BASIC CONCEPTS

"The history of cosmology shows that in every age devout people believe that they have at last discovered the true nature of the Universe."[1]

Cosmology aims to determine the contents of the entire Universe, explain its origin and evolution, and thereby obtain a deeper understanding of the laws of physics assumed to hold throughout the Universe.[2]

This is an ambitious goal for a species of primates on planet Earth, and you may well wonder whether it is indeed possible to know the whole Universe. As a matter of fact, humans have made huge strides in observational and theoretical cosmology over the last twenty-thirty years in particular. We can rightly claim that we have succeeded in measuring the fundamental cosmological parameters that describe the content, and the past, present and future history of our Universe with percent precision—a task that was beyond astronomers' most optimistic expectations only half a century ago.

This course will take you through the latest developments in the subject to show you how we have arrived at today's 'consensus cosmology'.

1.1 Fundamental Observers

Just as we can describe the properties of a gas by macroscopic quantities, such as density, pressure, and temperature—with no need to specify the behaviour of its individual component atoms and molecules, thus cosmologists treat matter in the universe as a smooth, idealised fluid which we call the *substratum*.

An observer at rest with respect to the substratum is a *fundamental observer*. If the substratum is in motion, we say that the class of fundamental observers are *comoving* with the substratum.

[1]E. R. Harrison (1981): Cosmology: the Science of the Universe.
[2]Adapted from Matts Roos, *Introduction to Cosmology*, ©2003 John Wiley & Sons, Ltd.

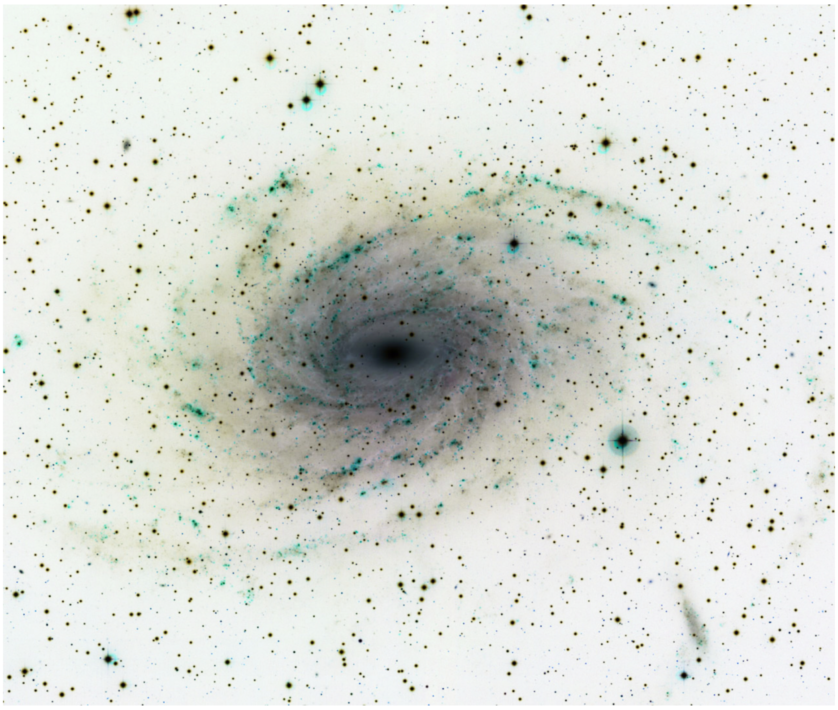

Figure 1.1: The nearby galaxy NGC 6744 observed with the Wide Field Imager on the MPG/ESO 2.2-metre telescope at La Silla. This large spiral galaxy is similar to the Milky Way and is at a distance of about 10 Mpc in the southern constellation of Pavo (The Peacock).

Are we fundamental observers? No, because:

- The Earth rotates around its axis and orbits the Sun.
- The Sun moves relative to the *Local Standard of Rest* (the local reference frame defined by the motions of the stars nearest the Sun).
- The Sun is one of $\sim 10^{11}$ stars in a spiral galaxy, the Milky Way. The LSR orbits the centre of the Milky Way Galaxy with a period $P \simeq 240$ million years.
- The Milky Way moves relative to the centre of mass of the Local Group.
- The Local Group is infalling into the Virgo Cluster of galaxies.
- The Virgo Cluster, along with several other large clusters, is in turn speeding towards a great concentration of mass known as 'The Great Attractor'.

