-Hadron transition**), is when quarks can no longer exist on their own; they combine into hadrons (baryons and mesons), glued together by gluons (strong force bosons). Quark confinement commences. Some types of WIMPS (Weakly Interacting Massive Particles, a generic name for a yet to be discovered Dark Matter particle) could have been made at this epoch.

7.2 The Universe at $t > 1$ s

The period from $t \sim 10^{-6}$ to $t \sim$ a few s is sometimes referred to as the **the Lepton era** because of two important events that involve neutrinos and electrons. Note that as the temperature falls below ~ 1 GeV we enter an energy regime where elementary particle physics is well understood and accessible to experimental verification with particle accelerators such as the Large Hadron Collider at CERN.

7.2.1 Decoupling of Neutrinos ($t \simeq 1$ s, $T \simeq 1$ MeV, $\simeq 10^{10}$ K)

$T \simeq 10^{12}$ K corresponds to ~ 100 MeV. This energy is much less that the rest-mass energy of protons and neutrons ($m_\mathrm{p} = 938.3\,\mathrm{MeV}/c^2$ and $m_\mathrm{n} = 939.6\,\mathrm{MeV}/c^2$ respectively); baryons are just too heavy to be produced

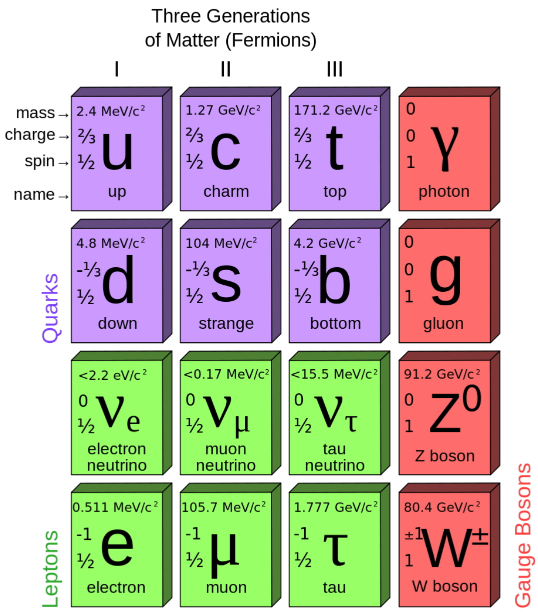

Figure 7.3: The Standard Model of elementary particles with the three generations of matter and gauge bosons in the fourth column.

(by pair production) at this temperature. Thus, all the baryons that exist today must have already been present when the Universe was one millionth of a second old. Of the leptons, only the electron and the neutrinos (and their anti-particles) have rest mass energies significantly below 100 MeV (see Figure 7.3). Thus, at $T \simeq 10^{12}$ K the relativistic species present and making up the radiation energy density, $u_{\rm rad}$, are electrons, neutrinos and their anti-particles, and photons. These species are kept in equilibrium by the following reactions:

$$\begin{aligned} e^{\pm} + \gamma \leftrightarrow e^{\pm} + \gamma &: \text{Compton scattering} \\ e^{+} + e^{-} \leftrightarrow \gamma + \gamma &: \text{pair production and annihilation} \\ \nu + \bar{\nu} \leftrightarrow e^{+} + e^{-} &: \text{neutrino} - \text{antineutrino scattering} \\ \nu + e^{\pm} \leftrightarrow \nu + e^{\pm} &: \text{neutrino} - \text{electron scattering} \end{aligned} \tag{7.15}$$

so long as the reaction rate is faster than the expansion rate (section 7.1.2). The reactions involving neutrinos are mediated by the weak force, for which

$$\frac{\Gamma}{H} \simeq \left(\frac{T}{1.6 \times 10^{10}\,\text{K}} \right)^3 , \tag{7.16}$$

