

Origin of the Metallicity Dependence of Exoplanet Host Stars in the Protoplanetary Disk Mass Distribution

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ABSTRACT

The probability of a star hosting a planet that is detectable in radial velocity surveys increases $P_{\text{pl}}(Z) \propto (10^Z)^2$, where Z is stellar metallicity. Models of planet formation by core accretion reproduce this trend, since the protoplanetary disk of a high metallicity star has a high density of solids and so forms planetary cores which accrete gas before the primordial gas disk dissipates. This paper considers the origin of the form of the metallicity dependence of $P_{\text{pl}}(Z)$. We introduce a simple model in which detectable planets form when the mass of solid material in the protoplanetary disk, M_s , exceeds a critical value. In this model the form of $P_{\text{pl}}(Z)$ is a direct reflection of the distribution of protoplanetary disk masses, M_g , and the observed metallicity relation is reproduced if $P(M_g > M'_g) \propto (M'_g)^{-2}$. We argue that a protoplanetary disk's dust mass measured in sub-mm observations is a relatively pristine indicator of the mass available for planet-building and find that the disk mass distribution derived from such observations is consistent with the observed $P_{\text{pl}}(Z)$ if a solid mass $M_s > 0.5M_J$ is required to form detectable planets. Any planet formation model which imposes a critical solid mass for detectable planets to form would reproduce the observed metallicity relation, and core accretion models are empirically consistent with such a threshold criterion. While the outcome of planet formation in individual systems is debatable, we identify 7 protoplanetary disks which, by rigid application of this criterion, would be expected to form detectable planets and may provide insight into the physical conditions required to form such planets. A testable prediction of the model is that the metallicity dependence should flatten both for $Z > 0.5$ dex and as more distant and lower mass planets are discovered. Further, combining this model with one in which the evolution of a star's debris disk is also influenced by the solid mass in its protoplanetary disk, results in the prediction that debris disks detected around stars with planets should be more infrared luminous than those around stars without planets in tentative agreement with recent observations.

Key words: circumstellar matter – stars: planetary systems: formation – stars: planetary systems: protoplanetary discs – stars: pre-main-sequence.

1 INTRODUCTION

The study of how planetary systems form and evolve was revolutionised when the first extrasolar planet was discovered in radial velocity studies of the star 51 Peg (Mayor & Queloz 1995). Over 200 extrasolar planets are now known (Butler et al. 2006), and studying these planets has yielded enormous advances in our understanding of how they formed (Papaloizou & Terquem 2006; Udry et al. 2007). Perhaps the most telling discovery was that of a correlation in the probability of a star hosting a planet, P_{pl} , which is found to

increase with stellar metallicity (Gonzalez 1997). Fischer & Valenti (2005; hereafter FV05) found that, for stars with a metallicity $Z = [\text{Fe}/\text{H}]$ between -0.5 and 0.5 dex, the metallicity dependence of the fraction of stars with planets with orbital periods < 4 years and with amplitudes in radial velocity studies in excess of $K > 30 \text{ m s}^{-1}$ (i.e., Saturn-Jupiter mass planets, depending on orbital period) is

$$P_{\text{pl}}(Z) = 0.03 \times 10^{2Z}, \quad (1)$$

which corresponds to a planet fraction which increases with the square of the number of iron atoms in the stellar atmosphere. Similar trends have been found to apply to all species including Si and Ni (e.g., Ecuivillon et al. 2004; Robinson et

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al. 2006; Gonzalez 2006). The origin of this metallicity dependence is thought to be intrinsic to the planet formation process (FV05), and not caused by contamination from planetesimals falling onto the star, as is believed to be the cause of the high metallicities of DAZ white dwarfs (Jura 2006; Kilic & Redfield 2007), although the recent discovery that planet hosting giant stars do not favour metal rich systems is currently reigniting this debate (Pasquini et al. 2007).

Since the discovery of the extrasolar planet metallicity correlation, much work has gone into considering how stellar metallicity could affect different aspects of the planet formation process in the various models (e.g., Livio & Pringle 2003). It has been found that forming planets by gravitational instability does not introduce any significant metallicity dependence (Boss 2002; Cai et al. 2006), whereas models of planet formation by core accretion seem to readily reproduce the observed trend (Ida & Lin 2004b; Kornet et al. 2005; Benz et al. 2006; Robinson et al. 2006). This is because, in the core accretion models, planetesimals grow into planet cores through collisions, subsequently accreting gas from the surrounding gas disk once they become large enough, and then interacting with that disk so as to migrate inward (e.g., Lin & Papaloizou 1986; Papaloizou et al. 2007). The core accretion models predict a metallicity dependence because a higher metallicity implies higher solid mass and hence faster core growth, which means that the critical core mass for gas accretion can occur before the gas disk dissipates on ~ 6 Myr timescales (Haisch, Lada & Lada 2001; Clarke, Gendrin & Sotomayer 2001). However, it remains to be explained why the metallicity dependence has a form $\propto 10^{2Z}$ as opposed to, e.g., $\propto 10^Z$. The origin of the dependence found in these models is hidden somewhere within the large number of model components of which they are comprised, although it has been shown that a large solid disk mass is required if planets are to form (Ida & Lin 2004b).