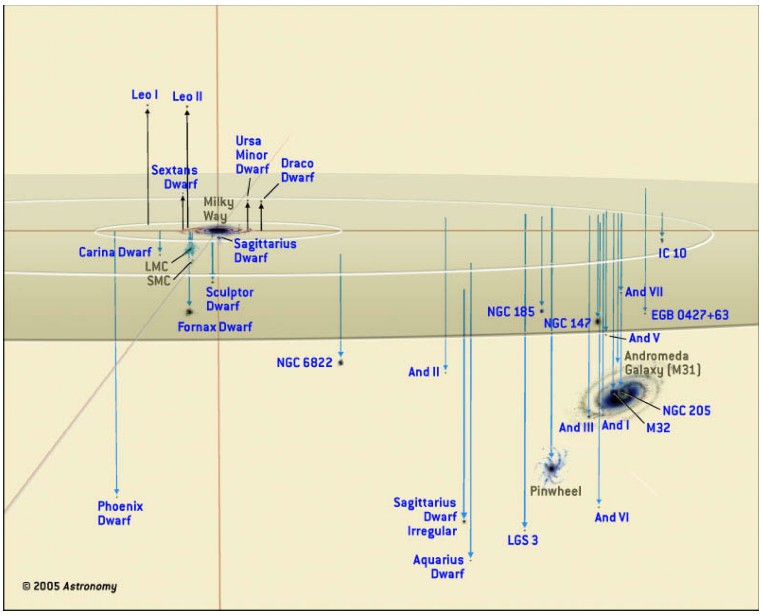

Figure 1.2: Relative locations of galaxies in the Local Group.

So, why the concept of a fundamental observer? Because, we believe/assume that, once we have corrected for all of these *peculiar motions*, we are at rest within the substratum and are receiving a fundamental observer's view of the universe.

1.2 Useful Astronomical Units

1.2.1 Distances

- Radius of Earth's orbit around the Sun $\equiv$ 1 Astronomical Unit (AU) $= 1.5 \times 10^{13}$ cm
- Distance at which 1 AU subtends an angle of 1 second of arc:

$$\frac{1.5 \times 10^{13} \times 360 \times 60 \times 60}{2\pi} = 3.1 \times 10^{18}\,\text{cm} \equiv 1\,\text{parsec} \simeq 3.3\,\text{light years} \tag{1.1}$$

- pc, kpc, Mpc are the most commonly used distance units in astronomy, e.g.:
- 1.3 pc = distance from the Sun to the nearest star (Proxima Centauri)
- 8.5 kpc = distance from the Sun to the Galactic Centre

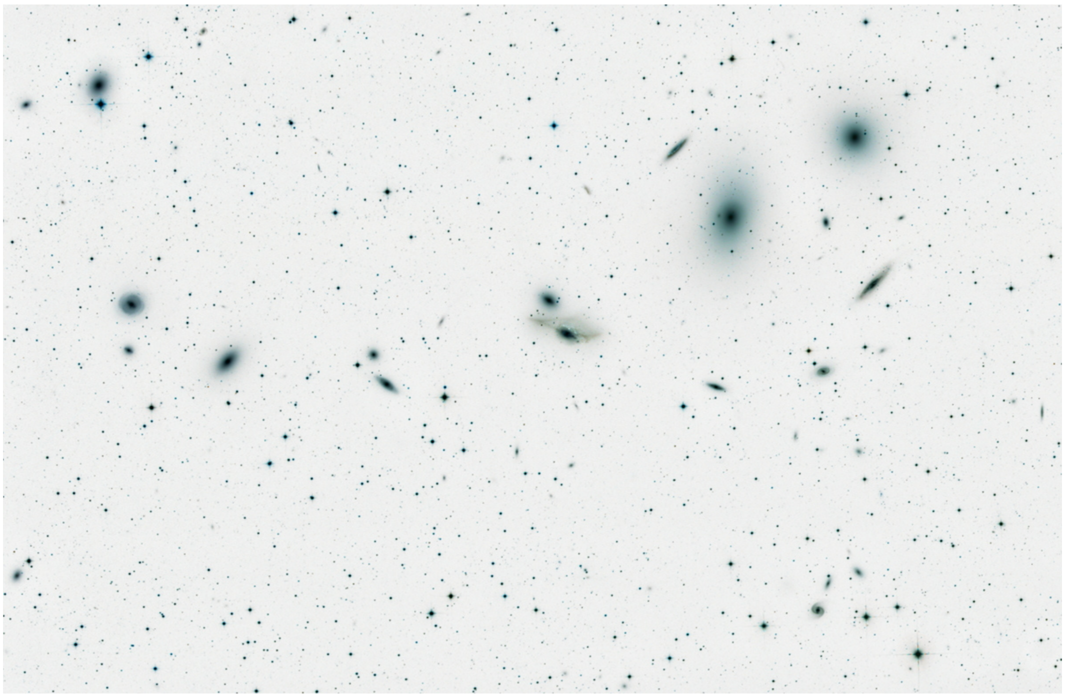

Figure 1.3: The closest cluster of galaxies is the Virgo cluster, at a distance of about 16.5 Mpc. It consists of more than 2000 galaxies, a mix of spirals and ellipticals; it forms the heart of the large Local supercluster, of which the Local Group is an outlying member.