so that when T falls below 10^{10} K the neutrinos are no longer in equilibrium and decouple from the rest of the plasma. At freeze-out, the neutrinos are still relativistic, with a thermal distribution at the same temperature as the electrons and photons that remained in mutual equilibrium. Cosmological neutrinos have been propagating through the Universe without further interactions since redshift $z \sim 10^{10}$, and have kept their thermal distribution to the present day, with the temperature decreasing as $T \propto 1/a$. Unfortunately, given the very low cross-section for interaction of neutrinos with matter, it is hard to think of how this low-energy neutrino background could be detected with current technologies. If it were possible, it would give us a snapshot of the Universe at much earlier times than the CMB ($t \simeq 1\,\mathrm{s}$ as apposed to $t \simeq 372\,000$ years).

7.2.2 Electron-Positron Annihilation

When the temperature falls below 500 keV (or $\sim 5 \times 10^9\,\mathrm{K}$ at $t \simeq 5\,\mathrm{s}$), the number density of photons with energies above the pair production threshold of $m_\mathrm{e} = 511\,\mathrm{keV/c^2}$ is insufficient to maintain the second reaction in eq. 7.15 in equilibrium. The reaction now proceeds preferentially in the right direction (pair annihilation), leaving only a small number of electrons to balance the protons produced by baryogenesis (our Universe is electrically neutral).

Pair annihilation injects additional energy into the photon gas, corresponding to the kinetic and rest mass energies of the e^+, e^- pairs. This re-heats the photon population, but not the bulk of the neutrinos since they are no longer in thermal equilibrium with the photons.[5] From the thermodynamics of this process, we have (after annihilation):

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \tag{7.17}$$

which has been maintained to the present day, so that the blackbody spectrum of the cosmic neutrino background is predicted to have a temperature $T_\nu = 0.71 \times 2.73 = 1.95\,\mathrm{K}$.

[5]Because the decoupling of the neutrinos is not instantaneous, some of the high energy neutrinos are still in thermal equilibrium with the photons and will feel the re-heating leading to a slight excess of high energy neutrinos relative to a blackbody spectrum.

7.3 Equilibrium Thermodynamics

Before turning to consider baryons in the next lecture, we recall some concepts from statistical mechanics and we'll use them to derive an expression for the energy density of a fully relativistic plasma.

In statistical mechanics, it is convenient to describe an ensemble of particles in phase space, defined by six parameters, the cartesian coordinates x, y, z, and the corresponding momenta $p_{\rm x}$, $p_{\rm y}$, $p_{\rm z}$. We further distinguish between *fermions*—particles of half-integer spin which obey Fermi-Dirac statistics, such as quarks and leptons, and *bosons*—particles of integer spin that obey Bose-Einstein statistics, such as photons and the other force carriers in Figure 7.3. Fermions obey Pauli's exclusion principle which states that no two identical fermions can occupy the same quantum state, while bosons do not suffer any such restriction.

From Heisenberg uncertainty principle, $\Delta p \, \Delta x = h$, it follows that h^3 is the phase space volume occupied by a single particle (or, alternatively, the density of states in phase space is $1/h^3$). If a particle has g internal degrees of freedom (e.g. spin, polarization), the density of states becomes $g/(2\pi\hbar)^3$. It follows that the number density of particles (per unit volume in real space) with momentum states in the range of d^3p is:

$$dn = g \frac{1}{(2\pi\hbar)^3} \, f(p) \, \mathrm{d}^3 p \tag{7.18}$$

where $f(p)$ is the distribution function, such that $f(p)\,dp$ is number of particles with momentum between p and $p + dp$.[6] The number density of particles (again in real space) at a given temperature T is found by integrating 7.18 over momentum:

$$\begin{aligned} n(T) &= \frac{g}{(2\pi\hbar)^3} \int_0^\infty f(p) \, \mathrm{d}^3 p \\ &= \frac{g}{(2\pi\hbar)^3} \int_0^\infty f(p) \, 4\pi p^2 \, \mathrm{d}p \end{aligned} \tag{7.19}$$

using spherical coordinates, and the energy density, $u = \rho(T)c^2$, is the

[6] We have assumed isotropy, which requires that the momentum dependence is only in terms of the magnitude of the momentum, i.e. $p \equiv |\boldsymbol{p}|$.