In this paper we consider the origin of the form of the metallicity dependence using a simple heuristic model in which detectable planets form as long as the solid mass of material in the protoplanetary disk exceeds a critical value (e.g., Greaves et al. 2007). That model is described in §2, where it is shown how the metallicity relation is then directly related to the initial disk mass distribution. This section also compares the disk mass distribution required to reproduce the observed planet-metallicity trend in this model with that inferred from sub-mm observations of star forming regions. The implications of this model are discussed in §3, along with a discussion of why the solid mass should provide such a strong constraint on whether a system goes on to form a detectable planet. The conclusions are given in §4.

2 CRITICAL SOLID MASS MODEL

This model assumes that stars form surrounded by a protoplanetary disk which is made up of both solids and gas. We denote the mass of each of these components by M_s and M_g , respectively. The gaseous component dominates the total mass of the disk, and it is assumed that the outcome of the star formation process results in some universal distribution of disk masses (i.e., gas masses), which we define by the probability of any given star having had a protoplanetary disk with a gas mass larger than M'_g as $P(M_g > M'_g)$.

The solid mass of any given disk is assumed to be directly related to the mass of the gaseous component through the final metallicity of the star (e.g., Greaves et al. 2007):

$$M_s = 0.01M_g 10^Z. \quad (2)$$

Here we have assumed that the ratio of gas to solids is 100 for stars formed in a $Z = 0$ environment, consistent with that seen in nearby star forming regions (James et al. 2006). Thus it is assumed that stellar metallicities are indicative of the conditions present prior to the formation of the star that continued to be reflected in the composition of the protoplanetary disk, and that exerted no influence over the resulting distribution of protoplanetary disk masses.

The most important assumption is then that all of the stars that have disks with M_g larger than some critical value $M_{s,crit}$ go on to form planets which can be detected in radial velocity surveys, i.e., $P_{pl} = P(M_s > M_{s,crit})$. The physical origin for this critical value is not part of this heuristic model, although it does have a physical motivation based on core accretion models (e.g., Ida & Lin 2004b), as discussed in §1 and in more detail in §3.

2.1 Analytical solution

Since the probability of forming a planet depends only on the solid mass, the critical mass above which the total disk mass (i.e., gas mass) must be to form a planet is dependent on metallicity:

$$M_{g,crit} = 100M_{s,crit} 10^{-Z}. \quad (3)$$

The gas mass distribution is assumed to be independent of metallicity, and so the probability of any star forming a planet is metallicity dependent, since $P_{pl} = P(M_g > M_{g,crit})$. Thus to reproduce equation (1) requires a gas mass distribution in which:

$$P(M_g > M'_g) = 0.03(100M_{s,crit}/M'_g)^2, \quad (4)$$

where the critical solid mass required to form a planet, $M_{s,crit}$, is some as yet undefined constant. Since the probability of any star hosting a planet given in equation (1) is only known to apply for $P_{pl} < 0.25$ (due to the lack of surveys at higher Z), it follows that the distribution given in equation (4) is also only valid for $P(M_g > M'_g) < 0.25$, and so for $M_g > \sqrt{1200}M_{s,crit}$. Thus, in this model the observed $P_{pl}(Z)$ in equation (1) is telling us about the mass distribution of the most massive 25% of disks.

2.2 Gas disk distribution from observations

The gas mass distribution required by this model in order to match the observed $P_{pl}(Z)$ (equation 4) can now be compared with the observed gas mass distribution. The gas mass distribution of protoplanetary disks is not well-known because the majority of that mass is in molecular hydrogen which is difficult to detect, especially in the cold outer regions of the disks where most of the mass resides (Thi et al. 2001; Sheret, Ramsay-Howat & Dent 2003). Species such as CO are easier to detect (e.g., Dent, Greaves & Coulson 2005; Dutrey, Guilloteau & Ho 2007), however there is uncertainty in the CO/H₂ ratio because some of this gas

ends up frozen onto dust grains or photo-dissociated (Dullemond et al. 2007; Najita et al. 2007). On the other hand, the dust mass distribution of protoplanetary disks is well characterised, since this can be measured with relatively few uncertainties from sub-mm and mm wavelength observations (André & Montmerle 1994; Beckwith, Henning, & Nakagawa 2000).