- 1 Mpc = size of the Local group of galaxies
- 16 Mpc = distance to the Virgo cluster of galaxies
- 4450 Mpc = Hubble radius (radius of the observable universe) $= c/H_0$, where H_0 is the value of the Hubble constant today $H_0 = 67.5\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$

1.2.2 Masses

- 1 solar mass ($1\,M_\odot$) $= 2 \times 10^{33}\,$g
- Stellar mass of the Milky Way $\simeq 1 \times 10^{11}\,M_\odot$
- Dynamical mass of the Milky Way $\simeq 1 \times 10^{12}\,M_\odot$
  (measured from the motion of its satellites $\Longrightarrow$ Dark Matter)
- Mass of giant elliptical galaxy $\simeq 1 \times 10^{13}\,M_\odot$
- Mass of the Virgo cluster $\approx 10^{15}\,M_\odot$

1.2.3 Luminosity and Magnitudes

Astronomers measure the brightness of celestial objects on a log scale which goes backwards!

$$F_1/F_2 = 10^{0.4(m_2-m_1)} \tag{1.2}$$

where F are the fluxes and m the *apparent magnitudes*.

- $\Delta m = +0.75 \;\rightarrow\; (F_1/F_2) = 2$
- $\Delta m = +1 \;\rightarrow\; (F_1/F_2) = 2.5$
- $\Delta m = +2.5 \;\rightarrow\; (F_1/F_2) = 10$
- $\Delta m = +5 \;\rightarrow\; (F_1/F_2) = 100$

The *absolute magnitude* M of an object is the magnitude that object would have if placed at a distance of 10 pc, from which we define a *distance modulus* (in magnitudes)

$$m - M = 2.5\log(d/10)^2 \;= 5\log(d/10) \;= 5\log d - 5 \tag{1.3}$$

where d is the distance in parsec.

Astronomers can measure apparent magnitudes of celestial sources with great precision; however, distances (particularly to objects at cosmological distances) and therefore absolute magnitudes are notoriously difficult to measure.

1.3 The Cosmological Principle

The Cosmological Principle states that the Universe is **Homogeneous** and **Isotropic**. These two terms are not the same:

Homogeneous: The same at every point.
Isotropic: The same in all direction,

To appreciate the difference, consider a spherically symmetric distribution of matter. When viewed from its central point, such a distribution would be isotropic, but not necessarily homogeneous.

Conversely, a universe permeated by a uniform magnetic field may be homogeneous but is certainly not isotropic, as directions along field lines can be distinguished from those perpendicular to them.

A universe isotropic about *every* point would then also be homogeneous.

To what extent is our Universe homogeneous and isotropic?

On scales of up to 100s of Mpc it is not—we see structure in the form of galaxies, clusters of galaxies, filaments and voids (see Figure 1.5). But on larger scales the Universe does seem to be roughly isotropic. For example, if we counted galaxies in two cubes 100 Mpc on the side, in opposite directions on the celestial sphere, we would measure the same mean density of matter to within a few percent.

As for uniformity, we'll never know for sure. But if we apply the **Coper-**

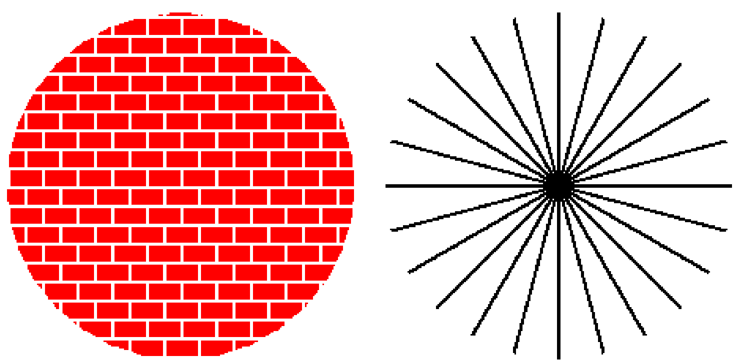

Figure 1.4: The pattern on the left of the figure is homogeneous but not isotropic, while the one on the right is isotropic but not homogeneous. (Figure reproduced from http://www.astro.ucla.edu/∼wright/A275.pdf).

nican principle—which says that the Earth does not occupy a special place in the universe—then homogeneity follows from the observation of isotropy.