integral of the distribution function weighted by energy:

$$u = \frac{g}{(2\pi\hbar)^3} \int_0^\infty f(E)E(p)\, 4\pi p^2\, \mathrm{d}p \tag{7.20}$$

The distribution function in energy terms, $f(E)$, is given generally by:

$$f(E) = \frac{1}{e^{E/kT} \pm 1} \tag{7.21}$$

where the + sign refers to fermions and the − sign to bosons. As usual, the total energy has contributions from the rest mass and the momentum:

$$E^2 = m^2c^4 + p^2c^2\,.$$

In the relativistic limit (i.e. $kT \gg mc^2$), $E = pc$. Using the substitution $y = pc/kT$, we have:

$$p^3 = y^3 \left(\frac{kT}{c}\right)^3; \quad \mathrm{d}p = \frac{kT}{c}\,\mathrm{d}y$$

so that eq. 7.20 can be re-written as:

$$\begin{aligned} u \equiv \rho(T)c^2 &= \frac{g}{(2\pi\hbar)^3} \int_0^\infty \frac{pc}{e^{pc/kT} \pm 1}\, 4\pi p^2 \mathrm{d}p \\ &= \frac{g}{(2\pi\hbar)^3}\, 4\pi c \left(\frac{kT}{c}\right)^4 \int_0^\infty \frac{y^3}{e^y \pm 1}\, \mathrm{d}y\,. \end{aligned} \tag{7.22}$$

Now (and we are almost there!), it can be shown that the integrals have the following values:

$$\int_0^\infty \frac{y^3}{e^y - 1}\, \mathrm{d}y = \frac{\pi^4}{15} \quad \text{(Bose-Einstein statistics), and}$$

$$\int_0^\infty \frac{y^3}{e^y + 1}\, \mathrm{d}y = \frac{7}{8}\frac{\pi^4}{15} \quad \text{(Fermi-Dirac statistics)}\,.$$

Defining:

$$a \equiv \frac{8\pi^5 k^4}{15 c^3 (2\pi\hbar)^3}\,, \tag{7.23}$$

we conclude that:

$$\begin{aligned} u &= \frac{g}{2}\, a\, T^4 \quad \text{for bosons, and} \\ &= \frac{7}{8}\,\frac{g}{2}\, a\, T^4 \quad \text{for fermions,} \end{aligned} \tag{7.24}$$

in equilibrium when $kT \gg mc^2$.

Note that for photons $g = 2$ (two polarizations), so that we recover the familiar relation for the energy density of a radiation field $u = a\,T^4$. The total energy density of the mixture of photons, electrons, positrons, neutrinos and antineutrinos at time $t \sim 1\,\mathrm{s}$ is thus:

$$u = \rho(T)c^2 = c^2 \sum \rho_{\mathrm{i}}(T) \; = \; \frac{1}{2} g_* \, a\, T^4 \tag{7.25}$$

where g_* is the effective number of degrees of freedom:

$$g_* = \sum_{\text{bosons}} g_{\mathrm{i}} + \frac{7}{8} \sum_{\text{fermions}} g_{\mathrm{j}} \, . \tag{7.26}$$

For photons $g = 2$ (two polarization states), for electrons and positrons $g = 2$ also (two spin states), and for neutrinos $g = 1$, assuming that neutrinos are purely left-handed (only one helicity state). Thus:

$$u = \frac{1}{2} a\, T^4 \left[2 + 4 \cdot \frac{7}{8} + 2 \cdot \frac{7}{8} \mathcal{N}_\nu \right] \tag{7.27}$$

or

$$u = a\, T^4 \left[1 + \frac{7}{4} + \frac{7}{8} \mathcal{N}_\nu \right] \tag{7.28}$$

where $\mathcal{N}_\nu$ is the number of neutrino families.