Here we make the assumption that dust mass can be used as a proxy for the total gas mass in protoplanetary disks (for a fixed Z), and we derive the gas mass distribution from the dust mass distribution in Taurus-Auriga, which was measured using sub-mm photometry of 153 pre-main sequence stars by Andrews & Williams (2005; hereafter AW05). Since the stars in the AW05 sample are at a range of evolutionary stages we chose to use only the disk masses of the 75 class II objects (i.e., T Tauri stars) in their sample to ensure that the disk mass distribution is indicative of that at the epoch of planet formation. Class I sources were omitted because of a potential contribution to the sub-mm flux from a remnant circumstellar envelope. Class III sources were omitted because of the possibility that their currently low disk masses are a consequence of the disks being at an advanced evolutionary stage, and so are not necessarily indicative of a low mass present at the planet forming epoch. To obtain the gas mass distribution, the gas/dust ratio was assumed to be 100 for all stars, based on the metallicities in nearby star forming regions being close to solar with a small dispersion for each region (Padgett 1996; Vuong et al. 2003; James et al. 2006). The mass distribution of class II objects is shown in Fig. 1. Ten objects from this sample have only upper limits to their disk masses, which were set to zero in Fig. 1. Since these upper limits are $\leq 1M_J$, we infer that the disk mass distribution is accurate for the most massive 69% (52/75) of disks that are above this limit ($\geq 1M_J$).¹

The critical solid mass model (§2.1) was used to determine the metallicity relation predicted from the observed gas mass distribution:

$$P_{\text{pl}} = [N(M_g > M'_g) \pm \sqrt{N(M_g > M'_g)}] / N_{\text{tot}}, \quad (5)$$

where Poisson counting statistics were used to determine the uncertainty in the number of disks larger than a given limit in the distribution and $N_{\text{tot}} = 75$. The probability determined from equation (5) could be assigned a corresponding metallicity, Z' , from the relation $M'_g = M_{g,\text{crit}}$. Equation (3) means that

$$Z' = -\log 0.01M'_g / M_{s,\text{crit}}. \quad (6)$$

The value of $M_{s,\text{crit}}$ was constrained to achieve a mean planet probability for the metallicity range $Z = 0.25 - 0.5$ dex in agreement with that found by FV05, i.e., $P_{\text{pl}} = 14.8 \pm 3.5\%$, giving²

¹ We note that, even though 10 of the AW05 class II sources were not detected in individual sub-mm photometry observations, co-addition of this data-set leads to a net positive detection of 2.7 ± 0.9 mJy, corresponding to a mean disk mass of $0.14M_J$ which is consistent with that expected from the log-normal distribution plotted in Fig. 1.

² The value derived in equation 7 differs slightly from the $0.24M_J$ derived by Greaves et al. (2007) because that paper included disks from AW05 of both class II and III in their primordial gas mass distribution.

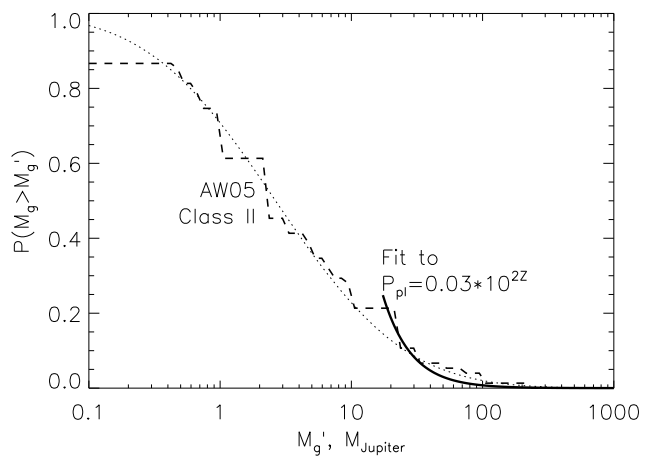


Figure 1. Distribution of protoplanetary disk gas masses. The gas mass distribution inferred from the dust mass distribution of class II objects in Taurus-Auriga (Andrews & Williams 2005) is shown with a dashed line. The dotted line is a log-normal fit to this distribution centred on $2.5M_J$ of width 0.77 dex. The distribution required in the critical solid mass model to fit the extrasolar planet metallicity relation (equations 1 and 4) is shown with a solid line, assuming $M_{s,\text{crit}} = 0.5M_J$.