As an aside, the first clear statement of the Copernican principle seems to be due to the Italian philosopher Giordano Bruno, burnt at the stake in Rome on 17 February 1600 on charges of heresy. Bruno was born in 1548, five years after Copernicus died. Between the years 1582 and 1592 he was the one who persistently, openly and actively spread the news about the "universe" that Copernicus had charted. In his book *"De la Causa, Principio et Uno"* he wrote:

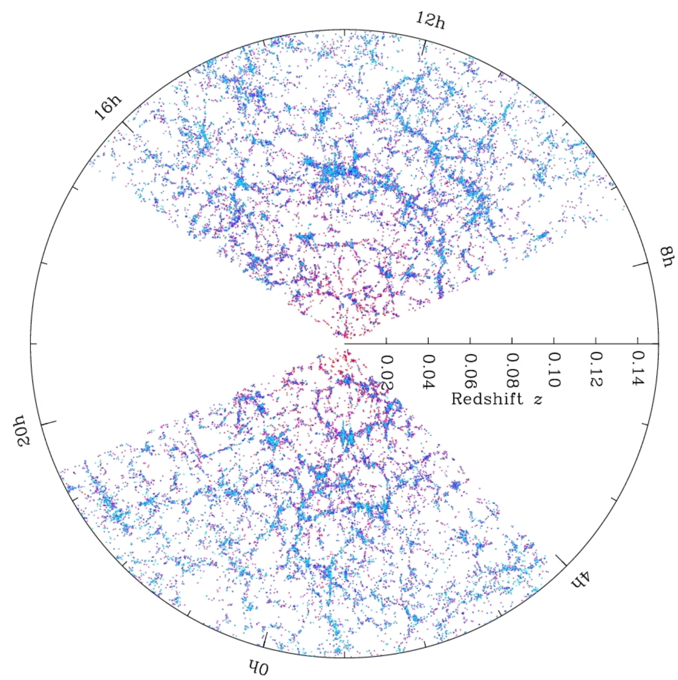

Figure 1.5: Two-dimensional projection of the distributions on the sky of galaxies mapped by the Sloan Digital Sky Survey (SDSS); each dot is one galaxy. Galaxies are colour-coded according to the ages of their stars, with the blue dots indicating older stellar populations (elliptical galaxies). The outer circle corresponds to a distance of $\sim 625\,\mathrm{Mpc}$; the light from these galaxies has taken two billion years (1/7 of the age of the Universe) to travel from their stars to Earth. (M. Blanton and the Sloan Digital Sky Survey: http://www.sdss.org/).

> “There is no absolute up or down, as Aristotle taught; no absolute position in space; but the position of a body is relative to that of other bodies. Everywhere there is incessant relative change in position throughout the universe, and the observer is always at the centre of things.”

The power of the hypothesis of homogeneity is that our own observations from Earth are then all we need to test a cosmological model. We therefore have to hope that homogeneity is a reasonable approximation on the largest scales, bearing in mind that so far we have not developed a successful explanation of why this idealised state of affairs should hold.

An immediate consequence of homogeneity is the existence of a universal *cosmic time, t*. Since all fundamental observers see the same sequence of events in the universe, they can synchronise their clocks by means of these events.

1.4 Redshifts

By redshift we mean the shift to longer wavelengths (perceived as red by the human eye) of light waves as a result of the relative motions (apart) of emitter and receiver. A few points of note:

- In astronomy, in order to measure redshifts, we need to record the spectra of astronomical sources and measure the wavelength(s) of well-defined spectral feature(s), such as emission or absorption lines (see Figure 1.6). Colours are not sufficient, normally, because stars and galaxies can appear red because they are cool, or because their light is reddened by interstellar dust.
- When we measure the velocities of nearby stars and galaxies, we are dealing with Doppler, or kinematic, redshifts:

$$z = \frac{\lambda_{\rm obs} - \lambda_0}{\lambda_0} \tag{1.4}$$

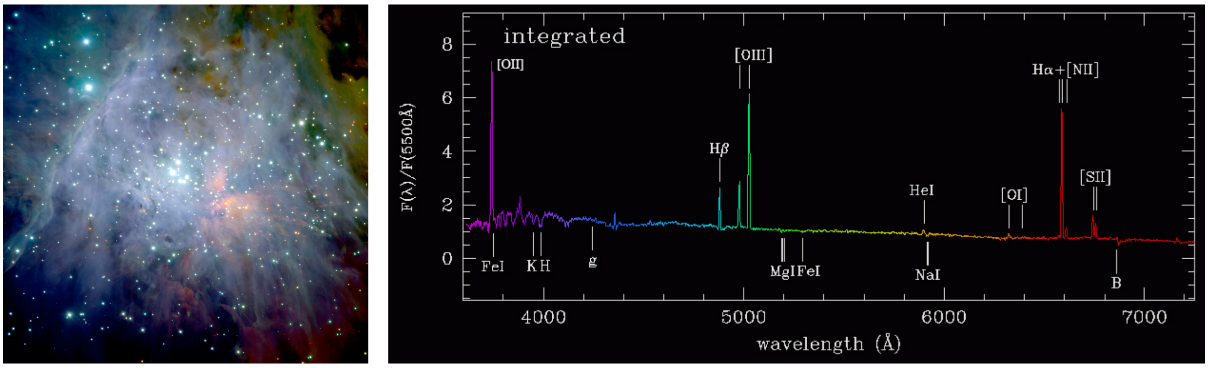

Figure 1.6: Left: Near-infrared image of the central part of the Orion nebula, obtained with the Very Large Telescope (VLT) of the European Southern Observatory (ESO) in Chile. The nebula is diffuse gas ionised by hot stars: the famous Trapezium stars can be seen near the centre together with the associated cluster of about 1,000 stars, approximately one million years old. Right: Typical emission line spectrum of an ionised (H II) region such as the Orion Nebula. The most important spectral features are labelled.

or

$$1 + z = \frac{\lambda_{\rm obs}}{\lambda_0} \tag{1.5}$$

where $\lambda_{\rm obs}$ is the observed wavelength of a given spectral line, and λ_0 is the rest-frame wavelength of the same atomic transition, as measured in the laboratory.

- With the Doppler redshift is associated a speed of recession (if z is positive):

$$v = c \cdot z \tag{1.6}$$

- Eq. 1.6 is the limiting (for $v \ll c$) case of the special relativity result

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{1.7}$$

Eq. 1.7 *is* used in astronomy when, for example, measuring the ejection velocity of gas clouds ejected from Active Galactic Nuclei.

- However, in cosmology we are generally concerned with another type of redshift, the expansion redshift

$$1 + z = \frac{\lambda_{\rm obs}}{\lambda_0} = \frac{a_{\rm obs}}{a_{\rm em}} \tag{1.8}$$

where a is the scale factor of the universe at the time of observation and emission. It is important not to confuse the expansion redshift with a kinematic redshift.

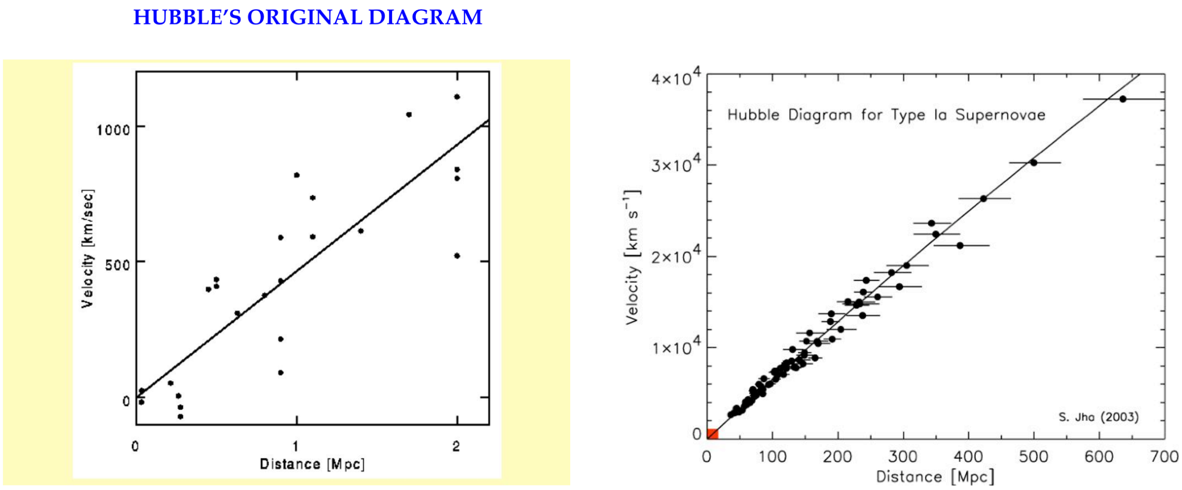

Figure 1.7: *Left*: Hubble diagram 1929. *Right*: Hubble diagram 2003. The original Hubble diagram is contained within the red square.