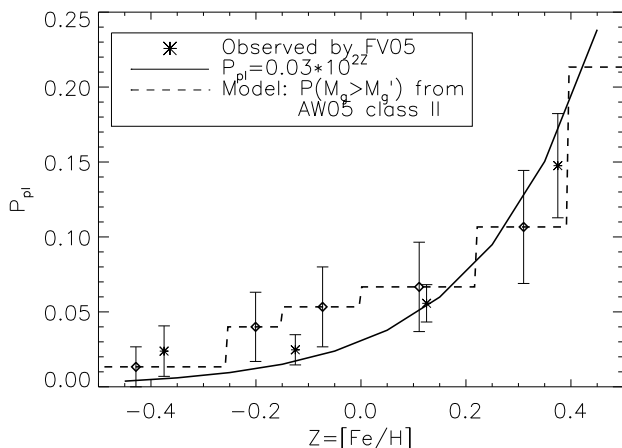


Figure 2. Probability that a star of given metallicity has an extrasolar planet that is detectable in the current radial velocity surveys. The predictions of the critical solid mass model based on the Andrews & Williams (2005) distribution of dust masses of class II protoplanetary disks in Taurus-Auriga is shown with a dashed line, with errors indicated by diamonds with \sqrt{N} error bars. The asterisks show the results of the radial velocity survey of Fischer & Valenti (2005) with \sqrt{N} error bars. The fit to the FV05 data (equation 1) is shown with a solid line.

$$M_{s,\text{crit}} = (0.5 \pm 0.1)M_J. \quad (7)$$

The extrasolar planet-metallicity relation predicted by this model is plotted in Fig. 2, and shows good agreement with the observed relation (equation 1).

We have also inverted the problem by deducing the *required* disk mass distribution that would lead to the solid line in Figure 2 (i.e., $P_{\text{pl}}(Z)$ parameterised according to

equation 1). In Figure 1 we compare this required distribution with the observed gas mass distribution. Noting that this comparison can only be made over the upper quartile of disk masses (since current planet detection statistics only extend to metallicities < 0.5 and, in the model, it is only this range of disk masses which can form planets in this metallicity regime), it is evident that there is also good agreement between the model and observed distributions when plotted in this way. To quantify this, we performed a one sided Kolmogorov-Smirnov test to compare the distribution of gas masses inferred from AW05, when converted into metallicity (equation 6), with that inferred from equation (1) for the range $Z' = -0.5$ to 0.5 dex. We found that discrepancies as large as or greater than those observed occur in 69% of samples of 75 members drawn from a population with a cumulative distribution function in which $P(Z < Z') = 0.03 \times 10^{2Z'}$; i.e., we conclude that the gas mass data are not unlikely to be drawn from such a distribution, since at least 2 out of 3 times one would expect data at least as discrepant as observed.

3 DISCUSSION

We have shown, under the assumption that a critical solid mass in the protoplanetary disk is required to form a planet that is detectable in radial velocity surveys, that the observed frequency of planet detections as a function of metallicity, $P_{\text{pl}}(Z)$, is compatible with the observed disk mass distribution (as derived from sub-mm dust mass measurements of Classical T Tauri stars in local star forming regions). We now discuss the physical basis for this simple model and further observational tests.

3.1 Comparison with core accretion models

To consider the physical basis for the outcome of planet formation being determined solely by dust mass, we appeal to the core accretion models of Ida & Lin (2004a, 2004b; hereafter IL04). The IL04 models are *local*, in the sense that planet formation depends on local quantities such as gas and solid surface density. Therefore we expect any threshold effect to involve surface density rather than mass. We first assess whether the results of IL04 are compatible with the hypothesis that planet formation requires a critical *metallicity independent* solid surface density and return to a discussion of the relationship between solid surface density normalisation and dust mass in §3.2. We can assess this hypothesis in two ways. Firstly, we can simply take the distribution of disk surface densities assumed by IL04 (a log-normal distribution of width 1.0 dex that is centred on the surface density of the minimum mass solar nebula and truncated at $> 1.48\sigma$), apply a threshold solid surface density for planet formation that is independent of metallicity and see whether we can reproduce their numerical results. Figure 3 shows that this is indeed the case: the nominal model from IL04b is well reproduced by assuming a critical solid surface density of 8 times the minimum mass solar nebula, whereas their variant models where the rate of core accretion is enhanced or reduced by a factor of three are well reproduced by models in which the critical solid surface density is respectively 4 and

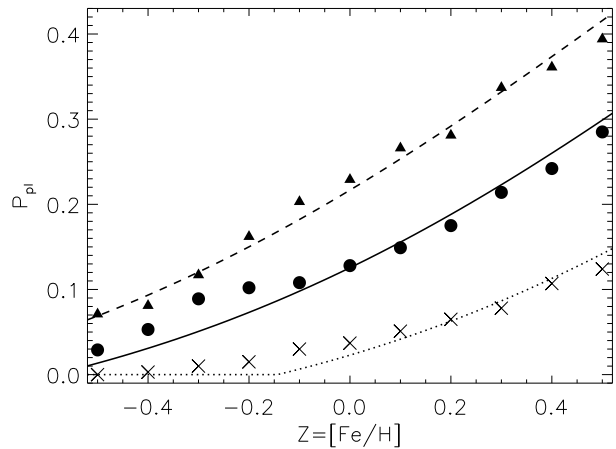


Figure 3. Prediction of the critical solid mass model for the probability that a star of given metallicity has an extrasolar planet that is detectable in the current radial velocity surveys assuming the disk mass distribution used in Ida & Lin (2004b) with critical solid masses of 4 (dashed line), 8 (solid line) and 22 (dotted line) times the minimum mass solar nebula. The numerical results of the nominal model in IL04b are shown with filled circles, and the results of their models in which the core accretion rate is 3 times faster and slower than the nominal model shown with triangles and crosses (see their fig. 2b).

22 times the minimum mass solar nebula. We stress that the IL04 models contain a large number of ingredients and do not explicitly impose a threshold criterion. Nevertheless, we see that their results are empirically equivalent to the imposition of a simple threshold.

In a second approach, we can now attempt to understand *why* the IL04 models behave in this way. Examination of these models shows that the formation of gas giant planets hinges on rocky cores being able to grow to a critical mass (a few M_{\oplus}) before the gas disk is dispersed. The requirement of sufficiently rapid core growth implies that they have to form inside a critical radius, a_{ig} , which depends on both gas and solid surface densities. On the other hand, inward of a second critical radius, a_{tg} (which depends on solid surface density), a critical core mass is not achievable because the required core mass exceeds the local isolation mass (at which point the core has consumed all the material in its local feeding zone). Evidently, the formation of gas giant planets is possible only for the case $a_{\text{tg}} < a_{\text{ig}}$ and we can derive a condition on the gas and solid surface densities corresponding to the critical case where $a_{\text{tg}} = a_{\text{ig}}$. This translates into a condition on the minimum surface density of solids as a function of metallicity. We find that the critical surface density of solids scales as $10^{-0.06Z}$ (assuming, as in IL04, that a disk's surface density scales $\Sigma \propto r^{-p}$ where r is radius and $p = 1.5$). This very weak dependence on metallicity results from the fact that the growth rate of solid cores is much more strongly dependent on orbital radius than on the gas column density and hence a_{ig} is only very weakly dependent on gas column density. Therefore, the threshold criterion $a_{\text{ig}} = a_{\text{tg}}$ is nearly independent of gas column density and thus the dependence of critical dust column on metallicity is extremely weak. It is this extremely weak dependence of the critical solid surface

density on metallicity which we believe to account for the excellent correspondence between the numerical results of IL04 and the application of our simple threshold hypothesis (see Fig. 3).

A further test of this hypothesis would be to examine how $P_{\text{pl}}(Z)$ predicted by the core accretion models depends on the assumed distribution of disk surface densities, since if the outcome is governed by a critical surface density of solids for planet formation then using a narrower distribution of disk surface densities as input would result in a steeper metallicity dependence (since in the critical solid surface density model the metallicity dependence simply reflects the disk surface density distribution used as input). In contrast to IL04, Robinson et al. (2006) did vary this quantity and indeed found that $P_{\text{pl}}(Z)$ rose more gently when a larger range of disk surface densities was employed.