1.5 Universal Expansion

When the velocities of external galaxies are plotted against their distances, we find a linear relationship between velocity and distance, the Hubble law:

$$v = H_0 \cdot d \tag{1.9}$$

proposed by Edwin Hubble in 1929, when he combined his Cepheids[3] measurements to those of Slipher.

A recessional velocity proportional to distance is the natural result of an expansion which is both isotropic and homogeneous. All fundamental observers see other observers moving away with a velocity which obeys Hubble's law.

H_0 is the Hubble constant. Hubble himself overestimated the value of H_0 by one order of magnitude (a clear demonstration of the difficulty astronomers have in measuring distances). Until about 25 years ago, the value of H_0 was a highly contentious issue, with two 'camps' claiming values which differed by a factor of two. The issue is now 'resolved' and most people agree that:

$$H_0 \simeq 70\,\mathrm{km\ s^{-1}Mpc^{-1}} = 100 \cdot h\,\mathrm{km\ s^{-1}Mpc^{-1}} \tag{1.10}$$

[3] Cepheids are variable stars with a tight Period-Luminosity relation.

with an accuracy of better than 10% (h is conventionally taken as a short-hand for the Hubble constant in units of 100 km $\mathrm{s}^{-1}\mathrm{Mpc}^{-1}$). In cgs units:

$$H_0 = 3.24 \times 10^{-18} h \ \mathrm{s}^{-1} \tag{1.11}$$

Clearly, the time for a galaxy at a distance d, moving at velocity v, to move apart by a further distance d is, by eq. 1.9, $d/v = 1/H_0$, independent of distance. Thus, the Hubble time, $t_H \equiv 1/H_0$, is a measure of the expansion time of the universe, the time for the Universe to double its size expanding at the present rate.

A seemingly inevitable consequence of the universal expansion is that in the past everything in the universe must have been much closer together than it is today. Trace the history back far enough and all the galaxies (*and all of space*) will simultaneously converge to one 'point'. Models of the evolution of the universe from such a beginning are known as Big Bang Cosmologies.

1.6 Components of the Universe

The behaviour of the Universe depends on the properties of its constituents.

For a particle, the crucial question is whether it moves relativistically or not. There are two contributions to a particle's energy, its rest-mass energy and its kinetic energy, which combine to give:

$$E_{\mathrm{tot}} = \sqrt{m^2 \cdot c^4 + p^2 \cdot c^2} \tag{1.12}$$

where m is the particle's rest mass and p its momentum.

If the first term on the right-hand side dominates, the particle will be moving at much less than the speed of light, and we say that it is non-relativistic. With $p^2c^2 \ll m^2c^4$, we can expand eq. 1.12:

$$E_{\mathrm{tot}} = mc^2 \left(1 + \frac{p^2}{m^2c^2}\right)^{1/2} \approx mc^2 + \frac{1}{2}\frac{p^2}{m} \tag{1.13}$$

where the last term in (1.13) is the particle kinetic energy $\frac{1}{2}mv^2$ ($p = mv$ in the non-relativistic limit).

If the first term in (1.12) does not dominate, the particle will be moving with a substantial fraction of c, and so it is relativistic.

Baryons — make up the familiar matter of our universe. By baryons we mean primarily protons and neutrons, the only stable elementary particles made up of quarks (two up quarks and one down quark for the proton and one up–two down quarks for the neutron).[4]

Electrons are charged leptons. Since the universe is charge neutral, there must be equal numbers of electrons and protons.

Astronomers generally group protons, neutrons and electrons together under the term baryons—from a mass point of view this is OK because $m_e/m_p < 10^{-3}$. The term *nucleon* is sometimes used to refer specifically to protons and neutrons. Baryons are non-relativistic.

Neutrinos — Very weakly interacting leptons, paired with the electron, muon, and tau in electroweak theory. Recent experiments suggest that they may have a very small, but non-zero, mass which makes them relativistic particles.

Radiation — Photons are relativistic particles with zero rest mass. Their energy is related to the frequency and wavelength of their wave-like manifestation by:

$$E = h\nu = hc/\lambda \tag{1.14}$$

where h is Planck's constant (first instance of a symbol ambiguity — earlier we used h to indicate the Hubble constant in units of $100\,\mathrm{km\ s^{-1}Mpc^{-1}}$).