3.2 Why sub-mm dust mass determines outcome

Regardless of the comparison with core accretion models, it is notable that the critical solid mass model fits the planet-metallicity relation found in nature. It is, however, surprising that sub-mm dust mass should be such a good indicator of whether planets are going to form in a disk, since sub-mm measurements probe the current mass in mm- to cm-sized dust and so are not necessarily representative of the primordial inventory of solid or gas mass. Indeed, class II objects in Taurus-Auriga have a range of ages and so we would expect the oldest stars to have already lost a significant quantity of gas through accretion onto the star (Clarke et al. 2001). We may also expect some loss of detectable dust mass with age through grain growth and accretion onto the star with the gas. However, there is no evidence that sub-mm dust mass changes with age on the pre-main sequence (e.g., Wyatt et al. 2003) suggesting that the mass in mm- to cm- sizes is constant. This is to be expected, since the total dust mass $M_{\text{dust}} \propto r_{\text{out}}^{2-p}$, where r_{out} is the disk outer edge, so that as long as $p < 2$ the sub-mm dust mass is concentrated in the outer regions of the disk. Since typically observed values for protoplanetary disks are $p \approx 0.85$ and $r_{\text{out}} \approx 200$ AU (Andrews & Williams 2007), the timescale for grains containing most of the disk mass to grow to larger than 1 metre, and so become invisible in the sub-mm, may be expected to be longer than the 10Myr period over which planet formation (in the inner regions) must take place (e.g., Dullemond & Dominik 2005). Indeed some disks cannot harbour significant quantities of "unseen" dust mass (i.e., with particle sizes either much larger or smaller than 1 mm), since, even in the absence of such unseen contributions, the gas mass inferred from mm dust measurements is in some cases already ~ 0.2 times the central star's mass, and thus close to the limit for gravitational instability. Given the evidence that grain growth to mm and cm scales has occurred in the outer regions of disks (Wilner et al. 2006), we are confident that this grain size scale contains the majority of the disk solid mass at these radii, and thus, by implication, the majority of the solid mass in the disk. Thus, while the dust seen in the sub-mm is not contributing to the planet formation process (because it is mainly at radii where it has not had time to grow to large - greater than metre - size scales), we are suggesting that it is nevertheless a good measure of the primordial inventory of solids in the disk.

The fact that the sub-mm dust mass distribution fits the observed planet-metallicity relation so well is because there is an order of magnitude difference between the highest and lowest masses of the top $\sim 25\%$ most massive gas disks (e.g., Figs 1 and 2). This result is not specific to the Taurus-Auriga star forming region, since class II disks in ρ Oph also exhibit an order of magnitude range for the most massive 25% of those disks (see Fig. 9 of André & Montmerle 1994). If this distribution had been much narrower or broader then we would have been able to rule out the critical solid mass model.

One further requirement of nature for the critical solid mass model to work is for a disk's outer radius to be less important than its solid mass in setting the outcome of planet formation. As noted in §3.1, models such as those in IL04 rely on a critical surface density (rather than mass). For the surface density profile assumed by IL04, the surface density normalisation (f_d , where $\Sigma \propto f_d$), disk outer radius (r_{out}) and total solid mass (M_s) are related via $M_s \propto f_d r_{\text{out}}^{0.5}$. Thus the mapping between critical surface density and critical mass is (weakly) dependent on r_{out} . While disk radii have been measured using sub-mm interferometry (Kitamura et al. 2002; Andrews & Williams 2007), these samples are biased toward the most massive disks so that it is not clear how representative the observed distribution is of the population as a whole. However, there is no evidence that the distribution of r_{out} is as broad as that of disk masses seen by AW05. We therefore expect the surface density of solids in the planet formation region to be mainly controlled by M_s rather than r_{out} , thus explaining the apparent success of sub-mm flux as a predictor of planet forming potential.

3.3 Disks forming detectable planets

One implication of this study is that we can predict which of the disks in the AW05 sample will go on to form planets like those detected in the current radial velocity surveys. The class IIs in their sample with more than $0.5M_J$ of dust are 04113+2758, DL Tau, GG Tau and GO Tau. However, we disqualify GG Tau as a planet-forming candidate, since its disk is circumbinary (Guilloteau, Dutrey & Simon 1999), and so its high sub-mm flux does not equate with a high surface density of solids in the inner disk. Massive circumbinary disks are rare (Jensen, Mathieu & Fuller 1996), so the majority of the more massive disks are not circumbinary disks and so would not be unsuitable for forming planets. Applying the same $0.5M_J$ dust mass limit to the ρ Oph study of André & Montmerle (1994) indicates that of the class IIs in this region, AS205, EL24, GSS39 and SR24S may go on to form detectable planets.

While we do not claim that we can unambiguously predict the outcome of planet formation for any one of these systems, we do suggest that studying the disks that are predicted to form planets, the characteristics of which we can constrain at least statistically, may provide a valuable way of probing the environments in which such planets form. The fact that it is the most massive disks which go on to form detectable planets means that these disks must be close to being gravitationally unstable, since the ratio $M_{\text{disk}}/M_* > 0.05$ for $Z = 0$ and $M_* = 1M_\odot$. This suggests that instability could play a role in the formation process. However, this cannot be the only determining factor, since

the gravitational instability process itself is not affected by metallicity (Cai et al. 2006), and there would be no metallicity dependence if $M_{g,crit}$ is a constant and not dependent on metallicity. Thus, this suggests that some degree of instability may help speed up the core accretion process, e.g., through concentration of particles in spiral structures (Rice et al. 2004) or instability in a thin dust layer (Youdin & Shu 2002).

3.4 Observational tests

Here we suggest three observational tests of the critical solid mass model:

Firstly, if the model is correct, we would expect $P_{pl}(Z)$ to rise much less steeply with Z at metallicities above 0.5 dex than implied by an extrapolation of equation (1), since at higher metallicities the model predicts that planets would be able to form in lower mass disks, and that $P_{pl}(Z)$ in this regime would reflect the disk mass distribution of intermediate mass disks. A discrepancy between the observed disk mass distribution and that resulting from an extrapolation of equation (1) to $Z > 0.5$ dex is readily apparent by considering how the solid curve on Figure 1, if extrapolated to lower disk masses, would compare with the dashed line on that Figure. Whether a suitable high metallicity sample can be found to test this prediction remains to be seen (e.g., Laughlin 2000; Valenti & Fischer 2005; Taylor 2006).

Secondly, one of the key assumptions of the model was that the distribution of protoplanetary disk masses is universal in that it is independent of metallicity. This can be tested by measuring the distribution of dust masses in low (or high) metallicity star forming regions using sub-mm photometry, since these masses should be correspondingly lower (or higher) than those of nearby regions like Taurus-Auriga where $Z \approx 0$. While ALMA can detect the brightest known class II disks out to 20 kpc, we are not aware of any young (< 10 Myr) cluster within the Milky Way which has a measured metallicity that is sufficiently sub- or super-solar for the predicted difference in disk mass distribution in comparison with Taurus-Auriga to be confidently detected, although star forming clusters such as those found by Santos et al. (2000) and Yun et al. (2007) may be suitable candidates if their large Galactocentric distances (15-16.5 kpc) are indicative of a low metallicity as suggested by observations Cepheids which indicate a metallicity gradient in the Milky Way of -0.06 dex/kpc (Luck et al. 2006).

Thirdly, we will be able to test in due course an adjunct hypothesis, i.e., that the incidence of planets of lower masses (and at greater orbital distances) is also regulated by a (lower) critical solid mass threshold. For example, extrapolation of the exoplanet semi-major axis distribution to 20 AU suggests that surveys able to detect planets to that distance would double the fraction of stars known to have planets to 12% (Marcy et al. 2005). The simplest hypothesis we can apply to this population would simply be that the progenitor disks corresponded to the top 12% of the disk mass distribution, implying a critical solid mass of $\sim 0.3M_J$. We plot in Figure 4 the predicted dependence of planet frequency on metallicity in this case. Although this "prediction" will eventually be compared with observational data, we emphasise that it is not entirely clear how this adjunct hypothesis (i.e., that the critical solid mass is lower for planets located

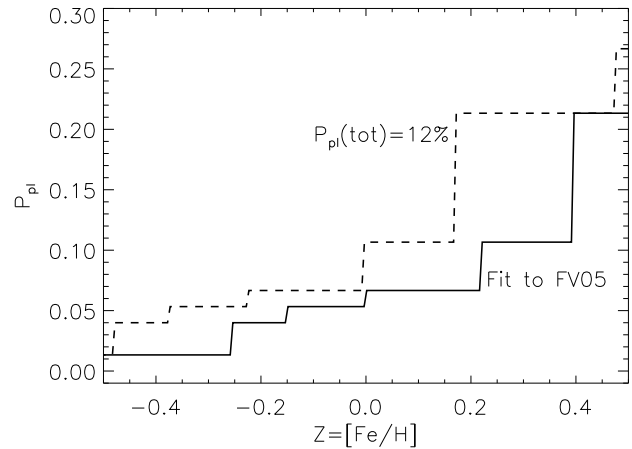


Figure 4. Prediction of the critical solid mass model for the probability that a star of given metallicity has an extrasolar planet. Assuming that surveys with different detection thresholds correspond to different critical solid masses, the prediction for planets that are detectable in the current radial velocity surveys is shown with a solid line, and the prediction for a survey with an overall planet detection frequency of 12% for the metallicity range -0.5 to 0.5 dex is shown with a dashed line.

at larger distances) can be squared with the expectations of core accretion models.

4 CONCLUSIONS

We have presented a simple analytical model which can be used to predict the outcome of planet formation, in which the formation of a planet that is detectable in radial velocity studies depends only on the mass of solids in the protoplanetary disk. We showed that this model predicts that the observed planet-metallicity relation is a reflection of the disk mass distribution. We also argued that the sub-mm dust mass seen in the protoplanetary disk phase is a good tracer of the initial mass budget available close to the star for planet formation, and showed that the observed planet-metallicity relation is consistent with the disk mass distribution estimated from sub-mm observations of protoplanetary disks if the critical solid mass required to form detectable planets is $0.5M_J$.