Photons, through the full spectrum of electromagnetic radiation, are the only means by which we are aware of the universe. Thus, epochs in the Universe history before recombination at $z \simeq 1090$, when the Universe was optically thick to photons, will remain fundamentally inaccessible to us until we develop alternative astronomies based on the detection of neutrinos

[4] If the proton is not stable, then its lifetime must be much longer than the present age of the universe. The neutron is only stable (in this sense) when bound in nuclei. A free neutron will decay to a proton, an electron, and an anti-electron neutrino with a half-life of $610.2 \pm 0.8\,\mathrm{s}$.

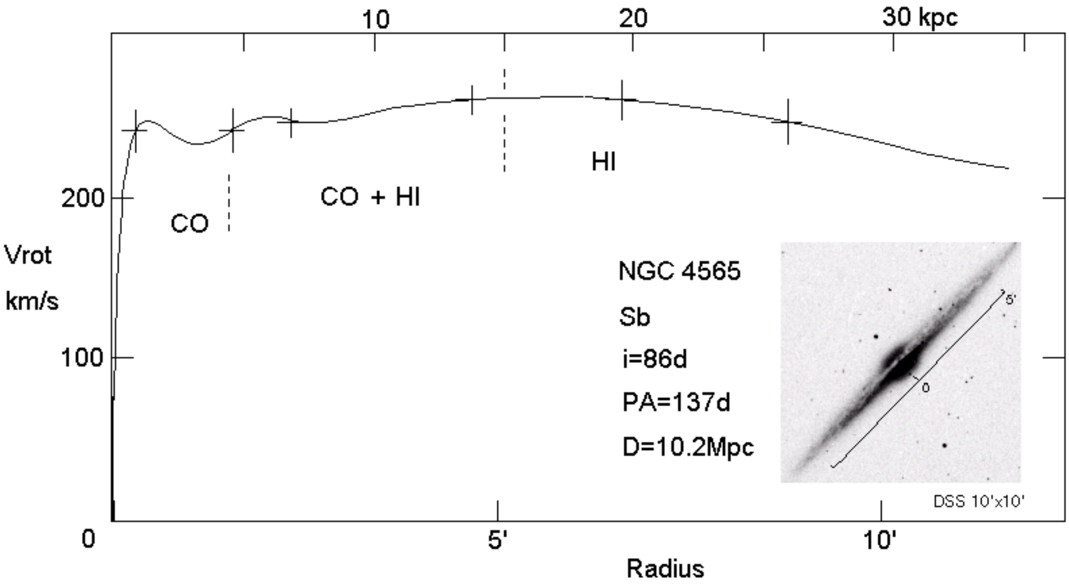

Figure 1.8: The rotation curves of spiral galaxies like NGC 4565 remain flat out to large distances, well beyond their optical dimensions, indicating the existence of massive halos of dark matter.

or gravitational waves.

Photons interact with baryons and electrons. Photons are absorbed/emitted when electrons move between energy levels within an atom (thus giving us the absorption/emission lines which we use to measure the redshifts of galaxies). Photons can ionise atoms by imparting sufficient energy to electrons. Photons can scatter off free electrons in a plasma, a process known as Thomson scattering when the electrons are non-relativistic ($h\nu \ll m_e c^2$), or Compton scattering in the relativistic case.

Dark Matter — The existence of non-baryonic dark matter is inferred from its gravitational manifestations through the flat rotation curves of galaxies, the mass-to-light ratios in clusters of galaxies, and the bending of space-time producing gravitational lensing of background sources. 'Cold' (i.e. non-relativistic), non-baryonic, dark matter is favoured by current models of structure formation. Its nature remains unknown at present, despite many efforts by particle physicists to identify it.

On galactic scales, consider Figure 1.8. With gravity supplying the centripetal force required to keep a test particle of mass m at distance r in orbit around the centre of the spiral galaxy NGC 4565, we have:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \tag{1.15}$$

where M is the mass contained within radius r and v is the orbital velocity. Solving for $v = f(r)$:

$$v = \left(\frac{GM}{r}\right)^{1/2} \tag{1.16}$$

Thus, we would expect the rotation curve of NGC 4565 to show that v decreases with increasing r *if most of the mass is contained within* r. In reality, the rotation curve remains flat well beyond the optical dimensions of the galaxy, indicating that M continues to increase with r. Such observations (and other lines of evidence) have led to the conclusion that the stars and the gas that make up a galaxy are confined within a more massive halo of *dark matter*.