We suggested that the detailed physics of the IL04 core accretion models boil down to a critical solid mass required to form detectable planets, although it needs to be confirmed that the good empirical agreement with the IL04 models is more than a coincidence. However, the value of this model is not just in its relevance to specific core accretion models, but in its general applicability, since it shows how the observed planet-metallicity relation would be reproduced by any planet formation model which imposes a critical solid mass for the formation of detectable planets. Other reasons for imposing a threshold on a disk's solid mass before detectable planets can form include the possibility that such conditions are required for the formation of $>$ km-sized planetesimals through gravitational instabilities (e.g., Johansen, Klahr & Henning 2006).

The value of this model is also in its simplicity, since

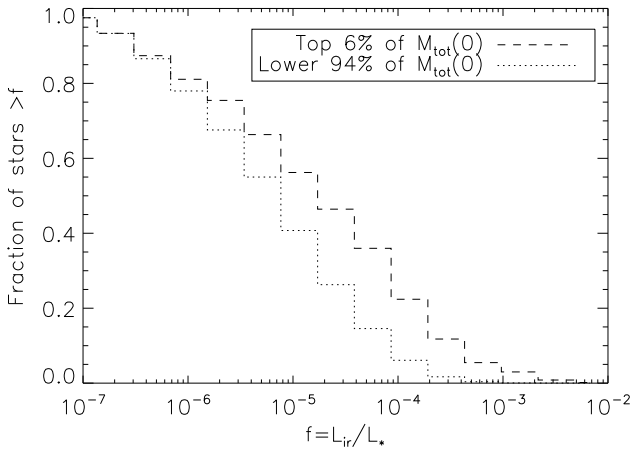


Figure 5. Distribution of infrared luminosities ($f = L_{\text{ir}}/L_{\star}$) of the debris disks of A stars in the model of Wyatt et al. (2007). The distribution for the debris disks formed from the most massive 6% of protoplanetary disks (the *planet bearers*) is compared with those formed from the least massive 94% of protoplanetary disks (the *non-planet bearers*).

this means that it can be readily applied to predict other observable properties of stars with and without detectable planets, should those properties also depend on the solid mass of the protoplanetary disk. For example, the statistics for the incidence of debris disks around A stars as a function of stellar age (e.g., Rieke et al. 2005) can be explained by a model in which all stars form planetesimal belts, the initial mass of which is determined by the solid mass in the protoplanetary disk (a distribution which is taken from AW05), and which are subsequently eroded by steady state collisional processing (Wyatt et al. 2007). We ran the A star debris disk model in order to predict the distribution of $f = L_{\text{ir}}/L_{\star}$ for the ensemble, computing separate distributions for the *planet bearers* (corresponding to the top 6% of the input mass distribution) and the *non-planet bearers* (corresponding to the remaining 94% of the population). Fig. 5 shows how the debris disks of the *planet bearers* are, on average, more luminous than those of the *non-planet bearers*; specifically, the mean luminosity of the most luminous 10% in both distributions (in L_{ir}/L_{\star}) differ by a factor of ~ 6 . While we do not know whether it is only the top 6% of A star protoplanetary disks that form detectable planets, because the A star exoplanet population is poorly known at present, here we predict that if A star planets form in a similar manner to those of sun-like stars (i.e., with a threshold solid mass criterion), then this will be seen in the luminosity distributions of their debris disks (Fig. 5). Similarly, while we do not know whether the luminosity distributions of the debris disks of sun-like stars behave as shown in Fig. 5, because the model of Wyatt et al. (2007) has yet to be applied to that population, here we predict that if the luminosities of sun-like star debris disks are governed by steady state processes, then the distributions of those luminosities will exhibit a trend similar to that in Fig. 5. Indeed observations of the debris disks of sun-like stars both with and without planets do show a trend in their luminosities of comparable magnitude to that suggested by Fig. 5 (Bryden et al. 2007).

Application of this model to known systems implies that the disks of 04113 + 2758, DL Tau, GO Tau, AS205, EL24, GSS39 and SR24S will form (or have formed) gas giant planets. While the outcome of planet formation in individual systems is uncertain, we suggest that studying these disks may help constrain the physical conditions of disks in which we know, at least statistically, what the outcome of planet formation will be. Observational tests of the model include a flattening of the metallicity relation for $Z > 0.5$ dex and also a flattening as planet searches continue.

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