Dark Energy — Responsible for the acceleration of the universe since $z \sim 1$. Its existence was discovered at the end of the twentieth century by careful measures of by the Hubble diagram (luminosity vs. distance) of Type Ia supernovae. Its nature is still totally unclear.

One of the aims of observational cosmology is to determine the relative contributions of these different components to the total mass-energy density of the Universe. In Table 1.1 I have collected the current best estimate of each, expressed as a fraction of the critical density (to be defined later)

$$\rho_{\rm c} = \frac{3H_0^2}{8\pi G} = 1.36 \times 10^{11}\,{\rm M}_\odot\ {\rm Mpc}^{-3} \ \simeq\ 5\,{\rm H\ atoms\ m}^{-3} \tag{1.17}$$

(For a comprehensive discussion of the "Cosmic Energy Inventory" see Fukugita & Peebles, 2004, ApJ, 616, 643).

The remarkable situation which we find ourselves in, in today's 'consensus cosmology', is that we are essentially ignorant of the nature of 95% of what accounts for the mass-energy budget of the universe. Our familiar baryons make up less than 5% of the critical density. Furthermore, most of these baryons appear to be in hard-to-detect warm intergalactic plasma—all visible matter in galaxies accounts for less than 10% of the total baryon rest mass and for only 3× as much energy density as that contributed by neutrinos.

But this is not all that is unsatisfying about the current consensus cosmology. Even more surprising, and difficult to accept, is that fact that we

Table 1.1: Cosmic Inventory

Component	$\mathbf{\Omega}$ (ρ/ρ_c)
Dark Energy	$\mathbf{0.691 \pm 0.006}$
Matter (baryonic and non-baryonic)	$\mathbf{0.312 \pm 0.009}$
Baryons (Total)	$\mathbf{0.0488 \pm 0.0004}$
Baryons in stars and stellar remnants	$\sim \mathbf{0.003}$
Neutrinos	$\sim \mathbf{0.001}$
Photons (CMB)	$\mathbf{5 \times 10^{-5}}$

seem to live at a very special cosmic epoch, when the first two entries in Table 1.1 have roughly comparable values—this was not always case nor will it always be. The fraction of the critical density contributed by a given component varies with the scale factor of the universe, $a(t)$, as shown in Figure 1.9 for Ω_Λ (the 'Dark Energy' component).

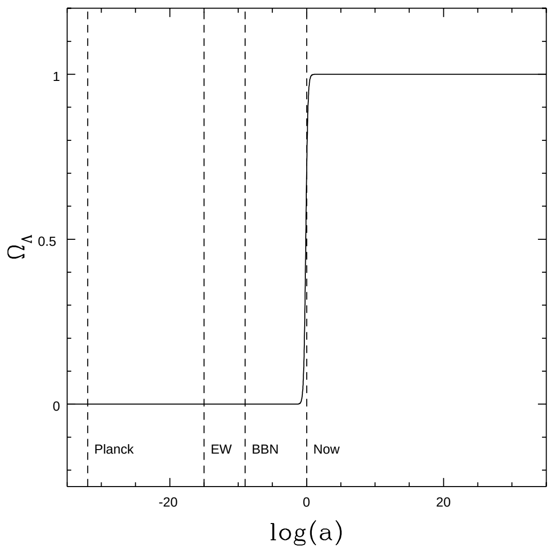

Figure 1.9: Ω_Λ as a function of the scale factor a, for a universe in which $\Omega_{m,0} = 0.3$ and $\Omega_{\Lambda,0} = 0.7$. Indicated are the scale factors corresponding to the Planck era, the electroweak phase transition, and Big Bang Nucleosynthesis.

At early times, Ω_Λ would have been negligible, while at later times the density of matter will be essentially zero and the universe will be empty. We happen to live in that brief era, cosmologically speaking, when both matter and vacuum energies are of comparable magnitude. Within the matter component, there are apparently contributions from baryons and from a non-baryonic source, both of which are also comparable (although at least their ratio is independent of time). This scenario staggers under the burden of its unnaturalness, but nevertheless crosses the finish line well ahead of any competitors by agreeing so well with the data.

In the words of Sean M. Carroll (2001, *Living Rev. Relativity*, 4 – astroph/0004075) we live in a 'Preposterous Universe'. Apart from confirming (or disproving) this picture, a major challenge to cosmologists and physicists in the years to come will be to understand whether these apparently distasteful aspects of our Universe are simply surprising coincidences, or actually reflect a beautiful underlying structure we do not as yet comprehend. If we are fortunate, what appears unnatural at present will serve as a clue to a deeper understanding of fundamental physics.

With this goal in mind, we shall proceed with the course!